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THE DISTRIBUTION OF BETAS IN PRESENCE OF NONTRADED ASSETS

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ABSTRACT

This study considers a capital assets pricing model (CAPM) in an incomplete financial market wherein not all risky assets are traded and the risk from non-traded assets is not orthogonal to that of the existing or traded assets. The model shows the extent of the divergence of the CAPM betas (true betas) from the traditional CAPM betas (perceived betas) in market equilibrium conditions in an incomplete market. Specifically, it implies that the more incomplete a financial market is, the wider is the discrepancy between the true and perceived betas, and the distribution of the perceived betas tends to centre more around 1 in an incomplete market than that of true betas. Empirical evidence in various settings support these results.

Keywords: CAPM beta, incomplete market, nontraded asset, portfolio choice, diversification

JEL classification numbers: G11, G12

I. INTRODUCTION

Most finance literature on portfolio choice generally assumes that all sources of risk are traded or can be perfectly hedged by traded assets in a complete financial market. For example, the capital asset pricing model (CAPM) developed by Sharpe (1964) and Lintner (1965) describes beta as a measure of an asset or portfolio's systematic risk compared to the market as a whole, which is implicitly assumed to have some non-traded assets and homogeneous investors. Specifically, beta is the covariance of an asset return with the return on the market portfolio relative to the variance of the return on the market portfolio, and it measures the risk of holding an asset or a portfolio. Based on beta, the traditional CAPM derives a security market line (SML) representing how the risk of holding an asset or a portfolio is rewarded in the financial market.

However, the real-word market seems to be incomplete, and nontraded assets do exist.[1] For example, investors cannot trade explicit claims on human capital and may not be able to hedge

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[1]Market incompleteness is a state in which a certain risk is not traded or cannot be completely hedged by any combination of existing assets. In reality, researchers often consider market incompleteness as a measure of financial development and vice versa. However, it should be noted that the degree of market incompleteness is not equivalent to the level of financial development.

the risk from human capital perfectly. Though market incompleteness may deteriorate investors' welfare, it does not always influence investors' portfolio choices. Non-traded assets will not influence investors' decisions if their returns are orthogonal to the returns from traded assets. In contrast, investors are exposed to some risk factors that they can only imperfectly hedge with traded assets if they are not orthogonal. Consequently, the risk factors will affect portfolio choices in the market.

This study examines whether and how the CAPM beta on the SML should be reconsidered in light of risk factors, both non-traded and non-orthogonal to the traded or existing assets in the market. In particular, we develop an incomplete market version of SML in the CAPM and compare it with the traditional SML in the finance literature. In an incomplete market version of SML, a higher (true) CAPM beta for an individual asset implies that the investor requires more compensation when the individual asset is more correlated with the risk from the portfolio consisting of both traded and non-traded assets.

In addition, we investigate the characteristics of individual assets' CAPM betas (true betas) from an incomplete market version of SML betas compared to the traditional CAPM betas (perceived betas). Our model shows that the true betas tend to diverge from the perceived betas by around 1, and thus the distribution of perceived betas tends to centre more around 1 than that of true betas. This suggests that the more incomplete a financial market is, the wider is the discrepancy between true and perceived betas. We also provide empirical experiments based on US financial market data to complement our model implications. We find that our empirical results largely support the model implications in various settings.

This study relates to earlier works investigating the effect of non-traded assets on an investor's portfolio choice and equilibrium asset prices (Constantinides and Duffee, 1996; Koo, 1991; Losq, 1978; Lucas, 1994; Mayers, 1972; Svensson and Werner, 1993; Telmer, 1993; Weil, 1994). In particular, our study is closely related to two papers: Fama and Schwert (1977) and Oh (1996). The former study investigates the extent to which non-traded human capital affects the SML. They find a weak correlation between returns on human capital and the market portfolio, concluding that the SML has only a marginal effect. The latter study investigates not only how asset prices are determined in an incomplete market but also how the prices of existing assets change with the market structure of the economy. More importantly, he also shows a positive SML in the CAPM framework in the presence of non-traded assets. However, our study differs from these because we specifically examine the cross-sectional dispersion of the CAPM betas implied in the incomplete market version of SML compared to that of the traditional CAPM betas. In addition, our strong empirical results suggest that practitioners should use the traditional CAPM betas carefully because they are widely employed in diverse financial decisions, such as estimating firms' cost of equity capital and evaluating the performance of managed portfolios.

The remainder of the paper proceeds as follows. Section II presents the model and derives an SML in an incomplete market. Section III discusses the settings for the empirical experiments, and Section IV provides the results. Section V concludes the paper.

II. MODEL

Consider an economy in which N risky assets and 1 risk-free asset are traded. Each investor holds an initial wealth of \$1 and allocates the fund among the N risk assets and the risk-free asset. The market also contains non-tradable risk Y. Hence, the investor should consider not only the correlations among the N risky assets, but their correlation with Y when composing a portfolio.

Mayers (1972) derives an SML by aggregating the first order conditions (FOCs) of individual investors, assuming that non-traded risk varies among them. This feature seems realistic in that human capital is heterogeneous and non-tradable. Similar to Mayers (1972), we assume that the j^{th} individual has different exposures to risk Y by $\theta_j Y$ (θ_j follows a uniform distribution on $[-1, 1]$[2] and $j \in [0, 1]$). Such heterogeneity originates from the investor's economic ability or status. Since θ_i is not observable, Y cannot be traded in the market. Beyond deriving the SML, we assess the divergence between the true and perceived CAPM betas and discuss its implications for the distribution of the betas.

Denoting the rate of returns from the nth asset as Z_i, $i = 1, 2, 3, \ldots, N$, we represent a mean-variance portfolio problem of the j^{th} investor as follows:

$$\min \frac{1}{2} w_j' V w_j + w_j' \theta_j Q \text{ s.t. } w_j e + (1 - w_j' \ell) r_f = E$$

$e = N \times 1$ vector of expected returns
$V = N \times N$ covariance matrix among traded assets
$Q = N \times 1$ covariance vector between the non-tradable risk factor Y and the returns on the traded assets
$\ell = N \times 1$ vector of 1s
$w = N \times 1$ vector of portfolio weights on the risky assets

In the objective function above, we add the covariance between $\theta_j Y$ and the rate of return from portfolio w_j to $\frac{1}{2} w_j' V w_j$. In a traditional CAPM model, it would suffice to minimize the volatility of portfolio return ($w_j' V w_j$). However, the presence of Y requires that the investor also consider reducing the covariance ($w_j' \theta_j Q$) between the portfolio and the inherent risk $\theta_j Y$.

We transform the constrained minimization program above as follows by introducing a Lagrange multiplier, λ_j:

$$\mathcal{L} \equiv \frac{1}{2} w_j' V w_j + w_j' \theta_j Q + \lambda (E - w_j' e - (1 - w_j' \ell) r_f)$$

The FOCs are

$$\frac{\partial \mathcal{L}}{\partial w} = V w_j + \theta_j Q - \lambda (e - r_f \ell) = 0.$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = E - w_j' e - (1 - w_j' \ell) r_f = 0.$$

Solving the two FOCs, we obtain an explicit solution for w_j^* and λ_j^*.

$$\lambda_j^* = \frac{(E - r_f)}{H} + \frac{\theta_j Q' V^{-1} (e - r_f \ell)}{H}$$

$$w_j^* = w_0 + x_j$$

$$w_0 \equiv \frac{(E - r_f) V^{-1} (e - r_f \ell)}{H},$$

$$x_j \equiv \frac{G_j}{H} V^{-1} (e - r_f \ell) - V^{-1} \theta_j Q,$$

$$H \equiv (e - r_f \ell)' V^{-1} (e - r_f \ell)$$

$$G_j \equiv (e - r_f \ell)' V^{-1} \theta_j Q.$$

[2] We could adopt any type of distributions with zero mean.

w_0 is a mean-variance frontier portfolio with no untradeable risk. By construction,[3] $\int_0^1 \theta_j \mathrm{dj} = 0$ and $\mathrm{w}^* \equiv \int_0^1 \mathrm{w}_\mathrm{j}^* \mathrm{dj} = \mathrm{w}_0$. Likewise, by aggregating individual portfolio choices, we calculate the variance of returns from w* and its covariance with any other portfolio[4] q as follows:

$$Var\left(r_w^*\right) = w^{*\prime} V w^* = (w_0 + x)' V (w_0 + x) = w_0{}' V w_0 + 2 w_0{}' V x + x\,' V x$$

$$Cov\left(r_w^*,\, r_q\right) = (w_0 + x)'\; V(w_q + x) = w_0{}' V w_q + w_0{}' V x + w_q{}' V x + x' V x$$

