

# Transmission power and altitude design for energy-efficient mission completion of small-size unmanned aerial vehicle

M. Umair, J. Joung<sup>✉</sup> and Y.S. Cho

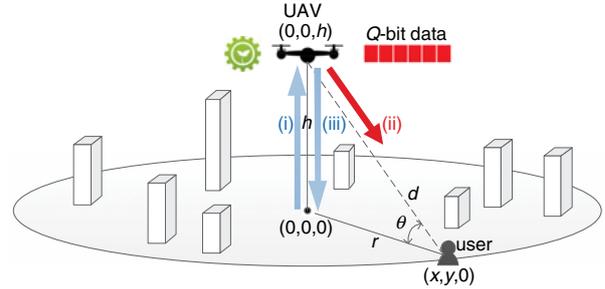
Owing to the rapid and facile deployment of unmanned aerial vehicles (UAVs), the third-generation partnership programme has recently shown considerable interest in UAVs for assisting current terrestrial base stations. Hence, the feasibility of using UAVs has been vigorously investigated for enhancing the performance of traditional ground-based communications, e.g. coverage and quality-of-service enhancement under natural disaster or overwhelming data requirement conditions. Here, however, the energy-efficient operation of the UAV is necessary to obtain the essential benefits. To strengthen the base of operation and management of UAVs in cellular systems, the authors consider in this Letter a single mission in which a small-size UAV transmits a given data volume to a cell boundary user. They design the transmit power and altitude of the UAV to complete the mission with high-energy efficiency. From the numerical results, it is verified that the UAV transmitting data with the designed transmit power at the designed altitude can complete the mission with near-optimal, i.e. maximum, energy efficiency.

**Introduction:** Recently, unmanned aerial vehicles (UAVs) have been vigorously studied as a substitute for or in support of the traditional terrestrial base stations in cellular communications [1, 2]. For example, UAVs can be deployed in various emergent cases, such as cases of failure of terrestrial base station owing to natural disasters or cases of the sharp increase of data demand in hot spot areas. Here, to cover the service area of wireless communications, either a fixed- or a rotary-wing UAV can be deployed. However, because demand can occur sporadically for short- or long-term periods and the data demands vary in time and geographically, rotary-wing UAVs, which can hover over a target area (i.e. a cell coverage area) while sustaining suitable communication links, are preferable to fixed-wing UAVs for such dynamic environments. Therefore, to enhance wireless communication performance, there have been many studies on rotary-wing UAVs. For example, the energy of a rotary-wing UAV was minimised for wireless communication using a rigorous power consumption model in a rotary-wing UAV [3]. In [4], considering cross-tier interference in space-air-ground heterogeneous networks, a two-stage joint hovering altitude and a power control solution were proposed for UAV networks. The coverage, throughput, and energy efficiency obtained by the UAV were studied for cellular [5] and vehicular networks [6]. Considering the essential limit of available on-board energy of UAVs, energy-efficient UAV operation, in such processes as charging-and-discharging, hovering, and flying, should be carefully managed. To this end, power consumption for propulsion, electronic hardware, and communication needs to be fully analysed. Here, communication-related power consumption has typically been ignored because it is relatively smaller than the power consumption for propulsion, e.g. ascending, flying, hovering, and descending. However, if the UAV size is very small, e.g. a Black Hornet PRS micro-UAV has a hovering power consumption of around 0.5 W [7], ~ 10% of total energy consumption of the UAV is used for the communications; this should be considered in the design of UAV operation.

In this Letter, we consider a small-size UAV that needs to transmit fixed and given data volume to a cell boundary user, i.e. a mission defined in this study. We then jointly design the transmit power (for communications) and altitude so that the UAV can complete the mission energy-efficiently. We first formulate the total energy consumption as a function of the transmit power and altitude of the UAV. We then design the transmit power and altitude to minimise the total energy consumption, i.e. equivalently maximising energy efficiency. Using the numerical results, the designed transmit power and altitude of UAV are verified. Furthermore, we examine the impact on energy efficiency of the UAV altitude, the fixed data volume of the mission, and the cell radius. The results obtained in this study provide an effective way for network operators to find better solutions for problems of coverage, quality of service, and operational conditions of UAVs.

