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Channel Sounding for Multi-User Massive MIMO in Distributed Antenna System Environment

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Abstract: We propose a generation scheme for a sounding reference signal (SRS) suitable for supporting a large number of users in massive multi-input multi-output (MIMO) system with a distributed antenna system (DAS) environment. The proposed SRS can alleviate the pilot contamination problem which occurs inherently in the multi-user system due to the limited number of orthogonal sequences. The proposed SRS sequence is generated by applying a well-chosen phase rotation to the conventional LTE/LTE-A SRS sequences without requiring an increased amount of resource usage. We also propose using the correlation-aided channel estimation algorithm as a supplemental scheme to obtain more reliable and refined channel estimation. It is shown that the proposed SRS sequence and the supplemental channel estimation scheme improve significantly the channel estimation performance in multi-user massive MIMO systems.

Keywords: massive multi-input multi-output (MIMO); distributed antenna systems (DAS); sounding reference signal (SRS); channel estimation

1. Introduction

It is expected that the amount of mobile wireless traffic in 2020 will be 1000 times higher than that of 2010 [1–3]. Along with the dramatic growth in the demand for wireless communications, performance requirements for data rate, spectral efficiency and energy efficiency are also getting higher [4–6]. To meet these growing demand and performance requirements for wireless communications, massive multi-input multi-output (m-MIMO) technology was proposed as one of the key technologies for the next generation cellular networks, known as fifth generation (5G) systems [7–10]. It is known that m-MIMO systems, whose transmitter or receiver is equipped with massive number of antennas, can improve the spectral efficiency and save energy in wireless communication systems [10–12]. As a result, m-MIMO systems have recently attracted many researchers and engineers in many aspects. Multi-user m-MIMO technology, in which a base station (BS) uses a large number of antennas to serve many pieces of user equipment (UE) simultaneously on the same time-frequency resource, is one example that is actively studied to be practically adopted in 5G systems [13–15]. The distributed antenna system (DAS) has also been considered a key technology for feasible deployment of 5G systems [16–18]. In DAS configuration, there exist a digital unit (DU) and multiple radio units (RUs) in a cell, where each RU is connected to DU via fiber optic links. The DU manages RUs in a centralized manner, by which RUs can transmit and receive signals in a cooperative manner.

The key requirement for enjoying the benefit of m-MIMO technology is to obtain the accurate channel state information (CSI) for each link at the BS, or at RUs in DAS environment. In the frequency-division duplex (FDD) approach, UEs estimate downlink (DL) channels by using DL pilot signals, or sounding reference signals (SRS), transmitted from the BS [19]. The required number

of DL pilots in an FDD based approach is proportional to the number of BS antennas multiplied by the number of served UEs, which complicates the adoption of such a DL channel estimation in massive MIMO environments. Thus, the m-MIMO systems typically employ the time-division duplex (TDD) approach to estimate the DL channel. In the TDD approach, UEs estimate DL channels by sending mutually orthogonal uplink (UL) pilot signals (SRS) to BS based on the DL and UL channel reciprocity within the channel coherence interval [9]. The total number of UL pilots required in such a TDD based approach is proportional to the number of served UEs irrespective of the number of BS antennas [7,9].

For a given length of sequence, say M , we may generate at most M orthogonal sequences. Conventionally, pilot sequences are mutually orthogonal, so the maximum number of pilot sequences is limited by M . If the number of UEs in simultaneous service exceeds M , we need to reuse some or all of the already generated orthogonal sequences. This results in the so-called pilot contamination problem [20] due to the violation of orthogonality between pilot sequences. The pilot contamination mainly limits the performance improvement of m-MIMO systems [7,15]. Most of the prior works that proposed to resolve this problem have considered the use of mutually orthogonal SRS sequences. They include pilot signal coordination [21], blind channel estimation with data samples [22] and cooperative multi-cell precoding in m-MIMO systems [23]. The number of mutually orthogonal SRS sequences is mainly limited by the length of the base sequence. In Long Term Evolution (LTE)/Long Term Evolution-Advanced (LTE-A) systems, the number of orthogonal SRS sequences is 16, which is not suitable for serving a large number of users in m-MIMO environments [24,25]. Using long pilot sequences may reduce the pilot contamination, but they replace data sequences and reduce channel spectral efficiency and throughput. If pilot sequences are made too long, they may occupy even the restricted band, which must be prevented. Thus, it may be desirable to generate a larger number of SRS sequences without increasing the sequence length.

For this purpose, we propose a mechanism for generating SRS sequences with a lower level of pilot contamination which is suitable for serving a large number of UEs. In the proposed mechanism, a phase rotation is applied to the base sequence without increasing the sequence length. The resultant SRS sequences may be mutually correlated and thus still incur a pilot contamination as the number of UEs grows and exceeds the length of base sequence. This results in high channel estimation error when a linear estimation based on the orthogonality of SRS sequences is used. To resolve this problem, we propose a two-step channel estimation algorithm by which the least square (LS) estimation [26] is first applied and the minimum mean squared error (MMSE) estimation [27] is additionally applied only to the group of UEs using mutually correlated SRS sequences. The proposed *correlation-aided channel estimation* shows the improved performance in the channel estimation. It is shown that the proposed SRS combined with the supplemental channel estimation algorithm guarantees lower mean squared error (MSE) in channel estimation, which alleviates the pilot contamination problem.

The rest of the paper is organized as follows. In Section 2, we introduce the system model in consideration. In Section 3, a brief review of SRS sequences in conventional LTE/LTE-A systems is provided. We introduce the generation of proposed SRS sequences and the analysis on the resultant correlation in Section 4. In Section 5, we propose the two-step channel estimation algorithm composed of an LS estimation followed by a supplemental MMSE estimation. We also formulate and analyze the MSE obtained for SRS sequences with an LS estimation. We verify the performances in various aspects by computer simulations in Section 6 and conclude this paper in Section 7.

2. System Model

Consider a cell having a DU and R RUs each of which serves K UEs as shown in Figure 1, where each RU has N_T transmit and receive antennas. We index RU by $r \in \{0, \dots, R-1\}$, and index the UE served by RU r as $r(k)$, $k \in \{0, \dots, K-1\}$. We consider orthogonal frequency division multiplexing (OFDM) communications with N_c subcarriers between UE and RU. We suppose the channel reciprocity, by which the DL channel from RU to UE can be estimated by using the UL pilots sent from UEs under the constraint that the time delay from the UL channel estimation to the DL

transmission is less than the coherence time of the channel [7]. Then, the UL channel estimated by sending SRS sequence from each UE to RU is used as the DL channel estimation. Let $\mathbf{s}_{r(k)} = [s_{r(k)}[0] \cdots s_{r(k)}[M-1]]^T$ denote the SRS sequence of UE $r(k)$, where the superscript T denotes a transpose of a vector. We also let $\mathbf{h}_{r(k),r'}[m] \in \mathcal{C}^{N_T \times 1}$ denote the channel gain between UE $r(k)$ and RU r' over the m -th subcarrier. Then, the channel gains corresponding to M subcarriers, $\mathbf{h}_{r(k),r'}[m]$, $m = 0, \dots, M-1$, are estimated by using an SRS sequence.

