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Robust Passivity Cascade Technique-Based Control Using RBFN Approximators for the Stabilization of a Cart Inverted Pendulum

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Abstract: This paper proposes a novel passivity cascade technique (PCT)-based control for nonlinear inverted pendulum systems. Its main objective is to stabilize the pendulum's upward states despite uncertainties and exogenous disturbances. The proposed framework combines the estimation properties of radial basis function neural networks (RBFNs) with the passivity attributes of the cascade control framework. The unknown terms of the nonlinear system are estimated using an RBFN approximator. The performance of the closed-loop system is further enhanced by using the integral of angular position as a virtual state variable. The lumped uncertainties (NN—Neural Network approximation, external disturbances and parametric uncertainty) are compensated for by adding a robustifying adaptive rule-based signal to the PCT-based control. The boundedness of the states is confirmed using the passivity theorem. The performance of the proposed approach was assessed using a nonlinear inverted pendulum system under both nominal and disturbed conditions.

Keywords: passivity cascade control; RBFN approximator; nonlinear systems; inverted pendulum

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1. Introduction

Inverted pendulums have long been considered to be interesting case studies for nonlinear control design. They owe their popularity to their inherently unstable and highly nonlinear dynamics [1–3]. Additionally, they have a vast range of applicability in various practical systems such as seismographs, humanoid robots, omni-wheel robots, satellite, etc. [4,5]. Various kinds of inverted pendulum systems can be found in the literature. The cart, rotational single arm and double inverted pendulum are a few examples [6,7]. In this paper, we implement our design for the cart inverted pendulum.

The stabilization of the cart inverted pendulum is a challenging task, especially when the system is subjected to model uncertainties and unknown exogenous disturbances. Various control techniques were proposed in the literature for the control and stabilization of inverted pendulum. The approach proposed in [8] combined a linear quadratic regulator (LQR) approach with a PID controller to control an inverted pendulum system. The gains of both controllers were tuned using an ant colony optimization (ANO) technique. Two fractional order PID controllers were raised in [9] for the control of a cart inverted pendulum. In [10], a sliding mode control (SMC) method was developed to achieve balance in a cart inverted pendulum system. Similarly, a second-order SMC approach was proposed in [11] for the same system. Intelligent control methods were explored in [12,13] to ensure the stability of the inverted pendulum. Besides the control approaches discussed above, in fact, the robust control of nonlinear inverted pendulum systems with matched and unmatched disturbances, which are common to many applications, remains an open problem of high practical relevance.

Cascade control has recently emerged as a potential easy to implement approach whose main property is its hierarchical structure. With the aid of cascade control, balancing of the inverted pendulum system was realized in [14]. In addition to this, the robust behavior of the closed-loop system is obtained based on active disturbance rejection technique. In [15], a cascade fuzzy controller was proposed to control the inverted pendulum system.

All the previous schemes were developed based on the Lyapunov theorem. Recently, passivity-based control has been gaining interest as a powerful tool to achieve system stabilization by passivation of the closed-loop system. Furthermore, with the use of passive property, the process of controller design is simpler. It is worth noting that some researchers have tried to find a suitable way to turn a non-passive system into a passive one, because of the value of passivity property [16,17]. Composition of passivity property and proposed controllers was reported in the literature in order to construct a robust passivity-based controller [18–22]. However, this control technique was seldom considered for the inverted pendulum [23].

Motivated by the above discussion, we propose a new robust passivity cascade technique, (PCT)-based control for a nonlinear inverted pendulum system subject to uncertainties. The framework of the proposed method is constructed based on cascade-RBFN technique. The main contributions of this paper are as follows:

- A robust model-free control design formulated using RBFN approximators and passivity framework.
- A design that enhances the performance of the closed-loop system by augmenting its dynamics with virtual states and an adaptive robustifying signal.
- An approach that guarantees the boundedness of all the states via the output of strictly passive (OSP) property.

The remainder of the paper is organized as follows. Section 2 briefly summarizes NN approximators and the passivity theorem. The proposed controller is detailed in Section 3. Simulation results are discussed in Section 4. Some concluding remarks are finally provided in Section 5.