**Proposed energy consumption model of rotary-wing UAV:** Fig. 1 shows the system model considered in this study, in which a UAV performs a

communication mission to transmit  $Q$ -bit data to a user who is located at cell boundary with radius  $r$ . The UAV mission completion procedure is as follows: UAV (i) ascends from cell centre with  $x$ - $y$ - $z$  coordinates  $(0, 0, 0)$  to a position  $(0, 0, h)$  with height  $h$  where the elevation angle is denoted by  $\theta$ , (ii) hovers over the cell centre while transferring  $Q$ -bit volume of data to cell boundary user at  $(x, y, 0)$ , and (iii) descends back to ground  $(0, 0, 0)$  after completing the data transmission.



**Fig. 1** Scenario of mission in urban cellular environment. UAV (i) ascends from cell centre, (ii) hovers and transmits  $Q$ -bit data to cell boundary user, and (iii) returns to cell centre

To complete the mission, the total energy consumption of the UAV is modelled as follows:

$$E_{\text{tot}} = E_{\text{asc}} + E_{\text{hov}} + E_{\text{com}} + E_{\text{des}} \quad (1)$$

$$= 2P_{\text{mov}}t_{\text{mov}} + (P_{\text{hov}} + P_{\text{com}})t_{\text{com}},$$

where  $E_{\text{asc}}$ ,  $E_{\text{hov}}$ ,  $E_{\text{com}}$ , and  $E_{\text{des}}$  are the energy consumption of UAV to ascend to height  $h$ , hover, communicate, and descend back to the ground, respectively;  $P_{\text{mov}}$ ,  $P_{\text{hov}}$ , and  $P_{\text{com}}$  are power consumptions for movement (i.e. ascending or descending), hovering, and communication, respectively; and  $t_{\text{mov}}$  and  $t_{\text{com}}$  are the time of movement and communication, respectively. Here, note that the communication time is equivalent to the hovering time as stated in the mission scenario, i.e. UAV communicates while it is hovering.

In (1),  $P_{\text{mov}}$  is obtained from the propulsion power consumption of a rotary-wing UAV flying with velocity  $v$  as follows [3]:

$$P(v) = \frac{\delta}{8} \rho s A \Omega^3 R^3 \left( 1 + \frac{3v^2}{U_{\text{tip}}^2} \right) \quad (2a)$$

$$+ (1+k) \frac{w^{3/2}}{\sqrt{2\rho A}} \left( \sqrt{1 + \frac{v^4}{4v_0^4}} - \frac{v^2}{2v_0^2} \right)^{1/2} \quad (2b)$$

$$+ \frac{1}{2} d_0 \rho s A v^3, \quad (2c)$$

where  $U_{\text{tip}}$ ,  $v_0$ ,  $d_0$ ,  $\rho$ ,  $s$ ,  $A$ ,  $k$ ,  $\Omega$ , and  $w$  are a tip speed of the rotor blade, mean rotor-induced velocity in hover state, fuselage drag ratio, air density, rotor solidity, rotor disc area, incremental correction factor to induced power, blade angular velocity in radians/s, and UAV weight in Newtons, respectively. Here, the blade profile power in (2a) and parasite power in (2c) are proportional to the UAV speed,  $v$ , while the induced power in (2b) is inversely proportional to  $v$ .