The signal received by RU r over the subcarrier m is denoted by $\mathbf{y}_r[m] \in \mathcal{C}^{N_T \times 1}$ and obtained by

$$\mathbf{y}_r[m] = \sum_{k=0}^{K-1} \mathbf{h}_{r(k),r}[m]s_{r(k)}[m] + \sum_{r'=0, r' \neq r}^{R-1} \sum_{k=0}^{K-1} \mathbf{h}_{r'(k),r}[m]s_{r'(k)}[m] + \mathbf{n}_r[m], \quad (1)$$

where $\mathbf{n}_r[m] \in \mathcal{C}^{N_T \times 1}$ is the zero-mean additive white Gaussian noise vector with covariance matrix $\sigma_n^2 \mathbf{I}_{N_T \times N_T}$. Note that $\mathbf{h}_{r(k),r'}[m] = \beta_{r(k),r'} \mathbf{g}_{r(k),r'}[m]$, where $\beta_{r(k),r'}$ represents the large scale fading while each entry of $\mathbf{g}_{r(k),r'}[m] \in \mathcal{C}^{N_T \times 1}$ denotes the small scale fading represented by an independent and identically distributed (i.i.d.) zero mean complex Gaussian random variable with unit variance. Note that the large scale fading factor $\beta_{r(k),r'}^2 = d_{r(k),r'}^{-\alpha}$, where $d_{r(k),r'}$ is a distance between UE $r(k)$ and RU r' , and α is an attenuation factor.

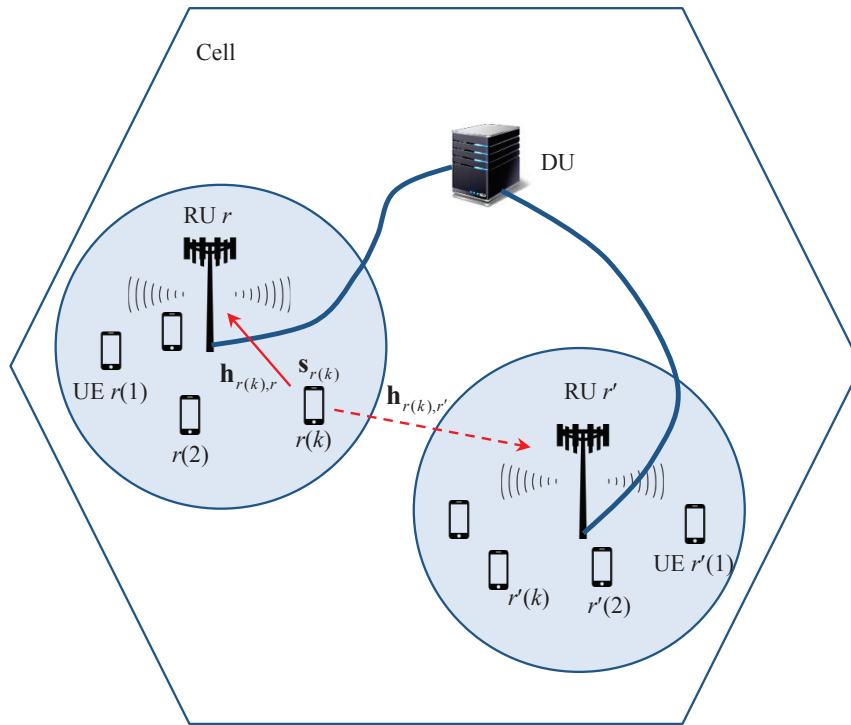


Figure 1. Multi-user m-MIMO configuration in DAS environments.

3. Conventional Channel Sounding Reference Signal in LTE/LTE-A Systems

In the following, we briefly introduce the generation of SRS in conventional LTE/LTE-A systems [24,25]. The structure of SRS symbol is illustrated in Figure 2. Basically, the SRS sequence is generated by a cyclic shift of a base sequence, which is obtained from Zadoff–Chu sequence [24,25] as presented below. Let N_{sc}^{RB} be the number of subcarriers per RB, where $N_{sc}^{RB} = 12$ in LTE/LTE-A systems, and L be the number of assigned subcarriers for SRS or sounding bandwidth. Let D be the decimation factor which is the number of SRS sequences sharing the allocated sounding bandwidth, where the length of SRS sequence is $M = L/D$. Note that L is the multiple of N_{sc}^{RB} , i.e., $L = n \cdot N_{sc}^{RB}$, $1 \leq n \leq N_{RB}^{UL}$, where N_{RB}^{UL} is the uplink system bandwidth in terms of RBs. Let L_z denote the length of

Zadoff–Chu sequence used to generate the base sequence of length M , where L_z is given by the largest prime number such that $L_z < M$.

Let $x_q[m]$ denote the q -th root Zadoff–Chu sequence defined by

$$x_q[m] = \exp \left\{ -j \frac{\pi q m(m+1)}{L_z} \right\}, \quad 0 \leq m < L_z, \tag{2}$$

where

$$\begin{aligned} q &= \lfloor \bar{q} + 0.5 \rfloor + v(-1)^{\lfloor 2\bar{q} \rfloor}, \\ \bar{q} &= L_z(u+1)/31, \end{aligned} \tag{3}$$

with $u \in \{0, 1, \dots, 29\}$ and $v = 0$ if $M = nN_{sc}^{RB}$, $n \leq 5$ and $v = 0, 1$ if $M = nN_{sc}^{RB}$, $n \geq 6$. The base sequence $\bar{x}[m]$ is obtained by

$$\bar{x}[m] = x_q[m \bmod L_z], \quad 0 \leq m < M. \tag{4}$$

Note that base sequences are divided into groups, where u is the group number associated with the physical cell ID and the length of the SRS sequence, and v is the base sequence number within the group. The SRS sequence $x^{(\alpha)}[m]$ of length M is defined by applying a cyclic shift α , $\alpha \in \{0, 1, \dots, 7\}$, to the base sequence $\bar{x}[m]$ as

$$x^{(\alpha)}[m] = e^{j2\pi \frac{\alpha}{L_c} m} \bar{x}[m], \quad 0 \leq m < M, \tag{5}$$

where $L_c > 7$ to obtain distinct values of $e^{j2\pi \frac{\alpha}{L_c}}$ for different $\alpha \in \{0, 1, \dots, 7\}$. For any α_i and α_j chosen from $\{0, \dots, 7\}$, two sequences $\mathbf{x}^{(\alpha_i)} = [x^{(\alpha_i)}[0] \dots x^{(\alpha_i)}[M-1]]^T$ and $\mathbf{x}^{(\alpha_j)} = [x^{(\alpha_j)}[0] \dots x^{(\alpha_j)}[M-1]]^T$ are orthogonal if $\frac{1}{M} \mathbf{x}^{(\alpha_i)H} \mathbf{x}^{(\alpha_j)} = \delta_{i-j}$, where $\delta_{i-j} = 1$ if $i = j$ and 0 otherwise, and the superscript H denotes a conjugate transpose of a vector. This condition reduces to $\frac{1}{M} \sum_{m=0}^{M-1} e^{j\frac{2\pi}{L_c} (\alpha_j - \alpha_i) m} = \delta_{i-j}$ by using Equation (5) and the property $|\bar{x}[m]|^2 = 1$ for all m , which is clear from Equations (2)–(4).

