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Hongwon Kim and Seokhee Cho*
Korean Educational Development Institute, Seoul, Korea
and Doechee Ahn
Chung-Ang University, Seoul, Korea

development of mathematical creative problem solving ability test for identification of the gifted in math

Abstract

To identify the mathematically gifted, Mathematical Creative Problem Solving Ability Test (MCPSAT) was developed. The test was developed for 2 years. In the first year, mathematical creative problem solving ability was conceptualized through literature review. Exemplary items were developed and pilot tested. In the second year, the actual test was constructed and standardized. National and local norms were constructed. Statistical analyses verified that the MCPSAT was valid and reliable to be used in identification of the mathematically gifted. Types of problems, specific criteria for scoring and statistical analyses for verification on goodness of tests are described in this paper.

Introduction

In 1995, Presidential Commission for Educational Reform suggested the strengthening of gifted education in schools to maximize the development of the potential of the gifted. Since 1996, the Ministry of Education has been providing financial support to the National Research Center on Gifted and Talented Education (NRCGTE) at Korean Educational Development Institute (KEDI). According to the 5 year plan (1996-2000) for promotion of gifted education system, the NRCGTE research team had developed MCPSAT for identification of the mathematically gifted.

NRCGTE developed MCPSAT rather than an intelligence test, aptitude test, or basic math skills test. It was because mathematics was thought to be one of the basic subjects for the gifted who wish to be creative professionals either in math, engineering, or technology areas in the future. There were many traditional math tests, but it was difficult to find valid and standardized instruments that can be used for identification of the gifted in mathematics in Korea. Creativity is believed to be the important component of giftedness. However, traditional math tests used for identification of the mathematically gifted did not seem to measure creativity in mathematics. Traditional math tests generally measured either math computational skills, logical thinking skills, or simple application of mathematical formula to solve problems.

Definition of Mathematically Creative Problem Solving Ability

The mathematically gifted was defined as those who show superior ability in solving the math problems in a creative way. In other words, they have high potential to be a creative mathematician in the future and show superior Mathematical Creative Problem Solving Ability (MCPSA). MCPSA is an ability to produce new solutions by using existing knowledge base, principles, concepts, and various thinking strategies.

To formulate the conceptual frame of MCPSA represented in Figure 1, theories on creativity, problem-solving, and giftedness of many researchers (Balka, 1974; Haylock, 1984, 1985, 1987; Isaksen et al., 1994; Polya, 1957; Renzulli, 1978, 1985; Urban, 1995; Wallas, 1926) had been reviewed.

The process of math creative problem solving was regarded as composed of four stages, namely, understanding of problems, planning to solve the problems, execution of the plan, and reflection of the answer and the whole problem solving process. Throughout the four stages, mathematical thinking ability, mathematical creativity, mathematical task commitment, and knowledge base are utilized for mathematical creative problem solving. Divergent thinking and convergent thinking are concurrently operated during mathematical creative problem solving. So MCPSA can be measured best when the tasks require both of the convergent and divergent thinking.

Characteristics of Instruments for Mathematical Creative Problem Solving Ability Test

Target Population

The test was developed to be used for identification of the mathematically gifted. Therefore, the target population of this test is those in upper 15-20% in each grade level from Grade 2-11 in terms of either intelligence or achievement in math.