Appropriately replacing w_0, w_q, and x in the above equations, we change their representation:

$$Var\left(r_w^*\right) - x' V x \;= \frac{\left(E[r_w^*] - r_f\right)^2}{H} + 2\frac{(E[r_w^*] - r_f)(e - r_f \ell)' x}{H}$$

$$Cov\left(r_w^*,\, r_q\right) - x' V x = \frac{\left(E\left[r_w^*\right] - r_f\right)(E[r_q] - r_f)}{H} + \frac{\left(E\left[r_w^*\right] - r_f\right)(e - r_f \ell)' x}{H} + \frac{(E[r_q] - r_f)(e - r_f \ell)' x}{H}$$

$$x' V x = \theta^2 Q' V^{-1} Q - \frac{G^2}{H}$$

Applying $(e - r_f \ell)' x = 0$, we present the ratio of the two equations above in terms of the following SML:

$$\frac{Cov\left(r_w^*,\, r_q\right) - x' V x}{Var\left(r_w^*\right) - x' V x} = \frac{(E[r_q] - r_f)}{\left(E\left[r_w^*\right] - r_f\right)}$$

$$E[r_q] -\; r_f = \frac{Cov\left(r_w^*,\, r_q\right) - x' V x}{Var\left(r_w^*\right) - x' V x}\;\left(E\left[r_w^*\right] - r_f\right)$$

On the other hand,

$$Cov\;\left(r_w^*,\; Y\right) = w^{*\prime} Q = \frac{\left(E\left[r_w^*\right] - r_f\right) G}{H} + \frac{G^2}{H} - \theta^2 Q' V^{-1} Q,$$

$$Cov\;(r_q, Y) = w_q' Q = \frac{(E[r_q] - r_f) G}{H} + \frac{G^2}{H} - \theta^2 Q' V^{-1} Q.$$

Accordingly, the ratio of the latter two becomes

$$\frac{Cov\left(r_w^*,\; Y\right)}{Cov\left(r_q,\; Y\right)} = \frac{\left(E\left[r_w^*\right] - r_f\right) G + G^2 - \theta^2 Q' V^{-1} Q H}{(E[r_q] - r_f) G + G^2 - \theta^2 Q' V^{-1} Q H}.$$

Summing up, we derive the SMLs in an incomplete market as follows:

$$E[r_i] - r_f = \frac{Cov\left(r_w^*,\, r_i\right) - x' V x}{Var\left(r_w^*\right) - x' V x}\left(E\left[r_w^*\right] - r_f\right)$$

$$E[r_i] - r_f = \frac{Cov(r_i,\; Y) + x' V x}{Cov\left(r_w^*,\; Y\right) + x' V x}\left(E\left[r_w^*\right] - r_f\right).$$

The first equation shows how the SML differs from the original in an incomplete market. The second shows that the risk premium of an individual asset is determined by the ratio of its

[3]Hereafter, $x \equiv \int_0^1 x_j \mathrm{dj}$ for any variable $x_j,\;\; j \in [0, 1]$.

[4]Portfolio q does not have to be on a mean-variance frontier. For the ease of derivation, we match q with the portfolio weight vector of $w_q + x$ (not w_q).

covariance with the untradeable risk factor to that of a market portfolio with an untradeable risk factor.

III. EMPIRICAL EXPERIMENTS

III.1 Testable Hypotheses

Based on the results from the model in Section II, we propose the following two testable hypotheses:[5]

Hypothesis I The more incomplete a financial market is, the wider the discrepancy between β_i^{True} and $\beta_i^{Perceived}$ becomes.

$$\beta_i^{True} \equiv \frac{E[r_i] - r_f}{E\left[r_w^*\right] - r_f} \geq \beta_i^{Perceived} \equiv \frac{Cov\left(r_i,\ r_w^*\right)}{Var\left(r_w^*\right)}, \quad \text{if } \beta_i^{Perceived} \geq 1;$$

$$\beta_i^{True} \equiv \frac{E[r_i] - r_f}{E[r_w^*] - r_f} < \beta_i^{Perceived} \equiv \frac{Cov\left(r_i,\ r_w^*\right)}{Var\left(r_w^*\right)}, \quad \text{if } \beta_i^{Perceived} < 1.$$

Hypothesis II The distribution of the perceived betas tends to center more around 1 in an incomplete market than that of true betas.

III.2 Test strategies and data

In this section, we empirically test the hypotheses and examine the behaviours of betas in an incomplete market. For the empirical tests, it is crucial to identify the rate of return from a portfolio of nontraded assets in an incomplete market. Relatedly, Fama and Schwert (1977) investigate the extent to which non-traded human capital affects the SML and find a weak correlation between returns on human capital and the market portfolio, concluding that the effect on the SML is only marginal. This finding may be due to the well-known fact that only a minority of households concentrated in the top of the income distribution own stocks. Only 21 per cent of households directly own stocks, while over 50 per cent of households in the top income decile directly hold stocks, according to the 2004 Survey of Consumer Finances. Much research also shows that considering the participation constraints is critical to understand the link between risk and return in the stock market better. Mankiw and Zeldes (1991) show that recognizing the difference in consumption between stockholders and non-stockholders improves the explanatory power of the consumption CAPM. Hence, simply relying on aggregate labour income as in Fama and Schwert (1977) may not account for the relationship between risk and return well in an incomplete market.

As an alternative to aggregate labour income as the nontraded asset, we consider highly illiquidity assets in an incomplete market because this helps to mitigate the participation constraints in Fama and Schwert (1977). Some assets are not traded frequently in the market and are thus much less liquid than other assets are (Easley *et al.*, 1996). For example, the market contains a financial asset called non-traded real estate investment trusts (non-traded REITs). They are registered with the Securities and Exchange Commission (SEC) and file regular SEC reports, but are not listed on an exchange or publicly traded. Hence, an investment in a non-traded REIT poses risks that differ from those of investments in a publicly traded REIT. Non-traded REITs are illiquid investments, meaning that investors cannot readily sell them in the market. Instead,

[5]Appendix 1 provides the detailed deductions for the hypotheses.

investors generally must wait until the non-traded REIT lists its shares on an exchange or liquidates its assets to achieve liquidity. These liquidity events, however, might not occur until more than 10 years after the investment. Therefore, investors are likely to view highly illiquid financial assets as non-traded assets, and the market should also consider the importance of the related illiquidity risk. In particular, our model suggests an effect on asset betas and prices when investors with non-traded assets (or highly illiquid assets) cannot achieve the most efficient portfolio investment. Similarly, Munk (2000) also considers the optimal consumption and portfolio choice of an investor with an uncertain stream of income from non-traded assets, showing that liquidity constraints from non-traded assets mainly explain the implicit value of the assets. Relatedly, Officer (2007) examines the setting of acquisitions and finds that liquidity from the sale of previously non-traded assets is priced. Consequently, we consider that risk from significantly illiquid assets could be highly and positively correlated with risk from unobservable non-traded assets, as very illiquid assets are often untradeable.

Specifically, we compute the difference between the returns on a portfolio of highly illiquid assets and those on a portfolio of highly liquid assets and denote it by $\tilde{Y}$. By assuming that $Y = E[Y] + \epsilon_Y$, and ϵ_Y and $\epsilon_{\tilde{Y}}$ are positively correlated,[6] we consider $\tilde{Y}$ a proxy for the non-traded risk factor.

In addition, in our empirical setting, we define r_i and r_w^* as the return on an individual traded asset and the value-weighted rate of return from a portfolio of traded assets, respectively. These together enable us to identify $Var(r_w^*)$, $Cov(r_w^*, Y)$, $Cov(r_i, Y)$, and $Cov(r_i, Y)$, which are determinants of the true betas in an incomplete market.

Our tests focus on the US stock market from 1981 to 2010 and our sample comprises all stocks traded on the NYSE, AMEX, and NASDAQ. To decompose the stocks into highly liquid and illiquid portfolios, we classify individual stocks into five quintiles based on Amihud's (2002) illiquidity ratio each year.[7] Amihud's illiquidity ratio is the ratio of absolute stock returns to the dollar value of the trading volume. It captures the price impact of trading and is widely considered a good proxy for a firm's liquidity. Amihud (2002), Acharya and Pedersen (2005), Brennan *et al.* (2013), and Chordia *et al.* (2009) show that the ratio is significant in a cross section of stock returns. Thus, the ratio is appropriate as an experimental tool for decomposition into highly illiquid and relatively liquid stocks, and for subsequent examination of their effects on betas in market equilibrium. Using daily stock returns and trading volumes provided by the Center for Research in Security Prices (CRSP), we measure the yearly average of the daily Amihud illiquidity ratio as follows:

$$Amihud_i = \frac{\sum_{d=1}^{t} \frac{|r_{i,d}|}{Vol_{i,d}}}{D_i}$$

where $r_{i,d}$ is the return of stock i on day d, $Vol_{i,d}$ is the trading volume of stock i on day d in dollar value, and D_i is the number of days over the year. The higher the ratio, the more illiquid the stock is over the year. Therefore, stocks in the highest quintile serve as proxies for non-traded assets and those in the remaining quintiles are proxies for traded assets.

[6] At an aggregate or a group level, the positive correlation tends to persist. However, it is not the only reason we use the risk from the illiquid assets. The non-traded and illiquid assets could be grouped, especially when they are compared with liquid assets. Furthermore, the risk from the non-traded assets is hardly observable in most cases. Hence, we exploit this property and use $\tilde{Y}$ to represent Y.

