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Asset allocation efficiency from dynamic and static strategies in underfunded pension funds

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Abstract

This study attempts to conduct a comparative analysis between dynamic and static asset allocation to achieve the long-term target return on asset liability management (ALM). This study conducts asset allocation using the *ex ante* expected rate of return through the outlook of future economic indicators because past economic indicators or realized rate of returns which are used as input data for expected rate of returns in the "building block" method, most adopted by domestic pension funds, does not fully reflect the future economic situation. Vector autoregression is used to estimate and forecast long-term interest rates. Furthermore, it is applied to gross domestic product and consumer price index estimation because it is widely used in financial time series data. Based on asset allocation simulations, this study derived the following insights: first, economic indicator filtering and upper-lower bound computation is needed to reduce the expected return volatility. Second, to reach the ALM goal, more stocks should be allocated than low-yielding assets. Finally, dynamic asset allocation which has been mirroring economic changes actively has a higher annual yield and risk-adjusted return than static asset allocation.

Keywords Dynamic and static asset allocation, Shortfall risk, Expected rate of return by asset class, Input data, Building block, Target rate of return, VAR model, Filtering

Paper type Research paper

1. Introduction

In any financial institution, there might be no dissenting opinion regarding that asset allocation policies are crucial. Brinson *et al.* (1986) analyze the performance of pension funds in the US between 1974 and 1983 and report that the effect of asset allocation on management can explain 93.6% of the fund's return volatility. Ibbotson and Kaplan (2000) and Hensel *et al.* (1991) report similar results. Furthermore, Jeong and Won (2005) conduct domestic research and conclude that asset allocation explains 90.48–98% of the monthly operating return. Won *et al.* (2013), as shown in Table 1, estimate that strategic asset allocation (SAA) contributes 6.04% toward a total annual average return of 6.01%, while active investment performance amounts to 3 bps. Although, the contribution will vary depending on which evaluation period is examined, as in overseas cases, asset allocation largely explains investment performance in domestic asset management. Accordingly, asset allocation, both at home and abroad, is a very important decision-making task that determines most of the portfolio's returns and risks.

Asset allocation should be implemented from the asset liability management (ALM) perspective. While duration-matching ALM is effective for insurance companies to hedge the interest rate risk of liability, asset allocation to maximize returns within a given constraint,

JEL Classification — G110, G230

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such as shortfall risk, is recommended to Korean public pension funds because most of them are underfunded and to rather compensate for this.

Thus, in this study, we attempt to conduct a comparative analysis between dynamic asset allocation, which executes asset allocation annually, and static asset allocation, which maintains a reference or normal portfolio for a long time regarding accumulated annual return and risk-adjusted return (Shape ratio) to achieve the long-term target return on ALM.

The remainder of this paper is organized as follows: in Section 2, the literature review of the efficiency between dynamic and static asset allocation methods is described, and the mean-variance optimization (MVO) model is explained. Section 3 covers the research methodology *vis-à-vis* how to estimate major economic indicators for calculating the expected rate of return for each asset class. In Section 4, we empirically analyze the comparative efficiency between the dynamic and static asset allocation methodologies based on estimates of expected returns (*ex ante* or looking-forward returns). Section 5 presents the conclusions of this study and discusses future research directions.

2. Literature review

2.1 Literature review of comparative efficiency between dynamic and static asset allocation

Asset allocation methodologies can be divided into static and dynamic asset allocation. Static asset allocation is known as a constant mix or fixed portfolio, which can be defined as "maintenance of the initial weight of asset classes by the end of investment period" and it requires regular rebalancing to maintain the initial asset allocation (see Fleten *et al.*, 2002). In investment industry practice, SAA, which is implemented every five years, or a long-term portfolio presenting the direction of SAA at the fund's 70-year finance projection [1] is generally called static asset allocation. A typical static asset allocation method in investment practice is SAA via the MVO model.

Meanwhile, Maillard (2011) describes dynamic asset allocation as "the process of continuously (at least theoretically) adjusting the portfolio structure," and it uses stochastic or mathematical programming, such as the multistage stochastic linear programming and Cholesky decomposition method. Instead of relying on such a statistical and mathematical model, this study aims to use a more practical model by conducting an in-sample empirical analysis using tactical asset allocation (TAA) based on the MVO model.

Many overseas studies have examined the performance of dynamic versus static asset allocation but few have been conducted domestically. In addition, the domestic empirical analysis has mainly focused on the analysis of out-of-sample data through the use of a multistage stochastic programming technique and only a few liquidity securities, including stocks and bonds, except alternative investments. Fleten *et al.* (2002) conduct three-stage stochastic dynamic programming and conclude that dynamic asset allocation is superior to

Classification		Rate of return	Contribution to revenue
Tactical asset allocation (TAA)	Benchmark control	0.06%	1.0%
	Proportion control	−0.24%	−3.9%
	Subtotal	−0.18%	−2.9%
Stock selection		0.13%	2.2%
Strategic asset allocation (SAA)		6.04%	100.6%
Error term		0.01%	0.1%
Total rate of return		6.01%	100.0%