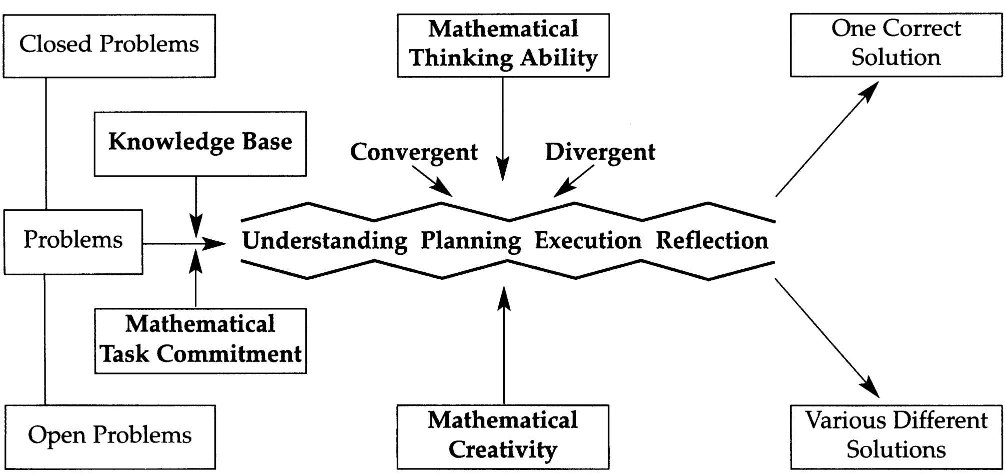


Figure 1: The Conceptual Frame of MCPSA

Structure of Tests: Part 1 and Part 2

The MCPSAT is composed of two parts: Part 1 and Part 2. Test items in Part 1 are characterized as open and requiring various different answers, meanwhile those in Part 2 are characterized as closed and requiring only one correct answer. Even when a mathematical task requires one correct response, it can be considered as creative problem solving task if it requires new ideas, approaches and principles to be solved.

Each of Part 1 and Part 2 is composed of 4 different difficulty levels from grade 2 to grade 11(grade 2-3, grade 4-6, grade 7-9, grade 10-11). In each level, two identical tests (Type A and Type B) are constructed.

Part 1: Test on Mathematical Creativity

Mathematical creativity is measured in Part 1 of test. Mathematical creativity means an ability that can produce various solutions for a math problem. Mathematical creativity is composed of four sub-factors as follows:

- Fluency refers to generation and creation of many responses and ideas. The more the number of correct answers, the more fluent the person is.
- Flexibility refers to generation of many different categories of responses and ideas overcoming the fixedness. The more the number of different categories of correct answers, the more flexible the person is.
- Originality refers to generation of responses and ideas different from other persons. It means rarity and uniqueness of answers.
- Elaboration refers to extension of a simple design to a more complex or intricate design. However, elaboration is not measured by MCPSAT.
- Problem-solving refers to the ability to produce many solutions.

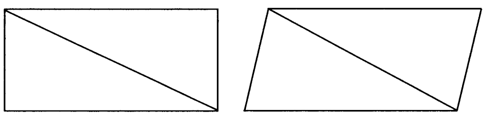
Example of problem (Haylock, 1984)

1. What do you think the numbers on cards A, B, C might be? List as many different possibilities as you can think of.

(A+B)x(C)=9

Example of problem (Haylock, 1984)

- Redefinition refers to ability to give up previous, existing interpretation of familiar objects in order to use them in some new ways.



Write down as many answers as you can think of;
These two figures are common in.................
These two figures are different in................

Part 2: Test on Mathematical Thinking Ability

Mathematical thinking ability is measured in Part 2 of test. It is regarded as composed of seven sub-abilities as follows:

- Intuitive insight refers to figuring out the relationship and structure of given information and conditions, find out the critical cues of problem solving.
- Organization of knowledge refers to collecting and manipulating the necessary information for solving problems.
- Space perception and visualization refers to imagery ability that transforms the given information into the space information.
- Abstraction refers to representing the ill-structured mathematical problems into concept, symbol, formula, and figure.
- Reasoning refers to systematic reasoning in terms of inductive thinking and deductive thinking.
- Generalization and application refers to

generalizing and applying the mathematical relationships.

- Reflective thinking refers to a kind of metacognitive processes on his/her own problem solving process and its relevancy with the problem.

Problems in Part 2 require one correct answer, but it is difficult for students solve the problems by applying knowledge and strategies simply and directly that they learn in school.

Taxonomy of Sub-categories of Abilities and Contents of MCPSAT

Example of sub-thinking abilities and sub-contents of MCPSAT(Part 1 and Part 2) of Type A for grade 2-3 are shown in Table 1. Because of limitation of the number of problems, all sub-abilities and contents in the taxonomy matrix could not be included in the tests. The number of problems in Part 1 of each level and type is 8 or 9. The number of problems in Part 2 is 16 for each of the level and type. Time needed for implementation of a test varies from 50 to 80 minutes depending on the level of the tests.

Procedures of Test Development

The test was developed for 2 years. In the first year, conceptualization of MCPSA through literature review was carried out, the types of exemplary items were developed, and then pilot test was carried out. In the second year, actual test items were developed and standardization was carried out and national/local norms were made.