2. Preliminaries and Control Objectives

2.1. RBFN Approximator

In order to approximate the unknown nonlinear terms, in this study, we consider an RBFN algorithm. This latter is composed of three layers, i.e., inputs $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$, hidden nodes and one output. Consider the Gaussian function:

$$h_j(\zeta) = \exp\left(\frac{\| \zeta - c_j \|^2}{2b_j^2}\right) \tag{1}$$

where $j = 1, 2, , \ldots, n$ and $C = [c_1, \ldots, c_n]$. $h(\zeta)$ is also defined as $\left[h_j(\zeta)\right]^T$. Hence, the output of RBFN can be described as:

$$y = \theta^T h(x) + w \tag{2}$$

where $\theta = [\theta_1, \ldots, \theta_n]^T$ is derived using an adaptation mechanism.

Consider a nonlinear system in which the unknown values of g and f are estimated by a neural network system.

Lemma 1 [24]**:** *For a set Ω, on any continuous function $f(\zeta)$, there is an NN system to satisfy:*

$$\sup_{x \in \Omega} \left| f(\zeta) - \theta^T h(\zeta) \right| \leq w \ , \forall\ w > 0 \tag{3}$$

2.2. Passivity Theorem

Definition 1 [25]**:** *For the below nonlinear affine system:*

$$\begin{gathered} \dot{\zeta} = S(\zeta) + T(\zeta)u \, , \, \zeta \in R^n \\ y = R(\zeta) \end{gathered} \tag{4}$$

Passivity property is expressed as:

$$\dot{\pi} \leqslant y^T u \tag{5}$$

where π, y and u are, respectively, positive semidefinite function, the output and input vectors of the system. If there exists a storage function π that satisfies (6), the system (4) output is strictly passive (OSP).

$$\dot{\pi} \leqslant y^T u - y^T K y \tag{6}$$

where K is a symmetric matrix with all positive eigenvalues.

Definition 2 [25]**:** *The system (4) is zero-state observable (ZSO), if the solution $\zeta(t) \equiv 0$ is the only answer of equation $\dot{\zeta} = S(\zeta)$ and $R(\zeta) = 0$.*

Our control objective is to design a robust model-free controller based on the cascade technique and propose the passivity theorem framework to ensure that all closed-loop variables are bounded and the upward states are well balanced and stabilized in the presence of uncertainties.

3. Proposed PCT-Based Control Design

It is well known that the system's relative degree is one of the restrictive conditions for the passivation procedure [17,26]. Hence, in the proposed approach, we consider a cascade control technique to remove this problem.

3.1. State-Space Model of the System

In what follows, the integral of the angular joint position is added to as a virtual state to further improve the effectiveness of the system. The state-space model of the augmented system is as follows:

$$\begin{gathered} \dot{\zeta}_1 = \zeta_2 \\ \dot{\zeta}_2 = \zeta_3 \\ \dot{\zeta}_3 = f + gu + \phi \end{gathered} \tag{7}$$

where ζ_1 is the integral of angular position, ζ_2 denotes the angular position and ζ_3 is the angular velocity of the system. ϕ is regarded as external disturbances. f and g are also two nonlinear terms expressed as:

$$\begin{gathered} f = \frac{sin(\zeta_2)\left(g(M+m) - ml\zeta_3^2 cos(\zeta_2)\right)}{\frac{4}{3}l(M+m) - mlcos^2(\zeta_2)} \\ g = \frac{cos(\zeta_2)}{\frac{4}{3}l(M+m) - mlcos^2(\zeta_2)} \end{gathered} \tag{8}$$

To achieve the control objectives set forth, we propose the PCT-based control approach detailed in the next section.

3.2. The PCT-Based Control

Based on the cascade technique, the passivation procedure is achieved using three loops. Define the tracking errors as follows:

$$\begin{aligned} \nu_1 &= \zeta_1 - \zeta_{1d} \\ \nu_2 &= \zeta_2 - \dot{\zeta}_{1d} \\ \nu_3 &= \zeta_3 - \ddot{\zeta}_{1d} \end{aligned} \tag{9}$$

where the desired signal ζ_{id} is set to zero to ensure a balanced system.

