

Received May 4, 2020, accepted May 27, 2020, date of publication June 1, 2020, date of current version June 10, 2020. Digital Object Identifier 10.1109/ACCESS.2020.2998787

Approximation-Based Event-Triggered Control Against Unknown Injection Data in Full States and Actuator of Uncertain Lower-Triangular **Nonlinear Systems**

SUNG JIN YOO⁽¹⁾, (Member, IEEE) School of Electrical and Electronics Engineering, Chung-Ang University, Seoul 06974, South Korea e-mail: sjyoo@cau.ac.kr

This work was supported in part by the National Research Foundation of Korea (NRF) Grant funded by the Korean Government under Grant NRF-2019R1A2C1004898, and in part by the Human Resources Development of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) Grant funded by the Korean Government Ministry of Trade, Industry and Energy under Grant 20174030201810.

ABSTRACT This paper addresses an approximation-based adaptive event-triggered control problem against unknown injection data in full state measurements and an actuator of systems with unknown strict-feedback nonlinearities. It is assumed that full state variables measured for state-feedback control are corrupted by unknown injection data that denote cyber attacks or fault signals, and all system nonlinearities are unknown. Owing to the corrupted state feedback information, error surfaces using exactly measured state variables become unknown during the recursive control design procedure for strict-feedback nonlinear systems. Thus, they cannot be used to implement the adaptive event-triggered controller. To address this problem, an approximation-based adaptive recursive event-triggered control design using the corrupted state variables is established to ensure that error surfaces using exactly measured state variables converge to an adjustable neighborhood of the origin in the Lyapunov sense. The adaptive controller and its event-triggering law using corrupted states are designed under uncertain injection data where the adaptive injection data compensators using the neural networks are constructed to deal with the unknown injection data effects. The stability of the closed-loop systems and the exclusion of Zeno behavior are analyzed.

INDEX TERMS Event-triggered control, corrupted full state measurements, unknown injection data, dynamic surface design, unknown strict-feedback nonlinearities.

I. INTRODUCTION

The development of information and communication technology has stimulated the tight interaction of control systems and cyber components [1], [2]. Under the network-based control environment, the exact transmission of measured state variables of physical systems is an important problem for ensuring the performance of controllers in the network. During the network transmission of the state information for the feedback control, unexpected time-varying injection data can be added to the state information measured by sensors. The sources of the injection data in the resilient control studies are largely divided into two categories: (i) measurement

The associate editor coordinating the review of this manuscript and approving it for publication was Zheng H. $Zhu^{\textcircled{U}}$.

faults and (ii) adversarial cyber attacks. In the first category, the sensor faults influence the measurement information for the feedback control. Many studies have addressed the resilient control problems of linear and nonlinear systems with measurement faults [3]-[6]. Especially, some limited studies have appeared for the recursive control design of lower-triangular nonlinear systems with measurement faults where backstepping [7] and dynamic surface designs [8], [9] have been used to construct the resilient control systems. In [10], [11], the output functions were regarded as sensor faults, and output-feedback control approaches were presented for nonlinear systems. In [12], an output-feedback stabilizer design problem using a dual-domination approach was addressed for lower-triangular nonlinear systems with unknown measurement sensitivity. In [13], an adaptive

compensation method of sensor failure was proposed for parametric strict-feedback systems. In [14], a fuzzy adaptive sensor fault compensation approach was studied for nonlinear strict-feedback systems. In [15], an adaptive control method was presented for nonlinear systems with time-varying sensor sensitivities. However, the existing works [10]-[15] for lower-triangular nonlinear systems considered only an output measurement fault. That is, they cannot be applied to state-feedback recursive control design and stability analysis problems in the presence of full state measurement faults because corrupted state variables, instead of exactly measured state variables, should be used to implement virtual and actual controllers. In the second category, the measured state information is stealthily monitored by adversarial attackers and deception attack signals for attackers' desire are maliciously injected into the measured states. Thus, the deception attack signals may depend on the state variables. For known cyber attacks, some resilient control approaches have been presented for linear and nonlinear systems [16]-[21]. To consider unknown cyber attack signals in state variables and an actuator, some adaptive control studies have recently been investigated. In [22], sensor attacks of uncertain linear systems were considered to design adaptive control architectures. An adaptive control methodology was presented in [23] where the time-varying parameters in deception attacks were estimated via the projection algorithm. In [24], an adaptive resilient control approach using Nussbaum functions was studied in the presence of unknown control direction. However, these research results [22]-[24] for compensating for unknown deception attacks in the adaptive control framework are only available for *linear* systems. They cannot be applied to the recursive design problem against unknown injection data in full state measurements of lower-triangular nonlinear systems.

Adaptive control problems of nonlinear lower-triangular systems with uncertainties unmatched to a control input have received great attention in the control design field because of their application to many practical systems such as robot manipulators, biological systems, power systems, flight systems, and traffic control systems. Thus, adaptive recursive control design strategies have been actively developed to deal with unmatched parametric or nonparametric uncertainties. During the early stages of the research, the adaptive technique was combined with the recursive designs to estimate unmatched parametric uncertainties online (see [25]-[31] and references therein). Function approximation techniques using neural networks or fuzzy logic systems have been applied to design approximation-based adaptive control systems, in an attempt to deal with unmatched nonparametric uncertainties (i.e, nonlinear uncertainties) (see [32]-[39] and references therein). In addition, distributed adaptive control approaches have been presented for uncertain multi-agent nonlinear systems in the strict-feedback form [40]-[44]. In these studies, unknown nonlinear functions derived from the recursive control design steps were estimated via radial basis function neural networks (RBFNNs) or fuzzy logic systems.

The uncorrupted state feedback information was used as the input for these function approximators. If the state feedback information is corrupted by unknown injection data, the corrupted state feedback information should be used as the input of the function approximators in the adaptive control framework. This problem still persists in the full-state-feedback-based adaptive control field of strict-feedback nonlinear systems.

The event-triggered control technique is particularly popular in the network-based control field of linear and nonlinear systems, for ensuring the efficient use of the network bandwidth [45]-[48]. In event-triggered control, a control input is updated when a triggering law is satisfied. Thus, the computational and communicational resources required for implementing the controller can be conserved. Therefore, event-triggered control problems have been studied for systems with nonlinearities unmatched to the control input. In [49]-[51], adaptive event-triggered control designs were developed for lower-triangular nonlinear systems with parametric uncertainties. In [52], the finitetime stabilization problem of uncertain nonlinear systems was addressed in the event-triggered control framework. In [53]-[56], approximation-based adaptive control techniques for estimating unknown nonlinear uncertainties were combined with event-triggered control designs. Despite these efforts, no studies have been reported thus far on the eventtriggered control problem against unknown injection data in full state measurements and an actuator of lower-triangular nonlinear systems.

On the basis of the above discussion, the main difficulties in designing an adaptive event-triggered resilient control scheme against the aforementioned unknown injection data are as follows.

(D1) Because full state measurements for feedback control are corrupted by unknown injection data, the corrupted state variables can be only used in the adaptive control scheme using backstepping or dynamic surface techniques and thus the error surfaces using the exactly measured state variables for the recursive design are unknown for the control design. Accordingly, the first difficulty is *how to design an adaptive controller using the corrupted full state variables to ensure the convergence of the error surfaces using the exactly measured state variables* for achieving the control objective in Lyapunov-based recursive stability analysis.

(D2) Since the exactly measured state variables are not available, the second difficulty is *how to design the triggering law using corrupted state variables* in the adaptive event-triggered control framework. Furthermore, the stability of the closed-loop system and the exclusion of Zeno behavior should be analyzed by the triggering law using the corrupted state variables.

The objective of this paper is to propose a remedy for difficulties (D1) and (D2), that is, to establish an adaptive resilient event-triggered control design strategy using corrupted full state measurements for uncertain nonlinear strict-feedback systems with unknown injection data in full

state measurements and an actuator. It is assumed that full state measurements corrupted by unknown injection data are available for feedback, and system nonlinearities are unknown. To overcome difficulties (D1) and (D2), auxiliary signals using corrupted full state variables are designed, and an approximation-based adaptive event-triggered controller and its triggering law are recursively constructed using the auxiliary signals. For the proposed control scheme, dynamic injection data compensators using RBFNNs are designed to compensate for unknown injection data effects. Although the proposed adaptive event-triggered controller is based on corrupted full state variables, it is shown that the convergence of the error surfaces using the exactly measured state variables is ensured for achieving the control objective in the Lyapunov-based stability sense. Finally, simulation examples including a practical application are given for testifying the validity of the proposed theoretical result.

The rest of this paper is outlined as follows. An adaptive resilient event-triggered control problem of uncertain nonlinear strict-feedback systems with unknown injection data in full state measurements and an actuator is formulated in Section 2. The proposed event-triggered control design using corrupted state variables and its stability analysis are presented in Sections 3. Simulation studies are given in Section 4. Finally, we conclude in Section 5.

II. PROBLEM FORMULATION

Let us consider the following uncertain nonlinear strictfeedback systems with unknown injection data in full state measurements and an actuator

$$\begin{aligned} \dot{x}_k &= x_{k+1} + h_k(\bar{x}_k) \\ \dot{x}_n &= u + \kappa_a(t, \bar{x}_n) + h_n(\bar{x}_n) \\ x_{i,a} &= x_i + \kappa_{i,s}(t, x_i) \end{aligned} \tag{1}$$

where k = 1, ..., n - 1, i = 1, ..., n, $\bar{x}_k = [x_1, x_2, ..., x_k]^\top \in \mathbb{R}^k$ are state variable vectors, $h_i(\bar{x}_i) : \mathbb{R}^i \mapsto \mathbb{R}$ are unknown C^1 nonlinear functions, κ_a and $\kappa_{i,s}$ denote injection data in the actuator and the *i*th state measurement, respectively, $x_{i,a}$ is the corrupted state variables, and $u \in \mathbb{R}$ is an event-triggered control input that is intermittently updated by a triggering law to be designed later. The injection data are represented by $\kappa_{i,s}(t, x_i(t)) = \zeta_i(t)x_i(t)$ and $\kappa_a(t, \bar{x}_n(t)) = \xi(t)\delta(\bar{x}_n(t))$ where ζ_i and ξ are unknown time-varying signals and δ is a continuous nonlinear function.

Assumption 1: Instead of the exactly measured state variables x_i , the corrupted state variables $x_{i,a}$ are available only for the feedback control design.

Assumption 2: [23], [24] There exist unknown constants $\bar{\zeta}_i, \bar{\zeta}_{i,d}$, and $\bar{\xi}$ such that $|\zeta_i| \leq \bar{\zeta}_i, |\dot{\zeta}_i| \leq \bar{\zeta}_{i,d}$, and $|\xi| \leq \bar{\xi}$.