Table 1: An example of taxonomy of abilities and content domains for grades 2-3 in primary school(Type A)

			Number and Estimation	Operation	Measurement	Relations	Set and Logic	Equation and Inequalities	Function	Calculus	Plain Algebra	Solid Algebra	Combinatory	Probability	Statistics
Math Thinking Ability	Intuitive Insight			•	•	•					•	•			
	Organizing of information		•	•		•		•			•				
	Visualization				•			•			•	•			
	Abstraction			•		•		•			•				
	Reasoning	Induction	•	•	•	•		•							
		Deduction	•		•			•							
	Generalization			•	•	•		•				•			
	Reflection		•	•	•	•		•				•			
Math Creativity	Fluency		•	•	•	•		•			•	•			
	Flexibility		•	•	•	•		•			•	•			
	Originality		•	•	•	•		•			•	•			

* Grade 2-3 students don't learn probability and statistics in regular school curriculum.

Pilot Test

Three types of test A , B, and C were made and the pilot test was carried out.
The Subjects were 1,214 primary school students, 722 middle school students, 672 high school students, and 89 science high school students.

Verification of goodness of test and each test item was carried out through analysis of reliability (i.e., Cronbach α coefficients); analysis of fitness and difficulty level of each item based on item response theory; analysis of correlation coefficients between item score and total score; analysis of mean differences among grade levels, and between high-achievers and low-achievers; and analysis of easiness for making scoring criteria for measuring math creative problem solving ability. Based on the results of statistical analyses, inappropriate test items were taken out or revised .

Development of Actual Test Items and Standardization

Actual test items were developed based on the results of the pilot test. The goodness of each test item was analyzed and only those good items were selected and actual test instruments were constructed. As a result, two identical types of tests, Type A and Type B were constructed.

Subjects. The subjects for verification of the goodness of the actual test were recruited by stratified random sampling techniques from metropolitan cities, mid-sized cities, and rural areas. The number of subjects was 4,991 primary school students, 3,367 middle school students, 2,345 high school students, and 116 science high school students. Total number of subjects was 10,819.

Scoring. Part 1: Math Creativity. Math creativity in one problem was measured by a sum of scores on fluency, flexibility, and originality in one problem. Fluency was scored according to the number of correct answers on one problem. Flexibility was scored according to the number of categories of correct answers on one problem. Originality was scored according to a rarity or uniqueness of correct answers on one problem. An originality score for each subject was calculated by using the categories obtained for the flexibility scores from all the subjects. Based on the frequencies of the categories expressed by the entire sample, scores were awarded according to the following schedule.

Scores	Number of subjects who answered the same category
0	5% or more subjects
1	2% to 4.99% subjects
2	less than 2% subjects

The example of scoring criteria for math creativity is presented in Table 2.

Table 2: Example of scoring criteria for math creativity

Grade 2-3	Type A	No. of item	5	Patterns of number series

<u>Problem</u> : John arranged numbers in four succeeding parentheses according to a rule as shown in the example. Place four numbers according to your own rule and write down the rule that you used.
Example of problem and answers
Problem: ( ) - ( ) - ( ) - ( )
Answer: (1) - (2) - (3) - (4): increase the number by 1

(N = 999)

Category of Responses	Real Responses	grade 2 (%)	grade 3(%)	Total(%)
1. increase by ( )	• increase by 2, 3, 4	380 (38.0)	400 (40.0)	780 (78.0)
2. decrease by ( )	• decrease by 2, 3, 4	124 (12.4)	88 (8.8)	212 (21.2)
3. multiply by ( )	• multiply by 3, 6 (2)*	-	19 (1.9)	19 (1.9)
4. an odd (even) number	• 2 - 4 - 8 –10 • 2 – 10 – 16 - 18 • 1 - 3 - 5 – 7 • 1 - 5 - 11 - 17	- -	5 (0.5) 5 (0.5)	5 (0.5) 5 (0.5)
5. divide by ( )	• 16 - 8 - 4 – 2 • 81 - 27 - 9 - 3	-	5* (0.3)	5* (0.3)
6. increase by ( ) and decrease by ( )	• 2 - 3 - 4 - 1 (2)*	2* (0.2)	1* (0.1)	3* (0.3)
7. add all number before	• 1 - 2 - 3 - 6 (2)*	-	3* (0.3)	3* (0.3)
8. increase by 1, 2, 3, 4	• 1 - 3 - 6 - 10 (2)*	-	3* (0.3)	3* (0.3)
9. decrease by ( ), increase by ( )	• 3 - 2 - 1 - 4 (2)*	2* (0.2)	-	2* (0.2)
10. increase by ( ) and decrease by ( ) and increase by ( )	• 1 - 4 - 2 - 3 (2)*	1* (0.1)	- -	1* (0.1)
11. multiply by 1, 2, 3	• 1 - 1 - 2 - 6 (2)*	-	1* (0.1)	1* (0.1)
12. multiply by itself and write the number after	• 2 - 4 - 16 - 256 (2)*	-	1* (0.1)	1* (0.1)

• ( )* : originality score awarded

Part 2: Math Thinking Ability. Since only one correct answer was required, the score was given depending on whether the answer was right or wrong.