Inner Loop: The first subsystem can be represented by defining $\sigma_1 = \nu_1$ based on the cascade technique, as follows:

$$\begin{aligned} \dot{\sigma}_1 &= -k_1\sigma_1 + \sigma_2 \\ y_1 &= \sigma_1 \end{aligned} \tag{10}$$

in which σ_2 is a virtual input defined by: $\sigma_2 = k_1\sigma_1 + \nu_2 = u_1$, whereas $y_1 = \sigma_1$ is a virtual output. Thus, system (10) is both OSP and ZSO via π_1:

$$\pi_1 = 0.5\,\sigma_1^2 \tag{11}$$

The time derivative of the innermost storage function yields:

$$\dot{\pi}_1 = \sigma_1\dot{\sigma}_1 = -k_1\sigma_1^2 + \sigma_1\sigma_2 = -k_1y_1^2 + y_1u_1 \tag{12}$$

where k_1 is a positive constant.

Middle Loop: Based on the cascade technique, the second step of design is represented as:

$$\begin{aligned} \dot{\sigma}_1 &= -k_1\sigma_1 + \sigma_2 \\ \dot{\sigma}_2 &= -k_2\sigma_2 + \sigma_3 \\ y_2 &= [\sigma_1\ \sigma_2] \end{aligned} \tag{13}$$

in which $\sigma_3 = \nu_3 + k_1\nu_2 + k_2\sigma_2$, $u_2 = [\sigma_2\ \sigma_3]$ and y_2 are, respectively, the virtual input and output of (13). Note that (13) is also OSP. This property is proven by derivative of below storage function with respect to time in the following

$$\pi_2 = \pi_1 + 0.5\,\sigma_2^2 \tag{14}$$

$$\dot{\pi}_2 = -k_1\sigma_1^2 - k_2\sigma_2^2 + \sigma_1\sigma_2 + \sigma_2\sigma_3 = y_2^T u_2 - y_2^T K y_2 \tag{15}$$

where $K = diag(k_1, k_2)$. Based on Definition 2, It is obvious that (13) is ZSO.

Outer Loop: In the last step of design, the entire system can be introduced as:

$$\begin{aligned} \dot{\sigma}_1 &= -k_1\sigma_1 + \sigma_2 \\ \dot{\sigma}_2 &= -k_2\sigma_2 + \sigma_3 \\ \dot{\sigma}_3 &= \left(f - \hat{f}\right) + (g - \hat{g})u + (\phi - u_r) - k_3\sigma_3 + u_p \\ y_3 &= [\sigma_1\ \sigma_2\ \sigma_3] \end{aligned} \tag{16}$$

in which u_r is the augmented robustifying signal and u_p is the passivation control input, such that:

$$\begin{aligned} u_p = \hat{f} + \hat{g}u + u_r + k_3\sigma_3 - \dddot{\zeta}_{1d} + (k_1 + k_2)\nu_3 \\ + k_1k_2\nu_2 \end{aligned} \tag{17}$$

where $u_3 = [\sigma_2\ \sigma_3\ u_p]$ is the input vector of (16). In the outer loop, NN approximators are used to estimate the unknown terms. The unknown upper bound of lumped uncertainties is considered as η^*. Then, $\widetilde{\eta}$ is defined as:

$$\widetilde{\eta} = \eta^* - \eta \tag{18}$$

The NN approximation errors for f and g are represented as:

$$\begin{aligned} f - \hat{f} &= \widetilde{P}^T \vartheta_1 + w_1 \\ g - \hat{g} &= \widetilde{\theta}^T \vartheta_2 + w_2 \end{aligned} \tag{19}$$

where $\widetilde{p} = p - \hat{p}$ (neural regressor of f), $\widetilde{\theta} = \theta - \hat{\theta}$ and w_1 and w_2 are the neural network approximation errors.

