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The mediating role of number-to-magnitude mapping precision in the relationship between approximate number sense and math achievement depends on the domain of mathematics and age

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ABSTRACT

Approximate number sense (ANS) refers to the ability to approximately estimate and manipulate relatively large numerical quantity representations. An accurate ANS is hypothesized to facilitate a precise mapping between symbolic numbers and their corresponding magnitude and thereby can lead to an advantage in representing and working with symbolic numbers. This is referred to as the ANS mapping theory. ANS mapping is one of the mechanisms through which symbolic number meaning is thought to be learned. In the present study, we aimed to examine whether the mediating role of number-to-magnitude mapping precision differs depending on the domain of mathematics in adults and children. We found that mapping precision fully mediated the relationship between ANS acuity and math achievement in certain domains (Quantitative Reasoning in adults and Arithmetic in children). These results suggest that ANS acuity indirectly affects only certain domains of math achievement through its contribution to number-to-magnitude mapping precision, and that mapping precision differentially contributes to distinct domains of mathematics throughout development.

1. Introduction

What helps us to be good at math? There has long been much interest in the foundation of mathematical achievement. Recently, the basic ability to instantly determine and operate upon approximate numerical quantity (i.e., approximate number sense; hereafter ANS) has been proposed to serve as one of the critical building blocks of math achievement (Cantlon, Platt, & Brannon, 2009; Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004; Halberda, Mazzocco, & Feigenson, 2008). ANS has also been found in animals such as pigeons, monkeys, fish, dolphins, etc. and also in indigenous people who do not formally learn math (Agrillo, Piffer, & Bisazza, 2011; Brannon, Wusthoff, Gallistel, & Gibbon, 2001; Kilian, Yaman, von Fersen, & Güntürkün, 2003; Nieder & Dehaene, 2009; Pica, Lemer, Izard, & Dehaene, 2004). Therefore, ANS is thought to have evolved because of its importance in hunting, gathering, and territorial fight, etc. (Halberda et al., 2008; Pica et al., 2004). (But see Discussion for an alternative theory proposing that discrete and continuous magnitudes may be processed holistically due to inevitable correlations between them.)

Individual differences in ANS acuity can be expressed as a minimum ratio of the two numerical magnitudes that are readily distinguishable (Dehaene, 1997; Feigenson et al., 2004; Halberda & Feigenson, 2008). This ratio is called the Weber fraction. The Weber fraction represents one's accuracy in discriminating between two physical magnitudes (following Weber's law indicating that discriminability depends on the difference between magnitudes relative to the absolute magnitude of the stimuli.) The fact that numerosity discrimination follows the Weber's law which is common to the discrimination of physical magnitudes supports the hypothesis that numerosities are represented as approximate, analogue mental magnitudes (Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; Pica et al., 2004).

Dehaene (1997) proposed that the ANS reflects the ability to mentally represent and manipulate numerosities on a mental number line, an analogue mental representation of numbers. The ANS has been proposed to serve as a foundation for mathematics and to have a cerebral substrate (in an area known as the intraparietal sulcus (IPS)). In line with this hypothesis, children's understanding of exact, numerical symbols seems to build on their ANS (Brannon, 2006; Cantlon et al., 2009; Dehaene, 2007), therefore having good ANS acuity may help develop an accurate mapping between symbolic numbers and their corresponding quantity. A large body of neuroimaging and neuropsychological evidence suggests that the ANS (and the associated

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neural system in the IPS) is engaged during symbolic mathematical problem solving (Dehaene, 1997; Halberda et al., 2008), and more generally whenever the semantic (i.e., quantitative) meaning of symbolic numbers are accessed (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Kiefer & Dehaene, 1997; Pinel, Dehaene, Riviere, & LeBihan, 2001). Taken together, ANS and its neural representation in the IPS are theorized to contribute to mathematical cognition by providing the quantitative meaning of numbers.

1.1. Investigation of the relationship between ANS acuity and math achievement

The 'ANS theory' refers to the hypothesis that ANS acuity contributes to higher level mathematical achievement. The literature is mixed with reports that do (Bonny & Lourenco, 2013; DeWind & Brannon, 2012; Feigenson, Libertus, & Halberda, 2013; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Libertus, Feigenson, & Halberda, 2011; Libertus, Feigenson, & Halberda, 2013; Libertus, Odic, & Halberda, 2012; Lourenco, Bonny, Fernandez, & Rao, 2012; Lyons & Beilock, 2011; Mazzocco, Feigenson, & Halberda, 2011b; Starr, Libertus, & Brannon, 2013) and do not (Gilmore et al., 2013; Price, Palmer, Battista, & Ansari, 2012; Sasanguie, De Smedt, Defever, & Reynvoet, 2012) support the ANS theory. Several recent meta-analyses concluded that there exists a weak but significant link between ANS acuity and math achievement (Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Schneider et al., 2017). First, there have been a number of studies reporting concurrent and predictive relationships between ANS acuity and mathematical achievement in children (Gilmore, McCarthy, & Spelke, 2007; Halberda et al., 2008; Inglis, Attridge, Batchelor, & Gilmore, 2011; Libertus et al., 2011; Mazzocco et al., 2011b; Starr et al., 2013). Secondly, children with developmental dyscalculia (DD) show ANS impairment in behavioral performance and in brain activity related to numerical processing (Mazzocco, Feigenson, & Halberda, 2011a; Mussolin, De Volder, et al., 2010; Mussolin, Mejias, & Noël, 2010; Piazza et al., 2010; Price, Holloway, Räsänen, Vesterinen, & Ansari, 2007). Lastly, several studies report that ANS training leads to improvement in ANS acuity and also in math achievement in various populations (Hyde, Khanum, & Spelke, 2014; Käser et al., 2013; Obersteiner, Reiss, & Ufer, 2013; Park & Brannon, 2013). These findings together support the hypothesis that ANS may form the foundation for higher level mathematical cognition.

However, unbiased attention should be given to reports of null correlations between ANS acuity and math achievement (Price et al., 2012; Sasanguie et al., 2012; see De Smedt, Noël, Gilmore, & Ansari, 2013 for review) or the impact of executive function (e.g., inhibition) on this relationship (Fuhs & McNeil, 2013; Gilmore et al., 2013; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2014; Bugden & Ansari, 2016; Leibovich & Ansari, 2016; Leibovich, Katzin, Harel, & Henik, 2017). Efforts to understand the underlying cause of the discrepancy across studies will enable a deeper understanding of the role of ANS in the development of mathematical cognition. Recent approaches include testing whether the contribution of ANS acuity or inhibition ability to math achievement depends on how ANS acuity was tested (e.g., whether or not incongruent conditions were included), the domain of mathematics (Inglis et al., 2011; Jang & Cho, 2016; Lourenco et al., 2012; Park & Cho, 2017) and age (Inglis et al., 2011). Thus, until enough evidence is accumulated to construct a complete picture, we should cautiously hypothesize that "ANS acuity *may differentially* contribute to certain domains of math ability depending on the stage of development".