Defining the energy consumption per unit of ascending/descending distance in joules/m as in

$$\frac{P(v)}{v}, \frac{\text{Watt}}{\text{m/s}} = \frac{\text{Watt} \times \text{s}}{\text{m}} = \frac{\text{Joule}}{\text{m}}, \quad (3)$$

we assume that the UAV moves with the maximum range speed that minimises (3) as  $v^0 = \min_{v \geq 0} P(v)/v$ . The maximum range speed  $v^0$  is the optimal UAV speed that maximises the total flight distance with the given on-board energy. Accordingly, the moving time of the UAV between  $(0, 0, 0)$  and  $(0, 0, h)$  is derived as

$$t_{\text{mov}} = \frac{h}{v^0}. \quad (4)$$

From the power consumption model in (2), the hovering power consumption  $P_{\text{hov}}$  in (1) can be readily obtained as

$$P_{\text{hov}} = P(0). \quad (5)$$

On the other hand, since the transmit power for communication is a main portion of the communication power consumption, it is reasonably

assumed that  $P_{\text{com}} \simeq P_{\text{tx}}$  [8], where  $P_{\text{tx}}$  is the transmit power of the communication signals from the UAV. Therefore, the communication time for the given  $Q$  is derived as follows:

$$t_{\text{com}} = \frac{Q}{R}, \quad (6a)$$

$$R = B \log_2 \left( 1 + \frac{P_{\text{tx}} \bar{\Gamma}^{-1}}{B\sigma^2} \right), \quad (6b)$$

where  $B$  is the communication bandwidth,  $\bar{\Gamma}$  is the average path loss between UAV and user, and  $\sigma^2$  is the noise power spectral density. In (6b),  $\bar{\Gamma}$  is defined as follows [9, 10]:

$$\bar{\Gamma} = p_{\text{LoS}} \Gamma_{\text{LoS}} + p_{\text{NLoS}} \Gamma_{\text{NLoS}}, \quad (7)$$

where the probabilities of line-of-sight (LoS) and non-LoS (NLoS) are modelled as

$$p_{\text{LoS}} = \frac{1}{1 + a \exp(-b\theta - a)}, \quad (8a)$$

$$p_{\text{NLoS}} = 1 - p_{\text{LoS}}. \quad (8b)$$

In (8a),  $a$  and  $b$  are environment-dependent constants that are obtained from 3D surface fitting;  $\theta = \tan^{-1}(h/r)$  is the elevation angle (radian) between UAV and user; and the path losses are modelled as follows:

$$\Gamma_{\text{LoS}} = \frac{4\pi f d}{c} 10^{\frac{\eta_{\text{LoS}}}{20}} \quad (9a)$$

$$\Gamma_{\text{NLoS}} = \frac{4\pi f d}{c} 10^{\frac{\eta_{\text{NLoS}}}{20}} \quad (9b)$$

where  $d = \sqrt{r^2 + h^2}$  is the distance between UAV and user,  $f$  is the system frequency,  $c$  is the speed of light, and  $\eta_{\text{LoS}}$  and  $\eta_{\text{NLoS}}$  are the average excessive pass loss owing to the LoS and NLoS environment factors, respectively [9, 10].

Using (2)–(9b) in (0), the total energy consumption of the UAV to complete the mission is modelled as a function of the transmission power  $P_{\text{tx}}$  and altitude  $h$  as follows:

$$E_{\text{tot}}(P_{\text{tx}}, h) = 2P(v^0) \frac{h}{v^0} + \frac{Q(P(0) + P_{\text{tx}})}{B \log_2 \left( 1 + \frac{P_{\text{tx}} \bar{\Gamma}^{-1}(h)}{B\sigma^2} \right)}. \quad (10)$$

Here, it should be noted that  $P(v)$  is given in (2) and  $\bar{\Gamma}$  in (7) is denoted as a function of  $h$  as follows:

$$\bar{\Gamma}(h) = \left( \frac{1}{1 + a \exp(-b\theta - a)} \right) \frac{4\pi f \sqrt{r^2 + h^2}}{c} 10^{\frac{\eta_{\text{LoS}}}{20}} + \left( 1 - \frac{1}{1 + a \exp(-b\theta - a)} \right) \frac{4\pi f \sqrt{r^2 + h^2}}{c} 10^{\frac{\eta_{\text{NLoS}}}{20}}. \quad (11)$$