Assumption 3: [23] The time-varying signals ζ_i satisfy $\zeta_i + 1 \neq 0$ and the sign of $\zeta_i + 1$ is assumed to be positive.

Problem 1: Consider system (1). Our problem is to design an adaptive resilient event-triggered control law u using corrupted state variables so that system (1) is stabilized in the presence of unknown strict-feedback nonlinearities and unknown injection data in full state measurements and an actuator while all other signals remain bounded.

Remark 1 (The following statements are noted): (i) In contrast to the existing resilient control results [23], [24] for linear systems, uncertain nonlinear strict-feedback systems are considered in this paper. A recursive resilient event-triggered control design strategy using the corrupted state variables is established in the presence of unknown injection data in full state measurements and an actuator;

(ii) Assumption 3 is reasonable for ensuring the controllability of the system (1) with injection data in full state measurements [23]. This implies the existence of a nominal solution for Problem 1;

(iii) Contrary to the existing recursive designs [10]–[15] against output measurement faults, this paper considers full state variables corrupted by unknown injection data (i.e, $x_{i,a} = x_i + \kappa_{i,s}(t, x_i)$ in (1)) and the adaptive event-triggered control problem using the corrupted state variables. Thus, Problem 1 is the first trial in the adaptive control branch of uncertain lower-triangular nonlinear systems.

Remark 2: System (1) in the strict-feedback form can represent many nonlinear practical applications such as aircraft wing rock models, jet engines, flight systems, biochemical processes, and flexible-joint robots [7]. The state variables measured for the network-based feedback control of these practical systems can be corrupted by sensor faults or cyber attacks. The resilient event-triggered control strategy proposed in this paper can then be applied to these practical control problems.

III. MAIN RESULTS

A. FUNCTION APPROXIMATION USING RADIAL BASIS FUNCTION NEURAL NETWORKS

In this paper, RBFNNs are employed to approximate unknown continuous nonlinear functions to be specified in the adaptive event-triggered control design procedure. Consider continuous real-valued nonlinear functions $\Psi_i(\gamma_i)$: $\mathbb{R}^{q_i} \mapsto \mathbb{R}$ where i = 1, ..., n. RBFNNs can approximate $\Psi_i(\gamma_i)$ over a compact set $\Pi_{\gamma_i} \subset \mathbb{R}^{q_i}$ as [57], [58]

$$\Psi_i(\gamma_i) = \theta_i^{\top} G_i(\gamma_i) + \epsilon_i(\gamma_i) \tag{2}$$

where $i = 1, ..., n, \gamma_i$ is the input vector, ϵ_i is the approximation error, $\theta_i \in \mathbb{R}^{l_i}$ is the optimal weighting vector defined as $\theta_i = \arg \min_{\hat{\theta}_i} [\sup_{\gamma_i \in \Pi_{\gamma_i}} |\Psi_i(\gamma_i) - \hat{\theta}_i^\top G_i(\gamma_i)|]; \hat{\theta}_i$ is an estimate of θ_i , and $G_i(\gamma_i) = [g_{i,1}(\gamma_i), ..., g_{i,l_i}(\gamma_i)]^\top \in \mathbb{R}^{l_i}$; Gaussian functions $g_{i,j}(\gamma_i), j = 1, ..., l_i$, are defined as

$$g_{i,j}(\gamma_i) = \exp\left[\frac{-(\gamma_i - o_{i,j})^\top (\gamma_i - o_{i,j})}{r_{i,j}^2}\right]$$
(3)

with the center of the receptive field $o_{i,j} \in \mathbb{R}^{q_i}$ and the width of the Gaussian function $r_{i,j} \in \mathbb{R}$.

Assumption 4: [59] θ_i and ϵ_i are bounded as $\|\theta_i\| \leq \overline{\theta}_i$ and $|\epsilon_i| \leq \overline{\epsilon}_i$, respectively, where i = 1, ..., n, $\overline{\theta}_i > 0$ and $\overline{\epsilon}_i > 0$ are unknown constants. *Lemma 1:* [59] It holds that $||G_i(\gamma_i)|| \leq \overline{G}_i$ where i = 1, ..., n and \overline{G}_i is a constant.

B. RECURSIVE ADAPTIVE EVENT-TRIGGERED CONTROL DESIGN

In this section, a recursive design strategy using the corrupted state feedback information is presented for the adaptive eventtriggered control of system (1).

Let us consider the following coordinate transformation on the basis of the dynamic surface design technique

$$e_1 = x_1 \tag{4}$$

$$e_k = x_k - \bar{\nu}_k \tag{5}$$

$$\rho_k = \bar{\nu}_k - \nu_k \tag{6}$$

where $k = 2, ..., n, e_1$ and e_k are error surfaces using exactly measured state variables x_1 and x_k , ρ_k and ν_k denote boundary layer errors and virtual control signals, respectively, and $\bar{\nu}_k$ are signals provided by the first-order low-pass filters

$$\tau_k \dot{\bar{\nu}}_k + \bar{\nu}_k = \nu_k, \quad \bar{\nu}_k(0) = \nu_k(0)$$
 (7)

with small time constants $\tau_k > 0$. Since the exactly measured state variables x_i , i = 1, ..., n, are corrupted by unknown injection data $\kappa_{i,s}$, the error surfaces e_i , i = 1, ..., n, are not available for designing the virtual control laws v_k and the actual control law u. Thus, we propose auxiliary signals using corrupted state variables $x_{i,a}$, i = 1, ..., n, as follows:

$$\varpi_1 = x_{1,a} \tag{8}$$

$$\varpi_k = x_{k,a} - \bar{\nu}_k - \bar{\zeta}_k \bar{\nu}_k \tag{9}$$

where k = 2, ..., n, and $\hat{\zeta}_k$ is the estimate of ζ_k which is provided by the injection data compensator to be designed later.

Remark 3: It is well known that the convergence of the error surfaces e_i should be analyzed for stable control design based on the Lyapunov stability theorem in conventional dynamic surface control where i = 1, ..., n. However, the error surfaces e_i are unknown for our control design because the exactly measured state variables x_i are corrupted during the feedback procedure. Thus, we present a recursive event-triggered control design using the available auxiliary signals ϖ_i , instead of the unknown error surfaces e_i to ensure the convergence of the error surfaces e_i in the Lyapunov sense.

For the recursive design based on the Lyapunov stability analysis, (8) and (9) can be rewritten as

$$\varpi_1 = \frac{e_1}{\phi_1}$$

$$\varpi_k = e_k + \zeta_k x_k - \hat{\zeta}_k \bar{\nu}_k + \zeta_k \bar{\nu}_k - \zeta_k \bar{\nu}_k$$

$$e_k + \tilde{\zeta}_k \bar{\nu}_k - \zeta_k \bar{\nu}_k$$
(11)

 $= \frac{e_k}{\phi_k} + \tilde{\zeta}_k \bar{\nu}_k \tag{11}$

where k = 2, ..., n, $\tilde{\zeta}_k = \zeta_k - \hat{\zeta}_k$, and $\phi_k = 1/(1+\zeta_k)$. From Assumption 3, (10) and (11) are well defined. In addition, from Assumption 2, there exist unknown positive constants ϕ_i , $\bar{\phi}_i$, and $\bar{\phi}_{i,d}$ such that $\phi_i \leq |\phi_i| \leq \bar{\phi}_i$ and $|\dot{\phi}_i| \leq \bar{\phi}_{i,d}$ where i = 1, ..., n. From now on, the approximation-based adaptive eventtriggered stabilizer using the auxiliary signals ϖ_i is recursively designed. The control design procedure is based on the Lyapunov stability theorem [7]. This theorem provides a design methodology to deal with the adaptive control design and stability analysis, simultaneously. In the first design step, a Lyapunov function V_1 consisting of the error surface e_1 and the weight estimation error $\tilde{\theta}_1$ for RBFNN is chosen to analyze the convergence of the error surface e_1 and the boundedness of $\tilde{\theta}_1$. In the *k*th design step, a Lyapunov function V_k consisting of the error surface e_k , weight estimation error $\tilde{\theta}_k$ for RBFNN, and compensation error $\tilde{\zeta}_k$ for the injection data is selected to analyze the convergence of the error surface e_k and the boundedness of $\tilde{\theta}_k$ and $\tilde{\zeta}_k$ where k = 2, ..., n.

Step 1: Differentiating the first error surface e_1 with respect to time yields

$$\dot{e}_1 = x_2 + h_1(x_1).$$
 (12)

Consider the Lyapunov function $V_1 = e_1^2/(2\phi_1) + \tilde{\theta}_1^\top \tilde{\theta}_1/(2\lambda_1)$ where $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$. Its time derivative is

$$\dot{V}_{1} = -\frac{\dot{\phi}_{1}}{2\phi_{1}^{2}}e_{1}^{2} + \frac{1}{\phi_{1}}e_{1}(e_{2} + \rho_{2} + \nu_{2} + h_{1}(x_{1})) - \frac{1}{\lambda_{1}}\tilde{\theta}_{1}^{\top}\dot{\hat{\theta}}_{1}$$

$$\leq \frac{1}{\phi_{1}}e_{1}(e_{2} + \rho_{2} + \nu_{2} + \varphi_{1}) - \frac{1}{\lambda_{1}}\tilde{\theta}_{1}^{\top}\dot{\hat{\theta}}_{1} \qquad (13)$$

where $\varphi_1 = h_1(x_1) + (\bar{\phi}_{1,d}/(2\phi_1))e_1$.

Then, there exists a continuous nonlinear function $\Psi_1(\gamma_1)$ such that

$$\frac{1}{\phi_1} e_1 \varphi_1 \le \frac{1}{\phi_1} e_1 \Psi_1(\gamma_1)$$
 (14)

where $\Psi_1(\gamma_1) = h_1(x_1) + (\bar{\phi}_{1,d}/(2\phi_1))e_1$ and $\gamma_1 = e_1$. From (2), employing the RBFNN $\theta_1^\top G_1(\gamma_1)$ to estimate $\Psi_1(\gamma_1)$ yields

$$\Psi_1(\gamma_1) = \theta_1^\top G_1(\gamma_1) + \epsilon_1.$$
(15)

Based on the auxiliary signal ϖ_1 , the first virtual control law is derived as

$$\psi_2 = -(\beta_1 + m_1)\overline{\omega}_1 - \hat{\theta}_1^\top G_1(\gamma_{1,a})$$
 (16)

$$\hat{\theta}_1 = \lambda_1 \overline{\omega}_1 G_1(\gamma_{1,a}) - \lambda_1 \sigma_1 \hat{\theta}_1$$
(17)

where $\beta_1 > 0$, $m_1 > 0$, $\lambda_1 > 0$, and $\sigma_1 > 0$ are design constants, and $\gamma_{1,a} = e_{1,a}$ with $e_{1,a} = x_{1,a}$.