This condition is satisfied only if $(\alpha_j - \alpha_i) \frac{M}{L_c}$ is an even integer, or, in other words, L_c is a factor of $(\alpha_j - \alpha_i) \frac{M}{2}$, for $\alpha_i \neq \alpha_j \in \{0, \dots, 7\}$. Consequently, L_c must be an integer that is greater than 7 and is a factor of $\frac{1}{2}M$.

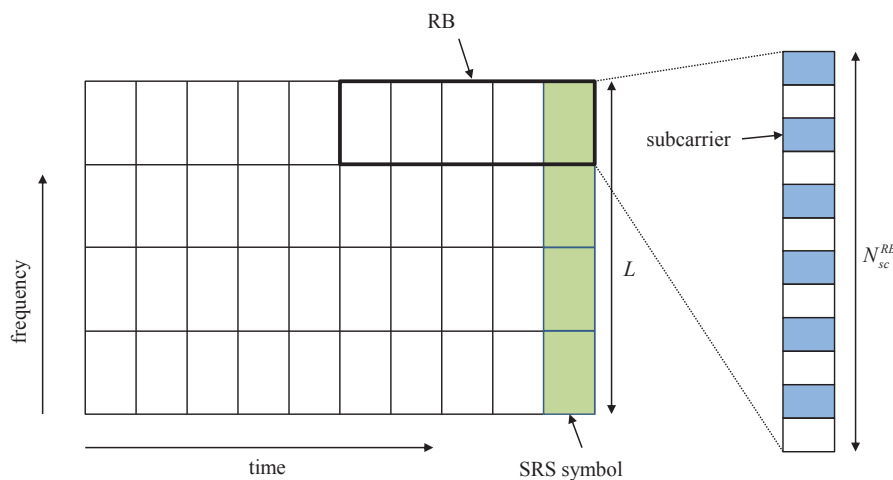


Figure 2. SRS symbol structure.

Multiple SRS sequences are defined from a single base sequence by using different values of α and the decimation factor. In LTE/LTE-A, the decimation factor of two is used and the signal occupies every second subcarrier within the allocated sounding bandwidth. By using distinct SRS

sequences obtained with different values of α and by using distinct sets of subcarriers as a result of decimation, multiple UEs can estimate their channel gains and can be served by RU simultaneously. In conventional LTE/LTE-A systems using $\alpha \in \{0, \dots, 7\}$ and the decimation factor of two, we can obtain only 16 orthogonal sequences. In the multi-user systems with higher number of UEs than 16, we need to design a larger set of SRS sequences having low cross-correlation.

4. Proposed Channel Sounding Reference Signal for Multi-User Systems

The lack of orthogonal SRS sequences may cause the pilot contamination problem. The easy way to alleviate this is using longer SRS sequences or wider sounding bandwidth. However, this approach may result in the degraded performance of channel estimation in the frequency selective environment and the lower spectral efficiency and throughput because SRS sequences replace data sequences. Moreover, if the sounding bandwidth is too wide, SRS sequences may occupy the restricted band, which must be prohibited. Thus, we aim to generate a new set of SRS sequences showing reduced pilot contamination without increasing the sounding bandwidth or sequence length. We apply phase rotation to the LTE/LTE-A SRS sequences, which is introduced in Section 3 to generate a new SRS sequence as

$$x^{(\alpha,s,p)}[m] \triangleq e^{j2\pi \frac{s}{M} m} e^{j2\pi \frac{p}{L_p} m} x^{(\alpha)}[m] = e^{j2\pi \left(\frac{s}{M} + \frac{p}{L_p} + \frac{\alpha}{L_c} \right) m} \bar{x}[m], \quad 0 \leq m < M, \quad (6)$$

where $0 \leq s < \frac{M}{L_c}$ and $0 \leq p < L_p$, and the last equality comes from Equation (5). Note that L_c is an integer greater than 7 which can divide $\frac{1}{2}M$ as introduced in Section 3 and L_p is a prime number which is smaller than L_c . For a given SRS sequence length M , we may generate up to M orthogonal sequences. However, by using Equation (5), we can generate only eight orthogonal sequences with varying $\alpha = 0, \dots, 7$. Thus, we use the phase rotation $e^{j2\pi \frac{s}{M} m}$, $0 \leq s < \frac{M}{L_c}$, together with $e^{j2\pi \frac{\alpha}{L_c} m}$, $0 \leq \alpha < L_c$, to generate M orthogonal SRS sequences without incurring pilot contamination. If the number of UEs exceeds M , we need to generate extra SRS sequences instead of reusing already generated ones. For this purpose, we apply additional phase rotation $e^{j2\pi \frac{p}{L_p} m}$, $0 \leq p < L_p$, where L_p needs to be coprime with M and thus be also coprime with L_c in order to make the resultant sequences distinct from already generated M sequences. We empirically found that a prime number L_p smaller than L_c results in a good performance.

By Equations (2)–(4), we can rewrite Equation (6) as

$$x^{(\alpha,s,p)}[m] = \begin{cases} \exp \left\{ j2\pi \left(\left(\frac{s}{M} + \frac{p}{L_p} + \frac{\alpha}{L_c} \right) m - \frac{qm(m+1)}{2L_z} \right) \right\}, & \text{for } 0 \leq m < L_z, \\ \exp \left\{ j2\pi \left(\left(\frac{s}{M} + \frac{p}{L_p} + \frac{\alpha}{L_c} \right) m - \frac{q(m-L_z)(m-L_z+1)}{2L_z} \right) \right\}, & \text{for } L_z \leq m < M. \end{cases} \quad (7)$$