1.2. Theories on how children learn the meaning of symbolic numbers

There is also controversy on how symbolic number meaning is learned (the symbol grounding problem) and the mechanism through which it contributes to math achievement (Leibovich et al., 2017; Leibovich & Ansari, 2016). The ANS mapping theory states that the semantic meaning of symbolic numbers is initially learned by being mapped onto the ANS (Barth, La Mont, Lipton, & Spelke, 2005; Barth, Starr, & Sullivan, 2009; Brannon, 2006; De Smedt et al., 2013; Dehaene, 2007; Izard & Dehaene, 2008; Libertus, 2015; Libertus, Odic, Feigenson, & Halberda, 2016; Lipton & Spelke, 2005; Verguts & Fias, 2004). Based on the ANS mapping theory, a person with an accurate ANS is expected to establish an accurate understanding of symbolic numbers based on a precise number-to-magnitude mapping. Having an accurate understanding of symbolic numbers will lead to better ability to work with numbers in general. This developmental chain of achievements, starting from good ANS acuity to accurate symbolic number representations and then to better ability to manipulate numbers is believed to be one mechanism through which ANS contributes to high level math achievement. This idea is currently debated given inconsistent findings in the literature (Fuhs & McNeil, 2013; Gilmore et al., 2013; Leibovich et al., 2017; Leibovich & Ansari, 2016; Price et al., 2012; Sasanguie et al., 2012; Szűcs et al., 2014). Some studies propose an alternative explanation of the symbol grounding problem. According to Carey (2001), the meaning of small number words may initially be learned via mapping through the object tracking system (OTS) and the counting routine, followed by gradual acquisition of the principles of the number system (e.g., ordered relations, etc.) (Carey, 2001). Afterwards, the meaning of larger number symbols is hypothesized to be learned through symbol-to-symbol relations (including ordinality) and application of learned principles (Reynvoet & Sasanguie, 2016). Researchers in favor of this alternative account argue that representations of symbol-to-symbol order contributes to math achievement. Some studies reported that symbol-to-symbol number processing is more efficient compared to symbol-to-ANS mapping (Lyons, Ansari, & Beilock, 2012) and that not all measures of mapping precision were correlated with math ability (Lyons, Price, Vaessen, Blomert, & Ansari, 2014). In line with this alternative account, Lyons and Beilock (2013) reported that symbolic and non-symbolic order processing were associated with distinct brain regions (Lyons & Beilock, 2013). Given these contradicting theories each with their own supporting evidence, we believe that these two theories may not be mutually exclusive. With regards to the learning of the meaning of small numbers within subitizing range, it is highly likely that children map small number words to its exact cardinality through the OTS. For symbolic numbers that are beyond subitizing range, it is possible that both ANS mapping and symbol-to-symbol associations are involved. Regardless of whether learning the semantic meaning of symbolic numbers occurs through symbol-to-ANS or symbol-to-symbol mapping, Leibovich and Ansari (2016) emphasizes that cognitive control is a critical component (Leibovich & Ansari, 2016). That being said, the present study focused on testing the ANS mapping account controlling for the confounding influence of children's inhibition ability and to examine whether the domain of mathematics matters to this issue.

1.3. Examination of the ANS mapping account in children

Behavioral evidence in support of the ANS mapping account comes from studies demonstrating that the precision of (or fluency in) the mapping between symbolic and nonsymbolic magnitude representations are correlated with or predict symbolic math skills (Brankaer, Ghesquière, & De Smedt, 2014; De Smedt, Verschaffel, & Ghesquière, 2009; Holloway & Ansari, 2009; Libertus et al., 2016; Mundy & Gilmore, 2009; Pinheiro-Chagas et al., 2014; Rousselle & Noël, 2007). Mundy and Gilmore (2009) measured mapping between symbolic and nonsymbolic magnitudes by using a two-alternative forced-choice quantity matching task and showed that children's accuracy of mapping was related to math achievement over and above the influence of symbolic and nonsymbolic comparison performance (Mundy & Gilmore, 2009). Recently, the variability of children's verbal estimation was found to predict formal math abilities, and to mediate the relationship between ANS and overall math achievement (Libertus et al., 2016). Similarly, Pinheiro-Chagas et al. (2014) reported that children's verbal estimation ability partly mediated the relation between ANS and symbolic arithmetic (Pinheiro-Chagas et al., 2014). In addition, children's number line estimation accuracy was correlated with symbolic mathematical abilities (Booth & Siegler, 2008) and mental arithmetic (Lyons et al., 2014). Taken together, the relationships among ANS, number-to-magnitude mapping and math achievement may depend on how ANS and mapping ability was measured (Brankaer et al., 2014; De Smedt et al., 2013; Holloway & Ansari, 2009; Libertus et al., 2016; Mazzocco et al., 2011a), which domain of mathematics was tested (Holloway & Ansari, 2009; Libertus et al., 2016) and the range of numbers studied (Brankaer et al., 2014).

1.4. Examination of the ANS mapping account in adults

Reports of the association between mapping precision and symbolic math achievement in the adult population also do not converge onto a consistent conclusion as to whether the quality of mapping between symbolic and nonsymbolic magnitudes relates to math achievement. Castronovo and Göbel (2012) measured both ANS acuity and mapping precision and examined their respective relationships to symbolic math abilities (i.e., symbolic number knowledge and four basic arithmetic operations) in adults (Castronovo & Göbel, 2012). These researchers measured mapping precision using numerosity production and numerosity perception tasks which required participants to map quantities between a dot array and an Arabic numeral. In this study, mapping precision (but not ANS acuity) was correlated with symbolic arithmetic ability. The authors interpreted their results as supporting the idea that the precision of symbolic number representations and arithmetic ability is associated with the quality of the mapping between symbolic numbers and their corresponding magnitude. Sasanguie and Reynvoet (2014) measured mapping precision using (audio-to-visual) 'number word-to-dots' vs. 'number word-to-numeral' matching task, which required participants to determine whether a number word matched the numerosity of a dot array or an Arabic numeral, respectively. In this study, the precision of the mapping between 'number word-to-numeral', but not 'number word-to-dots' was correlated with arithmetic ability. These results were interpreted as indicating that adults' math competence is based on the precision of symbolic number representations which have been detached from the underlying nonsymbolic magnitude representation (Lyons et al., 2012). As such, there exists inconsistency in reports of the relationship between ANS acuity and math achievement in adults (Agrillo, Piffer, & Adriano, 2013; DeWind & Brannon, 2012; Gilmore, Attridge, & Inglis, 2011; Guillaume, Nys, & Mussolin, 2013; Halberda et al., 2012; Inglis et al., 2011; Jang & Cho, 2016; Leibovich & Ansari, 2016; Libertus et al., 2012; Lourenco et al., 2012; Price et al., 2012).

1.5. Possible explanations for inconsistencies across studies

These discrepancies may reflect differences among distinct domains of math achievement tested and how ANS acuity or mapping precision was measured (including the range of numerical magnitudes tested) across studies. For instance, gaining expertise in arithmetic through efficient retrieval of arithmetic facts in well-educated adults is expected to be more dependent on the verbal mathematical system rather than the analogue magnitude system (Ansari, 2008; Cho, Ryali, Geary, & Menon, 2011; Dehaene, 2009; Grabner, Ansari, Koschutnig, Reishofer, & Ebner, 2013; Grabner et al., 2009; Taillan et al., 2015). In other words, the mechanism of retrieval-based, drilled arithmetic is reported to be qualitatively different from effortful, strategic, quantitative reasoning. Quantitative reasoning[1] refers to the ability to analyze quantitative information and to devise a step-by-step strategy for mathematical problem solving based on understanding of quantitative information and knowledge of mathematical concepts (Dwyer, Gallagher, Levin, & Morley, 2003; Karaali, Villafane Hernandez, & Taylor, 2016; National Council of Teachers of Mathematics, 2000; Sons, 1996). Quantitative reasoning, but not drilled arithmetic engages the fronto-parietal quantity processing networks because it requires thinking about the semantic meaning of numbers and their inter-relationships (Cho et al., 2011; Dehaene, 2009; Dehaene & Cohen, 1995; Dehaene, Piazza, Pinel, & Cohen, 2003; Delazer et al., 2005; Grabner et al., 2009; Grabner et al., 2013; Jang & Cho, 2016; Taillan et al., 2015; Zamarian, Ischebeck, & Delazer, 2009). Several neuroimaging studies also demonstrated that as arithmetic training progresses, activity of the verbal fact retrieval system including the angular gyrus increases, while activation of the fronto-parietal quantity system decreases (Delazer et al., 2005; Taillan et al., 2015; Zamarian et al., 2009). Furthermore, Jang and Cho (2016) reported that ANS acuity is correlated with mathematical problem solving ability only when it involves strategic processing of quantitative information and not when overlearned arithmetic facts are effortlessly retrieved (e.g., as in Arithmetic Fluency tests) (Jang & Cho, 2016). In this sense, the non-significant correlations between ANS acuity and symbolic arithmetic ability in previous studies may have been due to the inclusion of basic arithmetic problems in the measurement of math achievement (Castronovo & Göbel, 2012; Sasanguie & Reynvoet, 2014).