Then, substituting (14) and (15) into (13) gives

$$\dot{V}_{1} \leq \frac{1}{\phi_{1}}e_{1}(e_{2} + \rho_{2} + \nu_{2} + \theta_{1}^{\top}G_{1}(\gamma_{1}) + \epsilon_{1}) - \frac{1}{\lambda_{1}}\tilde{\theta}_{1}^{\top}\dot{\theta}_{1}.$$
(18)

From (10), it holds that $e_1 = \phi_1 \overline{\omega}_1$. Using (16) and $e_1 = \phi_1 \overline{\omega}_1$, (18) becomes

$$\dot{V}_{1} \leq \frac{1}{\phi_{1}}e_{1}(e_{2}+\rho_{2}) + \varpi_{1}(-(\beta_{1}+m_{1})\varpi_{1}+\tilde{\theta}_{1}^{\top}G_{1}(\gamma_{1,a}) + \theta_{1}^{\top}\psi_{1}+\epsilon_{1}) - \frac{1}{\lambda_{1}}\tilde{\theta}_{1}^{\top}\dot{\hat{\theta}}_{1} \quad (19)$$

VOLUME 8, 2020

where $\psi_1 = G_1(\gamma_1) - G_1(\gamma_{1,a})$. Adding and subtracting the term $(\beta_1/\phi_1^2)e_1^2$ to (19), and using $e_1 = \phi_1 \varpi_1$ and (17) yield

$$\dot{V}_{1} \leq \frac{1}{\phi_{1}} e_{1}(e_{2} + \rho_{2}) + \overline{\omega}_{1}(-m_{1}\overline{\omega}_{1} + \varsigma_{1}) - \frac{\beta_{1}}{\phi_{1}^{2}} e_{1}^{2} + \sigma_{1}\tilde{\theta}_{1}^{\top}\hat{\theta}_{1} \quad (20)$$

where $\varsigma_1 = \theta_1^\top \psi_1 + \epsilon_1$.

Step k (k = 2, ..., n - 1): The time derivative of the kth error surface (5) is given by

$$\dot{e}_k = x_{k+1} + h_k(\bar{x}_k) - \dot{\bar{\nu}}_k$$
 (21)

where k = 2, ..., n - 1.

Consider the Lyapunov function $V_k = e_k^2/(2\phi_k) + \tilde{\zeta}_k^2/2 + \tilde{\theta}_k^\top \tilde{\theta}_k/(2\lambda_k)$ where k = 2, ..., n - 1, $\tilde{\zeta}_k = \zeta_k - \hat{\zeta}_k$, and $\tilde{\theta}_k = \theta_k - \hat{\theta}_k$. Its time derivative along (6) and (21) is represented by

$$\dot{V}_{k} = -\frac{\phi_{k}}{2\phi_{k}^{2}}e_{k}^{2} + \frac{1}{\phi_{k}}e_{k}(e_{k+1} + \rho_{k+1} + \nu_{k+1} - \dot{\bar{\nu}}_{k} + h_{k}(\bar{x}_{k})) + \tilde{\zeta}_{k}(\dot{\zeta}_{k} - \dot{\bar{\zeta}}_{k}) - \frac{1}{\lambda_{k}}\tilde{\theta}_{k}^{\top}\dot{\bar{\theta}}_{k} \leq \frac{1}{\phi_{k}}e_{k}(e_{k+1} + \rho_{k+1} + \nu_{k+1} - \dot{\bar{\nu}}_{k} + \varphi_{k}) - \frac{1}{\phi_{k-1}}e_{k-1}e_{k} + \frac{1}{\phi_{k}}e_{k}(\beta_{k} + \iota_{k})\bar{\nu}_{k}\zeta_{k} + \tilde{\zeta}_{k}(\dot{\zeta}_{k} - \dot{\bar{\zeta}}_{k}) - \frac{1}{\lambda_{k}}\tilde{\theta}_{k}^{\top}\dot{\bar{\theta}}_{k}$$
(22)

where $\varphi_k = h_k(\bar{x}_k) + (\bar{\phi}_{k,d}/(2\phi_k))e_k + (\phi_k/\phi_{k-1})e_{k-1} - (\beta_k + \iota_k)\bar{\nu}_k\zeta_k; \ \beta_k > 0 \text{ and } \iota_k > 0 \text{ are constants.}$

From the boundedness of ζ_k , ϕ_k and ϕ_{k-1} , there exists a continuous function $\Psi_k(\gamma_k)$ such that

$$\frac{1}{\phi_k} e_k \varphi_k \le \frac{1}{\phi_k} e_k \Psi_k(\gamma_k) + a_k \tag{23}$$

where $a_k > 0$ is a constant and $\gamma_k = [\bar{x}_k^\top, e_{k-1}, e_k, \bar{\nu}_k, b_k]^\top$ with a constant $b_k > 0$. By employing the RBFNN $\theta_k^\top G_k(\gamma_k)$ to estimate $\Psi_k(\gamma_k)$, we have

$$\Psi_k(\gamma_k) = \theta_k^\top G_k(\gamma_k) + \epsilon_k.$$
(24)

An adaptive injection data compensator is presented as

$$\dot{\hat{\boldsymbol{\xi}}}_{k} = \hat{\boldsymbol{\theta}}_{k}^{\top} \boldsymbol{G}_{k}(\boldsymbol{\gamma}_{k,a}) \bar{\boldsymbol{\nu}}_{k} - (\boldsymbol{\beta}_{k} - \boldsymbol{m}_{k} - (\boldsymbol{\beta}_{k} + \boldsymbol{\iota}_{k})^{2} \bar{\boldsymbol{\nu}}_{k}^{2}) \bar{\boldsymbol{\nu}}_{k} \boldsymbol{\varpi}_{k} - (\boldsymbol{\beta}_{k} + \boldsymbol{\iota}_{k}) \bar{\boldsymbol{\nu}}_{k}^{2} \hat{\boldsymbol{\zeta}}_{k} - \boldsymbol{\alpha}_{k} \hat{\boldsymbol{\zeta}}_{k}$$
(25)

where $k = 2, ..., n, \beta_k > 0, m_k > 0, \iota_k > 0$, and $\alpha_k > 0$ are design parameters, $\gamma_{k,a} = [\bar{x}_{k,a}^{\top}, e_{k-1,a}, e_{k,a}, \bar{\nu}_k, b_k]^{\top}; \bar{x}_{k,a} = [x_{1,a}, ..., x_{k,a}]^{\top}, e_{k-1,a} = x_{k-1,a} - \bar{\nu}_{k-1}$ with $\bar{\nu}_1 = 0$, and $e_{k,a} = x_{k,a} - \bar{\nu}_k$.

The *k*th virtual control law using the auxiliary signal ϖ_k is designed as

$$\nu_{k+1} = -(\beta_k + m_k + (\beta_k + \iota_k)^2 \bar{\nu}_k^2) \overline{\omega}_k
- \hat{\theta}_k^\top G_k(\gamma_{k,a}) + \frac{\nu_k - \bar{\nu}_k}{\tau_k}$$
(26)

$$\dot{\hat{\theta}}_k = \lambda_k \overline{\omega}_k G_k(\gamma_{k,a}) - \lambda_k \sigma_k \hat{\theta}_k$$
(27)

where $\lambda_k > 0$ and $\sigma_k > 0$ are design constants.

Using $e_k = \phi_k(\overline{\omega}_k - \tilde{\zeta}_k \overline{\nu}_k)$ in (11) and ν_{k+1} in (26), (22) becomes

$$\begin{split} \dot{\mathbf{v}}_{k} \\ &\leq \frac{1}{\phi_{k}} e_{k}(e_{k+1} + \rho_{k+1}) - \frac{\beta_{k}}{\phi_{k}^{2}} e_{k}^{2} + \frac{\beta_{k}}{\phi_{k}^{2}} e_{k}^{2} \\ &+ (\varpi_{k} - \tilde{\zeta}_{k} \bar{v}_{k}) \{-(\beta_{k} + m_{k} + (\beta_{k} + \iota_{k})^{2} \bar{v}_{k}^{2}) \varpi_{k} \\ &+ \tilde{\theta}_{k}^{\top} G_{k}(\gamma_{k,a}) + \theta_{k}^{\top} \psi_{k} + \epsilon_{k} \} \\ &+ \iota_{k} \tilde{\zeta}_{k}^{2} \tilde{v}_{k}^{2} - \iota_{k} \tilde{\zeta}_{k}^{2} \bar{v}_{k}^{2} - \frac{1}{\phi_{k-1}} e_{k-1} e_{k} + \frac{1}{\phi_{k}} e_{k}(\beta_{k} + \iota_{k}) \bar{v}_{k} \zeta_{k} \\ &+ \tilde{\zeta}_{k} (\dot{\zeta}_{k} - \dot{\zeta}_{k}) - \frac{1}{\lambda_{k}} \tilde{\theta}_{k}^{\top} \dot{\theta}_{k} + a_{k} \end{split} \\ &= \frac{1}{\phi_{k}} e_{k}(e_{k+1} + \rho_{k+1}) \\ &+ \varpi_{k} \{-(\beta_{k} + m_{k} + (\beta_{k} + \iota_{k})^{2} \bar{v}_{k}^{2}) \varpi_{k} + \tilde{\theta}_{k}^{\top} G_{k}(\gamma_{k,a}) + \varsigma_{k} \} \\ &- \tilde{\zeta}_{k} \bar{v}_{k} \{-(\beta_{k} + m_{k} + (\beta_{k} + \iota_{k})^{2} \bar{v}_{k}^{2}) \varpi_{k} + \tilde{\theta}_{k}^{\top} G_{k}(\gamma_{k,a}) + \varsigma_{k} \} \\ &- \frac{\beta_{k}}{\phi_{k}^{2}} e_{k}^{2} + \beta_{k} (\varpi_{k}^{2} - 2 \tilde{\zeta}_{k} \bar{v}_{k} \varpi_{k}) + (\beta_{k} + \iota_{k}) \tilde{\zeta}_{k}^{2} \bar{v}_{k}^{2} - \iota_{k} \tilde{\zeta}_{k}^{2} \bar{v}_{k}^{2} \\ &- \frac{1}{\phi_{k-1}} e_{k-1} e_{k} + \frac{1}{\phi_{k}} e_{k}(\beta_{k} + \iota_{k}) \bar{v}_{k} \zeta_{k} + \tilde{\zeta}_{k} (\dot{\zeta}_{k} - \dot{\zeta}_{k}) \\ &- \frac{1}{\lambda_{k}} \tilde{\theta}_{k}^{\top} \dot{\theta}_{k} + a_{k} \end{aligned} \\ &= \frac{1}{\phi_{k}} e_{k}(e_{k+1} + \rho_{k+1}) \\ &+ \varpi_{k} \{-m_{k} \varpi_{k} - (\beta_{k} + \iota_{k})^{2} \bar{v}_{k}^{2} \varpi_{k} + \tilde{\theta}_{k}^{\top} G_{k}(\gamma_{k,a}) + \varsigma_{k} \} \\ &- \tilde{\zeta}_{k} \bar{v}_{k} \{\beta_{k} \varpi_{k} - m_{k} \varpi_{k} - (\beta_{k} + \iota_{k})^{2} \bar{v}_{k}^{2} \varpi_{k} + (\beta_{k} + \iota_{k}) \bar{v}_{k} \dot{\zeta}_{k} \\ &+ \tilde{\theta}_{k}^{\top} G_{k}(\gamma_{k,a}) + \varsigma_{k} \} - \frac{\beta_{k}}{\phi_{k}^{2}} e_{k}^{2} + (\beta_{k} + \iota_{k}) \bar{v}_{k}^{2} \zeta_{k} \zeta_{k} \\ &- \iota_{k} \tilde{\zeta}_{k}^{2} \bar{v}_{k}^{2} - \frac{1}{\phi_{k-1}} e_{k-1} e_{k} + \frac{1}{\phi_{k}} e_{k}(\beta_{k} + \iota_{k}) \bar{v}_{k} \dot{\zeta}_{k} \\ &+ \tilde{\zeta}_{k} (\dot{\zeta}_{k} - \dot{\zeta}_{k}) - \frac{1}{\lambda_{k}} \tilde{\theta}_{k}^{\top} \dot{\theta}_{k} + a_{k} \end{aligned} \tag{28}$$