**Problem formulation and proposed strategy:** In this section, we propose a transmit power and altitude for the UAV for energy-efficient completion of the mission, i.e. delivering data volume  $Q$  with the lowest possible UAV energy consumption. To this end, the optimisation problem is formulated as follows:

$$\{P_{\text{tx}}^0, h^0\} = \min_{\{P_{\text{tx}}, h\}} E_{\text{tot}}(P_{\text{tx}}, h) \quad (12a)$$

$$\text{s.t. } 0 < P_{\text{tx}} \leq P_{\text{max}}, \quad (12b)$$

$$0 \leq h \leq h_{\text{max}}, \quad (12c)$$

where  $P_{\text{max}}$  and  $h_{\text{max}}$  are the maximum transmit power and altitude of the UAV, respectively, determined from regulations and hardware capability (e.g. RF circuit and weight of UAV). Here, note that, for a given volume of transmitted data, minimisation of total energy consumption is equivalent to maximisation of energy efficiency, i.e.  $Q/E_{\text{tot}}(P_{\text{tx}}, h)$ . Since the convexity of the objective function in (12a) with respect jointly to  $P_{\text{tx}}$  and  $h$  is intractable, it is difficult to directly solve (12). Instead, to efficiently solve (12), the original problem is divided into two subproblems.

The first subproblem is to optimise  $P_{\text{tx}}$  for given  $h$  as follows:

$$P_{\text{tx}}^0 = \min_{P_{\text{tx}}} E_{\text{tot}}(P_{\text{tx}}, h) \quad (13a)$$

$$\text{s.t. } 0 < P_{\text{tx}} \leq P_{\text{max}}. \quad (13b)$$

Since the objective function in (13a) is convex with respect to  $P_{\text{tx}}$ , subproblem (13) can be readily solved from the first-order optimality condition, i.e.  $(\partial E_{\text{tot}}(P_{\text{tx}}))/(\partial P_{\text{tx}}) = 0$ , as follows:

$$P_{\text{tx}}^* = \sigma^2 \bar{\Gamma}(h) \left( \frac{b - \sigma^2 \bar{\Gamma}(h)}{\sigma^2 \bar{\Gamma}(h) W_0((e^{-1}(P_{\text{hov}} - \sigma^2 \bar{\Gamma}(h)))/(\sigma^2 \bar{\Gamma}(h)))} - 1 \right), \quad (14)$$

where  $W_0(\cdot)$  is a Lambert W function (a principle branch). Considering the feasibility of  $P_{\text{tx}}^*$  for constraint (13b), the optimal transmit power  $P_{\text{tx}}^0$  is determined as

$$P_{\text{tx}}^0 = \min\{P_{\text{max}}, P_{\text{tx}}^*\}. \quad (15)$$

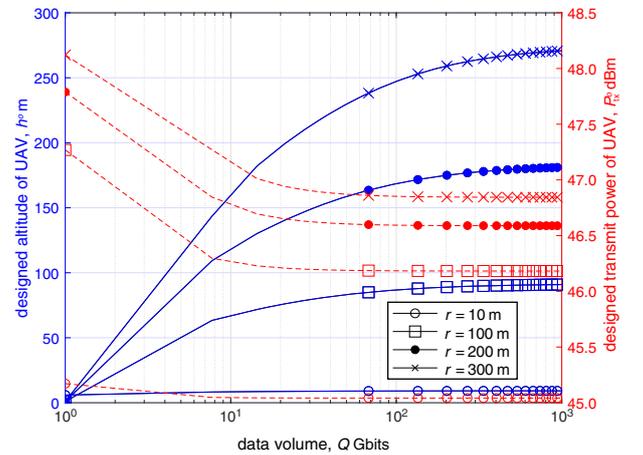
The second subproblem is to optimise  $h$  for given  $P_{\text{tx}}^0$  as follows:

$$h^0 = \min_h E_{\text{tot}}(h) \quad (16a)$$

$$\text{s.t. } 0 \leq h \leq h_{\text{max}}. \quad (16b)$$

Since the convexity of the objective function in (16a) with respect to altitude  $h$  is still intractable, the solution of (16) is obtained from a heuristic algorithm. In this study, a gradient descent method is employed to obtain a local optimal solution within the feasibility region in constraint (16b).