The correlation of SRS sequences $\mathbf{s}_{r(k)}$ and $\mathbf{s}_{r'(j)}$ is defined and expanded as

$$C_{r(k),r'(j)} \triangleq \frac{1}{M} \mathbf{s}_{r(k)}^H \mathbf{s}_{r'(j)} = \frac{1}{M} \sum_{m=0}^{M-1} s_{r(k)}^*[m] s_{r'(j)}[m] = \frac{1}{M} \sum_{m=0}^{M-1} e^{-j2\pi \left(\frac{s-s'}{M} + \frac{p-p'}{L_p} + \frac{\alpha-\alpha'}{L_c} \right) m}, \quad (8)$$

where $s_{r(k)}[m] = x^{(\alpha,s,p)}[m]$, $s_{r'(j)}[m] = x^{(\alpha',s',p')}[m]$ and the superscript $*$ represents the complex conjugate of a complex variable. The detailed derivation of Equation (8) is provided in Appendix A. It is clear that $C_{r(k),r'(j)}$ obtained with $s = s'$, $p = p'$ and $\alpha = \alpha'$ corresponds to the auto-correlation of $\mathbf{s}_{r(k)}$ because $\mathbf{s}_{r'(j)} = \mathbf{s}_{r(k)}$, where $C_{r(k),r(k)} = \frac{1}{M} \sum_{m=0}^{M-1} e^{j \cdot 0} = 1$. The cross-correlation $C_{r(k),r'(j)} = 0$ if $p = p'$ and either $s \neq s'$ or $\alpha \neq \alpha'$, while $C_{r(k),r'(j)} \neq 0$ if $p \neq p'$ by the following reason.

Consider $f(\phi) = \frac{1}{M} \sum_{m=0}^{M-1} e^{j2\pi \phi m}$, where $f(\phi) = 0$ if ϕM is a nonzero integer, and $f(\phi) \neq 0$, otherwise. If $p = p'$, the rightmost side of Equation (8) becomes $f\left(\frac{s'-s}{M} + \frac{\alpha'-\alpha}{L_c}\right)$. If $s \neq s'$ or $\alpha \neq \alpha'$, $\left(\frac{s'-s}{M} + \frac{\alpha'-\alpha}{L_c}\right)M$ is a nonzero integer because M is a multiple of L_c as introduced in Section 3, and thus

$C_{r(k),r'(j)} = f\left(\frac{s'-s}{M} + \frac{\alpha'-\alpha}{L_c}\right) = 0$. On the other hand, if $p \neq p'$, $\left(\frac{s'-s}{M} + \frac{p'-p}{L_p} + \frac{\alpha'-\alpha}{L_c}\right)M$ cannot be a nonzero integer because $(p' - p)M$ is not divisible by L_p , and, thus $C_{r(k),r'(j)} = f\left(\frac{s'-s}{M} + \frac{p'-p}{L_p} + \frac{\alpha'-\alpha}{L_c}\right) \neq 0$, where L_p and M are coprime and $|p - p'| < L_p$.

Consequently, for each p , we can generate a set of M orthogonal SRS sequences, where $0 \leq s < \frac{M}{L_c}$ and $0 \leq \alpha < L_c$ enables the generation of M orthogonal sequences. Applying the phase rotation $e^{j2\pi \frac{p}{L_p} m}$, $0 \leq p < L_p$, in Equation (7) enables to obtain L_p sets of M orthogonal sequences. Any two sequences obtained with different values of p are mutually correlated.

Distinct SRS sequences generated with different α , s and p by Equation (7) are assigned to different UEs. The first set of M orthogonal sequences generated with $p = 0$ are assigned to the first M UEs. Then, the next set of M orthogonal sequences generated with $p = 1$ are assigned to the next M UEs. This procedure is repeated by increasing p until all UEs are assigned SRS sequences.

5. Channel Estimation

5.1. Least Square Channel Estimation

Let $\hat{\mathbf{h}}_{r(k),r}[m]$ denote the estimation of the channel gain $\mathbf{h}_{r(k),r}[m]$. We apply the LS estimation algorithm [26] to the received signal of SRS sequence for the channel estimation. In this process, we assume a block fading with length M , by which the channel is considered invariant over consecutive M subcarriers. Then, the estimation of channel gain at the m -th subcarrier in a fading block, $\hat{\mathbf{h}}_{r(k),r}[m]$, can also be denoted by $\hat{\mathbf{h}}_{r(k),r}$ and obtained as

$$\hat{\mathbf{h}}_{r(k),r}[m] = \hat{\mathbf{h}}_{r(k),r} = \frac{1}{M} \sum_{i=0}^{M-1} s_{r(k)}^*[i] \mathbf{y}_r[i]. \tag{9}$$

By using Equation (1), we can rewrite Equation (9) as

$$\begin{aligned} \hat{\mathbf{h}}_{r(k),r} &= \frac{1}{M} \sum_{i=0}^{M-1} s_{r(k)}^*[i] \left\{ \sum_{j=0}^{K-1} \mathbf{h}_{r(j),r}[i] s_{r(j)}[i] + \sum_{r'=0, r' \neq r}^{R-1} \sum_{j=0}^{K-1} \mathbf{h}_{r'(j),r}[i] s_{r'(j)}[i] + \mathbf{n}_r[i] \right\} \\ &= \frac{1}{M} \sum_{i=0}^{M-1} \left\{ \mathbf{h}_{r(k),r}[i] + \sum_{j=0, j \neq k}^{K-1} \mathbf{h}_{r(j),r}[i] s_{r(k)}^*[i] s_{r(j)}[i] + \sum_{r'=0, r' \neq r}^{R-1} \sum_{j=0}^{K-1} \mathbf{h}_{r'(j),r}[i] s_{r(k)}^*[i] s_{r'(j)}[i] + s_{r(k)}^*[i] \mathbf{n}_r[i] \right\}, \end{aligned} \tag{10}$$

where $s_{r(k)}^*[i] s_{r(k)}[i] = |s_{r(k)}[i]|^2 = 1$ is used. If the channel is actually block faded with length M , the channel gain $\mathbf{h}_{r(k),r}[m]$ is identical for all $m = 0, \dots, M - 1$ and thus, we can represent the channel gain by $\mathbf{h}_{r(k),r}$. Then, Equation (10) can be rewritten as

$$\hat{\mathbf{h}}_{r(k),r} = \mathbf{h}_{r(k),r} + \sum_{j=0, j \neq k}^{K-1} \mathbf{h}_{r(j),r} C_{r(k),r(j)} + \sum_{r'=0, r' \neq r}^{R-1} \sum_{j=0}^{K-1} \mathbf{h}_{r'(j),r} C_{r(k),r'(j)} + \frac{1}{M} \sum_{i=0}^{M-1} s_{r(k)}^*[i] \mathbf{n}_r[i]. \tag{11}$$

In case that all SRS sequences of UEs in the cell are mutually orthogonal, we have zero cross-correlation between any two SRS sequences so that Equation (11) is simplified as

$$\hat{\mathbf{h}}_{r(k),r} = \mathbf{h}_{r(k),r} + \frac{1}{M} \sum_{i=0}^{M-1} s_{r(k)}^*[i] \mathbf{n}_r[i]. \tag{12}$$