where $\psi_k = G_k(\gamma_k) - G_k(\gamma_{k,a})$ and $\zeta_k = \theta_k^\top \psi_k + \epsilon_k$. Then, from $\tilde{\zeta}_k \bar{\psi}_k = \varpi_k - (e_k/\phi_k)$, we have

$$(\beta_{k} + \iota_{k})\bar{\nu}_{k}^{2}\tilde{\zeta}_{k}\zeta_{k} = (\beta_{k} + \iota_{k})\bar{\nu}_{k}\zeta_{k}\left(\varpi_{k} - \frac{e_{k}}{\phi_{k}}\right)$$

$$\leq (\beta_{k} + \iota_{k})^{2}\bar{\nu}_{k}^{2}\varpi_{k}^{2} + \frac{\bar{\zeta}_{k}^{2}}{4}$$

$$- \frac{1}{\phi_{k}}e_{k}(\beta_{k} + \iota_{k})\bar{\nu}_{k}\zeta_{k}.$$
(29)

Substituting (25), (27), and (29) into (28) yields

$$\dot{V}_{k} \leq \frac{1}{\phi_{k}} e_{k}(e_{k+1} + \rho_{k+1}) + \varpi_{k}(-m_{k}\varpi_{k} + \varsigma_{k})$$

$$-\tilde{\zeta}_{k}\bar{\nu}_{k}(\theta_{k}^{\top}G_{k}(\gamma_{k,a}) + \varsigma_{k}) - \frac{\beta_{k}}{\phi_{k}^{2}}e_{k}^{2} - \iota_{k}\tilde{\zeta}_{k}^{2}\bar{\nu}_{k}^{2}$$

$$-\frac{1}{\phi_{k-1}}e_{k-1}e_{k} + \tilde{\zeta}_{k}(\dot{\zeta}_{k} + \alpha_{k}\hat{\zeta}_{k}) + \sigma_{k}\tilde{\theta}_{k}^{\top}\hat{\theta}_{k}$$

$$+a_{k} + \frac{\tilde{\zeta}_{k}^{2}}{4}.$$
(30)

101751

Step *n*: Consider the Lyapunov function $V_n = e_n^2/(2\phi_n) + \tilde{\zeta}_n^2/2 + \tilde{\theta}_n^{\top} \tilde{\theta}_n/(2\lambda_n)$ with $\tilde{\zeta}_n = \zeta_n - \hat{\zeta}_n$ and $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$. From $\dot{e}_n = u + \xi \phi(\bar{x}_n) + h_n(\bar{x}_n) - \dot{v}_n$, the time derivative of \dot{V}_n is given by

$$\dot{V}_{n} \leq \frac{1}{\phi_{n}} e_{n} (u - \dot{\bar{v}}_{n} + \varphi_{n}) - \frac{1}{\phi_{n-1}} e_{n-1} e_{n} + \frac{1}{\phi_{n}} e_{n} (\beta_{n} + \iota_{n}) \bar{v}_{n} \zeta_{n} + \tilde{\zeta}_{n} (\dot{\zeta}_{n} - \dot{\tilde{\zeta}}_{n}) - \frac{1}{\lambda_{n}} \tilde{\theta}_{n}^{\top} \dot{\hat{\theta}}_{n} \quad (31)$$

where $\varphi_n = h_n(\bar{x}_n) + \xi \phi(\bar{x}_n) + (\bar{\phi}_{n,d}/(2\phi_n))e_n + (\phi_n/\phi_{n-1})e_{n-1} - (\beta_n + \iota_n)\bar{\nu}_n\zeta_n; \beta_n > 0$ and $\iota_n > 0$ are constants.

Due to the boundedness of ϕ_n , ϕ_{n-1} , ζ_n , and ξ , there exists a continuous function $\Psi_n(\gamma_n)$ such that

$$\frac{1}{\phi_n}e_n\varphi_n \le \frac{1}{\phi_n}e_n\Psi_n(\gamma_n) + a_n \tag{32}$$

where $a_n > 0$ is a constant and $\gamma_n = [\bar{x}_n^\top, e_{n-1}, e_n, \bar{\nu}_n, b_n]^\top$ with a constant $b_n > 0$. By employing the RBFNN $\theta_n^\top G_n(\gamma_n)$ to estimate $\Psi_n(\gamma_n)$, we have

$$\Psi_n(\gamma_n) = \theta_n^\top G_n(\gamma_n) + \epsilon_n.$$
(33)

The approximation-based adaptive event-triggered control law is presented as

$$u(t) = \bar{u}(t_l), \quad t \in [t_l, t_{l+1})$$
 (34)

$$t_{l+1} = \inf\{t > t_l | |S_u(t)| \ge m_{u,1} | \varpi_n(t)| + m_{u,2}\}$$
(35)

where for $l \in \mathbb{Z}^+$, t_l denotes the update time of the controller, $S_u = \bar{u}(t) - u(t)$ is the measurement error due to the triggering, ϖ_n is defined as $\varpi_n = x_{n,a} - \bar{v}_n - \hat{\zeta}_n \bar{v}_n$ in (9), and $m_{u,1}$ and $m_{u,2}$ are positive design constants. (34) and (35) imply that the control law u is set to $\bar{u}(t_l)$ for $t \in [t_l, t_{l+1})$, and when the triggering law (35) is satisfied, its value is updated at t_{l+1} . Here, the signal \bar{u} with the auxiliary signal ϖ_n and the injection data compensator $\hat{\zeta}_n$ is designed as follows:

$$\bar{\mu} = -(\beta_n + m_n + (\beta_n + \iota_n)^2 \bar{\nu}_n^2) \overline{\omega}_n - \hat{\theta}_n^\top G_n(\gamma_{n,a}) + \frac{\nu_n - \bar{\nu}_n}{\tau_n}$$
(36)

$$\dot{\hat{\theta}}_n = \lambda_n \varpi_n G_n(\gamma_{n,a}) - \lambda_n \sigma_n \hat{\theta}_n$$
(37)

$$\hat{\zeta}_n = \hat{\theta}_n^\top G_n(\gamma_{n,a})\bar{\nu}_n - (\beta_n - m_n - (\beta_n + \iota_n)^2 \bar{\nu}_n^2)\bar{\nu}_n \overline{\omega}_n - (\beta_n + \iota_n)\bar{\nu}_n^2 \hat{\zeta}_n - \alpha_n \hat{\zeta}_n$$
(38)

where $\overline{\omega}_n = x_{n,a} - \overline{v}_n - \hat{\zeta}_n \overline{v}_n$, $\gamma_{n,a} = [\overline{x}_{n,a}^\top, e_{n-1,a}, e_{n,a}, \overline{v}_n, b_n]^\top$; $\overline{x}_{n,a} = [x_{1,a}, \dots, x_{n,a}]^\top$, $e_{n-1,a} = x_{n-1,a} - \overline{v}_{n-1}$, and $e_{n,a} = x_{n,a} - \overline{v}_n$, $m_n > 0$ and $\alpha_n > 0$ are design parameters, and $\lambda_n > 0$ and $\sigma_n > 0$ are design constants.

Using $e_n = \phi_n(\varpi_n - \tilde{\zeta}_n \bar{\nu}_n)$ in (11), $S_u = \bar{u}(t) - u(t)$, and (36), (31) becomes

$$\begin{split} \dot{V}_n &\leq \varpi_n \{ -(\beta_n + m_n + (\beta_n + \iota_n)^2 \bar{\nu}_n^2) \varpi_n + \tilde{\theta}_n^\top G_n(\gamma_{n,a}) + \varsigma_n \} \\ &- \tilde{\zeta}_n \bar{\nu}_n \{ -(\beta_n + m_n + (\beta_n + \iota_n)^2 \bar{\nu}_n^2) \varpi_n + \tilde{\theta}_n^\top G_n(\gamma_{n,a}) \\ &+ \varsigma_n \} - \frac{\beta_n}{\phi_n^2} e_n^2 + \beta_n(\varpi_n^2 - 2\tilde{\zeta}_n \bar{\nu}_n \varpi_n) + (\beta_n + \iota_n) \tilde{\zeta}_n^2 \bar{\nu}_n^2 \end{split}$$

$$-\iota_n \tilde{\zeta}_n^2 \bar{\nu}_n^2 - \frac{1}{\phi_{n-1}} e_{n-1} e_n + \frac{1}{\phi_n} e_n (\beta_n + \iota_n) \bar{\nu}_n \zeta_n$$
$$+ \tilde{\zeta}_n (\dot{\zeta}_n - \dot{\tilde{\zeta}}_n) - \frac{1}{\lambda_n} \tilde{\theta}_n^\top \dot{\theta}_n + a_n - \frac{1}{\phi_n} e_n S_u.$$
(39)

Owing to $S_u(t_l) = 0$ for $l \in \mathbb{Z}^+$ and $\varpi_n(t) = \tilde{\zeta}_n \bar{\nu}_n + (e_n/\phi_n)$, we have

$$\frac{1}{\phi_n} e_n S_u \leq \frac{1}{\phi_n} |e_n| (m_{u,1} |\varpi_n(t)| + m_{u,2}) \\ \leq \frac{m_{u,1}}{\phi_n^2} e_n^2 + \frac{m_{u,1}}{\phi_n} |e_n| |\tilde{\zeta}_n \bar{\nu}_n| + \frac{m_{u,2}}{\phi_n} |e_n| \\ \leq \frac{2m_{u,1}}{\phi_n^2} e_n^2 + \frac{m_{u,1}}{4} \tilde{\zeta}_n^2 \bar{\nu}_n^2 + \frac{\lambda_u}{\phi_n^2} e_n^2 + \frac{m_{u,2}^2}{4\lambda_u} \quad (40)$$

where $\lambda_u > 0$ is a constant.