1.6. Aims and hypotheses of the present study

The aims of the present study were to examine whether Mapping Precision mediates the relationship between ANS acuity and multiple domains of math achievement in adults (Experiment 1) and children (Experiment 2). We hypothesize that ANS acuity and number-to-magnitude mapping precision (hereafter Mapping Precision) may contribute differently to distinct domains of math achievement. We expect that ANS acuity and Mapping Precision will be more related to Quantitative Reasoning rather than Arithmetic Fluency, especially in well-educated adults. Furthermore, we hypothesized that Mapping Precision will mediate the relationship between ANS acuity and Quantitative Reasoning in adults. In relation to children, however, we hypothesized that ANS acuity and Mapping Precision will correlate with overall math achievement given that they are just beginning to learn the quantitative meaning of numbers via number-to-magnitude mapping. In addition, we hypothesized that Mapping Precision will mediate the relationship between ANS acuity and children's overall math achievement.

2. Experiment 1

The main purpose of Experiment 1 was to test whether Mapping Precision mediates the relationship between ANS acuity and math achievement in adults. Based on previous studies, the domains of math achievement tested in adults were classified into two categories; 1) Quantitative Reasoning and 2) Arithmetic Fluency. The *Quantitative Reasoning* test assesses the ability to analyze and interpret quantitative information and to devise a strategy to solve problems based on knowledge of mathematical concepts. In contrast, the **Arithmetic Fluency** test measures the efficiency of drilled arithmetic based on four basic operations (addition, subtraction, multiplication, division). We hypothesized that ANS acuity will be correlated with Quantitative Reasoning and that this relationship will be mediated by Mapping Precision. Secondly, we hypothesized that Arithmetic Fluency is

[1] Quantitative reasoning includes 1) understanding and interpreting quantitative information presented in various formats; 2) drawing inferences based on quantitative information; 3) strategically solving problems using arithmetic, algebraic, geometric, or statistical methods, etc.

unlikely to be associated with ANS acuity because of reduced engagement of quantitative processing in well-educated adults. As such, we expected no mediating effect of Mapping Precision in the relationship between ANS acuity and Arithmetic Fluency.

2.1. Materials and methods

2.1.1. Participants

Fifty-three undergraduate and graduate students (23 females; age range from 19 to 27, mean age = 22.434, SD = 2.099, 33 in Social Science/Humanities, 20 in Natural Science/Engineering majors) participated in the study. Our sample size was determined by a power analysis using G power software based on pilot data (see Supplementary Methods). All participants had normal or corrected to normal vision. Written informed consent was obtained from all participants. All participants received monetary reward upon completion of the experiment.

2.1.2. The dots vs. number (DN) task (nonsymbolic vs. symbolic number comparison)

The DN task required subjects to compare the numerosity of a dot array and an Arabic numeral. Each trial started with a fixation period of 1000 ms, and then a dot array and a numeral appeared side by side simultaneously on the computer screen for 200 ms (Fig. 1). The subject was asked to choose the stimulus representing the larger magnitude within 10 s. Participants responded by pressing one of two keyboard buttons (press "3" for stimulus on left and "8" for stimulus on right). Each button was labeled with a sticker labeled "Left" or "Right". The left-right order of the stimuli was counterbalanced across trials, resulting in equal numbers of Dots (left) vs. Number (right) and Number (left) vs. Dots (right) trials. The font size of the symbolic number was 30 points. The dot array appeared within a 217 × 290 pixel rectangular frame. No corrective feedback was given after five practice trials. The magnitude of each stimulus (numerosity and numeral) ranged from 6 to 49. The ratio between the magnitude of the stimuli varied from 1:2 to 10:11 (1:2, 3:4, 4:5, 5:6, 6:7, 7:8, 8:9, 9:10, and 10:11). Each subject performed 10 trials in each ratio bin for each condition (ND and DN) adding up to a total of 180 trials (10 trials × 9 ratio bins × 2 conditions = 180 trials). The order of trials from each ratio/condition was randomly intermixed. Accuracy (ACC) and Response Time (RT) (rather than *w*) were used as main dependent variables because they have been reported to be a more reliable measure of numerical comparison ability (Geary & vanMarle, 2016; Inglis & Gilmore, 2014; see Supplementary Methods for further details.).

2.1.3. The dots vs. dots (DD) task (nonsymbolic number comparison)

DD task required subjects to compare between the numerosity of a pair of dot arrays (Fig. 1). The procedure of the task was otherwise the same as in the DN task. As already noted by many researchers, it is practically impossible for any one study to control for the influence of all possible continuous visual variables using the DD task format (Dietrich, Huber, & Nuerk, 2015; Gebuis & Reynvoet, 2012; Leibovich & Henik, 2013). Therefore, we chose to actively control for cumulative area and average dot size and randomized the influence of other visual variables such as density (average spacing between dots) and convex hull (the smallest convex envelope that contains all dots). Each dot was randomly placed within a fixed rectangular frame ensuring equal average inter-dot spacing between arrays (Halberda et al., 2008; Halberda & Feigenson, 2008). In order to control for the possible confounding influence of cumulative area and average size of dots, we used two control conditions that effectively controlled for each of these variables. The average performance of both control conditions was taken as a measure of one's ANS acuity as in previous studies (Halberda et al., 2008; Halberda & Feigenson, 2008; Libertus et al., 2012; Lourenco et al., 2012; Odic, Libertus, Feigenson, & Halberda, 2013). In the condition controlling for cumulative area (hereafter AREA-controlled condition), the cumulative area of dots was equivalent between arrays but the array with more dots inevitably had a smaller average dot size (dot size was negatively correlated with numerosity). Average diameter of individual dot size ranged from 8 to 23 pixels. The purpose of this control condition was to prevent the subject from using the total area of dots as a cue for estimating numerosity. In the condition controlling for average dot size (hereafter SIZE-controlled condition), the average diameter of dot size was equivalent (approximately 8 pixels) between arrays but the array with more dots inevitably had larger cumulative area (total area was positively correlated with numerosity). This control condition prevents the subject from using average dot size as a cue for numerosity estimation. Trials from each control condition were randomly intermixed so that subjects could not consistently rely on any continuous visual cue to guess numerosity (Maloney, Risko, Preston, Ansari, & Fugelsang, 2010).

The ratio between the numerosities of the dot arrays, the total number of trials and the range of numerosities were the same as in the DN task. After five trials of practice, corrective feedback was not provided during the main experiment. As in the DN task, ACC and RT (rather than *w*) were used as dependent variables.

2.1.4. Mathematical achievement test

Two tests were administered to measure math achievement in adults. First, one section of the Quantitative Reasoning subtest of the Graduate Record Examination was used to measure Quantitative Reasoning ability (ETS, 2010). This test includes 25 questions on algebra, probability, statistics, geometry and data analysis, etc. Secondly, an Arithmetic Fluency test based on the Calculation subtest of the Work-Net Job Aptitude Test by the Korean Employment Information Service was administered (Park, Kim, Song, Jeong, & Jeong, 2008). This test included problems that require fast and accurate arithmetic computations using natural numbers, decimal numbers, fractions, square roots, etc. The time limit was 40 min for the Quantitative Reasoning and 8 min for the Arithmetic Fluency test.

2.1.5. Fluid intelligence test

An abbreviated version of the Raven's Advanced Progressive Matrices test (Arthur, Tubre, Paul, & Sanchez-Ku, 1999) was used. This test consisted of 15 questions including 3 practice questions. The time

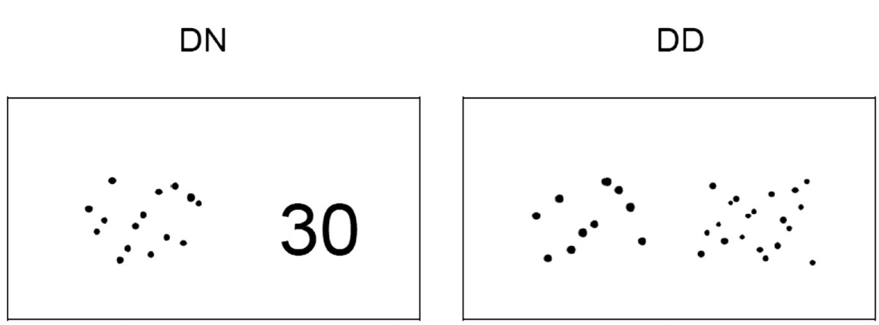

Fig. 1. Example stimuli of the DN (Dots vs. Number) & DD (Dots vs. Dots) tasks.