**Simulation results and discussion:** For the simulation, we considered a UAV cellular network whose operation frequency is  $f = 2$  GHz for downlink communications with bandwidth  $B = 10$  MHz. The noise power spectral density ( $\sigma^2$ ) is set to  $-174$  dBm/Hz. The weight ( $w$ ) of the UAV is 1 kg. The tip speed of the rotor blade ( $U_{\text{tip}}$ ), mean rotor-induced velocity in hover state ( $v_0$ ), fuselage drag ratio ( $d_0$ ), air density ( $\rho$ ), rotor solidity ( $s$ ), rotor disc area ( $A$ ), incremental correction factor to induced power ( $k$ ), and blade angular velocity in radians/s ( $\Omega$ ) are set as given in [3], i.e.  $U_{\text{tip}} = 120$  m/s,  $v_0 = 4.03$ ,  $d_0 = 0.6$ ,  $\rho = 1.225$  kg/m<sup>3</sup>,  $s = 0.05$ ,  $A = 0.503$  m<sup>2</sup>,  $k = 0.1$ , and  $\Omega = 300$  radians/s. Based on these parameters, the maximum range velocity  $v^0$  of the rotary-wing UAV is  $\sim 27$  m/s. For an urban environment, the environmental constants  $a$  and  $b$  are set at 9.61 and 0.16, respectively, and average excess losses  $\eta_{\text{LoS}}$  and  $\eta_{\text{NLoS}}$  are 1 and 20 dB, respectively [9, 10].

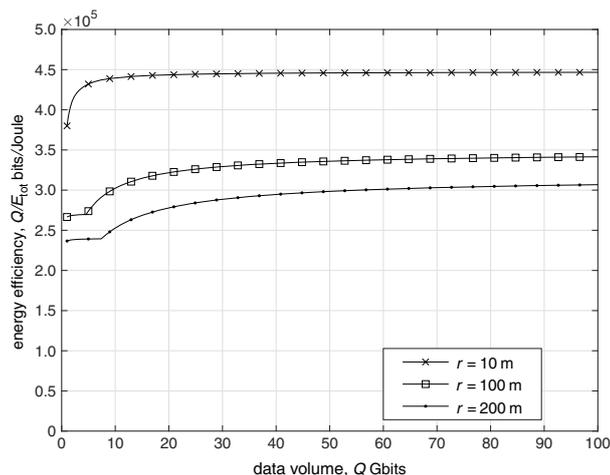


**Fig. 2** Proposed energy-efficient transmit power  $P_{\text{tx}}^0$  and altitude  $h^0$  across data volumes  $Q$  for various cell radii  $r$