Using (40) and $(\beta_n + \iota_n)\tilde{\zeta}_n^2 \bar{\nu}_n^2 = (\beta_n + \iota_n)\tilde{\zeta}_n \bar{\nu}_n^2 (\zeta_n - \hat{\zeta}_n)$, we obtain

$$\begin{split} \dot{V}_{n} &\leq \varpi_{n} \{-m_{n} \varpi_{n} - (\beta_{n} + \iota_{n})^{2} \bar{v}_{n}^{2} \varpi_{n} + \tilde{\theta}_{n}^{\top} G_{n}(\gamma_{n,a}) + \varsigma_{n} \} \\ &- \tilde{\zeta}_{n} \bar{v}_{n} \{\beta_{n} \varpi_{n} - m_{n} \varpi_{n} - (\beta_{n} + \iota_{n})^{2} \bar{v}_{n}^{2} \varpi_{n} \\ &+ (\beta_{n} + \iota_{n}) \bar{v}_{n} \hat{\zeta}_{n} + \tilde{\theta}_{n}^{\top} G_{n}(\gamma_{n,a}) + \varsigma_{n} \} \\ &- (\beta_{n} - 2m_{u,1} - \lambda_{u}) \frac{e_{n}^{2}}{\phi_{n}^{2}} + (\beta_{n} + \iota_{n}) \bar{v}_{n}^{2} \tilde{\zeta}_{n} \zeta_{n} \\ &- \left(\iota_{n} - \frac{m_{u,1}}{4}\right) \tilde{\zeta}_{n}^{2} \bar{v}_{n}^{2} - \frac{1}{\phi_{n-1}} e_{n-1} e_{n} \\ &+ \frac{1}{\phi_{n}} e_{n} (\beta_{n} + \iota_{n}) \bar{v}_{n} \zeta_{n} + \tilde{\zeta}_{n} (\dot{\zeta}_{n} - \dot{\zeta}_{n}) \\ &- \frac{1}{\lambda_{n}} \tilde{\theta}_{n}^{\top} \dot{\theta}_{n} + a_{n} + \frac{m_{u,2}^{2}}{4\lambda_{u}} \end{split}$$

$$(41)$$

where $\psi_n = G_n(\gamma_n) - G_n(\gamma_{n,a})$ and $\varsigma_n = \theta_n^\top \psi_n + \epsilon_n$. Then, using $\tilde{\zeta}_n \bar{v}_n = \overline{\omega}_n - (e_n/\phi_n)$ gives

$$(\beta_n + \iota_n)\bar{\nu}_n^2 \tilde{\zeta}_n \zeta_n \le (\beta_n + \iota_n)^2 \bar{\nu}_n^2 \overline{\omega}_n^2 + \frac{\zeta_n^2}{4} - \frac{1}{\phi_n} e_n (\beta_n + \iota_n) \bar{\nu}_n \zeta_n.$$
(42)

Substituting (38), (37), and (42) into (41) yields

$$\dot{V}_{n} \leq \overline{\varpi}_{n}(-m_{n}\overline{\varpi}_{n}+\varsigma_{n})-\tilde{\zeta}_{n}\bar{\nu}_{n}(\theta_{n}^{\top}G_{n}(\gamma_{n,a})+\varsigma_{n})$$

$$-(\beta_{n}-2m_{u,1}-\lambda_{u})\frac{e_{n}^{2}}{\phi_{n}^{2}}-\left(\iota_{n}-\frac{m_{u,1}}{4}\right)\tilde{\zeta}_{n}^{2}\bar{\nu}_{n}^{2}$$

$$-\frac{1}{\phi_{n-1}}e_{n-1}e_{n}+\tilde{\zeta}_{n}(\dot{\zeta}_{n}+\alpha_{n}\hat{\zeta}_{n})$$

$$+\sigma_{n}\tilde{\theta}_{n}^{\top}\hat{\theta}_{n}+a_{n}+\frac{\bar{\zeta}_{n}^{2}}{4}+\frac{m_{u,2}^{2}}{4\lambda_{u}}.$$
(43)

Remark 4: Compared with the existing event-triggered control results [53]–[55], this paper considers the event-triggered control problem against unknown injection data in full state measurements and an actuator. Moreover, the adaptive controller (34) and its event-triggering law (35) are designed using the auxiliary signal ϖ_n with the corrupted state variable $x_{n,a}$ and the injection data compensation $\hat{\zeta}_n$.

101752

Although the corrupted state variables are used in the proposed adaptive event-triggered control scheme, the convergence of the error surfaces e_i using the exactly measured state variables x_i , the boundedness of all closed-loop signals, and the exclusion of Zeno behavior are successfully analyzed in the following section.

C. STABILITY ANALYSIS

For the stability analysis, the dynamics of the boundary layer errors (6) are considered as

$$\dot{\rho}_{k+1} = -\frac{\rho_{k+1}}{\tau_{k+1}} + \eta_{k+1}(\bar{e}_{k+1}, \rho_2, \dots, \rho_{k+1}, \bar{\hat{\theta}}_k, \bar{\hat{\zeta}}_k, \Delta_k) \quad (44)$$

where k = 1, ..., n - 1, $\bar{e}_{k+1} = [e_1, ..., e_{k+1}]^\top$, $\bar{\hat{\theta}}_k = [\hat{\theta}_1, ..., \hat{\theta}_k]^\top$, $\bar{\hat{\zeta}}_k = [\hat{\zeta}_1, ..., \hat{\zeta}_k]^\top$ with $\hat{\zeta}_1 = 0$, $\Delta_k = [\zeta_1, ..., \zeta_k, \dot{\zeta}_1, ..., \dot{\zeta}_k]^\top$, and $\eta_2 = (\beta_1 + m_1)\dot{\varpi}_1 + \dot{\zeta}_1$ $\hat{\theta}_1^\top G_1 + \hat{\theta}_1^\top (\partial G_1 / \partial \gamma_{1,a}) \dot{\gamma}_{1,a}$ and $\eta_{k+1} = (\beta_k + m_k + m_k)$ $(\beta_k + \iota_k)^2 \bar{\nu}_k^2) \dot{\varpi}_k - 2(\beta_k + \iota_k)^2 \bar{\nu}_k (\rho_k / \tau_k) \overline{\omega}_k + \hat{\theta}_k^\top G_k +$ $\hat{\theta}_{k}^{\top}(\partial G_{k}/\partial \gamma_{k,a})\dot{\gamma}_{k,a} + \dot{\rho}_{k}/\tau_{k}$ are continuous functions.

A total Lyapunov function V is selected as $V = \sum_{i=1}^{n} V_i +$ $\sum_{i=1}^{n-1} \rho_{i+1}^2 / 2.$

Theorem 1: Consider uncertain nonlinear strict-feedback systems (1) with unknown injection data in full state measurements and an actuator. The proposed adaptive event-triggered controller (34)–(38) with an event-triggering condition (35) achieves that for any initial conditions satisfying $V(0) < \mu$ with a constant $\mu > 0$,

(i) all signals of the closed-loop system are semi-globally uniformly ultimately bounded;

(ii) the error surfaces e_i , i = 1, ..., n, converge to an adjustable neighborhood of the origin;

(iii) the minimum inter-event time $t_l > 0$ satisfying $|t_{l+1} - t_l| \ge \underline{t}_l$ for $l \in \mathbb{Z}^+$ exists.

Proof: The time derivative of V using (20), (30), and (43) is

$$\dot{V} \leq \sum_{i=1}^{n} \left(-\frac{\beta_{i}}{\phi_{i}^{2}} e_{i}^{2} - m_{i} \varpi_{i}^{2} + \varpi_{i} \varsigma_{i} + \sigma_{i} \tilde{\theta}_{i}^{\top} \hat{\theta}_{i} \right) + \sum_{i=2}^{n} \left(-\tilde{\zeta}_{i} \bar{\nu}_{i} \chi_{i} - \iota_{i} \tilde{\zeta}_{i}^{2} \bar{\nu}_{i}^{2} + \tilde{\zeta}_{i} (\dot{\zeta}_{i} + \alpha_{i} \hat{\zeta}_{i}) + \frac{\tilde{\zeta}_{i}^{2}}{4} \right) + \sum_{i=1}^{n-1} \left(-\frac{\rho_{i+1}^{2}}{\tau_{i+1}} + \bar{\eta}_{i+1} \rho_{i+1} \right) + (2m_{u,1} + \lambda_{u}) \frac{e_{n}^{2}}{\phi_{n}^{2}} + \frac{m_{u,1}}{4} \tilde{\zeta}_{n}^{2} \bar{\nu}_{n}^{2} + \frac{m_{u,2}^{2}}{4\lambda_{u}} + \sum_{i=2}^{n} a_{i}$$
(45)

where $\bar{\eta}_{i+1} = \eta_{i+1} + (1/\phi_i)e_i$ and $\chi_i = \theta_i^\top G_i(\gamma_{i,a}) + \varsigma_i$. Consider the sets $A_i := \{\sum_{j=1}^i ((e_j^2/\phi_j) + (\tilde{\theta}_j^\top \tilde{\theta}_j/\lambda_j)) + (\tilde{\theta}_j^\top \tilde{\theta}_j/\lambda_j)\}$ $\sum_{j=2}^{i} (\tilde{\zeta}_{j}^{2} + \rho_{j}^{2}) \leq 2\mu$ and $B_{i} = \{\sum_{j=1}^{i} (\zeta_{j}^{2} + \zeta_{j}^{2}) \leq d_{i}\}$ where $i = 1, \ldots, n$ and $d_{i} = \sum_{j=1}^{i} (\bar{\zeta}_{j}^{2} + \bar{\zeta}_{j,d}^{2})$. A_{i} and B_{i} are compact in $\mathbb{R}^{\dim(A_{i})}$ and \mathbb{R}^{2i} , respectively. Since $A_{i} \times B_{i}$ is also compact in $\mathbb{R}^{\dim(A_i)+2i}$, $|\bar{\eta}_{i+1}| \leq q_{i+1}$ on $A_i \times B_i$ where $q_{i+1} > 0$ are constants and dim (A_i) denotes the dimension of the set A_i .