Table 1
Correlations between performance on the DN/DD tasks and mathematical achievement.

Variable	1	2	3	4
1. DN_{acc}				
2. DD_{acc}	0.310*			
3. Quantitative reasoning	0.347*	0.302*		
4. Arithmetic fluency	0.105	0.131	0.529**	

N = 53.
* $p < 0.05$.
** $p < 0.01$.

limit for this test was 15 min. The total accuracy score was used in the analyses.

2.1.6. General procedure

Each participant performed computer-based ANS acuity tasks (DN and DD task), and two (paper and pencil) mathematical achievement tests in a quiet room. The order of task administration was counterbalanced across participants. The computer-based tasks were administered while the subject was seated approximately 60 cm away from the computer monitor. The entire procedure of the study was approved by the Institutional Review Board of the authors' institution.

2.1.7. Analysis

First, differences between performance of the DN and DD tasks and ratio effects from both tasks were tested with a repeated measures one-way ANOVA. The effects of gender (female, male) and major (Social Science/Humanities, Natural Science/Engineering) on math achievement were tested with a 2 by 2, two-way ANOVA. Pearson's correlation analyses were conducted between all measurements. When a significant correlation was found between DN or DD performance and math achievement, hierarchical regression analyses were conducted to test whether certain task performance predicts math achievement while controlling for age, gender, and fluid intelligence. Next, bootstrapping mediation analyses were conducted using AMOS 20.0 (10,000 re-samples, standardized Z scores were used for all variables) to test the mediating effect of Mapping precision in the relationship between ANS acuity and math achievement. We determined whether or not a "full-mediation" effect exists based on the following three criteria; (1) 95% BCa CI of the indirect effect does not contain 0, (2) the two-tailed p value of the indirect effect is lower than 0.05, and (3) the BCa CI of the direct effect contains 0 and the two-tailed p value of the direct effect is not lower than 0.05 (Baron & Kenny, 1986; Rucker, Preacher, Tormala, & Petty, 2011).

2.2. Results

2.2.1. Descriptive statistics of task performance

In the DN task, mean accuracy (hereafter DN_{acc}) was 0.711 (SD = 0.062) and mean correct trial RT (hereafter DN_{RT}) was 1100 ms (SD = 300 ms). Ratio effects from both DN_{acc} and DN_{RT} were highly significant (accuracy: $F(8, 416) = 46.911$, $p < 0.001$, $\eta^2 = 0.474$; RT: $F(8, 45) = 7.859$, $p < 0.001$, $\eta^2 = 0.583$). Mean accuracy (ACC) from each ratio were all above chance level ($ps < 0.001$).

In the DD task, mean correct trial RT (DD_{RT}) was 647 ms (SD = 149 ms) and mean ACC (hereafter DD_{acc}) was 0.758 (SD = 0.059). Ratio effects from DD_{acc} and DD_{RT} were both highly significant (DD_{acc}: $F(8, 45) = 123.218$, $p < 0.001$, $\eta^2 = 0.956$; DD_{RT}: $F(8, 45) = 12.357$, $p < 0.001$, $\eta^2 = 0.687$). DD_{acc} were all above chance level ($ps < 0.001$).

2.2.2. Comparison between DN vs. DD task performance

Repeated measures ANOVA was conducted to test for the difference between DD vs. DN task performance. The main effect of task was significant from both ACC and RT revealing better performance on the DD task (ACC: $F(1, 52) = 26.209$, $p < 0.001$, $\eta^2 = 0.335$; RT: $F(1, 52) = 124.809$, $p < 0.001$, $\eta 2 = 0.706$). Performance measures from the DN and DD task were respectively correlated with each other (ACC: $r(52) = 0.310$, $p = 0.024$; RT: $r(52) = 0.280$, $p = 0.042$).

2.2.3. The results of ANOVA and correlation tests with mathematical achievement

The mean of the Quantitative Reasoning score was 18.358 (± 2.675 SD, range: 12–24). Two-way ANOVA (gender × major) on Quantitative Reasoning scores revealed no significant main or interaction effects ($ps > 0.05$). The correlation between age and Quantitative Reasoning scores was also not significant ($r(52) = -0.110$, $p = 0.431$). Fluid intelligence scores were significantly correlated with Quantitative Reasoning scores ($r(52) = 0.478$, $p < 0.001$).

The mean score of the Arithmetic Fluency test was 19.622 (± 4.133 SD, range: 10–25). The same two-way ANOVA on Arithmetic Fluency scores revealed no significant main effects of gender or major ($ps > 0.3$). The two way interaction effect was also not significant ($p > 0.5$). The correlation between age and Arithmetic Fluency scores was not significant ($p > 0.1$). Fluid intelligence was significantly correlated with Arithmetic Fluency scores ($r(52) = 0.280$, $p = 0.042$).

2.2.4. The correlations between DN/DD performance and mathematical achievement

The correlations between Quantitative Reasoning scores and DN_{acc} or DD_{acc} were both significant (DN_{acc}: $r(52) = 0.347$, $p = 0.011$; DD_{acc}: $r(52) = 0.302$, $p = 0.024$; Table 1). On the other hand, Arithmetic Fluency scores did not correlate with either DN_{acc} or DD_{acc} (DN_{acc}: $r(52) = 0.105$, $p = 0.453$; DD_{acc}: $r(52) = 0.131$, $p = 0.349$; Table 1). DN_{RT} was significantly correlated with Quantitative Reasoning ($r(52) = -0.362$, $p = 0.008$), but not Arithmetic Fluency ($r(52) = -0.249$, $p = 0.072$) scores (Table S1). DD_{RT} was not correlated with any math achievement score ($ps > 0.2$; Table S1).

2.2.5. Hierarchical regression analyses on math achievement

Hierarchical regression analyses were carried out to investigate the predictive effects of DN/DD performance on math achievement after controlling for the effects of age, major, and fluid intelligence. Thus, age, major and fluid intelligence were first entered into the model. To compare the predictive strength of Mapping Precision and ANS acuity on Quantitative Reasoning, DN_{acc} and DD_{acc} were entered into the model as the second or third predictor in two separate analyses (Table 2 model 1 vs. model 2). Regardless of the order, entering DN_{acc} significantly changed explained variance (R^2) of the model, while DD_{acc} did not (Table 2).

2.2.6. Mediation analysis

As shown in Fig. 2, DN_{acc} fully mediated the relation between DD_{acc} and Quantitative Reasoning. The indirect effect of DD_{acc} on Quantitative Reasoning through mediation by DN_{acc} was significant (BCa CI: 0.004 to 0.246, $p = 0.033$; $P_M = 0.289$) while the direct effect of DD_{acc}

Table 2
Hierarchical regression predicting Quantitative Reasoning scores controlling for age, major and fluid intelligence.

Model		Predicting variables	β	ΔR^2	Significance of ΔR^2
1	(1)	Age	−0.121	0.288	0.001
		Major	0.241		
		Fluid intelligence	0.398		
	(2)	DN_{acc}	0.286	0.078	0.019
	(3)	DD_{acc}	0.120	0.012	0.337
2	(2)	DD_{acc}	0.191	0.035	0.124
	(3)	DN_{acc}	0.252	0.056	0.046

N = 53. Outcome variable = Quantitative Reasoning scores.