[7] We also classify the stocks into 7 and 10 quintiles based on the illiquidity ratio to have more pronounced illiquid stocks, but our empirical results are not qualitatively different from those based on five quintiles. However, we use a sample based on five quintiles because having a sufficient number of non-traded assets in the market is more suitable for our empirical experiment to observe the impact of non-traded risk on the market.

Our empirical results may be sensitive to liquidity measures. Therefore, we consider other liquidity measures from the literature to classify stocks. Prior studies show that the illiquidity of stocks is significantly pronounced in firms with a high level of information asymmetry among investors (Attig *et al.*, 2006; Bhide, 1993; Easley *et al.*, 1996; Glosten and Milgrom, 1985). This line of thinking stems from the premise that the presence of informed traders in a financial market may affect its trading process. Specifically, discrepancies in information between informed and uninformed traders can lead to a loss of stock liquidity due to adverse selection costs, since it is reasonable that market participants in closer touch with a firm and its business are those who possess better information about that firm and trade on it. Amihud (2002) uses stock illiquidity as a proxy for information asymmetry because illiquidity reflects the price impact of informed trading. Chae (2005) documents that the cumulative trading volume decreases prior to earnings announcements and the effect could be related to the extent of information asymmetry around announcements. He also finds that a decrease in trading volume prior to an earnings announcement is more evident in small firms. In addition, Roll (1988) argues that idiosyncratic price movements are attributable to informed trading rather than impounding of public information. Ferreira and Laux (2007) consider idiosyncratic volatility as a proxy for information asymmetry and find that anti-takeover provisions are negatively related to idiosyncratic volatility. They argue that investors may have greater incentives to collect private information on firms with fewer anti-takeover provisions since these firms are more likely to be targets of other firms or investors. Following this literature, we use firm size and idiosyncratic volatility to gauge the extent of firm-level information asymmetry and consider them as proxies for stock liquidity. To construct the variables, we obtain data from the CRSP and COMPUSTAT databases. Specifically, we measure firm size for each quarter by market capitalization and calculate it as the natural log of the number of shares outstanding multiplied by the market price per share. We then compute the yearly average of firm sizes on a quarterly basis to reflect changes in firm size over a year. The smaller the firm size, the more illiquid the stock is over the year. Following Ang *et al.*'s (2006) study, we measure idiosyncratic volatility using the Fama-French three-factor model. In particular, we obtain idiosyncratic volatility as the yearly sum of the squared residuals of the regression of daily excess returns on the three Fama-French factors: market excess return, size, and book-to-market ratio. The higher the idiosyncratic volatility, the more illiquid the stock is over the year.

Further, our last metric for firm-level liquidity is the level of ownership held by institutional investors. Many studies suggest that institutional ownership is highly associated with a firm's liquidity. Bennett *et al.* (2003), Del Guercio (1996), Falkenstein (1996), and Gompers and Metrick (2001) show that institutional investors exhibit a strong preference for stocks with large market capitalizations and thick markets since their portfolios require frequent turn over. Schwartz and Shapiro (1991) argue that institutional investors are sensitive to transaction costs from trading illiquid stocks. The literature also shows that institutional ownership leads to stock liquidity. Schwartz and Shapiro (1991) document that institutional investors account for more than 70 per cent of market trading volume, suggesting that stock liquidity has increased with the growth in institutional ownership. Chordia *et al.* (2001) and Bennett *et al.* (2003) show that growth in institutional ownership increases a firm's share turnover. These studies imply that firms with large (small) shareholdings by institutional investors should have high (low) stock liquidity. We obtained the quarterly institutional holdings for all common stocks from March 1981 through December 2010 from Thomson Reuters, which are derived from institutional investors' 13F filings. The SEC requires that all institutional investors with US$100 million or more under management in exchange-traded or NASDAQ-quoted equity securities report all equity positions greater than 10,000 shares or US$200,000 in market value at the end of each quarter. They must file 13F reports within 45 days of the end of the calendar quarter.

TABLE 1
Descriptive statistics

Variables	*No. of Obs*	*Mean*	*Std. Dev.*	*Lower Quartile*	*Median*	*Upper Quartile*
Amihud's Illiquidity	7,023	10.9232	134.1242	0.0103	0.1712	1.8323
Idiosyncratic Volatility	6,601	0.0022	0.0171	0.0003	0.0007	0.0019
Size	7,171	11.9562	1.9066	9.5397	11.7278	13.1689
IO	6,517	0.2440	0.2555	0.0283	0.1473	0.4016

Notes: This table reports the descriptive statistics for the time-series mean, standard deviation, 25th percentile, median, and 75th percentile values of the annual cross-sectional averages for the firm characteristics from 1981 to 2010. Firm characteristics are obtained from the CRSP and COMPUSTAT databases. Institutional ownership is obtained from Thomson Reuters Institutional 13F Holdings. No. of Obs denotes the average number of firm-year observations in each year. Firms' illiquidity measures are obtained from Amihud's (2002) study. Idiosyncratic volatility is computed following Ang *et al.* (2006). IO is the total institutional ownership computed as the number of shares held by all institutions divided by shares outstanding. Size is the market capitalization computed as log of share price times the number of shares outstanding. Individual stocks are classified into five quintiles based on Amihud's illiquidity, idiosyncratic volatility, firm size, and institutional ownership each year. Stocks in the highest quintile serve as proxies for non-traded assets and those in the remaining quintiles are proxies for traded assets.

We eliminate any position for which we cannot observe the institution's holdings at the beginning and end of the quarter (i.e., when an institution's 13F holdings information is missing for two consecutive quarters). Institutional ownership is the ratio of the number of shares held by institutions to the total number of shares outstanding for stock i in each quarter. We then calculate the yearly average of quarterly institutional ownership to accommodate variation in ownership over a year.

Similar to Amihud's illiquidity ratio, we classify individual stocks into five quintiles for each year based on idiosyncratic volatility, (negative) firm size, and (negative) institutional ownership, and consider stocks in the highest and lowest quintiles as proxies for highly illiquid and liquid portfolios, respectively. Table 1 reports the descriptive statistics for the time-series mean, standard deviation, 25th percentile, median, and 75th percentile values of annual cross-sectional averages for Amihud's illiquidity ratio, idiosyncratic volatility, firm size, and institutional ownership from 1981 to 2010. The table shows sufficient cross-sectional variations in each proxy variable, suggesting that the choice of variables for the empirical experiments by specifying highly illiquid and liquid portfolios based on the extent of asset liquidity seems reasonable. Depending on the data availability for each variable, we have a different number of observations for the variables each year. However, the observations for all variables are extensive; we have more than 6,000 sample firms for each year. For example, we have more than 7,000 sample firms based on Amihud's illiquidity ratio and firm size for each year, implying that we use more than 210,000 firm-year observations for our empirical tests during the sample period. With these samples, we investigate the impact of non-traded risk on market equilibrium and the characteristics of individual risky assets from various aspects.

IV. RESULTS

Based on four proxy variables, we begin by testing our first hypothesis that the more incomplete a financial market is, the wider the discrepancy between β_i^{True} and $\beta_i^{Perceived}$ becomes. Each year, we compute β_i^{True} and $\beta_i^{Perceived}$ from $\frac{Cov(r_i,r_w^*)+Cov(r_i,Y)}{Var(r_w^*)+Cov(r_w^*,Y)}$ and $\frac{Cov(r_i,r_w^*)}{Var(r_w^*)}$, respectively, based on daily r_i, r_w^*, and Y values as well as daily portfolio weights. In particular, the first hypothesis implies that if $\beta_i^{Perceived} \geq (<)1$, $\beta_i^{True} \geq (<)\beta_i^{Perceived}$. Hence, we decompose the sample into two groups: one with $\beta_i^{Perceived} \geq 1$ and the other with $\beta_i^{Perceived} < 1$. Subsequently, we identify whether the corresponding β_i^{True} satisfies the conditions. Table 2 reports the results. We find strong evidence that $\beta_i^{True} \geq \beta_i^{Perceived}$ when $\beta_i^{Perceived} \geq 1$ with firm classifications based on all illiquidity measures. For instance, about 87% and 80% of firms with $\beta_i^{Perceived} \geq 1$ based on Amihud's illiquidity ratio satisfied the $\beta_i^{True} \geq \beta_i^{Perceived}$ condition in 1987 and 1996, respectively. In addition, the results are generally pronounced throughout the sample period.[8]

Our study is mainly concerned with the impact of non-traded risk on the aggregate market, and should thus also consider the relative importance of assets in the market. Therefore, we compute the value-weighted differences between β_i^{True} and $\beta_i^{Perceived}$ ($\beta_i^{True} - \beta_i^{Perceived}$) when $\beta_i^{Perceived} \geq 1$ and $\beta_i^{Perceived} < 1$ for each year. We expect that the value-weighted differences are positive and negative when $\beta_i^{Perceived} \geq 1$ and $\beta_i^{Perceived} < 1$, respectively. Table 3 reports the results. In general, we find evidence supporting our expectation. Based on Amihud's illiquidity ratio, size, and institutional ownership, the differences between β_i^{True} and $\beta_i^{Perceived}$ tend to be positive (negative) when $\beta_i^{Perceived} \geq 1$ ($\beta_i^{Perceived} < 1$), with a few exceptions in some years. However, we do not find consistent results based on idiosyncratic volatility throughout the sample period. Overall, based on the results in Tables 2 and 3, we find some evidence supporting our first hypothesis that the more incomplete a financial market is, the wider the discrepancy between β_i^{True} and $\beta_i^{Perceived}$ becomes.