Regarding the final storage function π_3, the OSP property of (16) is verified between π_3 and u_3.

$$\pi_3 = \pi_2 + 0.5\,\sigma_3^2 + \frac{1}{2r_1}\widetilde{P}^T\widetilde{P} + \frac{1}{2r_2}\widetilde{\theta}^T\widetilde{\theta} + \frac{1}{2r_3}\widetilde{\eta}^2 \tag{20}$$

The time derivative of π_3 is given by:

$$\dot{\pi}_3 = \dot{\pi}_2 + \sigma_3\dot{\sigma}_3 + \frac{1}{r_1}\widetilde{P}^T\dot{\widetilde{P}} + \frac{1}{r_2}\widetilde{\theta}^T\dot{\widetilde{\theta}} + \frac{1}{r_3}\widetilde{\eta}\dot{\widetilde{\eta}} \tag{21}$$

Based on (16)–(21), $\dot{\pi}_3$ can be simplified as:

$$\begin{aligned} \dot{\pi}_3 \leqslant \dot{S}_2 - k_3\sigma_3^2 + \sigma_3 u_p + \widetilde{P}^T\left(\tfrac{1}{r_1}\dot{\widetilde{P}} + \sigma_3\vartheta_1\right) + \widetilde{\theta}^T\left(\tfrac{1}{r_2}\dot{\widetilde{\theta}} + \sigma_3\vartheta_2 u\right) + \sigma_3\eta - \sigma_3 u_r \\ + \widetilde{\eta}\left(\tfrac{1}{r_3}\dot{\widetilde{\eta}} + \sigma_3\right) \end{aligned} \tag{22}$$

Therefore, one can choose the adaptive rules as follows:

$$\begin{aligned} \dot{\hat{P}} &= r_1\sigma_3\vartheta_1 \\ \dot{\hat{\theta}} &= r_2\sigma_3\vartheta_2 u \\ \dot{\eta} &= r_3\sigma_3 \end{aligned} \tag{23}$$

The value of $\sigma_3\eta - \sigma_3 u_r$ should lead to the negative domain. Hence,

$$\sigma_3\eta - \sigma_3 u_r \leqslant 0 \longrightarrow \sigma_3\eta \leqslant \sigma_3 u_r \tag{24}$$

It is clear that:

$$\sigma_3\eta \leqslant \parallel \sigma_3 \parallel \eta \tag{25}$$

Thus, one can write the robustifying signal as

$$u_r = norm(\sigma_3)\eta \tag{26}$$

Finally, (22) can be represented by the following equation:

$$\dot{\pi}_3 \leqslant \dot{\pi}_2 - k_3\sigma_3^2 + \sigma u_p \tag{27}$$

Substituting (12) and (15) into (27), one can conclude that the outer loop (16) is OSP.

$$\dot{\pi}_3 \leqslant y_3^T u_3 - y_3^T K' y_3 \tag{28}$$

where $K' = diag(k_1, k_2, k_3)$. It is noteworthy to point out that (16) is ZSO due to the NN systems identifying unknown values acceptably. Moreover, the robust signal is utilized in compensation of the NN estimation errors. The block diagram of the proposed controller is shown in Figure 1.