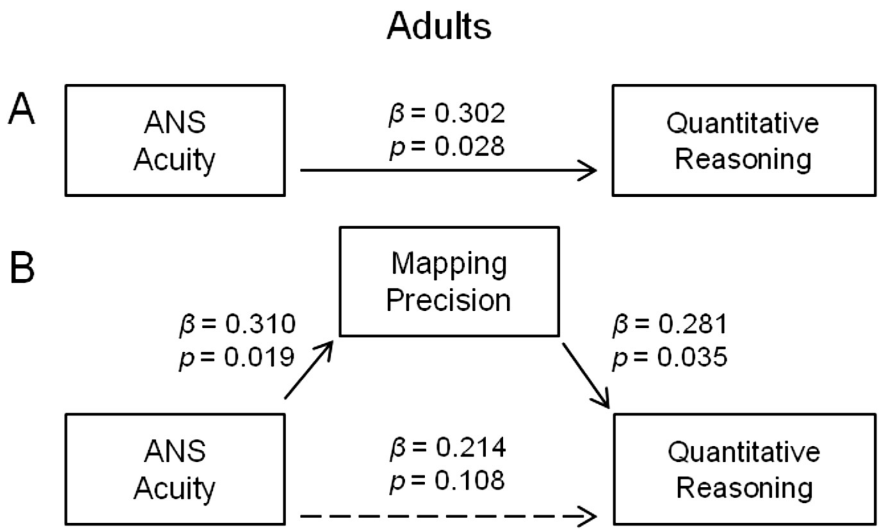

Fig. 2. A mediation model of the relationship between DD task performance (ANS acuity) and Quantitative Reasoning through mediation by DN task performance (Mapping Precision). Note, β denotes regression coefficient. (A) The direct effect of DD_{acc} (ANS acuity) on Quantitative Reasoning. DD_{acc} significantly predicted Quantitative Reasoning ($\beta = 0.302$, $t = 2.259$, $p = 0.028$). (B) DD_{acc} significantly predicted DN_{acc} (mapping precision) ($\beta = 0.310$, $t = 2.329$, $p = 0.019$). When DN_{acc} and DD_{acc} were simultaneously entered as predictors, the effect of DD_{acc} was no longer significant ($\beta = 0.214$, $t = 1.575$, $p = 0.108$; BCa CI of direct effect: -0.028 to 0.502, $p = 0.086$), while that of DN_{acc} remained significant ($\beta = 0.281$, $t = 2.063$, $p = 0.035$). The strength of the direct relationship between DD_{acc} and Quantitative Reasoning was significantly weaker than that of the relationship between DN_{acc} and Quantitative Reasoning (BCa CI: 0.044 to 0.246, $p = 0.033$, $P_M = 0.289$). This test revealed that DN_{acc} significantly mediated the relationship between DN_{acc} and Quantitative Reasoning. This result supports the interpretation that DD_{acc} indirectly predicts Quantitative Reasoning through the mediating effect of DN_{acc}.

on Quantitative Reasoning became nonsignificant (BCa CI: -0.028 to 0.502, $p = 0.086$). Since the BCa CI of the indirect effect did not include zero, the results of mediation analysis indicates full mediation, given that the relationship between DD_{acc} and Quantitative Reasoning was not significant with the mediation effect (Baron & Kenny, 1986; Hayes, 2013; Rucker et al., 2011; Fig. 2). This mediation effect was still significant when the influence of Arithmetic Fluency was controlled for (BCa CI: 0.004 to 0.215, $p = 0.031$, $P_M = 0.320$; Fig. S1). Note, the indirect effect of DN_{acc} on Quantitative Reasoning through the mediating effect of DD_{acc} was not significant (BCa CI: -0.009 to 0.595, $p = 0.069$; $P_M = 0.191$). DD_{RT} was not used in a mediation analysis given its null correlation with math achievement scores. Likewise, the mediation analysis using Arithmetic Fluency as the dependent variable was not conducted given its null correlation with DN/DD task performance.

2.3. Discussion

In Experiment 1, we investigated whether ANS acuity and Mapping Precision respectively correlate with mathematical achievement in young adults. In addition, we examined whether Mapping Precision mediates the relationship between ANS acuity and math achievement and whether this mediation effect depends on the domain of mathematics (Quantitative Reasoning vs. Arithmetic Fluency).

Correlation and regression analyses revealed that both ANS acuity and mapping precision were correlated with Quantitative Reasoning, but not Arithmetic Fluency scores. Although ANS acuity was correlated with Quantitative Reasoning ability, when Mapping Precision was considered together as a predictor in hierarchical regression, there was no significant relationship between ANS acuity and Quantitative Reasoning ability (Table 2). Moreover, the mediation analysis confirmed that the relationship between ANS acuity and Quantitative Reasoning ability was fully mediated by Mapping Precision (Fig. 2B). These findings support the idea that ANS acuity may serve as one of the foundational abilities for Quantitative Reasoning by contributing to an accurate mapping between symbolic numbers and their corresponding magnitude in adults (Barth et al., 2005; Brannon, 2006; Dehaene, 2007).

Arithmetic Fluency was not correlated with any measure of DD or DN task performance. These findings confirm our hypothesis that ANS may contribute to Quantitative Reasoning ability, but not Arithmetic Fluency in adults. Many items of the Arithmetic Fluency test required understanding of the relationships between symbolic numbers (rather than their corresponding magnitude) for efficient problem solving. For example, in order to solve "$0.1 - 0.1^2 - 0.1^3 - 0.1^4$", knowing that 0.1^2 equals 0.01 and that 0.1^3 equals 0.001, etc. greatly helps solve the problem. Thus, the Arithmetic Fluency test is likely to be more dependent on the verbal, symbolic math system and knowledge of symbol-to-symbol associations rather than the ANS. Our results and interpretation are consistent with those of Castronovo and Göbel (2012) and Sasanguie and Reynvoet (2014) which also reported nonsignificant correlations between ANS acuity and symbolic arithmetic ability. It is possible that the Arithmetic Fluency in well-educated adults is better explained by symbol-to-symbol associations rather than by ANS acuity or Mapping Precision (Goffin & Ansari, 2016; Lyons & Ansari, 2015; Lyons, Vogel, & Ansari, 2016; Reynvoet & Sasanguie, 2016).

3. Experiment 2

Several studies reported that a precise mapping between numerical magnitude and symbolic number predicts math achievement in children (Brankaer et al., 2014; Holloway & Ansari, 2009; Mundy & Gilmore, 2009). However, it remains to be verified whether Mapping Precision mediates the relationship between ANS acuity and math achievement in young elementary school children, and whether the contribution of Mapping Precision to math achievement differs depending on the specific domain of mathematics (Park & Cho, 2017). Mundy and Gilmore (2009) measured math achievement with the composite scores from symbolic number knowledge, arithmetic (addition, subtraction, & multiplication), and word problems and found that the composite scores correlate with mapping accuracy measured from a dot-array estimation task (Mundy & Gilmore, 2009). Holloway and Ansari (2009) examined 6 to 8 year-old children's math achievement using Mathematics Fluency and Calculation subtests of the Woodcock-Johnson III Tests of Achievement. They found that individual differences in symbolic (but not non-symbolic) numerical distance effect significantly predicted Mathematics Fluency (but not Calculation). Other studies showed that children's Mapping Precision measured with verbal estimation (Libertus et al., 2016) or digit-to-array mapping task (Brankaer et al., 2014) was correlated with overall math achievement (Brankaer et al., 2014; Libertus et al., 2016) or arithmetic fluency (Brankaer et al., 2014). These results suggest that ANS acuity and Mapping Precision is related to children's fluency in manipulating symbolic numbers. However, the contribution of children's ANS acuity and Mapping Precision may depend on the domain of mathematics and the specific pattern of results may differ from that of adults. It remains to be verified whether ANS acuity and Mapping Precision relates to multiple (but only certain) domains of mathematical achievement and whether Mapping Precision mediates this relationship, since previous studies examined mostly symbolic arithmetic ability or the composite math achievement score in children (Brankaer et al., 2014; Holloway & Ansari, 2009; Libertus et al., 2016; Mundy & Gilmore, 2009; Peng, Yang, & Meng, 2017). Considering previous reports of the influence of inhibition ability on the relationship between ANS acuity and children's math achievement (Fuhs & McNeil, 2013; Gilmore et al., 2013; Szűcs et al., 2014, but see Keller & Libertus, 2015), we controlled for the influence of inhibition ability in a hierarchical regression in order to test whether ANS acuity/mapping precision uniquely contributes to math achievement. Thus, in Experiment 2, we investigated the relationship among ANS acuity, Mapping Precision and multiple domains of math achievement, while controlling for the influence of age, fluid intelligence and inhibition ability in children.