Our second hypothesis is that the distribution of perceived betas centers more around 1 in an incomplete market than that of true betas. To examine the second hypothesis, we first show the yearly distributions of perceived and true betas in an incomplete market as well as their kernel density estimates. To gain a better visibility of distributions, we winsorize the betas at the 1 per cent and 99 per cent levels for each year. Figure 1 represents the distributions and kernel density estimates for perceived and true betas with firm classifications based on Amihud's illiquidity ratio. We find that, with a few exceptions, perceived betas tend to cluster more than true betas during the sample period. We also find similar distribution characteristics based on idiosyncratic volatility, firm size, and institutional ownership, although these remain unreported for brevity.[9] Second, we compute equal-weighted variances for perceived and true betas for each year based on four firm classifications. Table 4 shows that, in most cases, the equal-weighted variances for perceived betas are smaller than those for true betas. Further, in the same table, we report generally significant statistical differences between the two variances for each year based on the F-test. Last, to account for the relative influence of assets on the market due to their size, Table 5 provides value-weighted variances for perceived and true betas along with their value-weighted means in an incomplete market for each year. We find that the value-weighted means for both betas are close to 1, supporting Lemma 1 above.[10] Moreover, the result shows that perceived betas centre around 1 more often than true betas do, since the value-weighted variances tend to be smaller for perceived betas than true betas during the sample period. Overall, we find solid evidence supporting our second hypothesis in several respects.

[8]In addition, we find that an equal-weighted $\beta_i^{True}$is smaller than an equal-weighted $\beta_i^{Perceived}$ is, and the difference is statistically significant. This implies that the difference between true beta and perceived beta could be larger than the difference that Fama and Schwert (1977) document. We report the results in Appendix 2.

[9]These results are available upon request.

[10]The value-weighted means for perceived and true betas are not exactly 1 because our empirical strategy relies on daily value-weighted market portfolio returns, which allows for daily changes in relative weights for individual stocks in the market portfolio. However, we compute the value-weighted means based on the weights at the end of the year after estimating the perceived and true betas.

TABLE 2

Proportion of firms satisfying $\beta_i^{True} \geq (<) \beta_i^{Perceived}$ *if* $\beta_i^{Perceived} \geq (<) 1$ *in an incomplete market*

	Amihud's Illiquidity		*Idiosyncratic Volatility*		*Size*		*IO*	
Year	$\beta_i^{Perceived} \geq 1$	$\beta_i^{Perceived} < 1$	$\beta_i^{Perceived} \geq 1$	$\beta_i^{Perceived} < 1$	$\beta_i^{Perceived} \geq 1$	$\beta_i^{Perceived} < 1$	$\beta_i^{Perceived} \geq 1$	$\beta_i^{Perceived} < 1$
1981	0.7797	0.0525	0.3326	0.3326	0.8798	0.0781	0.8339	0.0950
1982	0.6587	0.1084	0.2743	0.4851	0.8018	0.1042	0.7856	0.0990
1983	0.7591	0.1608	0.6290	0.3007	0.8508	0.1207	0.8327	0.0799
1984	0.8002	0.1398	0.6574	0.3095	0.8056	0.1396	0.8301	0.0809
1985	0.7242	0.2454	0.7029	0.2805	0.7701	0.2100	0.7985	0.1330
1986	0.7461	0.1523	0.6428	0.2394	0.7308	0.1861	0.7753	0.0969
1987	0.8565	0.0763	0.8522	0.0820	0.8717	0.0673	0.8390	0.0564
1988	0.7005	0.1438	0.7038	0.1475	0.6993	0.1361	0.7013	0.0663
1989	0.6222	0.1863	0.6011	0.1915	0.6652	0.1614	0.7059	0.0956
1990	0.7931	0.2357	0.7171	0.2194	0.7768	0.1685	0.8205	0.1246
1991	0.7637	0.207	0.7339	0.2121	0.7763	0.1783	0.8493	0.1356
1992	0.7803	0.2348	0.6280	0.2422	0.8133	0.2112	0.8580	0.1851
1993	0.7441	0.289	0.6279	0.2207	0.7602	0.2435	0.8459	0.2009
1994	0.7209	0.2137	0.7195	0.2433	0.8298	0.1850	0.8672	0.1433
1995	0.722	0.2827	0.6644	0.2496	0.8255	0.2269	0.8930	0.1727
1996	0.8005	0.2708	0.8038	0.1699	0.8831	0.2033	0.9076	0.2033
1997	0.7242	0.2056	0.7773	0.1253	0.8607	0.1246	0.8474	0.1339
1998	0.7937	0.2304	0.8514	0.1416	0.9038	0.1165	0.8847	0.1705
1999	0.5604	0.3256	0.7749	0.3358	0.6684	0.2366	0.7045	0.3492
2000	0.5069	0.3309	0.9275	0.3841	0.9436	0.3614	0.9517	0.4164

(*Continued*)

TABLE 2
Continued

	Amihud's Illiquidity		*Idiosyncratic Volatility*		*Size*		*IO*	
Year	$\beta_i^{Perceived} \geq 1$	$\beta_i^{Perceived} < 1$	$\beta_i^{Perceived} \geq 1$	$\beta_i^{Perceived} < 1$	$\beta_i^{Perceived} \geq 1$	$\beta_i^{Perceived} < 1$	$\beta_i^{Perceived} \geq 1$	$\beta_i^{Perceived} < 1$
2001	0.5504	0.2556	0.7581	0.3118	0.9104	0.2271	0.9281	0.1717
2002	0.5953	0.2875	0.6494	0.3380	0.8201	0.2275	0.8903	0.1121
2003	0.6813	0.3947	0.3662	0.4885	0.8276	0.2191	0.8950	0.1410
2004	0.6705	0.2253	0.4004	0.3078	0.7692	0.2425	0.9348	0.1899
2005	0.6834	0.3007	0.3560	0.3595	0.7361	0.1763	0.9346	0.2300
2006	0.6948	0.3503	0.4028	0.3747	0.7493	0.2608	0.9248	0.2300
2007	0.6044	0.3032	0.4297	0.3479	0.5798	0.2016	0.8974	0.2935
2008	0.6633	0.3032	0.4784	0.3405	0.6458	0.2041	0.8992	0.2056
2009	0.5742	0.4113	0.5588	0.3501	0.6963	0.2359	0.8750	0.2690
2010	0.5513	0.4534	0.3443	0.4064	0.8015	0.2115	0.8593	0.3042
N	65,773	84,907	52,744	105,684	66,578	105,537	64,849	91,577
Mean	0.6649	0.2644	0.5906	0.2748	0.7743	0.1955	0.8705	0.1763

Notes: This table reports the proportion of firms satisfying $\beta_i^{True} \geq (<)\beta_i^{Perceived}$ if $\beta_i^{Perceived} \geq (<)1$ each year from 1981 to 2010. Firm characteristics are obtained from the CRSP and COMPUSTAT databases. Institutional ownership is obtained from Thomson Reuters Institutional 13F Holdings. Firms' illiquidity measures are obtained from Amihud's (2002) study. Idiosyncratic volatility is computed following Ang *et al.* (2006). IO is the total institutional ownership computed as the number of shares held by all institutions divided by shares outstanding. Size is the market capitalization computed as log of share price times the number of shares outstanding. Individual stocks are classified into five quintiles based on Amihud's illiquidity, idiosyncratic volatility, firm size, and institutional ownership each year. Stocks in the highest quintile serve as proxies for nontraded assets and those in the remaining quintiles are proxies for traded assets. N is the number of observations over the sample period. Mean represents the time-series averages of the proportions each year.