We define the normalized mean squared error (MSE) of the channel estimation between UE $r(k)$ and RU r at subcarrier m as

$$\sigma_{MSE,r(k)}^2[m] = \frac{E \|\hat{\mathbf{h}}_{r(k),r} - \mathbf{h}_{r(k),r}[m]\|^2}{\beta_{r(k),r}^2 N_T}. \tag{13}$$

Then, we define the average MSE by

$$\bar{\sigma}_{MSE}^2 = \frac{1}{KRM} \sum_{k=0}^{K-1} \sum_{r=0}^{R-1} \sum_{m=0}^{M-1} \sigma_{MSE,r(k)}^2[m] \tag{14}$$

and use this as the performance metric of channel estimation. Note that, if the channel is actually block faded with length M , the normalized MSE can be denoted by $\sigma_{MSE,r(k)}^2$ and Equation (14) can be simplified as $\bar{\sigma}_{MSE}^2 = \frac{1}{KR} \sum_{k=0}^{K-1} \sum_{r=0}^{R-1} \sigma_{MSE,r(k)}^2$. Under the assumption of block fading with length M , the average MSE can be obtained from Equations (11), (13) and (14) as

$$\begin{aligned} \bar{\sigma}_{MSE}^2 &= \frac{1}{KR} \sum_{k=0}^{K-1} \sum_{r=0}^{R-1} \frac{E \|\hat{\mathbf{h}}_{r(k),r} - \mathbf{h}_{r(k),r}\|^2}{\beta_{r(k),r}^2 N_T} \\ &= \frac{1}{KR} \sum_{k=0}^{K-1} \sum_{r=0}^{R-1} \frac{1}{\beta_{r(k),r}^2} \left\{ \sum_{j=0, j \neq k}^{K-1} \beta_{r(j),r}^2 |C_{r(k),r(j)}|^2 + \sum_{r'=0, r' \neq r}^{R-1} \sum_{j=0}^{K-1} \beta_{r'(j),r}^2 |C_{r(k),r'(j)}|^2 + \frac{1}{M} \sigma_n^2 \right\}, \end{aligned} \tag{15}$$

where the detailed derivation is given in Appendix B.

Let us predict analytically the average MSE under the block fading environment. The large scale fading factors are assumed to be $\beta_{r(k),r} = \beta_1$ and $\beta_{r'(k),r} = \beta_2$ for all k, r and $r', r' \neq r$, which means that the large scale fading between UE and serving RU is represented by β_1 while the large scale fading between UE and other neighboring RUs are represented by β_2 . We suppose $\beta_2 < \beta_1$ because UE is usually served by a nearby located RU. Then, Equation (15) can be rewritten as

$$\bar{\sigma}_{MSE}^2 = \frac{1}{KR} \sum_{k=0}^{K-1} \sum_{r=0}^{R-1} \left\{ \sum_{j=0, j \neq k}^{K-1} |C_{r(k),r(j)}|^2 + \frac{\beta_2^2}{\beta_1^2} \sum_{r'=0, r' \neq r}^{R-1} \sum_{j=0}^{K-1} |C_{r(k),r'(j)}|^2 + \frac{\sigma_n^2}{\beta_1^2 M} \right\}. \tag{16}$$

We consider the reuse of M orthogonal SRS sequences to UEs repeatedly without applying the phase rotation $e^{j2\pi \frac{p}{L_p} m}$ in Equation (7). We suppose that each RU in the cell serves the equal number of UEs and distinct set of $\frac{M}{R}$ orthogonal sequences are assigned repeatedly to UEs in each RU. Then, we obtain a correlation matrix $\mathbf{C} \in \mathcal{C}^{KR \times KR}$ of SRS sequences whose (i, j) -th entry is defined by

$$\mathbf{C}_{ij} = \begin{cases} 1, & \text{if } |i - j| = kM, \quad k = 0, \dots, \lfloor KR/M \rfloor, \\ 0, & \text{else.} \end{cases} \tag{17}$$

Every other $\frac{M}{R}$ column and row of \mathbf{C} correspond to UEs in the same RU. Due to the reuse of $\frac{M}{R}$ orthogonal sequences for each RU, SRS sequences used for different RUs are always mutually orthogonal. It follows that $C_{r(k),r'(j)} = 0$ for all k, j and $r \neq r'$ and Equation (16) becomes

$$\bar{\sigma}_{MSE}^2 = \frac{1}{KR} \sum_{k=0}^{K-1} \sum_{r=0}^{R-1} \sum_{j=0, j \neq k}^{K-1} |C_{r(k),r(j)}|^2 + \frac{\sigma_n^2}{\beta_1^2 M}. \tag{18}$$

From Equation (17) and the allocation rule of orthogonal sequences to UEs introduced above, we obtain

$$\frac{1}{KR} \sum_{k=0}^{K-1} \sum_{r=0}^{R-1} \sum_{j=0, j \neq k}^{K-1} |C_{r(k),r(j)}|^2 = \begin{cases} 0, & \text{if } KR \leq M, \\ \frac{1}{KR} \sum_{i=1}^{\lfloor \frac{KR}{M} \rfloor} 2(KR - iM), & \text{if } KR > M. \end{cases} \tag{19}$$

Consequently, we can rewrite Equation (18) as

$$\bar{\sigma}_{MSE}^2 = \begin{cases} \frac{\sigma_n^2}{\beta_1^2 M}, & \text{if } KR \leq M, \\ \lfloor \frac{KR}{M} \rfloor \left(2 - \frac{M}{KR} \left(\lfloor \frac{KR}{M} \rfloor + 1 \right) \right) + \frac{\sigma_n^2}{\beta_1^2 M}, & \text{if } KR > M. \end{cases} \tag{20}$$

Note that $\bar{\sigma}_{MSE}^2$ is independent of β_2^2 while it depends on β_1^2 . Recall that β_2^2 is determined by the distance between a UE and its neighboring RU. Thus, the performance of channel estimation obtained by reusing orthogonal sequences repeatedly is not affected by how far neighboring RUs are located from UE. It is also clear from Equation (20) that the average MSE decreases as M grows. Since the number of orthogonal sequences is limited for a given sequence length, the number of UEs served simultaneously in a cell is limited. In the multi-user systems, a larger number of UEs must be accommodated, so we may need to use non-orthogonal sequences for channel sounding.