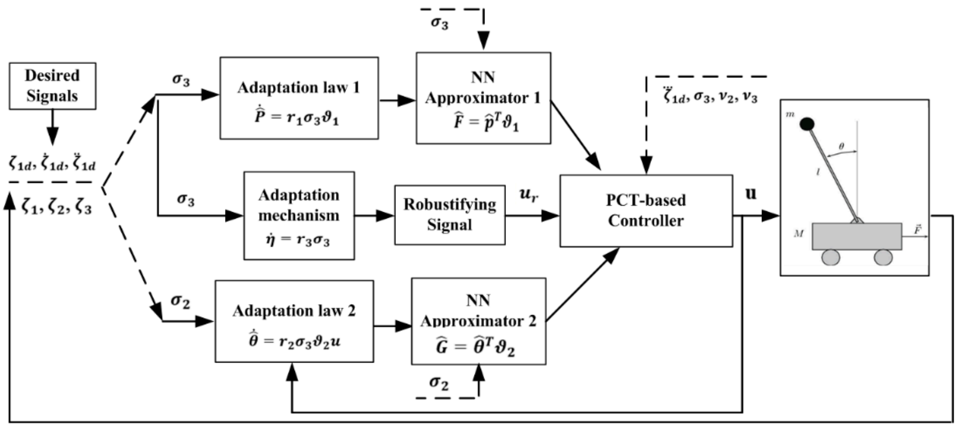

Figure 1. Schematic of the PCT-based control approach.

3.3. Stability Analysis

The stability conditions for output of strictly passive systems are illustrated using Lemma 2, as follows:

Lemma 2 [25]**:** *Consider system (4) for which the origin is assumed to be the equilibrium point. If (4) is output as strictly passive with positive definite storage function and zero-state observable (ZSO), then the origin of (4) is globally asymptotically stable for $u \equiv 0$.*

Based on Lemma 2, the control input for (7) is determined by replacing $u_p = 0$ in (17) as follows:

$$u = \hat{g}^{-1}\left(-\hat{f} + \dddot{\zeta}_{1d} - k_3\sigma_3 - (k_1 + k_2)\nu_3 - k_1k_2\nu_2 - u_r\right) \tag{29}$$

The asymptotic stability of (16) yields the convergence of σ_1, σ_2, σ_3 to zero. Hence, ν_1 converges to zero. The convergence of ν_2 to zero is also concluded, since $\nu_2 = \sigma_2 - k_1\sigma_1$. Moreover, ν_3 converges to zero, since $\nu_3 = \sigma_3 - k_1\nu_2 - k_2\sigma_2$. Then, the boundedness of ζ_1, ζ_2 and ζ_3 is also guaranteed.

Remark 1: *By defining the NN approximation errors ($w_1(\zeta) = f(\zeta) - \hat{f}(\zeta|p^*)$, $w_2(\zeta) = g(\zeta) - \hat{G}(\zeta|\theta^*)$) and also according to Lemma 1, the $|\phi + w_1 + w_2u|_{max}$ is equal to η^*, which is regarded as an optimal parameter of uncertainties.*

The upper band is unknown which was derived in (23).

4. Simulation Results

The performance of the proposed approach was assessed using the cart system inverted pendulum, the parameters of which are listed in Table 1. Two cases were considered: a normal case which assesses its performance under nomination conditions and an abnormal case which considers external disturbances (rectangular shape). In order to achieve proper tracking performance, the control parameters are fine-tuned using a trial and error approach. The obtained values are as follows:

$$\begin{array}{c} r_1 = 10,\ r_2 = 5,\ r_3 = 4 \\ k_1 = 12,\ k_2 = 9,\ k_3 = 6 \end{array} \tag{30}$$

Table 1. The inverted pendulum parameters.

Symbol	Parameter	Value
m	Mass of pendulum	0.2
M	Mass of cart	0.5
l	Half-length of pendulum	0.5
g	Gravity acceleration	9.8

4.1. Normal Case

The pendulum's tracking performance under nominal conditions is illustrated in Figure 2 through Figure 5. As shown in Figure 2, the pendulum's angle is able to perfectly track the desired position. Figures 3 and 4 show the angular velocity and phase plane, respectively. From Figure 4, it is understood that the system is stable since the trajectory tends from the initial conditions to the origin. The force (control effort) applied to the system is also depicted in Figure 5. Based on the above results, it can be concluded that the pendulum's upright states are perfectly balanced under nominal conditions.