TABLE 3
Value-weighted differences between β_i^{True} and $\beta_i^{Perceived}$ when $\beta_i^{Perceived} \geq 1$ and $\beta_i^{Perceived} < 1$ in an incomplete market

	Amihud's Illiquidity		*Idiosyncratic Volatility*		*Size*		*IO*	
Year	*VW_D1*	*VW_D2*	*VW_D1*	*VW_D2*	*VW_D1*	*VW_D2*	*VW_D1*	*VW_D2*
1981	0.3496	−0.3125	−0.0912	0.8395	−0.1711	0.1464	−0.0484	0.0428
1982	0.1649	0.1568	0.1127	−0.0337	0.1349	0.1426	−0.0505	0.0493
1983	0.2241	−0.1579	−0.1412	0.0567	−0.0625	0.0789	−0.0546	0.0550
1984	0.1431	−0.1034	−0.0740	0.0464	0.1509	−0.1128	−0.0816	−0.0613
1985	0.1699	−0.1517	−0.1148	0.1156	0.1225	−0.1134	−0.0585	0.0531
1986	−0.1308	0.1598	−0.1890	0.2088	0.0970	−0.1221	−0.0453	0.0389
1987	0.1119	−0.169	−0.0966	0.1359	−0.1425	0.2114	−0.0960	0.1425
1988	−0.2165	0.3089	−0.1485	0.2102	0.1938	0.2848	−0.1116	0.1551
1989	0.2417	0.2803	−0.2255	0.2894	−0.2691	−0.3347	−0.1627	0.1665
1990	0.0158	−0.0302	−0.1348	0.1418	0.0555	−0.0463	0.0024	−0.0147
1991	0.0133	−0.0508	−0.0742	0.1184	0.0448	−0.0879	0.0127	0.0033
1992	0.0242	−0.0866	−0.3274	0.4073	0.1322	−0.0464	0.0592	−0.0242
1993	0.0492	−0.0339	−0.2738	0.3343	0.1286	0.0161	0.1006	−0.0355
1994	0.1264	−0.1004	−0.0914	0.0543	−0.0417	0.0357	−0.0054	−0.0008
1995	0.1157	−0.0758	−0.1879	0.1288	0.1228	−0.0848	0.1989	−0.1372
1996	0.0761	0.056	−0.1435	0.0877	−0.0084	−0.0022	0.0564	−0.0443
1997	0.1026	0.1038	−0.1548	0.1539	−0.1520	0.1357	−0.0723	−0.0720
1998	0.0288	−0.0211	−0.0336	0.0214	0.0171	−0.0261	0.0084	−0.0086
1999	−0.3255	0.2268	0.1005	−0.0223	−0.1784	0.1804	0.0948	−0.0191
2000	0.2104	−0.2445	0.2809	−0.2807	0.3290	−0.2963	0.3179	−0.2783
2001	0.2595	−0.2283	0.0656	−0.0470	0.1292	−0.0857	0.0272	−0.0316
2002	0.0357	−0.004	0.0452	−0.0329	0.0791	−0.0777	0.0019	0.0003
2003	0.025	0.0037	−0.0002	0.0026	0.0321	−0.0002	0.0213	−0.0110
2004	0.0056	−0.0145	−0.0942	0.0688	0.0905	−0.0549	0.0706	−0.0410
2005	0.0053	0.0019	0.0323	−0.0179	0.0880	−0.0527	0.0699	−0.0496
2006	0.0292	−0.0144	0.0486	−0.0262	0.1617	−0.1014	0.0700	−0.0452
2007	−0.0079	0.0023	0.0290	−0.0116	0.0311	−0.0047	0.0135	−0.0184
2008	0.004	−0.0006	0.0036	−0.0132	0.0057	−0.0153	0.0081	−0.0075
2009	0.0266	−0.0069	0.0448	−0.0231	0.0474	−0.0262	0.0077	−0.0082
2010	0.0034	−0.0004	−0.0048	0.0047	0.0257	−0.0198	0.0182	−0.0156
Mean	0.0627	−0.0168	−0.0612	0.0972	0.0397	−0.0159	0.0124	−0.0072

Notes: This table reports the value-weighted differences between β_i^{True} and $\beta_i^{Perceived}$ ($\beta_i^{True} - \beta_i^{Perceived}$) when $\beta_i^{Perceived} \geq 1$ and $\beta_i^{Perceived} < 1$ each year from 1981 to 2010. VW_D1 and VW_D2 are the value-weighted ($\beta_i^{True} - \beta_i^{Perceived}$) when $\beta_i^{Perceived} \geq 1$ and $\beta_i^{Perceived} < 1$, respectively. Firm characteristics are obtained from the CRSP and COMPUSTAT databases. Institutional ownership is obtained from Thomson Reuters Institutional 13F Holdings. Firms' illiquidity measures are obtained from Amihud's (2002) study. Idiosyncratic volatility is computed following Ang *et al.* (2006). IO is the total institutional ownership computed as the number of shares held by all institutions divided by shares outstanding. Size is the market capitalization computed as log of share price times the number of shares outstanding. Individual stocks are classified into five quintiles based on Amihud's illiquidity, idiosyncratic volatility, firm size, and institutional ownership each year. Stocks in the highest quintile serve as proxies for nontraded assets and those in the remaining quintiles are proxies for traded assets. Mean represents the time-series averages of the differences each year.

As a robustness check on our main findings, we further consider empirical tests based on the alternative liquidity measure in Pástor and Stambaugh (2003). Specifically, following Pástor and Stambaugh (2003), we first compute the liquidity measure for stock i in month t as a cross-sectional estimate of $\gamma_{i,t}$ in the following model:

$$r^e_{i,d+1,t} = \theta_{i,t} + \phi_{i,t} r_{i,d,t} + \gamma_{i,t} sign\left(r^e_{i,d,t}\right) \cdot v_{i,d,t} + \varepsilon_{i,\mathrm{d}+1,t}, \quad d = 1, \ldots, D,$$

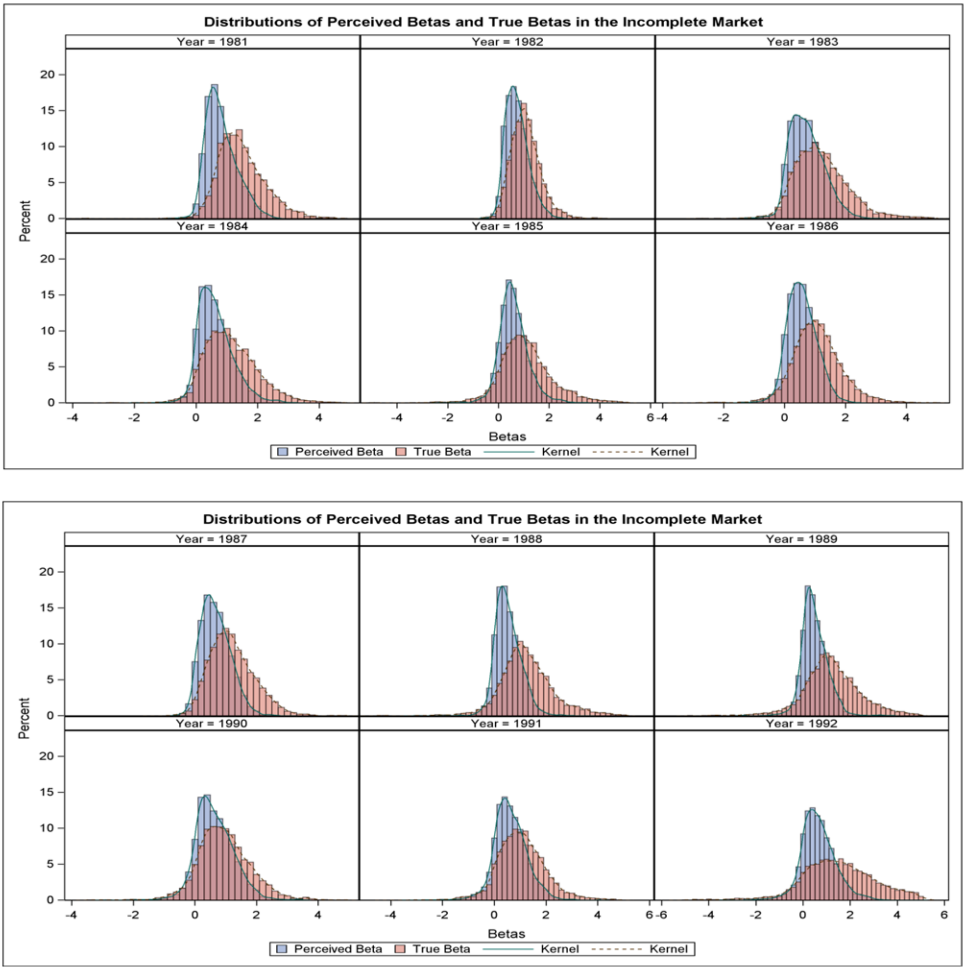

Fig. 1. Distributions of $\beta_i^{Perceived}$ and β_i^{True} in the incomplete market. [Colour figure can be viewed at wileyonlinelibrary.com]

Notes: This figure presents the distributions and kernel density estimates of $\beta_i^{Perceived}$ and β_i^{True} in an incomplete market each year from 1981 to 2010. Firms' illiquidity measures are computed from Amihud's (2002) study based on the CRSP and COMPUSTAT databases. Individual stocks are classified into five quintiles based on Amihud's illiquidity each year, and stocks in the highest quintile serve as proxies for nontraded assets and those in the remaining quintiles are proxies for traded assets.

where $\gamma_{i,d,t}$ is the return on stock i on day d in month t; $r^e_{(i,d,t)} = \gamma_{(i,d,t)} - \gamma_{(m,d,t)}$, where $\gamma_{m,d,t}$ is the return on the CRSP value-weighted market return on day d in month t; and $v_{i,d,t}$ is the dollar volume for stock i on day d in month t. $\gamma_{i,t}$ measures the firm's monthly liquidity, and we expect it to be negative and larger in absolute magnitude when liquidity is lower.