5.2. Supplemental Correlation-Aided Channel Estimation

As the number of UEs grows in the cell, it is impossible to assign orthogonal SRS sequences to all UEs even by using the proposed SRS sequences because the number of orthogonal sequences is limited to M . Thus, we propose using the supplemental channel estimation scheme to enhance the channel estimation performance even with correlated SRS sequences. Suppose DU knows SRS sequences and their cross-correlations. We rewrite Equation (11) as

$$\hat{\mathbf{h}}_{r(k),r} = \sum_{r'=0}^{R-1} \sum_{j=0}^{K-1} \mathbf{h}_{r'(j),r} C_{r(k),r'(j)} + \frac{1}{M} \sum_{i=0}^{M-1} s_{r(k)}^*[i] \mathbf{n}_r[i], \tag{21}$$

where $0 \leq r < R$ and $0 \leq k < K$. This can be expressed by using matrix forms as

$$\hat{\mathbf{H}} = \mathbf{H}\mathbf{C} + \mathbf{N}, \tag{22}$$

where $\hat{\mathbf{H}} \in \mathcal{C}^{N_T \times KR}$, $\mathbf{H} \in \mathcal{C}^{N_T \times KR}$ and $\mathbf{N} \in \mathcal{C}^{N_T \times KR}$ are matrices whose columns are $\hat{\mathbf{h}}_{r(k),r}$, $\mathbf{h}_{r(k),r}$ and $\mathbf{n}_{r(k),r}$, respectively, $0 \leq r < R$ and $0 \leq k < K$. We let $\mathbf{C} \in \mathcal{C}^{KR \times KR}$ denote a correlation matrix whose entries are $C_{r(k),r'(j)}$, where $0 \leq r, r' < R$ and $0 \leq k, j < K$. We define two classes of UEs. The first one is the set of UEs that are assigned mutually orthogonal SRS sequences. UEs assigned mutually correlated SRS sequences compose the second class. We reorder $\hat{\mathbf{H}}$, \mathbf{H} , \mathbf{C} and \mathbf{N} and partition them as $\hat{\mathbf{H}} = [\hat{\mathbf{H}}_u \ \hat{\mathbf{H}}_c]$, $\mathbf{H} = [\mathbf{H}_u \ \mathbf{H}_c]$, $\mathbf{C} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}$ and $\mathbf{N} = [\mathbf{N}_u \ \mathbf{N}_c]$, where submatrices with subscripts u and c correspond to the first and the second class of UEs, respectively, and \mathbf{A} is a non-diagonal matrix. Then, Equation (22) can be rewritten in partitioned forms as

$$\begin{bmatrix} \hat{\mathbf{H}}_u & \hat{\mathbf{H}}_c \end{bmatrix} = \begin{bmatrix} \mathbf{H}_u & \mathbf{H}_c \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} + \begin{bmatrix} \mathbf{N}_u & \mathbf{N}_c \end{bmatrix}, \tag{23}$$

which implies

$$\hat{\mathbf{H}}_u = \mathbf{H}_u + \mathbf{N}_u, \tag{24}$$

$$\hat{\mathbf{H}}_c = \mathbf{H}_c \mathbf{A} + \mathbf{N}_c. \tag{25}$$

It is clear from Equation (24) that $\hat{\mathbf{h}}_{r(k),r} = \mathbf{h}_{r(k),r} + \mathbf{n}_{r(k),r}$ for the first class of UEs. So, $\hat{\mathbf{h}}_{r(k),r}$ is considered to estimate sufficiently the actual channel $\mathbf{h}_{r(k),r}$ of UE $r(k)$ in the first class. On the other hand, as can be seen from Equation (25), for the second class of UEs, $\hat{\mathbf{h}}_{r(k),r}$ includes the linear combination of other UEs' channels as well. Thus, we need a supplemental procedure to obtain the more reliable channel estimations for UEs in the second class. For this purpose, we propose the *correlation-aided channel estimation* which applies the MMSE algorithm [27] to the estimated channels for the second class of UEs as the supplemental procedure. We obtain the refined channel estimation $\tilde{\hat{\mathbf{H}}}_c$ for UEs in the second class by multiplying the MMSE nulling matrix \mathbf{W} to $\hat{\mathbf{H}}_c$ as

$$\tilde{\hat{\mathbf{H}}}_c = \hat{\mathbf{H}}_c \mathbf{W}, \tag{26}$$

where

$$\mathbf{W} = \mathbf{A}^H \left(\mathbf{A}\mathbf{A}^H + \mathbf{B}^{-1}\mathbf{I} \right)^{-1} \quad (27)$$

and \mathbf{B} is a diagonal matrix whose diagonal entries are $\beta_{r(k),r}^2$ corresponding to UEs in the second class. Then, after refinement of $\hat{\mathbf{H}}_c$ to $\tilde{\mathbf{H}}_c$ by Equation (26), the channel estimation $\hat{\mathbf{H}}$ is replaced by $\hat{\mathbf{H}} = \begin{bmatrix} \hat{\mathbf{H}}_u & \tilde{\mathbf{H}}_c \end{bmatrix}$.

In summary, first, we apply the LS estimation to obtain $\hat{\mathbf{h}}_{r(k),r}$ for all UEs. Next, we define the first and the second classes of UEs based on the correlation matrix \mathbf{C} . The first class is defined by UEs whose SRS sequences are mutually orthogonal. The remaining UEs form the second class. Then, for the second class of UEs, we apply additionally the MMSE algorithm to the channel estimation obtained by LS scheme as the supplemental procedure. Finally, we use the result of LS estimation for the first class and the result of supplement estimation for the second class as the channel estimation. If the number of UEs is less than or equal to the number of mutually orthogonal SRS sequences, we only need to perform a LS estimation as introduced in Equation (9). Otherwise, we need to apply the proposed correlation-aided supplemental estimation after performing the LS estimation.

6. Numerical Results

We evaluate the performances of the proposed SRS and the proposed supplemental channel estimation algorithm in terms of MSE in the multi-user m-MIMO DAS environment. For comparison, the MSE obtained by using conventional LTE/LTE-A SRS scheme is also shown. As the proposed scheme, we consider the following three types:

- using repeatedly M orthogonal sequences generated by varying α and s without applying the phase rotation $e^{j2\pi\frac{p}{L_p}m}$ in Equation (7),
- using the phase rotation $e^{j2\pi\frac{p}{L_p}m}$ together with varying α and s for SRS generation, but not using the supplemental correlation-aided channel estimation,
- using the phase rotation $e^{j2\pi\frac{p}{L_p}m}$ together with varying α and s for SRS generation, and the supplemental correlation-aided channel estimation as well.

We consider the large scale fading factor between UEs and serving RU as β_1 and the large scale fading factor between UEs and other neighboring RUs as β_2 , i.e., $\beta_{r(k),r} = \beta_1$ and $\beta_{r(k),r'} = \beta_2$ for all k , r and $r' \neq r$. We let $\beta_2/\beta_1 = 0.7692$, which comes from the assumption that the distance of UE from serving RU is 1.3 times shorter than the distance from other neighboring RUs, where the attenuation factor is assumed to be $a = 2$. We let each RU serve the same number of UEs. We choose $L_c = 8$ in Equations (5) and (7), and $L_p = 3$ when applying the phase rotation $e^{j2\pi\frac{p}{L_p}m}$ in types (b) and (c) of the proposed scheme. We consider a block fading channel, International Telecommunication Union Radiocommunication Sector (ITU-R) Ped A and ITU-R Veh A channels [28], where ITU-R Ped A [29] and ITU-R Veh A channels are examples of frequency selective channels. The simulation parameters used for performance evaluation are listed in Table 1.