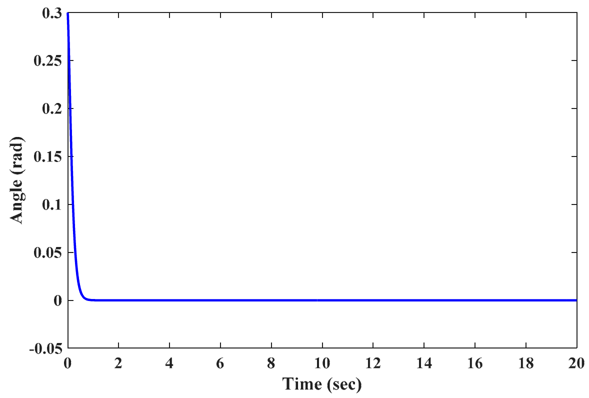

Figure 2. The pendulum angle (case 1).

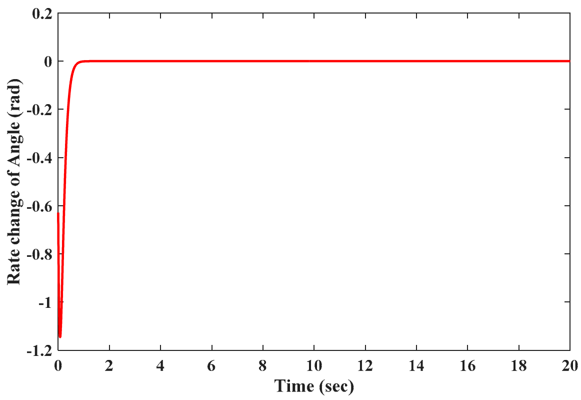

Figure 3. The angular velocity (case 1).

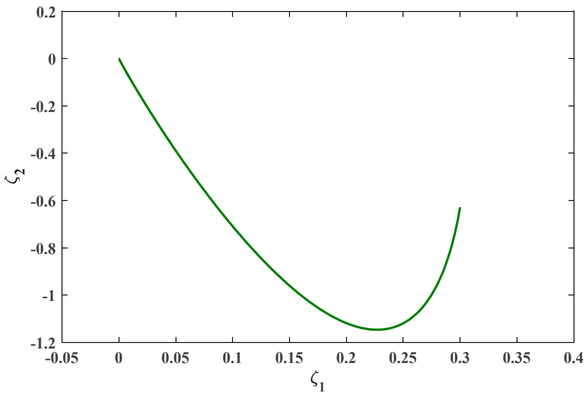

Figure 4. The phase plane trajectory (case 1).

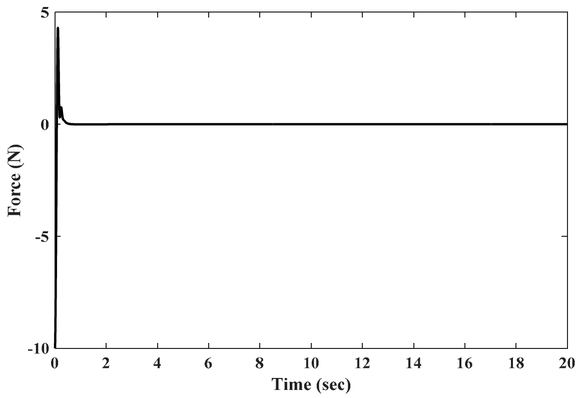

Figure 5. The control signal (case 1).

4.2. *Abnormal Case (Existence of External Disturbances)*

In this case, the system is subjected to the following external disturbance:

$$\phi(t) = \begin{cases} 10 & 2.5\text{s} < t < 7.5\text{s},\ 12.5\text{s} < t < 17.5\text{s} \\ 0 & o.w \end{cases} \tag{31}$$

The performance of the closed-loop system in this case is illustrated in Figures 6–9.

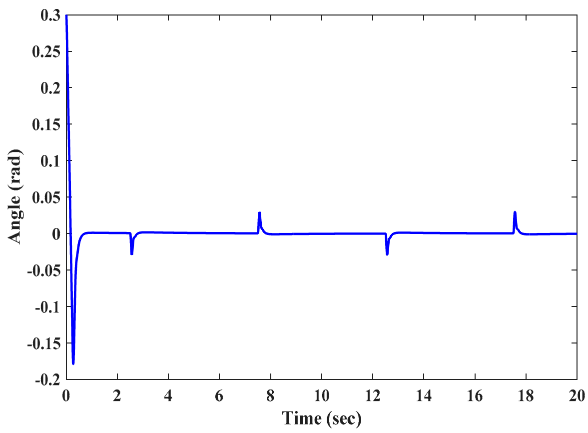

Figure 6. Pendulum angle (case 2).