From Assumption 4 and Lemma 1, there exist constants $\bar{\zeta}_i > 0$ and $\bar{\chi}_i > 0$ such that $|\zeta_i| \leq \bar{\zeta}_i$ and $|\chi_i| \leq \bar{\chi}_i$. Using $\overline{\omega}_i \varsigma_i \le m_i \overline{\omega}_i^2 + \overline{\varsigma}_i^2 / (4m_i), -\overline{\zeta}_i \overline{\nu}_i \chi_i \le \overline{\iota}_i \overline{\zeta}_i^2 \overline{\nu}_i^2 + \overline{\chi}_i^2 / (4\overline{\iota}_i)$ with a constant $\bar{\iota}_i > 0$, $\hat{\theta}_i^{\top} \hat{\theta}_i \le -(1/2) \hat{\theta}_i^{\top} \hat{\theta}_i + (1/2) \hat{\theta}_i^{\top} \bar{\theta}_i$, $\tilde{\zeta}_i \hat{\zeta}_i \le$ $\bar{\iota}_i \tilde{\zeta}_i^2 + \bar{\zeta}_{i,d}^2 / (4\bar{\iota}_i), \, \tilde{\zeta}_i \hat{\zeta}_i \le -(1/2)\tilde{\zeta}_i^2 + (1/2)\bar{\zeta}_i^2, \, \text{and} \, \bar{\eta}_{i+1}\rho_{i+1} \le$ $\bar{\iota}_{i+1}\bar{\eta}_{i+1}^2\rho_{i+1}^2 + 1/(4\bar{\iota}_{i+1})$, and choosing $\beta_i = 2m_{u,1} + \lambda_u + \beta_i^*$ with a constant $\beta_i^* > 0$, $\iota_i = \overline{\iota}_i + m_{u,1}/4$, $\alpha_i = 2\overline{\iota}_i + \alpha_i^*$ with a constant $\alpha_i^{\dot{i}} > 0$, and $1/\tau_{i+1} = \overline{\iota}_{i+1}q_{i+1}^2 + \tau_{i+1}^*$ with a constant $\tau_{i+1}^* > 0$ give

$$\dot{V} \leq \sum_{i=1}^{n} \left(-\frac{\beta_{i}^{*}}{\phi_{i}^{2}} e_{i}^{2} - \frac{\sigma_{i}}{2} \tilde{\theta}_{i}^{\top} \tilde{\theta}_{i} \right) - \sum_{i=2}^{n} \frac{\alpha_{i}^{*}}{2} \tilde{\zeta}_{i}^{2} + c_{0} + \sum_{i=1}^{n-1} \left(-\tau_{i+1}^{*} \rho_{i+1}^{2} - \left(1 - \frac{\bar{\eta}_{i+1}^{2}}{q_{i+1}^{2}} \right) \bar{\iota}_{i+1} q_{i+1}^{2} \rho_{i+1}^{2} \right)$$
(46)

where $c_0 = \sum_{i=1}^n (\bar{\zeta}_i^2 / (4m_i) + (\sigma_i / 2)\bar{\theta}_i^\top \bar{\theta}_i) + \sum_{i=2}^n a_i + \sum_{i=2}^n (\bar{\chi}_i^2 / (4\bar{\iota}_i) + \bar{\zeta}_{i,d}^2 / (4\bar{\iota}_i) + (\alpha_i / 2)\bar{\zeta}_i^2 + \bar{\zeta}_i^2 / 4 + 1 / (4\bar{\iota}_i)) + m_{u,2}^2 / (4\lambda_u).$

Owing to $|\bar{\eta}_{i+1}| \leq q_{i+1}$ on $V = \mu$, (46) becomes $\dot{V} \leq -c_1 V + c_0$ with $0 < c_1 < \min\{2(\beta_1^*/\bar{\phi}_1), \dots,$ $2(\beta_n^*/\overline{\phi}_n), 2\tau_2^*, \ldots, 2\tau_n^*, \sigma_1\lambda_1, \ldots, \sigma_n\lambda_n, \alpha_2^*, \ldots, \alpha_n^*\}$. Thus, $\dot{V} < 0$ on $V = \mu$ when $c_1 > c_0/\mu$. This implies that $V \le \mu$ is an invariant set, i.e., if $V(0) \leq \mu$, then $V(t) \leq \mu$ for all $t \ge 0$. Integrating both sides of $\dot{V} \le -c_1 V + c_0$ with respect to time yields $V(t) \le e^{-c_1 t} V(0) + (c_0/c_1)(1 - e^{-c_1 t})$. Using $(1/\bar{\phi}_M)\sum_{i=1}^n e_i^2/2 \le V(t)$ with $\bar{\phi}_M = \max_{i=1,\dots,n} \{\bar{\phi}_i\}$, the stabilization error vector $e = [e_1, \ldots, e_n]^{\top}$ is exponentially bounded to the compact set $\Omega = \{e | \|e\| \le \sqrt{2\bar{\phi}_M c_0/c_1}\}$ where the set Ω can be adjusted arbitrarily small by c_1 . Thus, Theorems 1-(i) and 1-(ii) are ensured. This completes the proofs of Theorems 1-(i) and 1-(ii).

The existence of the minimum inter-event time t_1 satisfying $|t_{l+1} - t_l| \ge t_l$ is checked to show the exclusion of Zeno behavior. Differentiating the measurement error $S_u(t), \forall t \in [t_l, t_{l+1})$, with respect to time, it holds that $d|S_u|/dt = \operatorname{sgn}(S_u)S_u \leq |\bar{u}|$ where $\operatorname{sgn}(S_u)$ is the sign of S_u and $\dot{\bar{u}}$ is defined as

$$\dot{\bar{u}} = -(\beta_n + m_n + (\beta_n + \iota_n)^2 2\bar{\nu}_n \dot{\bar{\nu}}_n) \overline{\sigma}_n - (\beta_n + m_n + (\beta_n + \iota_n)^2 \bar{\nu}_n^2) \dot{\overline{\sigma}}_n - \dot{\bar{\theta}}_n^\top G_n - \hat{\theta}_n^\top \dot{G}_n + \frac{\dot{\nu}_n - \dot{\bar{\nu}}_n}{\tau_n}.$$
(47)

From the proofs of Theorems 1-(i) and 1-(ii), all closed-loop signals are bounded. Therefore, we obtain $|\hat{u}| \le d_u$ where d_u is a constant. Let us integrate $|\tilde{u}| \leq d_u$ during $t \in [t_l, t_{l+1})$ and use the event-triggering law (35). Then, the inter-execution times $t_{l+1} - t_l$ satisfy $|t_{l+1} - t_l| \ge (m_{u,1}|\varpi_n(t)| + m_{u,2})/\bar{d}_u \ge$ $m_{u,2}/d_u$. Thus, the minimum inter-event time is defined as $t_1 = m_{u,2}/d_u$. This completes the proof of Theorem 1-(iii).

Remark 5: From the proof of Theorem 1, the stabilization performance can be improved by reducing the compact set Ω . From this standpoint, the design parameters can be selected. The choice of the design parameters is only a sufficient condition for Theorem 1. The guidelines for the choice of the design parameters are as follows.

(i) Increasing the design parameters β_i , i = 1, ..., n helps to increasing of the convergence rate of the error surfaces e_i (i.e., increasing c_1), and thus Ω can be reduced.

(ii) As the design parameters m_i , i = 1, ..., n and $\iota_j, j = 2, ..., n$ increase, c_0 is reduced. Therefore, Ω can be reduced.

(iii) As the design parameters λ_i denoting the tuning rate increase and the design parameters σ_i for the σ -modification are fixed as small values, the tuning speed of the adaptive parameters $\hat{\theta}_i$ can be increased where i = 1, ..., n.

(iv) According to the limited resources of the network, the number of released data during the transient and steady-state responses can be adjusted by choosing the design parameters $m_{u,1}$ and $m_{u,2}$ in the event-triggering law (35).

Remark 6: The control gain functions and the eventtriggered state variables are not considered in the strictfeedback nonlinear systems (1). The problems caused by the control gain functions reported in [60] and the event-triggered state variables reported in [61] should be reformulated under the state feedback information corrupted by unknown injection data. Thus, the proposed approach cannot be applied in a straightforward manner to these problems. Future work can explore the extension of the proposed approach to complex systems in the strict-feedback form.

IV. SIMULATION RESULTS

Two simulation studies including a wing rock model with ailerons are presented to illustrate the validity of the proposed theoretical result against unknown injection data in full states and an actuator. In these simulations, the triggering law (35) is checked periodically with the sampling time 1 ms.

Example 1: Consider the following second-order nonlinear system with unknown injection data in full state measurements and an actuator:

$$\dot{x}_{1} = x_{2} + h_{1}(x_{1})$$

$$\dot{x}_{2} = u + \kappa_{a}(t, \bar{x}_{2}) + h_{2}(\bar{x}_{2})$$

$$x_{i,a} = x_{i} + \kappa_{i,s}(t, x_{i})$$
 (48)

where $i = 1, 2, h_1 = x_1^2 \sin(x_1) + x_2$, and $h_2 = x_1x_2 + x_2 \cos(x_1)$. Then, the injection data are given by $\kappa_{1,s} = (0.5 + \cos(1.2t))x_1, \kappa_{2,s} = (0.5 + \sin(1.8t))x_2$, and $\kappa_a = x_1x_2 \cos(t)$. It is assumed that the inherent nonlinearities h_1 and h_2 , and the injection data $\kappa_{1,s}, \kappa_{2,s}$, and κ_a are unknown. The initial values are set to $x_1(0) = 1.5$ and $x_2(0) = -1$ where i = 1, 2. The design parameters are selected as $\beta_1 = \beta_2 = 1, m_1 = 1, m_2 = 4, \iota_2 = 0.1, \iota_2 = 0.1, \tau_2 = 0.001, \lambda_1 = \lambda_2 = 0.02, \sigma_1 = \sigma_2 = 0.001, \alpha_2 = 0.5, m_{u,1} = 0.4$, and $m_{u,2} = 0.04$.