We depict the average MSE of channel estimation with respect to the total number of UEs in the cell obtained for block fading channel, ITU-R Ped A channel and ITU-R Veh A channel in Figures 3–5, respectively. In each figure, we plot the average MSE obtained by using conventional SRS in LTE/LTE-A systems and the proposed scheme of three types.

We also plot the average MSE obtained by applying supplemental MMSE estimation to all UEs instead of applying MMSE selectively only to UEs belonging to the second class, where the phase rotation $e^{j2\pi\frac{p}{L_p}m}$ is applied and the corresponding curves are marked with ‘nonselective supplemental MMSE.’ In Figure 3, we include the average MSE predicted analytically by Equation (20) for the proposed scheme of type (a) over block fading channel.

Table 1. Parameters used in performance evaluation.

Parameters	Values
number of subcarriers (N_c)	1024
number of subcarriers per RB (N_{sc}^{RB})	12
SRS length (M)	48 (proposed scheme) 24 (LTE/LTE-A with a decimation factor of 2)
sampling time	0.1 μ s
average SNR (β_1^2 / σ_n^2)	0 dB
channel model	block fading, ITU-R Ped A, ITU-R Veh A
number of RUs (R)	4
number of RU antennas (N_T)	8

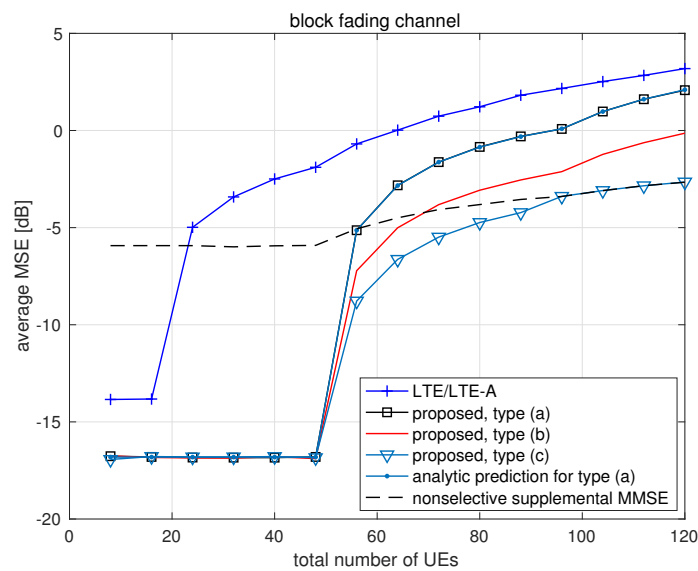


Figure 3. Average MSE of channel estimation with respect to the total number of UEs over block fading channel.

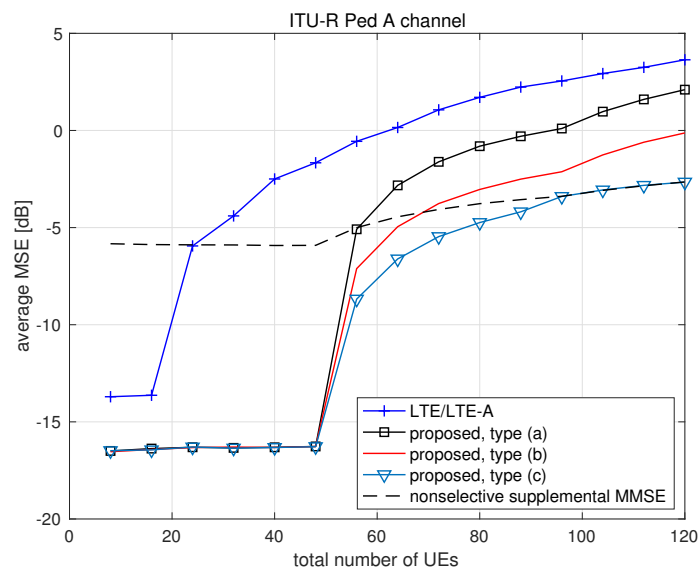


Figure 4. Average MSE of channel estimation with respect to the total number of UEs over ITU-R Ped A channel.

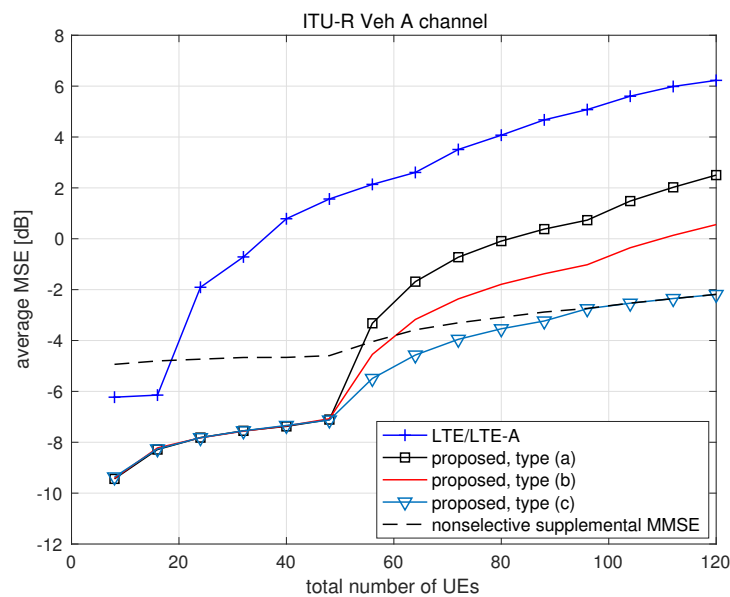


Figure 5. Average MSE of channel estimation with respect to the total number of UEs over ITU-R Veh A channel.