We classify individual stocks into five quintiles based on the yearly average of monthly $\gamma_{i,t}$. The lower the value, the more illiquid the stock is over the year. Therefore, stocks in the lowest quintile serve as proxies for non-traded assets and those in the remaining quintiles are proxies for traded assets. Based on the liquidity measure in Pástor and Stambaugh (2003), we

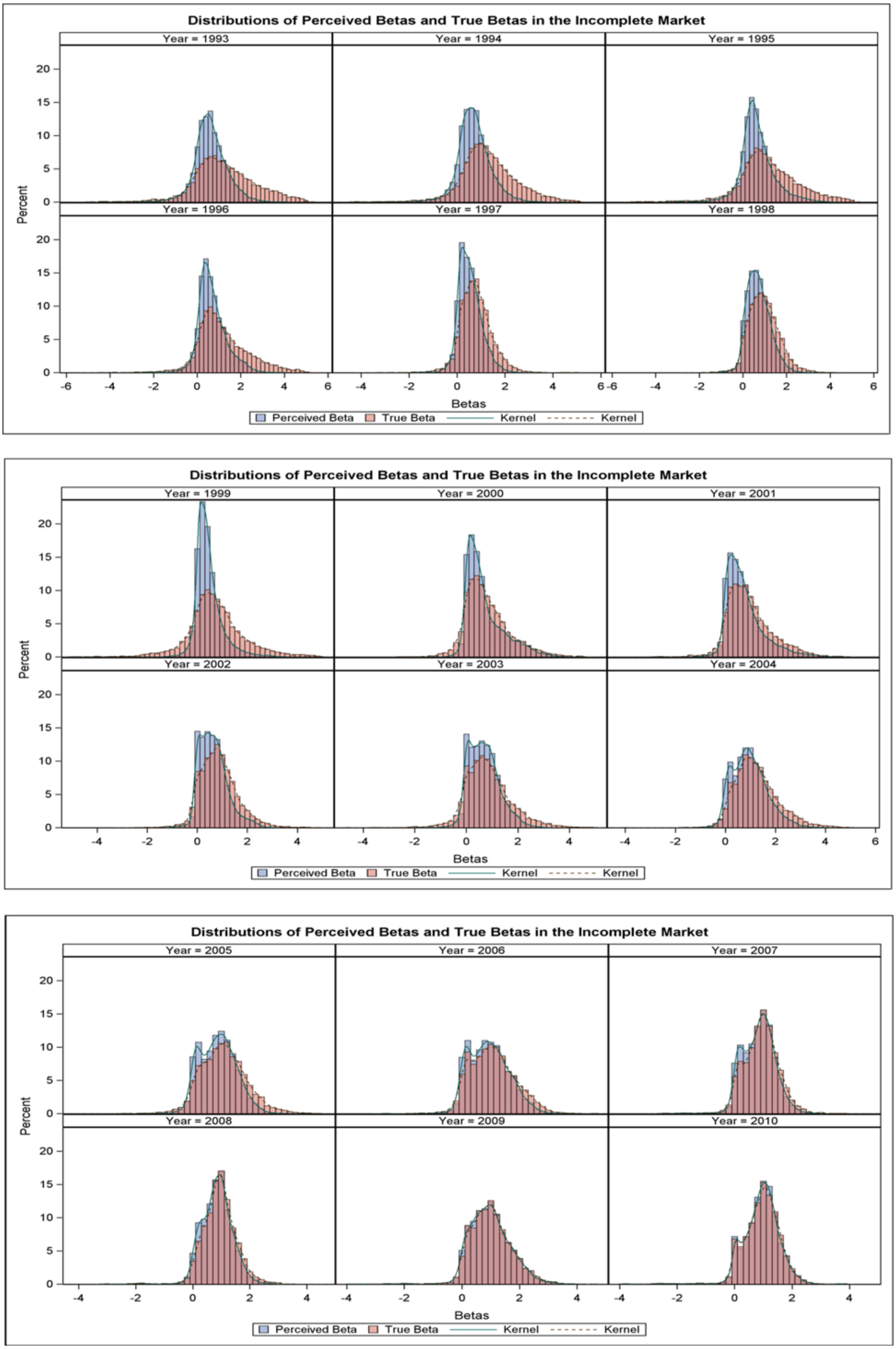

Fig. 1. *Continued.*

TABLE 4

Equal-weighted variances for $\beta_i^{Perceived}$ *and* β_i^{True} *in an incomplete market*

Panel A	*Amihud's Illiquidity*				*Idiosyncratic Volatility*			
Year	*EW_V1*	*EW_V2*	*Difference*	*F-test*	*EW_V1*	*EW_V2*	*Difference*	*F-test*
1981	0.2385	0.5604	−0.3219***	2.35 (p = 0.0001)	0.3268	0.7420	−0.4152***	2.27 (p = 0.0001)
1982	0.192	0.3172	−0.1252***	1.65 (p = 0.0001)	0.2192	0.3154	−0.0962***	1.44 (p = 0.0001)
1983	0.3016	0.7868	−0.4851***	2.61 (p = 0.0001)	0.2966	0.5697	−0.2731***	1.92 (p = 0.0001)
1984	0.3044	0.7066	−0.4022***	2.32 (p = 0.0001)	0.3091	0.6014	−0.2923***	1.95 (p = 0.0001)
1985	0.3309	1.199	−0.8682***	3.62 (p = 0.0001)	0.3352	1.1044	−0.7691***	3.29 (p = 0.0001)
1986	0.2231	0.6606	−0.4376***	2.96 (p = 0.0001)	0.2295	0.7427	−0.5132***	3.25 (p = 0.0001)
1987	0.2207	0.5139	−0.2932***	2.33 (p = 0.0001)	0.2170	0.4328	−0.2159***	2.00 (p = 0.0001)
1988	0.244	1.2526	−1.0086***	5.13 (p = 0.0001)	0.2425	0.5726	−0.3301***	2.36 (p = 0.0001)
1989	0.3085	2.0512	−1.7427***	6.65 (p = 0.0001)	0.2933	1.2412	−0.9479***	4.23 (p = 0.0001)
1990	0.3802	0.7709	−0.3907***	2.03 (p = 0.0001)	0.3670	1.0199	−0.6529***	2.78 (p = 0.0001)
1991	0.4248	0.9914	−0.5666***	2.33 (p = 0.0001)	0.4202	2.0187	−1.5985***	4.81 (p = 0.0001)
1992	0.6072	3.7234	−3.1162***	6.13 (p = 0.0001)	0.6126	17.3272	−16.7146***	28.28 (p = 0.0001)
1993	0.6405	3.3981	−2.7576***	5.31 (p = 0.0001)	0.6597	16.6807	−16.0210***	25.28 (p = 0.0001)

(Continued)

TABLE 4
Continued

Panel A	*Amihud's Illiquidity*				*Idiosyncratic Volatility*			
Year	*EW_V1*	*EW_V2*	*Difference*	*F-test*	*EW_V1*	*EW_V2*	*Difference*	*F-test*
1994	0.5134	1.7729	−1.2595***	3.45 (p = 0.0001)	0.5187	1.1115	−0.5929***	2.14 (p = 0.0001)
1995	0.6871	2.9429	−2.2559***	4.28 (p = 0.0001)	0.6977	2.4252	−1.7275***	3.48 (p = 0.0001)
1996	0.429	1.642	−1.2130***	3.83 (p = 0.0001)	0.4385	1.0927	−0.6541***	2.49 (p = 0.0001)
1997	0.2252	0.4661	−0.2408***	2.07 (p = 0.0001)	0.2264	0.4460	−0.2196***	1.97 (p = 0.0001)
1998	0.2741	0.4982	−0.2241***	1.82 (p = 0.0001)	0.2758	0.4665	−0.1907***	1.69 (p = 0.0293)
1999	0.2919	1.6698	−1.3779***	5.72 (p = 0.0001)	0.2944	0.8720	−0.5776***	2.96 (p = 0.0001)
2000	0.5274	0.785	−0.2576***	1.49 (p = 0.0001)	0.5486	1.4779	−0.9293***	2.69 (p = 0.0001)
2001	0.5396	0.8158	−0.2761***	1.51 (p = 0.0001)	0.5445	0.7078	−0.1633***	1.30 (p = 0.0001)
2002	0.3087	0.5971	−0.2884***	1.93 (p = 0.0001)	0.3111	0.3481	−0.0370***	1.12 (p = 0.0001)
2003	0.3411	0.9078	−0.5668***	2.66 (p = 0.0001)	0.3423	0.4095	−0.0671***	1.20 (p = 0.0001)
2004	0.4725	0.7974	−0.3249***	1.69 (p = 0.0001)	0.4918	0.5506	−0.0587***	1.12 (p = 0.0001)
2005	0.3829	0.6529	−0.2700***	1.71 (p = 0.0001)	0.3918	0.4732	−0.0815***	1.21 (p = 0.0001)
2006	0.4558	0.6072	−0.1514***	1.33 (p = 0.0001)	0.4684	0.6610	−0.1926***	1.41 (p = 0.0001)