3.1. Materials and methods

3.1.1. Participants

Fifty 1st grade elementary school students participated in Experiment 2. Our sample size was determined by a power analysis using G power software based on a meta-analysis by Schneider et al. (2017) (see Supplementary Methods). Data from three participants were excluded due to missing data points. Thus, data from 47 children (26 girls, mean age = 7.53, SD = 0.31) were included in the data analysis. The experiment was conducted during winter break during which all paricipants had completed the first grade curriculum. The entire procedure of the study was approved by the Institutional Review Board of the authors' institution. Written informed consent to participate was obtained from all children and their parents.

3.1.2. The DN task

In the DN task of Experiment 2, only the Dots vs. Number condition (in which the dot array was on the left and the Arabic numeral was on the right) was used given that the left vs. right position of the array and numeral was not found to influence performance in Experiment 1. The duration of stimulus presentation was 1000 ms. The total number of trials was 72 (6 ratio conditions (1:2, 3:4, 5:6, 6:7, 7:8, 8:9) × 12 trials). The procedure of the DN task was otherwise identical to that of Experiment 1.

3.1.3. The DD task

The total number of trials was 120 (6 ratio conditions (1:2, 3:4, 5:6, 6:7, 7:8, 8:9) × 2 control conditions (AREA-controlled, SIZE-controlled) × 10 trials). Except for the increased stimulus presentation time of 1000 ms, the procedure of the DD task was otherwise identical to that of Experiment 1.

3.1.4. Mathematical achievement test

Three subtests (i.e., Number Concepts, Arithmetic and Quantative Reasoning) of the standard mathematical achievement test for Korean elementary school students (KISE-BAAT; Park et al., 2008) were used (see Supplementary Materials for example questions). The Number Concepts test includes problems that measure children's knowledge of symbolic (Arabic) numbers (e.g., understanding of cardinality/ordinality principles, the ability to read large numbers, etc.). The Arithmetic test measures basic addition and subtraction abilities. Items of the Quantitative Reasoning test are described in words and require the child to think about the quantitative meaning of numbers and their inter-relationships and then to devise step by step stategies for problem solving. The difficulty level of the problems gradually increased after each correct response. All math problems were printed on paper and were provided to the children throughout the test. An experimenter verbally explained each problem to the child on a one-on-one setting. Children were asked to verbally state their answer to all questions. There was no time limit for any of the subtests. If a child made five incorrect responses in a row, the test was terminated. The total number of correct answers was recorded. Raw scores from each subtest was used in the analyses.

3.1.5. Fluid intelligence test

The number of correctly solved problems of the Raven's Advanced Progressive Matrices (Raven's APM) test was used to measure fluid intelligence (Raven & Lewis, 1996).

3.1.6. Measure of inhibition ability

In order to rule out possible confounding influence of children's inhibition ability, we measured performance on a computerized numerical Stroop test (N = 45; 25 girls, mean age = 7.533, SD = 0.314). (Note, we could not include data from two children who did not complete the task.) This task required subjects to choose the stimulus which was either numerically or physically larger within a pair of symbolic numbers. There were two factors; Congruency (Congruent vs. Neutral vs. Incongruent) and Instruction ("choose the *numerically* vs. *physically* larger stimulus"). In the Congruent (or Incongruent) condition, the numerically larger number was also physically larger (or smaller). In the Neutral condition, the two numbers were either equivalent in numerical or physical size. The range of numbers was from 2 to 9, and the total number of trials was 192. The Inverse Efficiency Score (IES) calculated by RT divided by proportion correct was used as a measure of inhibition ability (Bruyer & Brysbaert, 2011).

3.1.7. Analysis

All analysis procedures were identical to Experiment 1.

3.2. Results

3.2.1. Descriptive statistics of task performance

In the DN task, DN_{acc} was 0.603 (SD = 0.0719) and DN_{RT} was 1832 (SD = 580). Repeated measures ANOVA revealed a significant main effect of ratio from DN_{acc} ($F(5, 46) = 13.942$, $p < 0.001$, $\eta^2 = 0.233$). The ratio effect from DN_{RT} was not significant ($F(5, 46) = 2.015$, $p = 0.077$, $\eta^2 = 0.042$). (Given that there was a floor-like effect in DN task performance, we excluded data from the two conditions with the lowest performance for quality control). Therefore, DN_{acc} and DN_{RT} were recalculated based on the following ratios 1:2, 3:4, 5:6, and 7:8 (DN_{acc} = 0.628, SD = 0.082; DN_{RT} = 1804, SD = 560).

In the DD task, DD_{acc} was 0.748 (SD = 0.084) and DD_{RT} was 1535 (SD = 279). Repeated measures ANOVA revealed a significant ratio effect from both DD_{acc} ($F(5, 46) = 53.309$, $p < 0.001$, $\eta^2 = 0.537$) and DD_{RT} ($F(5, 42) = 7.942$, $p < 0.001$, $\eta^2 = 0.486$).

3.2.2. Comparison between DN vs. DD task performance

As in adults, the performance on the DN task was significantly worse than that of the DD task (Accuracy: $F(1, 46) = 72.385$, $p < 0.001$, $\eta^2 = 0.611$; RT: $F(1, 46) = 11.342$, $p = 0.002$, $\eta^2 = 0.198$). The correlations between performance measures from the DN and DD tasks were significant (RT: $r(46) = 0.293$, $p = 0.046$, Accuracy: $r(46) = 0.320$, $p = 0.028$).

3.2.3. The results of ANOVA and correlation tests with mathematical achievement

The mean standardized scores from the Number Concepts, Arithmetic, and Quantitative Reasoning subtests were 13.020 (SD = 2.642), 15.110 (SD = 3.453), and 14.960 (SD = 4.324), respectively. The main effect of gender on math achievement scores were not significant ($ps > 0.1$). Age was significantly correlated with Number Concepts ($r(46) = 0.357$, $p = 0.014$) and Arithmetic ($r(46) = 0.427$, $p = 0.003$), but not Quantitative Reasoning ($r(46) = 0.195$, $p = 0.190$) scores. Fluid intelligence was significantly correlated with all math achievement scores (Number Concepts: $r(46) = 0.477$, $p = 0.001$;

Table 3
Correlations between accuracies of DN/DD tasks and mathematical achievement.

Variable	1	2	3	4	5
1. DN_{acc}					
2. DD_{acc}	0.320*				
3. Number concepts	0.389**	0.198			
4. Arithmetic	0.472**	0.356*	0.709**		
5. Quantitative Reasoning	0.198	0.189	0.581**	0.545**	

N = 47.
* $p < 0.05$.
** $p < 0.01$.

Arithmetic: $r(46) = 0.447$, $p = 0.002$; Quantitative Reasoning: ($r(46) = 0.404$, $p = 0.005$)). Inhibition ability was negatively correlated with Arithmetic ($r(44) = -0.382$, $p = 0.010$), but not with Number Concepts ($r(44) = -0.132$, $p = 0.387$) and Quantitative Reasoning ($r(44) = -0.063$, $p = 0.681$) scores.