It is observed that the channel estimation MSE obtained by using any SRS scheme rapidly increases when the number of UEs in service exceeds the number of orthogonal SRS sequences, where the conventional LTE/LTE-A scheme and the proposed scheme generate 16 and $M (=48)$ orthogonal SRS sequences, respectively. For more than 48 UEs, the proposed scheme also provides much lower channel estimation MSE than LTE/LTE-A scheme. Even reusing repeatedly $M (=48)$ orthogonal SRS sequences generated in type (a) can lower significantly the channel estimation MSE for any number of UEs. It is observed that applying phase rotation $e^{j2\pi \frac{p}{L_p} m}$ to SRS generation further lowers the channel estimation MSE, and the use of supplemental selective MMSE estimation can even further improve the channel estimation performance at the cost of increasing complexity. Note that the computational complexity of supplemental selective MMSE estimation is, in general, $O(n^3)$ because it requires the matrix inversion, where n is the dimension of a square matrix \mathbf{A} . Considering, as a reference, the case that M orthogonal sequences are repeatedly used to generate SRS sequences, the dimension of \mathbf{A} is determined from Equation (17) as $n = \min(KR, 2 \max(0, KR - M))$. Applying supplemental MMSE estimation nonselectively to all UEs degrades the performance when the number of UEs is not high enough. In Figure 3, the analytic prediction of MSE for block fading channel is observed to match the numerical result very well. It is clear that the proposed scheme of type (c) shows the best channel estimation performance in terms of MSE for all numbers of UEs. The MSE gain of the proposed scheme of type (c) over LTE/LTE-A system is about 6 dB in block fading and ITU-R Ped A channels, and about 8 dB in ITU-R Veh A channel when serving 120 UEs through four RUs. The performance gain achieved by using the proposed scheme can be observed in block fading channel as well as frequency selective channel.

Conventional LTE/LTE-A systems may not effectively employ m-MIMO transmission for the service of a large number of UEs mainly due to the pilot contamination problem. However, the proposed SRS and the supplemental channel estimation can relieve the pilot contamination problem in a significant level. It is expected that the proposed scheme can be effectively adopted in multi-user m-MIMO systems.

7. Conclusions

In this paper, we proposed a generation of SRS sequences resulting in improved channel estimation performance without increasing the sequence length. The proposed SRS can easily be generated

by imposing a phase rotation to the base sequence. Even though the proposed SRS sequences are mutually correlated when the number of UEs is higher than the sequence length, it lowers significantly the MSE of channel estimation and thus alleviates the pilot contamination problem. We also proposed a supplemental correlation-aided channel estimation scheme to further improve the channel estimation performance of multi-user m-MIMO technology in DAS environments. It is expected that the proposed SRS scheme and the supplemental channel estimation scheme can be effectively adopted in m-MIMO systems.

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Appendix A. Derivation of Equation (8)

With $s_{r(k)}[m] = x^{(\alpha,s,p)}[m]$ and $s_{r'(j)}[m] = x^{(\alpha',s',p')}[m]$, we obtain Equation (8) by using Equation (7) as

$$\begin{aligned} C_{r(k),r'(j)} &\triangleq \frac{1}{M} \mathbf{s}_{r(k)}^H \mathbf{s}_{r'(j)} = \frac{1}{M} \sum_{m=0}^{M-1} s_{r(k)}^*[m] s_{r'(j)}[m] \\ &= \frac{1}{M} \sum_{m=0}^{L_z-1} e^{-j2\pi \left(\left(\frac{s}{M} + \frac{p}{L_p} + \frac{\alpha}{L_c} \right) m - \frac{qm(m+1)}{2L_z} \right)} e^{j2\pi \left(\left(\frac{s'}{M} + \frac{p'}{L_p} + \frac{\alpha'}{L_c} \right) m - \frac{qm(m+1)}{2L_z} \right)} \\ &\quad + \frac{1}{M} \sum_{m=L_z}^{M-1} e^{-j2\pi \left(\left(\frac{s}{M} + \frac{p}{L_p} + \frac{\alpha}{L_c} \right) m - \frac{q(m-L_z)(m-L_z+1)}{2L_z} \right)} e^{j2\pi \left(\left(\frac{s'}{M} + \frac{p'}{L_p} + \frac{\alpha'}{L_c} \right) m - \frac{q(m-L_z)(m-L_z+1)}{2L_z} \right)} \\ &= \frac{1}{M} \sum_{m=0}^{M-1} e^{-j2\pi \left(\frac{s-s'}{M} + \frac{p-p'}{L_p} + \frac{\alpha-\alpha'}{L_c} \right) m} \end{aligned}$$

Appendix B. Derivation of Equation (15)

By using Equations (11), (13) and (14), we obtain Equation (15) as

$$\begin{aligned} \bar{\sigma}_{MSE}^2 &= \frac{1}{KR} \sum_{k=0}^{K-1} \sum_{r=0}^{R-1} \frac{E \|\hat{\mathbf{h}}_{r(k),r} - \mathbf{h}_{r(k),r}\|^2}{\beta_{r(k),r}^2 N_T} \\ &= \frac{1}{KRN_T} \sum_{k=0}^{K-1} \sum_{r=0}^{R-1} \frac{1}{\beta_{r(k),r}^2} E \left\| \sum_{j=0, j \neq k}^{K-1} \mathbf{h}_{r(j),r} C_{r(k),r(j)} + \sum_{r'=0, r' \neq r}^{R-1} \sum_{j=0}^{K-1} \mathbf{h}_{r'(j),r} C_{r(k),r'(j)} + \frac{1}{M} \sum_{i=0}^{M-1} s_{r(k)}^*[i] \mathbf{n}_r[i] \right\|^2 \\ &= \frac{1}{KRN_T} \sum_{k=0}^{K-1} \sum_{r=0}^{R-1} \frac{1}{\beta_{r(k),r}^2} \left\{ \sum_{j=0, j \neq k}^{K-1} E \|\mathbf{h}_{r(j),r}\|^2 |C_{r(k),r(j)}|^2 + \sum_{r'=0, r' \neq r}^{R-1} \sum_{j=0}^{K-1} E \|\mathbf{h}_{r'(j),r}\|^2 |C_{r(k),r'(j)}|^2 \right. \\ &\quad \left. + \frac{1}{M^2} \sum_{i=0}^{M-1} E \|\mathbf{n}_r[i]\|^2 \right\} \\ &= \frac{1}{KRN_T} \sum_{k=0}^{K-1} \sum_{r=0}^{R-1} \frac{1}{\beta_{r(k),r}^2} \left\{ \sum_{j=0, j \neq k}^{K-1} N_T \beta_{r(j),r}^2 |C_{r(k),r(j)}|^2 + \sum_{r'=0, r' \neq r}^{R-1} \sum_{j=0}^{K-1} N_T \beta_{r'(j),r}^2 |C_{r(k),r'(j)}|^2 + \frac{1}{M^2} \sum_{i=0}^{M-1} N_T \sigma_n^2 \right\} \\ &= \frac{1}{KR} \sum_{k=0}^{K-1} \sum_{r=0}^{R-1} \frac{1}{\beta_{r(k),r}^2} \left\{ \sum_{j=0, j \neq k}^{K-1} \beta_{r(j),r}^2 |C_{r(k),r(j)}|^2 + \sum_{r'=0, r' \neq r}^{R-1} \sum_{j=0}^{K-1} \beta_{r'(j),r}^2 |C_{r(k),r'(j)}|^2 + \frac{1}{M} \sigma_n^2 \right\}. \end{aligned}$$

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