(*Continued*)

TABLE 4
Continued

Panel A	*Amihud's Illiquidity*				*Idiosyncratic Volatility*			
Year	*EW_V1*	*EW_V2*	*Difference*	*F-test*	*EW_V1*	*EW_V2*	*Difference*	*F-test*
2007	0.31	0.3742	−0.0642***	1.21 (p = 0.0001)	0.3209	0.3252	−0.0043	1.01 (p = 0.5911)
2008	0.3066	0.3739	−0.0673***	1.22 (p = 0.0001)	0.3156	0.3126	0.0030	1.01 (p = 0.7014)
2009	0.5579	0.6037	−0.0459***	1.08 (p = 0.0017)	0.5759	0.6414	−0.0655***	1.11 (p = 0.0001)
2010	0.432	0.4494	−0.0174	1.04 (p = 0.1183)	0.4408	0.4339	0.0069	1.02 (p = 0.5303)

Panel B	*Size*				*IO*			
Year	*EW_V1*	*EW_V2*	*Difference*	*F-test*	*EW_V1*	*EW_V2*	*Difference*	*F-test*
1981	0.3355	0.724	−0.3886***	2.16 (p = 0.0001)	0.2613	0.3324	−0.0710***	1.27 (p = 0.0001)
1982	0.226	0.5906	−0.3646***	2.61 (p = 0.0001)	0.1925	0.2277	−0.0353***	1.18 (p = 0.0001)
1983	0.3071	0.8473	−0.5402***	2.76 (p = 0.0001)	0.2332	0.3493	−0.1161***	1.50 (p = 0.0001)
1984	0.3111	0.8621	−0.5510***	2.77 (p = 0.0001)	0.2795	0.4174	−0.1379***	1.49 (p = 0.0001)
1985	0.334	1.4738	−1.1398***	4.41 (p = 0.0001)	0.2494	0.4880	−0.2386***	1.96 (p = 0.0001)
1986	0.2335	0.9823	−0.7488***	4.21 (p = 0.0001)	0.1644	0.2657	−0.1013***	1.62 (p = 0.0001)

(*Continued*)

TABLE 4
Continued

Panel B	Size				IO			
Year	EW_V1	EW_V2	Difference	F-test	EW_V1	EW_V2	Difference	F-test
1987	0.2254	0.5849	−0.3595***	2.59 (p = 0.0001)	0.1646	0.3283	−0.1637***	1.99 (p = 0.0001)
1988	0.2366	0.9166	−0.6800***	3.87 (p = 0.0001)	0.1908	0.2901	−0.0993***	1.52 (p = 0.0001)
1989	0.2943	2.065	−1.7707***	7.02 (p = 0.0001)	0.2058	0.3666	−0.1608***	1.78 (p = 0.0001)
1990	0.3699	1.0153	−0.6454***	2.74 (p = 0.0001)	0.2712	0.4876	−0.2164***	1.80 (p = 0.0935)
1991	0.4191	2.3725	−1.9534***	5.66 (p = 0.0001)	0.2780	0.4624	−0.1844***	1.66 (p = 0.0042)
1992	0.6346	5.7864	−5.1519***	9.12 (p = 0.0001)	0.4539	1.0048	−0.5509***	2.21 (p = 0.0001)
1993	0.659	9.1688	−8.5098***	13.91 (p = 0.0001)	0.4311	1.1844	−0.7533***	2.75 (p = 0.0001)
1994	0.5243	2.1966	−1.6723***	4.19 (p = 0.0001)	0.3362	0.6675	−0.3313***	1.99 (p = 0.0001)
1995	0.6909	3.9303	−3.2394***	5.69 (p = 0.0001)	0.5309	1.4957	−0.9649***	2.82 (p = 0.0001)
1996	0.4357	1.9036	−1.4678***	4.37 (p = 0.0001)	0.3595	1.1249	−0.7654***	3.13 (p = 0.0001)
1997	0.2279	0.7953	−0.5674***	3.49 (p = 0.0001)	0.1884	0.3250	−0.1367***	1.73 (p = 0.0001)
1998	0.2776	0.6828	−0.4051***	2.46 (p = 0.0001)	0.2431	0.3923	−0.1491***	1.61 (p = 0.0001)
1999	0.2886	5.3306	−5.0420***	18.47 (p = 0.0001)	0.2872	1.4791	−1.1920***	5.15 (p = 0.0001)

(*Continued*)

TABLE 4
Continued

Panel B	Size				IO			
Year	EW_V1	EW_V2	Difference	F-test	EW_V1	EW_V2	Difference	F-test
2000	0.525	2.0224	−1.4973***	3.85 (p = 0.0001)	0.5565	1.4540	−0.8974***	2.61 (p = 0.0001)
2001	0.5357	1.0112	−0.4756***	1.89 (p = 0.0001)	0.5672	0.7790	−0.2118***	1.37 (p = 0.0001)
2002	0.3114	0.5594	−0.2480***	1.80 (p = 0.0001)	0.2453	0.3158	−0.0705***	1.29 (p = 0.0001)
2003	0.3434	1.177	−0.8336***	3.43 (p = 0.0001)	0.2548	0.3828	−0.1280***	1.50 (p = 0.0001)
2004	0.4797	0.9235	−0.4438***	1.93 (p = 0.0001)	0.4046	0.5914	−0.1867***	1.46 (p = 0.0001)
2005	0.3868	0.6053	−0.2185***	1.57 (p = 0.0001)	0.3123	0.4400	−0.1277***	1.41 (p = 0.0001)
2006	0.4582	0.735	−0.2768***	1.60 (p = 0.0001)	0.4039	0.5402	−0.1364***	1.34 (p = 0.0001)
2007	0.3126	0.3543	−0.0417***	1.13 (p = 0.0001)	0.2316	0.3151	−0.0834***	1.36 (p = 0.0001)
2008	0.3099	0.3211	−0.0112	1.04 (p = 0.1464)	0.1977	0.2366	−0.0389***	1.20 (p = 0.0001)
2009	0.5634	0.6252	−0.0618***	1.11 (p = 0.0001)	0.4048	0.4606	−0.0559***	1.14 (p = 0.0006)
2010	0.4344	0.479	−0.0446***	1.10 (p = 0.0001)	0.2363	0.2736	−0.0373***	1.16 (p = 0.0001)

Notes: This table reports the equal-weighted variances for $\beta_i^{Perceived}$ and β_i^{True} each year from 1981 to 2010. EW_V1 and EW_V2 are the equal-weighted variances for $\beta_i^{Perceived}$ and β_i^{True}, respectively. Firm characteristics are obtained from the CRSP and COMPUSTAT databases. Institutional ownership is obtained from Thomson Reuters Institutional 13F Holdings. Firms' illiquidity measures are obtained from Amihud's (2002) study. The idiosyncratic volatility is computed following Ang *et al.* (2006). IO is the total institutional ownership computed as the number of shares held by all institutions divided by the shares outstanding. Size is the market capitalization computed as log of share price times the number of shares outstanding. Individual stocks are classified into five quintiles based on Amihud's illiquidity, idiosyncratic volatility, firm size, and institutional ownership each year. Stocks in the highest quintile serve as proxies for nontraded assets and those in the remaining quintiles are proxies for traded assets. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

TABLE 5
Value-weighted means and variances for $\beta_i^{Perceived}$ and β_i^{True} in an incomplete market