Analysis of Goodness. Means, Score Range on each test are presented in Table 3. The score range was very wide in all schools, even in the science high school where only those who are already identified as the scientifically gifted are studying. Thus, it was verified that MCPSAT was good in discriminating abilities in students.

Table 3: Score range and Means of MCPSAT

			Grade 2-3	Grade 4-6	Grade 7-9	Grade 10-11	
						General	Science
Part 1	Type A	Range	0-92	0-134	0-77	0-50	9-62
		Mean	18.20	31.46	27.67	14.10	32.68
	Type B	Range	0-79	0-115	0-79	0-59	20-82
		Mean	21.72	34.73	21.72	20.36	47.34
Part 2	Type A	Range	0-85	0-92	0-73	0-65	19-88
		Mean	22.72	21.28	12.53	11.44	50.81
	Type B	Range	0-101	0-92	0-89	0-75	12-98
		Mean	28.29	20.21	17.44	15.24	53.08

Note. * The maximum score of Part 2 of the test is 108.

Reliability. Reliability was analyzed by computing Cronbach α coefficients, as presented in Table 4. Cronbach α coefficients which are bigger than .5 means 'reliability exists.'; bigger than .7 'reliablity is good.'; and bigger than .9 'reliablity is very high.' As shown in the analyses of Cronbach α coefficients in Table 5, reliability of the MCPSAT was verified as good in general.

Table 4: Cronbach α coefficients of MCPSAT Grade level

	Part 1		Part 2	
	Type A	Type B	Type A	Type B
Grade 2-3	.62	.68	.73	.74
Grade 4-6	.76	.67	.73	.74
Grade 7-9	.60	.63	.73	.67
Grade 10-11	.55	.60	.72	.74

Internal Validity. Internal validity was analyzed based on item response theory. The coefficients on item adequateness were computed and analyzed. The infit and outfit coefficients of items were almost 1.0. Therefore, it was verified that the internal validity was good.

Difficulty. The item difficulty was analyzed based on item response theory. If the logit coefficient is 0.00, the difficulty is average, and if it is bigger than 0.00, it means the item is difficult, and if it is smaller than 0.00, the item is easy. The 1st part of the test showed the logit coefficients of items between -.73 and .73. The 2nd part of the test showed the logit coefficients of items between -2.51 and 4.35. The range of item difficulty of the 2nd part of the test was greater than that of 1st part of the test.

Discriminality. The item discriminality was analyzed based on item response theory. The point-biserial correlation coefficients were computed. The point-biserial correlation means correlation between item scores and total scores, and if the value is negative, the item does not discriminate adequately abilities of students. All items' values were positive in the point-biserial correlation analyses.

Correlation Coefficients among MCPSA Scores, Math Academic Achievements Scores, and IQ. Correlation coefficients among MCPSA scores, math academic achievements scores, and IQ of students in one primary, one middle, one high school are presented in Table 5. Table 6 shows that MCPSA scores, math academic achievements scores, and IQ are inter-related and at the same time each of them are composed of separated abilities. The correlation coefficients between Part 1 scores and Part 2 scores is .44-.60, and this shows that two constructs are somewhat different.

Table 5: Correlation coefficients among MCPSA scores, math academic achievements scores, and IQ(Type A)

Grade 2-3 (n=73)

	Part 2	IQ	MA
Part 1	.45**	.40**	.31**
Part 2		.42**	.43**
IQ		.30**	

Grade 4-6 (n=162)

	Part 2	IQ	MA
Part 1	.60**	.56**	.58**
Part 2		.39**	.42**
IQ			.59**

Grade 7-9 (n=162)

	Part 2	IQ	MA
Part 1	.56**	.48**	.46**
Part 2		.39**	.44**
IQ			.60**

Grade 10-11 (n=180)

	Part 2	IQ	MA
Part 1	.44**	.34**	.49**
Part 2		.40**	.53**
IQ			.52**

Note. ** $p<.001$ MA: Math Academic Achievement Score in school

MCPSA and National Math Olympiad. The Math Olympiad can be one of the good means to identify math giftedness. There were eleven high school students who were subjects of MCPSAT and participated in National Math Olympiad held by Korean Math Society and won medals. The results of these students in MCPSAT and Olympiad are presented in Table 6. At Part 2 of the test measuring math thinking abilities, when science high school students were excluded, eight students' scores exceeded the scores of ninety-nine percent of total subjects. At Part 1 of the test

measuring math creativity, when science high school students were excluded, seven students' scores exceeded the scores of ninety-eight percent of total subjects.
Two bronze-medal winning students' MCPSAT scores on Part 1 and Part 2 of the test were very high. These results showed that MCPSAT had a good validity.