Note(s): The table shows the performance of TAA and SAA for the NPS over a five-year period from 2008 to 2012
Source(s): *The data of Won *et al.* (2013) are reconstructed by the authors of this study

Table 1. Performance attribution on the NPS portfolio

static asset allocation, while also establishing that the in-sample study is more suitable than the out-of-sample procedure regarding dynamic asset allocation. Comparing the two methods based on utility functions, Maillard (2011) finds dynamic asset allocation to be moderately superior, which is significant to risk-averse investors. In addition, he insists that an asset allocation method reflecting new information is believed to have a greater impact than the transaction cost of dynamic asset allocation. Bernstein Global Management (2010) notes that in the case of static asset allocation, when volatility surges and inter-asset correlations increase suddenly during tail events, the diversification effect disappears and this method reaches its limit. Therefore, it is argued that dynamic asset allocation, which systematically detects changes in the financial market environment and finds the optimal balance between risk and return, is more efficient than static asset allocation. Furthermore, Ziemba (2003) also states that dynamic asset allocation is efficient.

However, Infanger (2008) claims that static asset allocation (fixed-mix portfolio strategy) is superior because "high-priced selling" and "low-priced buying" are practically impossible in financial markets when volatility pumping exists and sometimes decision-making can be done based on misinformation (Collomb, 2004). Tokat *et al.* (2006) state that dynamic asset allocation is effective only when investors are able to accurately identify changes in the financial market and update the expected return of the asset classes.

Cho *et al.* (2001) examine the effects of static asset allocation (MVO) and dynamic asset allocation (multistage asset allocation model: MAAM) over a 60-month period with a three-stage structure utilizing multistage stochastic linear programming. They construct the relationship between shortfalls and expected net wealth with the type of efficient frontier on the risk-return profile map and find that MAAM dominates MVO, which implies that dynamic asset allocation is more efficient than static asset allocation. Lee (2016) analyzes the efficiency of both methodologies according to economic and stock market's regime shift—an economic boom and recession phase as well as a period of the high and low stock market volatility—and conclude that dynamic asset allocation is particularly effective in high volatility markets.

Park (2017) states that even if the superiority of dynamic asset allocation is recognized, it is difficult to achieve actual results, so he withholds the assessment of superiority between the two methods. However, he argues that dynamic asset allocation complements the shortcomings of static asset allocation and increases the chances to achieve target returns if the former is operated systematically rather than fragmentarily or spontaneously. Conversely, Won *et al.* (2013) argue that SAA, as shown in Table 1, is superior to TAA upon analyzing the Korean National Pension Service (hereafter NPS)'s performance attribution over a five-year investment period.

2.2 Mean-variance optimization (MVO) model

Asset allocation generally proceeds through the following process.

① Set up investment objectives and target rate of return
→ ② Set up asset class and its benchmark return
→ ③ Conduct mid- to long-term market outlook → ④ Determine input data
→ ⑤ Set up constrains (shortfall, etc.) → ⑥ Implement asset allocation and execution
→ ⑦ Performance attribution and feedback

In the past, the standard deviation of returns or value-at-risk has been used as risk control indicators to achieve the target rate of return in asset allocation; nowadays, however, many financial institutions use shortfall risk or shortfall probability. Cho *et al.* (2001) define the operating rate of return as a probability of failing to meet the minimum target, which is used as a constraint in asset allocation. With the expected rate of return and risk by asset classes as well as the correlation between asset classes as input variables, an asset allocation approach that maximizes the portfolio's return or risk-adjusted return (Sharpe ratio) reflecting shortfall risk, investment policy, and constraints is selected as a final one.

This model is the most commonly used in the investment industry. It has the advantage of being easy to understand and convenient; however, it has the disadvantage that the weights of asset classes are highly dependent on the input variables, especially the expected return for each asset class. The efficient frontier concept was introduced by Markowitz (1952) and Tobin (1958) established its framework by adding a utility function. Samuelson (1970) completes the theoretical foundation of this model by loosening the hypothesis of Tobin's return distribution. Samuelson (1970) simplifies Tobin's hypothesis by disregarding moments and assumes that it follows a normal distribution under the preconditions that the future return of the portfolio follows a compact or small-risk distribution. According to the MVO model, the basic utility function and formula of the efficient frontier are as follows:

$$U = U[E(R_p), \quad \sigma_p], \frac{\partial U}{\partial E(Rp)} > 0, \quad \frac{\partial U}{\partial E(Rp)} < 0 \tag{1}$$

$$.Max\ E(R_p) = \left[\sum (w_i \times E(R_i)\right] \tag{2}$$

$$.\ s.t.\ \ \sigma_p = \sigma_{target}, \sum w_i = 1$$

$$\sigma_p^2 = \sum_i \sum_j \omega_i \omega_j \sigma_{ij} = \sum_i \omega_i^2 \sigma_i^2 + \sum_i \sum_j \omega_i \omega_j \sigma_{ij} (i \neq j)$$

In the absence of risk-free assets, the optimal portfolio is determined at the intersection of the efficient frontier and the indifference curve of each investor, such as a risk-seeking or risk-averse investor. Meanwhile, if a risk-free asset exists and borrowing is possible, the straight line connecting the risk-free assets to the market portfolio becomes an efficient frontier, and the optimal portfolio is located at the meeting point of the investor's utility function on this line. In this case, the efficient frontier becomes the capital market line.