Panel A	*Amihud's Illiquidity*				*Idiosyncratic Volatility*			
Year	*VW_A1*	*VW_A2*	*VW_V1*	*VW_V2*	*VW_A1*	*VW_A2*	*VW_V1*	*VW_V2*
1981	0.9789	0.9926	0.0646	0.1963	0.9852	0.9792	0.0075	0.0165
1982	0.9945	0.9972	0.2231	0.2747	1.0003	1.0441	0.0011	0.0016
1983	0.9915	0.9704	0.0359	0.0831	0.9966	0.9587	0.0333	0.0524
1984	0.9897	0.9802	0.0200	0.0337	0.9956	0.9876	0.0183	0.0266
1985	1.0058	1.0056	0.0302	0.0745	1.0185	1.0242	0.0387	0.0859
1986	1.0009	1.0028	0.0118	0.0192	1.0106	1.0021	0.0238	0.0427
1987	1.0194	1.0292	0.0047	0.0075	1.0227	1.0263	0.0065	0.0084
1988	1.0016	1.0087	0.0093	0.0262	1.0078	1.0094	0.0098	0.0116
1989	1.0089	1.0028	0.0083	0.0309	1.0154	1.0082	0.0083	0.0181
1990	1.0150	1.0098	0.0060	0.0090	1.0165	1.0057	0.0044	0.0087
1991	1.0243	1.0399	0.0053	0.0090	1.0271	1.0404	0.0056	0.0194
1992	1.0164	1.0478	0.0044	0.0250	1.0160	1.0567	0.0047	0.1263
1993	1.0094	1.0503	0.0059	0.0266	1.0083	1.0629	0.0066	0.1572
1994	1.0045	0.9997	0.0146	0.0470	1.0068	0.9937	0.0126	0.0232
1995	0.9918	0.9976	0.0228	0.0887	0.9982	1.0088	0.0245	0.0751
1996	1.0128	1.0103	0.0108	0.0343	1.0142	0.9994	0.0115	0.0234
1997	0.9989	1.0009	0.0071	0.0083	1.0027	1.0008	0.0073	0.0075
1998	1.0390	1.0337	0.0040	0.0053	1.0377	1.0302	0.0042	0.0050
1999	1.1257	1.0552	0.0134	0.0313	1.1347	1.1821	0.0144	0.0203
2000	0.8757	0.9772	0.1319	0.1919	0.8838	0.7819	0.1418	0.3804
2001	0.9319	0.9747	0.1876	0.2677	0.9314	0.9277	0.1981	0.2424
2002	0.9836	1.0013	0.2835	0.4105	0.9830	0.9847	0.3033	0.3051
2003	1.0122	1.0265	0.3853	0.8023	1.0131	1.0140	0.4022	0.4602
2004	0.9991	1.0100	0.4527	0.8144	1.0014	1.0040	0.4873	0.5446
2005	1.0072	1.0106	0.1995	0.3259	1.0095	1.0158	0.2100	0.2440
2006	1.0005	1.0035	0.1497	0.1979	1.0042	1.0100	0.1571	0.2183
2007	1.0035	1.0010	0.0232	0.0260	1.0110	1.0189	0.0247	0.0244
2008	0.9859	0.9874	0.0052	0.0060	0.9887	0.9832	0.0054	0.0052
2009	1.0747	1.0834	0.0161	0.0172	1.0838	1.0928	0.0167	0.0184
2010	1.0213	1.0193	0.3601	0.3717	1.0227	1.0225	0.3737	0.3688
Mean	1.0041	1.0109	0.0898	0.1487	1.0082	1.0092	0.0854	0.1181

Panel B	*Size*				*IO*			
Year	*VW_A1*	*VW_A2*	*VW_V1*	*VW_V2*	*VW_A1*	*VW_A2*	*VW_V1*	*VW_V2*
1981	0.9735	0.9817	0.0102	0.0225	0.9847	0.9867	0.0737	0.0894
1982	0.9960	0.9934	0.0015	0.0024	0.9998	1.0009	0.0582	0.0544
1983	0.9940	1.0056	0.0424	0.0967	0.9938	0.9964	0.0368	0.0487
1984	0.9892	0.9812	0.0242	0.0459	0.9895	0.9869	0.0587	0.0770
1985	1.0051	1.0075	0.0354	0.1033	1.0076	1.0089	0.0604	0.0923
1986	1.0012	1.0036	0.0141	0.0311	1.0035	0.9984	0.0082	0.0087
1987	1.0193	1.0305	0.0056	0.0095	1.0186	1.0258	0.0336	0.0540
1988	1.0012	1.0085	0.0112	0.0205	1.0034	1.0067	0.0039	0.0033
1989	1.0085	0.9964	0.0098	0.0357	1.0102	1.0038	0.0033	0.0032

(*Continued*)

TABLE 5
Continued

Panel B	*Size*				*IO*			
Year	*VW_A1*	*VW_A2*	*VW_V1*	*VW_V2*	*VW_A1*	*VW_A2*	*VW_V1*	*VW_V2*
1990	1.0144	1.0047	0.0053	0.0098	1.0130	1.0076	0.0287	0.0433
1991	1.0249	1.0402	0.0064	0.0264	1.0195	1.0278	0.0154	0.0212
1992	1.0162	1.0591	0.0052	0.0466	1.0157	1.0331	0.0174	0.0401
1993	1.0078	1.0754	0.0068	0.0902	1.0097	1.0374	0.0137	0.0371
1994	1.0043	1.0041	0.0133	0.0506	1.0057	1.0028	0.0349	0.0731
1995	0.9919	0.9843	0.0250	0.1407	0.9916	0.9803	0.0665	0.1954
1996	1.0127	1.0077	0.0120	0.0450	1.0105	1.0110	0.0399	0.1161
1997	0.9988	0.9928	0.0078	0.0136	0.9974	0.9974	0.0223	0.0222
1998	1.0392	1.0360	0.0043	0.0076	1.0425	1.0428	0.0070	0.0092
1999	1.1257	1.1132	0.0143	0.1004	1.1165	1.1594	0.0209	0.0499
2000	0.8757	0.7758	0.1374	0.5453	0.8767	0.7835	0.0092	0.0256
2001	0.9317	0.9276	0.1988	0.3551	0.9384	0.9296	0.1063	0.1524
2002	0.9835	0.9750	0.3063	0.3887	0.9896	0.9906	0.0997	0.1155
2003	1.0122	1.0281	0.4066	1.0328	1.0144	1.0189	0.1413	0.1994
2004	0.9990	1.0028	0.4747	0.9892	1.0007	1.0055	0.2649	0.4552
2005	1.0069	1.0180	0.2070	0.3210	1.0079	1.0123	0.1002	0.1613
2006	1.0004	1.0042	0.1528	0.2525	0.9993	0.9992	0.0871	0.1312
2007	1.0035	1.0155	0.0240	0.0247	0.9987	0.9954	3.2189	4.4787
2008	0.9859	0.9800	0.0054	0.0052	0.9872	0.9866	0.0104	0.0129
2009	1.0748	1.0828	0.0164	0.0178	1.0796	1.0785	0.0287	0.0344
2010	1.0211	1.0243	0.3688	0.3958	1.0197	1.0207	0.1283	0.1597
Mean	1.0039	1.0053	0.0851	0.1742	1.0048	1.0044	0.1599	0.2321

Notes: This table reports the value-weighted means and variances for $\beta_i^{Perceived}$ and β_i^{True} each year from 1981 to 2010. VW_A1 (VW_V1) and VW_A2 (VW_V2) are the value-weighted means (variances) for $\beta_i^{Perceived}$ and β_i^{True}, respectively. Firm characteristics are obtained from the CRSP and COMPUSTAT databases. Institutional ownership is obtained from Thomson Reuters Institutional 13F Holdings. Firms' illiquidity measures are obtained from Amihud's (2002) study. Idiosyncratic volatility is computed following Ang *et al.* (2006). IO is the total institutional ownership computed as the number of shares held by all institutions divided by shares outstanding. Size is the market capitalization computed as log of share price times the number of shares outstanding. Individual stocks are classified into five quintiles based on Amihud's illiquidity, idiosyncratic volatility, firm size, and institutional ownership each year. Stocks in the highest quintile serve as proxies for nontraded assets and those in the remaining quintiles are proxies for traded assets. Mean represents the time-series averages of the means and variances each year.

revisit our main Tables 3 and 5, and report the results in Appendix 3. The results show that the value-weighted differences between β_i^{True} and $\beta_i^{Perceived}$ ($\beta_i^{True} - \beta_i^{Perceived}$) when $\beta_i^{Perceived} \geq 1$ and $\beta_i^{Perceived} < 1$ for most sample years. This is consistent with our first hypothesis that the more incomplete a financial market is, the wider the discrepancy between β_i^{True} and $\beta_i^{Perceived}$ becomes. We also provide value-weighted variances for perceived and true betas along with their value-weighted means in an incomplete market for each year. We find that the value-weighted means for both betas are close to 1, supporting Lemma 1. Moreover, the result shows that perceived betas center around 1 more often than true betas do, because the value-weighted variances tend to be smaller for perceived betas than true betas during the sample period. This corroborates our second hypothesis that the distribution of perceived betas centres more around 1 in an incomplete market than that of true betas. In sum, the findings in Appendix 3 suggest that our main findings are not sensitive to the measure of stock illiquidity and are thus robust.

V. CONCLUSION

This study examines the extent to which market incompleteness affects an investor's portfolio choices and beta in the traditional CAPM. We consider the extended CAPM in an incomplete financial market in which not all risky assets are traded and the risk from non-traded assets is not orthogonal to that of the existing assets. The CAPM betas (true betas) from the market equilibrium conditions in an incomplete market diverge from the traditional CAPM betas (perceived betas), and the distribution of perceived betas in an incomplete market centre more around 1 than that of true betas. This implies that the more incomplete a financial market is, the wider the discrepancy between true and perceived betas become. In addition, we find empirical evidence supporting the theoretical implications in various settings. Overall, this study highlights the effect of non-traded risk in an incomplete market on an investor's portfolio choices and the CAPM beta on the SML. In particular, our results suggest that practitioners should use the traditional CAPM beta with caution due to its limitation in actual practice.

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SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article at the publisher's website.

Appendix 1. Proofs of propositions and lemma
Appendix 2. Equal-weighted means for β_i^{True} and $\beta_i^{Perceived}$ in an incomplete market
Appendix 3. Value-weighted differences between β_i^{True} and $\beta_i^{Perceived}$ when $\beta_i^{Perceived} \geq 1$ and $\beta_i^{Perceived} < 1$ and value-weighted means and variances for $\beta_i^{Perceived}$ and β_i^{True} in an incomplete market