Construction of Norms. Norms were constructed in accordance with test level (grade 2-3, grade 4-6, grade 7-9, grade 10-11), Parts (Part 1 and Part 2), gender, grade, fluency, flexibility, and originality. The student's raw score, frequency, and percentile are included in the norm.

Use of MCPSAT. MCPSAT is currently used by gifted education centers attached to universities and school boards and special classes for the gifted in regular schools to identify the mathematically gifted all over the country. These educational institutions go through at least three different stages of identification: Recommendation, MCPSAT, and oral interview or observation after placement in enrichment program. Later, the tests can be used not only for identification but also for evaluation of the effects of enrichment programs using two different types of MCPST for pre- and post-tests.

Table 6: Comparison of MCPSAT results and Math Olympiad results

Subjects	School	Grade	Gender	Type	Part 1		Part 2		Olympiad
					Score	Percentile	Score	Percentile	
1	Science	10	M	A	18	72.0 (69.4)	84	- (99.6)	EM
2	High School	10	F	A	35	98.1 (96.4)	98	- (100.0)	within 15% from the top
3		10	M	B	50	99.4 (97.6)	81	- (99.8)	Bronze Medal
4		11	M	B	82	100.0 (100.0)	88	- (100.0)	Bronze Medal
5	General	11	F	A	16	69.9 (62.4)	53	99.5 (97.1)	ME
6	High	11	M	A	46	99.7 (98.8)	38	96.7 (93.6)	ME
7	School	11	F	A	46	99.7 (98.8)	44	98.4 (95.3)	ME
8		11	M	A	24	87.6 (85.0)	31	92.8 (89.1)	ME
9		11	F	A	47	99.9 (99.3)	50	99.4 (96.9)	ME
10		11	M	B	25	72.5 (69.2)	48	99.2 (96.8)	ME
11		11	M	B	47	98.7 (96.6)	65	100.0(99.0)	ME

Note. * ME : Medal for Encouragement
* The number in parentheses is a percentile when science high school students are not included and the number out of parentheses is percentile when science high school students are included.

Implications

The implications acquired from this research are as follows.

First, when both tests of Part 1 and Part 2 are used, the identification of the mathematically gifted can be more valid and reliable.

Second, the good items to measure math creativity are unstructured and ill-defined problems to which students can respond with diversity of solutions. However, it was difficult to devise such items. It is, therefore, necessary to carry out more in-depth study in devising ill-defined or unstructured test items more to measure math creativity.

Third, in math creativity, the correlation between fluency and flexibility appeared to be very high, more than r = .95. This fact implies that it may be sufficient to measure only one variable. In addition, by requiring students to produce as many responses as possible within the limited time period to measure flexibility and fluency, students' ability to produce original solution could not be maximally executed. Therefore, further study about this is needed.

Fourth, elaboration of math creativity was not measured in this study considering the elaboration as rendering simple designs into more complex designs. However, elaboration in math problem solving can be redefined. Elaboration of mathematical thinking can result in an elegant formula for expressing various solutions or responses. If elaboration in math problem solving can be redefined in this way, it may become possible to measure elaboration and meaningfully use this in developing math creativity.

Finally, it is highly recommended that Part 1 of the test for mathematical creative problem solving ability be introduced in schools, since it can influence the modes of presentation of math problems in the direction of stimulating students' mathematical divergent thinking and enhance their interest in mathematics.

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An Ant

An Ant
Pauses
At a twig
On my mattress,
A temporary reprieve.
The stunted grass
Casts a shadow,
Superimposed on a mosaic
Of dappled shade,
Strewn over the abstract continuity
Of a muddy print.
Tiger Asics?

An Ant
Dashes on
Colliding.
Helter-skelter.
The shrivelled cocoon
Of a tortured Acacia leaf falls,
Rolls -
A broken parabola
Bounding in phase
With the chaos
Of avalanching folds.

An Ant
Sprawls with futility-catapulted into the grass.
How vast the scale of life is
Surveyed from a blue mattress
Under a tree.

SCOTT BREDIN
(age 14)