Fig. 1 shows the stabilization result and the event-triggered control input. Fig. 2 displays the outputs of RBFNNs and the injection data compensator. The inter-execution times and the cumulative number of events are displayed in Fig. 3. The total triggering number is 163 and thus only 1.087% of the total 15000 sampled data are used to implement the proposed adaptive controller. Thus, the control efforts can be reduced using the event-triggering laws. Although the



FIGURE 1. Control results and input for example 1 (a) x_1 and x_2 (b) u.



FIGURE 2. Outputs of RBFNNs and injection data compensator for example 1 (a) $\hat{\theta}_1^{\top} G_1$ and $\hat{\theta}_2^{\top} G_2$ (b) $\hat{\zeta}_2$.

corrupted state variables are only used in the controller and its event-triggering law, the proposed recursive event-triggered design against unknown injection data in full state measurements and an actuator achieves the stabilization of uncertain nonlinear strict-feedback systems.

Example 2: The stabilization problem of the wing rock model with ailerons in the presence of unknown injection data



FIGURE 3. Inter-execution times and the cumulative number of events for example 1 (a) inter-execution times $t_{l+1} - t_l$ (b) the cumulative number of events.



FIGURE 4. Control results and input for example 2 (a) x_i , i = 1, 2, 3 (b) u.

in full state measurements and an actuator is considered in this example. The model dynamics is described by

$$\dot{x}_1 = x_2 + h_1(x_1)$$

 $\dot{x}_2 = x_3 + h_2(\bar{x}_2)$



FIGURE 5. Outputs of RBFNNs and injection data compensators for example 2 (a) $\hat{\theta}_1^{\top} G_1$, $\hat{\theta}_2^{\top} G_2$, and $\hat{\theta}_3^{\top} G_3$ (b) $\hat{\zeta}_2$ and $\hat{\zeta}_3$.



FIGURE 6. Inter-execution times and the cumulative number of events for example 2 (a) inter-execution times $t_{l+1} - t_l$ (b) the cumulative number of events.

$$\dot{x}_{3} = u + \kappa_{a}(t, \bar{x}_{3}) + h_{3}(\bar{x}_{3})$$

$$x_{i,a} = x_{i} + \kappa_{i,s}(t, x_{i})$$
(49)

where $i = 1, 2, 3, x_1$ is the roll angle, x_2 is the roll rate, x_3 is the aileron deflection angle, $h_1 = 0$,

 $h_2 = p_1 x_1 + p_2 x_2 + p_3 |x_1| x_2 + p_4 |x_2| x_2 + p_5 x_1^3$, and $h_3 = -x_3$. Here, the model parameters of the delta wing for a 25° angle of attack are set to $p_1 = -0.01859521$, $p_2 = 0.015162375, p_3 = -0.06245153, p_4 = 0.00954708,$ and $p_5 = 0.02145291$ [62]. The unknown injection data $\kappa_{1,s} = (1 + \cos(t))x_1, \kappa_{2,s} = (1 + \sin(t))x_2, \kappa_{3,s} = (1 + \sin(t))x_3, \kappa_{3,s} = (1 + \sin(t)$ $\sin(t)x_3$, and $\kappa_a = (x_1^2)x_3 \sin(x_2t) \cos(t)$ influence the system (49). It is assumed that h_2 , h_3 , $\kappa_{i,s}$, i = 1, 2, 3, and κ_a are unknown for the adaptive event-triggered control design. In the uncontrolled system (49) with u = 0, the roll angle x_1 is divergent for the large initial condition [62]. Thus, the large initial conditions are chosen as $x_1(0) = 0.52359$ rad, $x_2(0) =$ 0.17453 rad, and $x_3(0) = 0$ rad. The design parameters for the proposed event-triggered controller are chosen as $\beta_i = 1$, $m_1 = 1, m_2 = m_3 = 5, \iota_2 = \iota_3 = 0.2, b_2 = b_3 = 0.1,$ $\tau_2 = \tau_3 = 0.001, \lambda_1 = 0.06, \lambda_2 = \lambda_3 = 0.02, \sigma_i = 0.001,$ $\alpha_2 = \alpha_3 = 0.5, m_{u,1} = 0.1, \text{ and } m_{u,2} = 0.01$ where i = 1, 2, 3.

The control result and event-triggered input are displayed in Fig. 4. Fig. 5 shows the outputs of RBFNN and the injection data compensators. Fig. 6 displays the inter-execution times and the cumulative number of events where the total triggering number is 243. Therefore, only 1.62% of the 15000 sampled data are used to achieve good stabilization performance against unknown injection data in full state measurements and an actuator, and unknown system nonlinearities.

V. CONCLUSION

An adaptive event-triggered control approach has been proposed for uncertain nonlinear strict-feedback systems with unknown injection data in full state measurements and an actuator. The approximation-based adaptive controller and its event-triggering condition have been constructed using state variables corrupted by unknown injection data in full state measurements. To this end, auxiliary signals using corrupted state variables and dynamic injection data compensators using neural networks have been designed to compensate for unknown injection data effects. It has been shown that the proposed event-triggered control scheme using corrupted state variables ensures the convergence of the error surfaces using exactly measured state variables. Further extension to the decentralized resilient event-triggered control problem of interconnected nonlinear systems with unknown injection data is conceivable. This remains as a meaningful subject for future research.

REFERENCES

- P. Antsaklis, "Goals and challenges in cyber-physical systems research editorial of the editor in chief," *IEEE Trans. Autom. Control*, vol. 59, no. 12, pp. 3117–3119, Dec. 2014.
- [2] S. Huda, S. Miah, M. Mehedi Hassan, R. Islam, J. Yearwood, M. Alrubaian, and A. Almogren, "Defending unknown attacks on cyber-physical systems by semi-supervised approach and available unlabeled data," *Inf. Sci.*, vol. 379, pp. 211–228, Feb. 2017.
- [3] G. H. Yang, J. L. Wang, and Y. C. Soh, "Reliable H_∞ controller design for linear systems," *Automatica*, vol. 37, no. 5, pp. 717–725, 2001.
- [4] B. Chen and J. Lam, "Reliable observer-based H_{∞} control of uncertain state-delayed systems," *Int. J. Syst. Sci.*, vol. 35, no. 12, pp. 707–718, 2004.

- [5] Y. Wang, D. Zhou, S. J. Qin, and H. Wang, "Active fault-tolerant control for a class of nonlinear systems with sensor faults," *Int. J. Control Autom. Syst.*, vol. 6, no. 3, pp. 339–350, 2008.
- [6] J. H. Richter, W. P. M. H. Heemels, N. van de Wouw, and J. Lunze, "Reconfigurable control of piecewise affine systems with actuator and sensor faults: Stability and tracking," *Automatica*, vol. 47, no. 4, pp. 678–691, Apr. 2011.
- [7] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, Nonlinear and Adaptive Control Design. Hoboken, NJ, USA: Wiley, 1995.
- [8] D. Swaroop, J. K. Hedrick, P. P. Yip, and J. C. Gerdes, "Dynamic surface control for a class of nonlinear systems," *IEEE Trans. Autom. Control*, vol. 45, no. 10, pp. 1893–1899, Oct. 2000.
- [9] P. P. Yip and J. K. Hedrick, "Adaptive dynamic surface control: A simplified algorithm for adaptive backstepping control of nonlinear systems," *Int. J. Control*, vol. 71, no. 5, pp. 959–979, Jan. 1998.
- [10] X.-J. Xie, Z.-J. Li, and K. Zhang, "Semi-global output feedback control for nonlinear systems with uncertain time-delay and output function," *Int. J. Robust Nonlinear Control*, vol. 27, no. 15, pp. 2549–2566, Oct. 2017.
- [11] Z. J. Li, X. J. Xie, and K. Zhang, "Output feedback stabilisation for nonlinear systems with unknown output function and control coefficients and its application," *Int. J. Control*, vol. 90, no. 5, pp. 1027–1036. 2016.
- [12] C.-C. Chen, C. Qian, Z.-Y. Sun, and Y.-W. Liang, "Global output feedback stabilization of a class of nonlinear systems with unknown measurement sensitivity," *IEEE Trans. Autom. Control*, vol. 63, no. 7, pp. 2212–2217, Jul. 2018.
- [13] D. Zhai, L. An, J. Dong, and Q. Zhang, "Output feedback adaptive sensor failure compensation for a class of parametric strict feedback systems," *Automatica*, vol. 97, pp. 48–57, Nov. 2018.
- [14] L. Zhang and G.-H. Yang, "Observer-based fuzzy adaptive sensor fault compensation for uncertain nonlinear strict-feedback systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 4, pp. 2301–2310, Aug. 2018.
- [15] D. M. Jeong, Y. H. Choi, and S. J. Yoo, "Adaptive output-feedback control of a class of nonlinear systems with unknown sensor sensitivity and its experiment for flexible-joint robots," *J. Elect. Eng. Technol.*, vol. 15, pp. 907–918, Feb. 2020.
- [16] D. Ding, Q. L. Han, Y. Xiang, X. Ge, and X. M. Zhang, "A survey on security control and attack detection for industrial cyber-physical systems," *Neurocomputing*, vol. 275, pp. 1674–1683, Jan. 2018.
- [17] V. S. Dolk, P. Tesi, C. De Persis, and W. P. M. H. Heemels, "Eventtriggered control systems under Denial-of-Service attacks," *IEEE Trans. Control Netw. Syst.*, vol. 4, no. 1, pp. 93–105, Mar. 2017.
- [18] W. He, X. Gao, W. Zhong, and F. Qian, "Secure impulsive synchronization control of multi-agent systems under deception attacks," *Inf. Sci.*, vol. 459, pp. 354–368, Aug. 2018.
- [19] D. Ding, Z. Wang, Q.-L. Han, and G. Wei, "Security control for discretetime stochastic nonlinear systems subject to deception attacks," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 48, no. 5, pp. 779–789, May 2018.
- [20] M. Zhu and S. Martinez, "On the performance analysis of resilient networked control systems under replay attacks," *IEEE Trans. Autom. Control*, vol. 59, no. 3, pp. 804–808, Mar. 2014.
- [21] A. J. Gallo, M. S. Turan, F. Boem, G. Ferrari-Trecate, and T. Parisini, "Distributed watermarking for secure control of microgrids under replay attacks," *IFAC-PapersOnLine*, vol. 51, no. 23, pp. 182–187, 2018.
- [22] T. Yucelen, W. M. Haddad, and E. M. Feron, "Adaptive control architectures for mitigating sensor attacks in cyber-physical systems," *Cyber-Phys. Syst.*, vol. 2, nos. 1–4, pp. 24–52, Oct. 2016.
- [23] X. Jin, W. M. Haddad, and T. Yucelen, "An adaptive control architecture for mitigating sensor and actuator attacks in cyber-physical systems," *IEEE Trans. Autom. Control*, vol. 62, no. 11, pp. 6058–6064, Nov. 2017.
- [24] L. An and G.-H. Yang, "Improved adaptive resilient control against sensor and actuator attacks," *Inf. Sci.*, vol. 423, pp. 145–156, Jan. 2018.
- [25] M. Krstic and M. Bement, "Nonovershooting control of strict-feedback nonlinear systems," *IEEE Trans. Autom. Control*, vol. 51, no. 12, pp. 1938–1943, Dec. 2006.
- [26] B. Yao and M. Tomizuka, "Adaptive robust control of SISO nonlinear systems in a semi-strict feedback form," *Automatica*, vol. 33, no. 5, pp. 893–900, May 1997.
- [27] S. S. Ge, F. Hong, and T. H. Lee, "Robust adaptive control of nonlinear systems with unknown time delays," *Automatica*, vol. 41, no. 7, pp. 1181–1190, 2005.
- [28] C. Wen, J. Zhou, Z. Liu, and H. Su, "Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance," *IEEE Trans. Autom. Control*, vol. 56, no. 7, pp. 1672–1678, Jul. 2011.