3.2.4. The relationship between DN/DD performance and mathematical achievement

DD_{acc} was significantly correlated with Arithmetic ($r(46) = 0.356$, $p = 0.014$; Table 3), but not Number Concepts ($r(46) = 0.198$, $p = 0.182$) or Quantitative Reasoning ($r(46) = 0.236$, $p = 0.110$; Table 3) scores. DN_{acc} was significantly correlated with Number Concepts ($r(46) = 0.389$, $p = 0.007$; Table 3) and Arithmetic ($r(46) = 0.472$, $p = 0.001$), but not Quantitative Reasoning ($r(46) = 0.198$, $p = 0.181$; Table 3) scores. DD_{RT} was not correlated with any math achievement score ($ps > 0.1$; Table S2). DN_{RT} was significantly correlated with Number Concepts ($r(46) = 0.345$, $p = 0.018$), Arithmetic ($r(46) = 0.356$, $p = 0.014$) and Quantitative Reasoning ($r(46) = 0.358$, $p = 0.014$) scores (Table S2).

3.2.5. Hierarchical regression analyses on math achievement

Hierarchical regression analysis was conducted on Arithmetic scores with DD and DN performance measures as predictors, while controlling for the influence of age, fluid intelligence and inhibition ability. Thus, age, fluid intelligence and inhibition ability were entered first and then DD_{acc} and DN_{acc} were added next. DN_{acc} reliably changed the significance of the model regardless of whether it was added before or after DD_{acc} (Table 4 model 1 vs. model 2). However, DD_{acc} did not make any significant change to any model (Table 4).

3.2.6. Mediation analysis

The mediation analysis revealed that the indirect effect of Mapping Precision on the relationship between ANS acuity and Arithmetic was significant (BCa CI of the indirect effect: 0.025 to 0.337, $p = 0.013$, $P_M = 0.359$; Fig. 3). The direct effect of ANS acuity on Arithmetic became non-significant when Mapping Precision was entered as a mediating variable (BCa CI: -0.052 to 0.429, $p = 0.096$). This reveals a full-mediation effect of Mapping Precision on the relationship between ANS acuity and Arithmetic (Fig. 3). The mediating effect of ANS acuity on the relationship between Mapping Precision and Arithmetic was not significant (BCa CI: -0.001 to 0.213, $p = 0.054$; $P_M = 0.155$).

Table 4
Hierarchical regression on Arithmetic after controlling for age, fluid intelligence and inhibition ability in Experiment 2.

Model		Predicting variables	β	ΔR^2	Significance of ΔR^2
1	(1)	Age	0.230	0.480	< 0.001
		Fluid intelligence	0.512		
		Inhibition ability	−0.402		
	(2)	DN_{acc}	0.290	0.078	0.012
	(3)	DD_{acc}	0.106	0.009	0.366
2	(2)	DD_{acc}	0.159	0.021	0.197
	(3)	DN_{acc}	0.271	0.065	0.020

N = 45. Outcome variable = Arithmetic score.

3.3. Discussion

In Experiment 2, we investigated whether children's ANS acuity and Mapping Precision relates to different domains of math achievement (i.e., Number Concepts, Arithmetic, and Quantitative Reasoning) and whether Mapping Precision mediates the relationship between ANS acuity and math achievement. The predictive effects of ANS acuity and Mapping Precision on Arithmetic were significant when each predictor was separately entered into a hierarchical regression model controlling for age, fluid intelligence and inhibition ability. However, the predictive effect of ANS acuity was non-significant when both ANS acuity and Mapping Precision were entered together as predictors for Arithmetic (Table 4). This result was confirmed by a mediation analysis demonstrating that Mapping Precision fully mediated the relationship between ANS acuity and Arithmetic (Fig. 3). These results not only support previous reports of the relationship between Mapping Precision and Arithmetic during childhood but reveals a more direct contribution of Mapping Precision (compared to that of ANS acuity) to children's symbolic math abilities. After controlling for age, fluid intelligence, inhibition ability and Mapping Precision, ANS acuity was no longer a significant predictor for Arithmetic. On the other hand, Mapping Precision was a significant predictor for Arithmetic regardless of the influence of ANS acuity, age, fluid intelligence and inhibition ability. The results of the present study serve as strong evidence that accurate mapping between numbers and their corresponding magnitude contribute to children's learning of symbolic mathematics (De Smedt et al., 2013; Libertus, 2015; Sasanguie et al., 2012).

A novel finding of the present study is that the contribution of ANS acuity and Mapping Precision differed depending on the domain of mathematics, despite strong correlations among all math subtest scores (Table 3). Unlike Arithmetic, Number Concepts was significantly correlated with Mapping Precision but not ANS acuity. An important difference between the Number Concepts and Arithmetic subtest is that the former measures the mastery level of mathematical principles such as cardinality or ordinality, whereas the latter measures the fluency of simple arithmetic. In all subtests, the difficulty level of the problems gradually increased towards the end. On the first few questions of the Number Concepts test, the child needed to apply the counting principle to answer how many items were in a picture (e.g., cardinality). Mastering the cardinality principle is one of the most important components of early quantitative competency (Geary & vanMarle, 2016; vanMarle, Chu, Li, & Geary, 2014). The later part of the Number Concepts test presented questions that require knowledge of symbol-symbol associations (e.g., ordinality) and large numbers (e.g., Fill in the blank to complete the pattern in "5 8 _ 14" or Read "231,654"). All children succeeded in solving earlier questions related to cardinality, thus most of the variance in the Number Concepts scores reflected individual differences in their understanding of symbolic numbers. Therefore, it is likely that Mapping Precision, but not ANS acuity itself, contributed to individual differences in Number Concept scores (vanMarle et al., 2014). In contrast, both ANS acuity and Mapping Precision are likely to contribute substantially to 1st grade children's basic symbolic arithmetic ability, given that they are just beginning to learn the quantitative meaning of symbolic numbers via number-to-magnitude mapping (Libertus, 2015).

Contrary to our hypothesis, neither ANS acuity nor Mapping Precision was correlated with children's Quantitative Reasoning ability. This is likely due to the fact that the Quantitative Reasoning test consisted of verbally described problems which made heavy demands on linguistic and domain-general cognitive abilities such as working memory, long-term memory, attention, language ability, reading and concept formation, thereby reducing the unique contribution of ANS

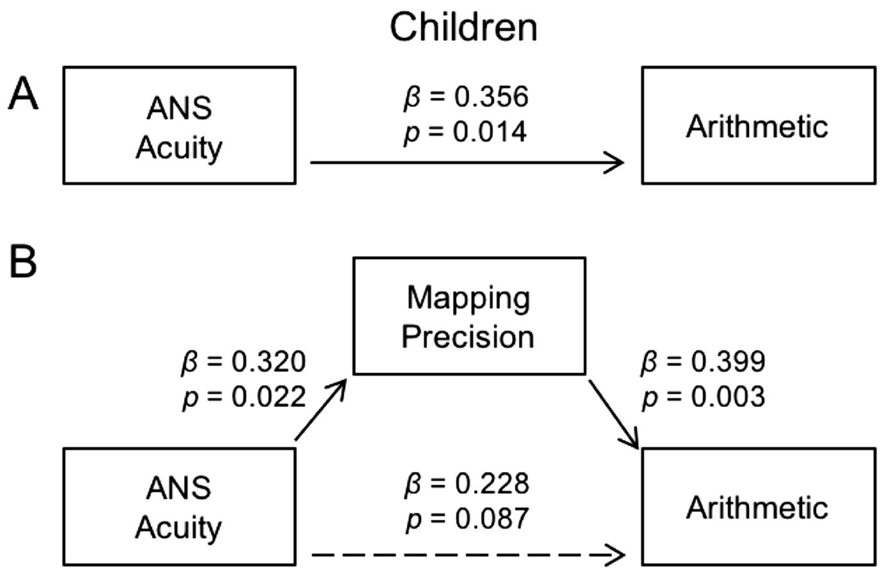

Fig. 3. A mediation model of the relationship between DD task performance (ANS acuity) and Arithmetic through mediation by DN task performance (Mapping Precision) in children. (A) DD_{acc} (ANS acuity) significantly predicted Arithmetic ($\beta = 0.356$, $t = 2.556$, $p = 0.014$). (B) The predictive effect of DD_{acc} (ANS acuity) on DN_{acc} (Mapping Precision) was significant ($\beta = 0.320$, $t = 2.266$, $p = 0.022$). When ANS acuity and Mapping Precision were simultaneously entered as predictors of Arithmetic, the effect of ANS acuity became non-significant ($\beta = 0.228$, $t = 1.571$, $p = 0.087$; BCa CI of direct effect: −0.052 to 0.429, $p = 0.096$), while that of Mapping Precision remained significant ($\beta = 0.399$, $t = 2.920$, $p = 0.003$). The strength of the direct relationship between DD_{acc} (ANS acuity) and Arithmetic was significantly weaker than that between DN_{acc} and Arithmetic (BCa CI of indirect effect: 0.025 to 0.337, $p = 0.013$, $P_M = 0.359$). Mapping Precision fully mediated the relationship between ANS acuity and math achievement, i.e., ANS acuity indirectly predicted Arithmetic through the mediating effect of Mapping Precision.

acuity/Mapping Precision (Fuchs et al., 2006; Swanson & Beebe-Frankenberger, 2004).