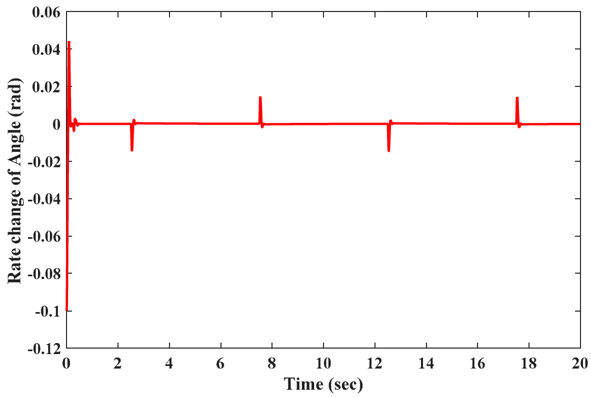

Figure 7. Angular velocity (case 2).

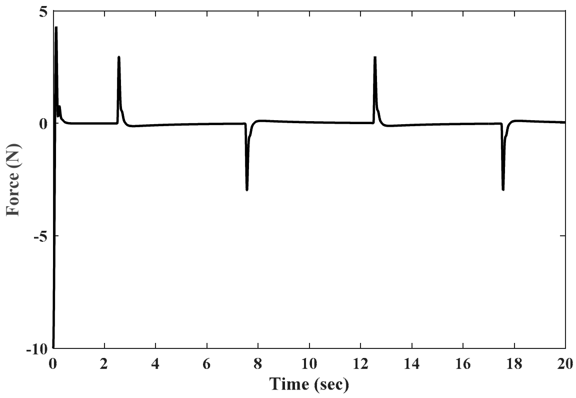

Figure 8. The control signal (case 2).

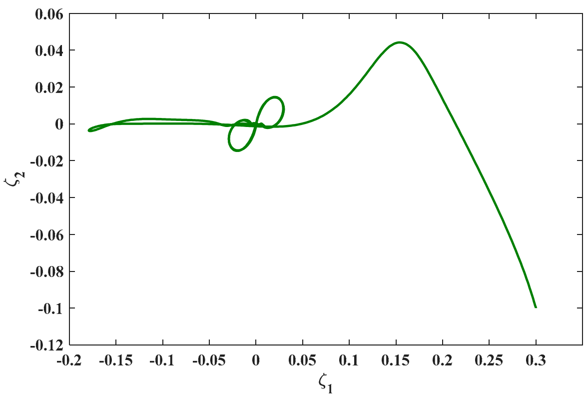

Figure 9. Phase plane trajectory (case 2).

Figures 6 and 7 show the angular position and velocity of the system, respectively. The control signal is depicted in Figure 8, where the abrupt jumps refer to the shape of external disturbance. As shown in Figure 9, the phase plane trajectory of the system moves away from the origin and moves again to the origin, which indicates that the closed-loop system is stable. Finally, it can be concluded that the system has a robust behavior in the presence of a large external disturbance.

5. Conclusions

A new passivity-based cascade neural network controller was proposed in this paper to stabilize the pendulum's upward states despite uncertainties and exogenous disturbances. The proposed approach uses RBFN approximators to estimate the system's unknown nonlinear terms. It considers the passivity framework to design a PCT-based control approach. The lumped uncertainties are compensated for by augmenting the PCT with a robustifying adaptive rule signal. The boundedness of all the states is guaranteed via the strictly passive (OSP) property output. The controller's performance was assessed using a nonlinear inverted pendulum system under both nominal and disturbed conditions. The obtained results confirmed the ability of the proposed approach to stabilize the pendulum's upward states despite uncertainties and exogenous disturbances. Robustness to disturbances, acceptable tracking performance and fast response are among the positive features of the proposed approach.

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