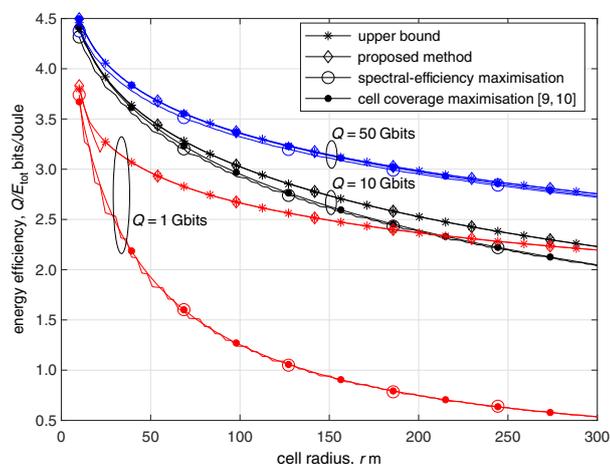
Fig. 2 shows the designed transmit power  $P_{\text{tx}}^0$  and altitude  $h^0$  across data volume  $Q$  for various cell radii  $r$ . In the results, it is observed that energy-efficient altitude  $h^0$  is proportional to data volume  $Q$ . In other words,  $h^0$  increases to support the increased  $Q$  with higher data rate  $R$ . Here, note that the average path loss in (11) decreases as  $h$  increases because  $\eta_{\text{NLoS}} \gg \eta_{\text{LoS}}$ , resulting in the increase of  $R$ . On the other hand, the UAV operates at a low altitude if  $Q$  is small because the low-rate data transmission is sufficient to complete the mission and UAV can save the energy used in ascending and descending. As a special case, it is noticeable that  $h = 0$  when  $Q = 1$  Mbits, which means that the UAV does not need to fly at all when using its maximum transmit power to complete the mission with very low data

volume. The energy-efficient altitude also depends on the cell radius  $r$ . As cell radius  $r$  increases, the higher altitude is required to sustain the relevant path loss to achieve data rate for mission completion. As expected, the transmit power  $P_{tx}$  increases as the cell radius increases. Interestingly,  $P_{tx}$  decreases up to a certain level as  $Q$  increases because compared to the case of increasing the transmit power of UAV, reducing the path loss by increasing the altitude of the UAV is more efficient to enhance  $R$  in (6b).

In Fig. 3, energy efficiency of the UAV exploiting the proposed communication power  $P_{tx}^o$  and altitude  $h^o$  was evaluated across the data volume  $Q$  for various cell radii. Here, we observed that (i) energy efficiency increases as  $Q$  increases and (ii) energy efficiency increases as the cell size decreases. From these observations, we surmise that communications using UAVs will become even more attractive when the data volume that needs to be delivered is high, and when the cell size is small.



**Fig. 3** Energy efficiency with proposed communication power  $P_{tx}$  and altitude  $h^o$  across data volume  $Q$  for various radii



**Fig. 4** Energy efficiency comparison for upper bound method, proposed method, spectral efficiency maximising method, and method in [9, 10] over cell radius  $r$  for various data volumes

In Fig. 4, energy efficiency, i.e.  $Q/E_{tot}$ , was evaluated over cell radius  $r$  for various data volumes  $Q$ . To justify our proposed method, the upper bound of the energy efficiency was numerically obtained and compared. Furthermore, two benchmarking schemes were also compared. One scheme is to maximise spectral efficiency. The other scheme uses a cell coverage maximisation strategy, in which the optimal elevation angle  $\theta$  was used to maximise coverage  $r$  by considering various environmental parameters [9, 10]. The resulting observations are the same as those shown in Fig. 3. Moreover, it was observed that

the cell coverage maximisation scheme in [9, 10] achieved the same performance as the maximum spectral efficiency scheme, but achieved maximum energy efficiency only if  $Q$  was sufficiently large. However, the proposed scheme achieved near-optimal (maximum) energy efficiency irrespective of both data volume  $Q$  and cell radius.

**Conclusion:** In this study, to transmit fixed and given data volume to a cell boundary user, energy-efficient small-size UAV operation was studied. Specifically, considering the physical parameters of the UAV and various parameters in the communication environment, energy-efficient transmit power and altitude of UAV were designed. In the numerical results, useful observations for energy efficiency were obtained and can be summarised as follows:

- The altitude of the UAV increases as data volume increases.
- If the data volume is too small, it is recommended that the UAV does not fly but rather communicate from the ground.
- The transmit power of the UAV decreases as the data volume increases.
- A smaller cell radius is preferable.
- A larger data volume is preferable.

From the observations obtained in this study, potential readers will gain insight into the energy-efficient operation of small-size UAV communications with specific data delivery missions.

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One or more of the Figures in this Letter are available in colour online.

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