- [29] J. Zhou, C. Wen, and G. Yang, "Adaptive backstepping stabilization of nonlinear uncertain systems with quantized input signal," *IEEE Trans. Autom. Control*, vol. 59, no. 2, pp. 460–464, Feb. 2014.
- [30] M. Elmi, H. A. Talebi, and M. B. Menhaj, "Robust adaptive dynamic surface control of nonlinear time-varying systems in strict-feedback form," *Int. J. Control, Autom. Syst.*, vol. 17, no. 6, pp. 1432–1444, Jun. 2019.
- [31] S. Zhang, W.-Y. Cui, and F. E. Alsaadi, "Adaptive backstepping control design for uncertain non-smooth strictfeedback nonlinear systems with time-varying delays," *Int. J. Control, Autom. Syst.*, vol. 17, no. 9, pp. 2220–2233, Sep. 2019.
- [32] S. J. Yoo, J. B. Park, and Y. H. Choi, "Adaptive neural control for a class of strict-feedback nonlinear systems with state time delays," *IEEE Trans. Neural Netw.*, vol. 20, no. 7, pp. 1209–1215, Jul. 2009.
- [33] P. Li and G.-H. Yang, "A novel adaptive control approach for nonlinear strict-feedback systems using nonlinearly parameterised fuzzy approximators," *Int. J. Syst. Sci.*, vol. 42, no. 3, pp. 517–527, Mar. 2011.
- [34] J. Ma, Z. Zheng, and P. Li, "Adaptive dynamic surface control of a class of nonlinear systems with unknown direction control gains and input saturation," *IEEE Trans. Cybern.*, vol. 45, no. 4, pp. 728–741, Apr. 2015.
- [35] Y. Li, S. Tong, and T. Li, "Composite adaptive fuzzy output feedback control design for uncertain nonlinear strict-feedback systems with input saturation," *IEEE Trans. Cybern.*, vol. 45, no. 10, pp. 2299–2308, Oct. 2015.
- [36] B. Xu, Z. Shi, C. Yang, and F. Sun, "Composite neural dynamic surface control of a class of uncertain nonlinear systems in strict-feedback form," *IEEE Trans. Cybern.*, vol. 44, no. 12, pp. 2626–2634, Dec. 2014.
- [37] Y. Li, S. Tong, Y. Liu, and T. Li, "Adaptive fuzzy robust output feedback control of nonlinear systems with unknown dead zones based on a smallgain approach," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 1, pp. 164–176, Feb. 2014.
- [38] Y. Ho Choi and S. Jin Yoo, "Simple adaptive output-feedback control of non-linear strict-feedback time-delay systems," *IET Control Theory Appl.*, vol. 10, no. 1, pp. 58–66, Jan. 2016.
- [39] S. Yin, P. Shi, and H. Yang, "Adaptive fuzzy control of strict-feedback nonlinear time-delay systems with unmodeled dynamics," *IEEE Trans. Cybern.*, vol. 46, no. 8, pp. 1926–1938, Aug. 2016.
- [40] S. Jin Yoo, "Distributed consensus tracking for multiple uncertain nonlinear strict-feedback systems under a directed graph," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 4, pp. 666–672, Apr. 2013.
- [41] K. Lu, Z. Liu, G. Lai, C. L. P. Chen, and Y. Zhang, "Adaptive consensus tracking control of uncertain nonlinear multiagent systems with predefined accuracy," *IEEE Trans. Cybern.*, early access, Sep. 2, 2019, doi: 10.1109/TCYB.2019.2933436.
- [42] Q. Zhou, W. Wang, H. Liang, M. Basin, and B. Wang, "Observer-based event-triggered fuzzy adaptive bipartite containment control of multi-agent systems with input quantization," *IEEE Trans. Fuzzy Syst.*, early access, Nov. 15, 2019, doi: 10.1109/TFUZZ.2019.2953573.
- [43] H. Liang, Y. Zhang, T. Huang, and H. Ma, "Prescribed performance cooperative control for multiagent systems with input quantization," *IEEE Trans. Cybern.*, vol. 50, no. 5, pp. 1810–1819, May 2020, doi: 10.1109/TCYB.2019.2893645.
- [44] W. Wang, H. Liang, Y. Pan, and T. Li, "Prescribed performance adaptive fuzzy containment control for nonlinear multiagent systems using disturbance observer," *IEEE Trans. Cybern.*, early access, Feb. 2, 2020, doi: 10.1109/TCYB.2020.2969499.
- [45] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Trans. Autom. Control*, vol. 52, no. 9, pp. 1680–1685, Sep. 2007.
- [46] M. Mazo and P. Tabuada, "Decentralized event-triggered control over wireless Sensor/Actuator networks," *IEEE Trans. Autom. Control*, vol. 56, no. 10, pp. 2456–2461, Oct. 2011.
- [47] F. Li, L. Gao, G. Dou, and B. Zheng, "Dual-side event-triggered output feedback H_∞ control for NCS with communication delays," *Int. J. Control, Automat. Syst.*, vol. 16, no. 1, pp. 108–119, 2018.
- [48] P. Tallapragada and N. Chopra, "On event triggered tracking for nonlinear systems," *IEEE Trans. Autom. Control*, vol. 58, no. 9, pp. 2343–2348, Sep. 2013.

- [49] L. Xing, C. Wen, Z. Liu, H. Su, and J. Cai, "Event-triggered adaptive control for a class of uncertain nonlinear systems," *IEEE Trans. Autom. Control*, vol. 62, no. 4, pp. 2071–2076, Apr. 2017.
- [50] L. Xing, C. Wen, Z. Liu, H. Su, and J. Cai, "Adaptive compensation for actuator failures with event-triggered input," *Automatica*, vol. 85, pp. 129–136, Nov. 2017.
- [51] L. Xing, C. Wen, Z. Liu, H. Su, and J. Cai, "Event-triggered output feedback control for a class of uncertain nonlinear systems," *IEEE Trans. Autom. Control*, vol. 64, no. 1, pp. 290–297, Jan. 2019.
- [52] C.-H. Zhang and G.-H. Yang, "Event-triggered global finite-time control for a class of uncertain nonlinear systems," *IEEE Trans. Autom. Control*, vol. 65, no. 3, pp. 1340–1347, Mar. 2020, doi: 10.1109/TAC.2019.2928767.
- [53] H. Ma, H. Li, H. Liang, and G. Dong, "Adaptive fuzzy event-triggered control for stochastic nonlinear systems with full state constraints and actuator faults," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 11, pp. 2242–2254, Nov. 2019.
- [54] J. Qiu, K. Sun, T. Wang, and H. Gao, "Observer-based fuzzy adaptive event-triggered control for pure-feedback nonlinear systems with prescribed performance," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 11, pp. 2152–2162, Nov. 2019.
- [55] J. Wang, Z. Liu, Y. Zhang, and C. L. P. Chen, "Neural adaptive eventtriggered control for nonlinear uncertain stochastic systems with unknown hysteresis," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 11, pp. 3300–3312, Nov. 2019.
- [56] H. Liang, X. Guo, Y. Pan, and T. Huang, "Event-triggered fuzzy bipartite tracking control for network systems based on distributed reducedorder observers," *IEEE Trans. Fuzzy Syst.*, early access, Mar. 23, 2020, doi: 10.1109/TFUZZ.2020.2982618.
- [57] J. Park and I. W. Sandberg, "Universal approximation using radial-basisfunction networks," *Neural Comput.*, vol. 3, no. 2, pp. 246–257, 1991.
- [58] A. J. Kurdila, F. J. Narcowich, and J. D. Ward, "Persistency of excitation in identification using radial basis function approximants," *SIAM J. Control Optim.*, vol. 33, no. 2, pp. 625–642, Mar. 1995.
- [59] C. Wang, D. J. Hill, S. S. Ge, and G. Chen, "An ISS-modular approach for adaptive neural control of pure-feedback systems," *Automatica*, vol. 42, no. 5, pp. 723–731, May 2006.
- [60] S. Tong, X. Min, and Y. Li, "Observer-based adaptive fuzzy tracking control for strict-feedback nonlinear systems with unknown control gain functions," *IEEE Trans. Cybern.*, early access, Mar. 18, 2020, doi: 10.1109/TCYB.2020.2977175.
- [61] Y.-X. Li and G.-H. Yang, "Observer-based fuzzy adaptive event-triggered control codesign for a class of uncertain nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1589–1599, Jun. 2018.
- [62] J. M. Elzebda, A. H. Nayfeh, and D. T. Mook, "Development of an analytical model of wing rock for slender delta wings," J. Aircr., vol. 26, no. 8, pp. 737–743, 1989.



SUNG JIN YOO (Member, IEEE) received the B.S., M.S., and Ph.D. degrees in electrical and electronic engineering from Yonsei University, Seoul, South Korea, in 2003, 2005, and 2009, respectively. He was a Postdoctoral Researcher with the Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Illinois, from 2009 to 2010. Since 2011, he has been with the School of Electrical and Electronics Engineering, Chung-Ang University,

Seoul, where he is currently a Professor. His research interests include nonlinear adaptive control, decentralized control, distributed control, faulttolerant control, and neural networks theories, and their applications to robotic, flight, nonlinear time-delay systems, large-scale systems, and multiagent systems.

...