4. General discussion

The primary purpose of the present study was to test the 'ANS mapping theory' based on multiple domains of mathematics in adults and children. First, we investigated whether ANS acuity and Mapping Precision respectively correlate with mathematical achievement in adults (Experiment 1) and children (Experiment 2) across multiple domains of mathematics. Next, we examined whether Mapping Precision explains unique variance in math achievement while controlling for the influence of ANS acuity, age, fluid intelligence and other related variables. Finally, we tested whether Mapping Precision mediates the relationship between ANS acuity and math achievement in adults (Experiment 1) and children (Experiment 2).

4.1. The mediating role of mapping precision in the relationship between ANS acuity and math achievement

In both adults and children, ANS acuity was correlated with math achievement (Quantitative Reasoning in adults and Arithmetic in children). However, this relationship was no longer significant when Mapping Precision was considered together in hierarchical regression (Tables 2, 4). Moreover, mediation analyses confirmed that the relationship between ANS acuity and math achievement was fully mediated by Mapping Precision (Figs. 2, 3). These results are consistent with the ANS mapping theory which states that the semantic meaning of symbolic numbers (beyond subitizing range) is first learned by being mapped onto underlying analogue magnitude representations. Given that ANS acuity may reflect the precision of the mental number line, individuals with better ANS acuity may be able to obtain a more accurate understanding of symbolic numbers. By extension, an accurate understanding of numbers will contribute to better ability to manipulate numbers in mathematical activities. Thus, the present study supports the idea that ANS acuity serves as one of the foundations for higher level math achievement by contributing to an accurate mapping between symbolic numbers and their corresponding magnitude (Barth et al., 2005; Brannon, 2006; Dehaene, 2007).

4.2. Developmental changes in the mediating role of mapping precision on the relationship between ANS and math achievement

Full mediation effects of Mapping Precision in children and adults demonstrate a developmental continuity in the mediating role of Mapping Precision in the relationship between ANS acuity and certain domains of math achievement. Specifically, Mapping Precision served a mediating role for the relationship between the ANS acuity and different domains of mathematics in adults vs. children. Mapping Precision contributed to adults' Quantitative Reasoning (but not Arithmetic Fluency) and children's Arithmetic (but not Quantitative Reasoning or Number Concepts) abilities. We believe that this difference reflects distinct cognitive processes engaged by math achievement tests depending on the level of mathematical learning and expertise. Adults are likely to process the quantitative meaning of symbolic numbers while they solve problems of the Quantitative Reasoning test (Jang & Cho, 2016). In contrast, 1st grade children's linguistic and general cognitive ability to comprehend verbal descriptions may have greatly affected their performance on the Quantitative Reasoning test. Thus, the unique contribution of Mapping Precision to children's quantitative reasoning ability may have been reduced. On the other hand, well-educated adults who are fluent in arithmetic are not likely to activate underlying quantity representations for simple arithmetic, but would rather use their verbal, symbolic math system (to access knowledge of arithmetic facts and symbol-symbol associations) for efficient retrieval-based problem solving (Lyons et al., 2012; Lyons & Beilock, 2011; Reynvoet & Sasanguie, 2016). In contrast, first-grade children are in the beginning stage of learning the quantitative meaning of symbolic numbers, thus are expected to heavily depend on number-to-magnitude mapping when working with symbolic numbers. These findings are consistent with the ANS mapping theory stating that ANS acuity and number-to-magnitude mapping is one of the foundations for early stages of learning symbolic math skills. After a certain level of mastery is reached, the contribution of ANS acuity and mapping precision to simple arithmetic seem to become secondary to knowledge of symbolic principles (e.g., ordinality) and symbol-symbol associations (Geary, 2013; Lyons et al., 2012; vanMarle et al., 2014). We carefully speculate that simple arithmetic skills may become less dependent on the ANS or mapping precision around mid-elementary school years during which a shift from cardinality to ordinality based processing seem to occur (Lyons et al., 2014). Interestingly, the present results revealed that ANS acuity continued to contribute to complex quantitative reasoning via number-to-magnitude mapping in adulthood.

4.3. Limitations and future directions

One limitation of the current study is that we could not more completely address methodological issues in relation to the validity and reliability of the DD task format (Dietrich et al., 2015; Gebuis, Kadosh, & Gevers, 2016; Gebuis & Reynvoet, 2012; Leibovich & Henik, 2013; Leibovich, Vogel, Henik, & Ansari, 2016). Although we implemented multiple control conditions and randomization procedures to minimize possible confounding influences, it would have been better to experimentally or statistically control for the influence of non-numerical visual characteristics of the dot arrays (such as convex hull size and inter-dot spacing). Recent studies report that convex hull size, especially, can have a stronger impact on numerosity processing compared to other visual properties of the dot arrays (Clayton, Gilmore, & Inglis, 2015; Gilmore, Cragg, Hogan, & Inglis, 2016). Thus, it would have been ideal to hold convex hull size constant across all arrays and to control for as many non-numerical visual magnitudes as possible and to statistically regress out any variables that could not be systematically controlled. In addition, we could not include data from two children in the hierarchical regression of Experiment 2, because they did not complete the numerical Stroop task. Finally, although our sample size exceeded the minimum number of participants determined by power analysis, it was not large enough to allow correction for multiple tests. We acknowledge these as limitations of the present study.

The present demonstration of the mediating role of mapping precision in the development of mathematical competence by no means excludes the possibility that other foundational abilities may contribute to mathematical achievement (Fazio et al., 2014; Geary & vanMarle, 2016; vanMarle et al., 2014). Although the present study only focused on ANS acuity and Mapping Precision, future studies would benefit from a multi-mediator structural equation modeling approach to uncover the complex relationship among multiple building blocks of mathematical cognition and math achievement across different stages of development.

Furthermore, we emphasize that future work should be directed towards testing an alternative theory proposing that discrete and continuous magnitudes are inevitably correlated and thus are holistically processed by a ‘sense of magnitude’ (Leibovich et al., 2017). This ‘sense of magnitude’ theory proposes that the ability to process continuous magnitudes is innate and that the development of language and cognitive control enables acquisition of number concept, thereby providing an alternative view on the symbol grounding problem. Though not mutually exclusive with the ANS theory, the ‘sense of magnitude’ theory raises many important questions that challenge basic assumptions of the ANS theory. We believe that efforts to test and reconcile these competing theories will bring about major advancement of our knowledge of numerical cognition.

5. Conclusions

Overall, the results of our study indicate that ANS acuity may contribute to children's symbolic arithmetic and adults' quantitative reasoning ability but only through the mediating effect of Mapping Precision. However, ANS acuity or Mapping Precision should not be taken as the only foundational factor that contributes to math achievement. For instance, understanding ordinality or symbol-symbol associations are known to be critical factors that contribute to higher-level mathematical abilities (Goffin & Ansari, 2016; Lyons & Ansari, 2015; Lyons & Beilock, 2011; Price & Fuchs, 2016). Multiple foundational abilities are likely to contribute to different domains of mathematical achievement at different stages of development. We look forward to continued efforts to discover other basic abilities that contribute to mathematical learning and how the interplay between them together enable mathematical learning throughout development.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.lindif.2018.05.005.

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