For an ALM-based investment approach, the actual portfolio return must be higher than the threshold return (0%, consumer price index [CPI], etc.). When using the CPI that is common in the Korean pension fund, the shortfall risk can be summarized as follows adding a standardization process on X and CPI on the right side of Equation (3) (Lee, 2015).

$$SRCPI = P(X < CPI) = \quad P\left[\left(Z = \frac{X - \mu}{\sigma p}\right) \leq \left(\frac{CPI - E(R_p)}{\sigma p}\right)\right] \tag{3}$$

This can be simplified as shown in Equation (4) applying a 10% shortfall risk.

$$SRCPI = [P(X < CPI) \leq 10\%] = \left[P\left(Z = \frac{CPI - E(R_p)}{\sigma_p}\right) \leq 10\%\right] \tag{4}$$

$$Z_{10\%} \geq \frac{CPI - E(R_p)}{\sigma_p}$$

$$CPI - E(R_p) \leq \sigma_p \cdot Z_{10\%} \rightarrow \quad E(R_p) \geq CPI + Z_{10\%} \cdot \sigma_p \tag{5}$$

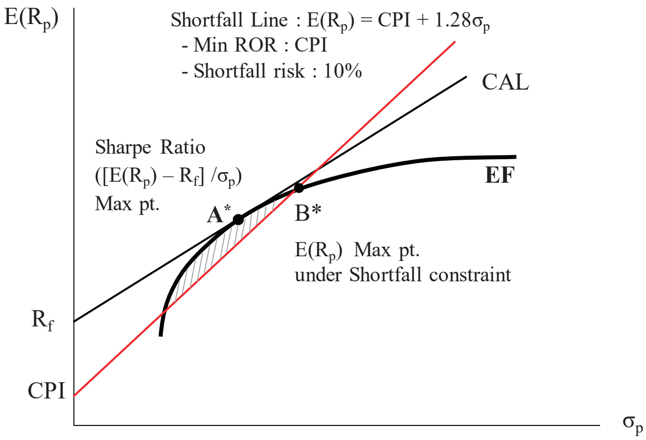
Equation (5) depicts the shortfall line with a 10% of shortfall risk, meaning that the asset allocation should be located above the shortfall line. Considering all the efficient frontiers, shortfall risks, and critical rate of returns discussed above, the result of applying the building process of an optimal portfolio into the mean-variance map is illustrated in Figure 1.

When using the critical rate of return as CPI and a 10% shortfall risk, a shortfall line is generated based on Equation (5) with a slope of 1.28 and a one-sided *Z*-value of an 80% probability. The optimal portfolio must be constructed within the investment area of the shaded portion above the shortfall line when the financial institution cannot borrow the money [2]. The optimal asset allocation, to be discussed again in Section 4, should be determined between (A), the one with the highest Sharpe ratio, and (B) that gives the maximum return under the shortfall constraint.

Extending the concept of the shortfall line based on shortfall probability and critical rate of return, the smaller the critical rate of return or the greater shortfall probability, the broader the optimal portfolio construction area that can be created, as shown in Figure 2.

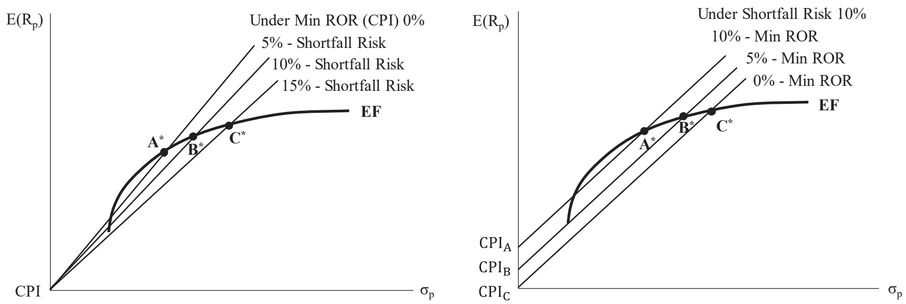
3. Research methodology

To compare the efficiency between dynamic and static asset allocation, the static (MVO) and dynamic (stochastic programming) [3] asset allocation methods introduced above may be used. Nevertheless, it is preferable to use the same MVO methodology; the annual TAA based on the MVO method is dynamic asset allocation and the long-term SAA based on the MVO model is static asset allocation rather than heterogeneous analysis methods, such as stochastic programming, to maintain the consistency of the analysis methodology. As it is difficult to find the solution at the working level via the stochastic programming method, it is not easy to apply the method in investment practice, and because the possible analysis period of the method is only 3–5 years, it is unsuitable for a long-term analysis of more than 10 years, which is used in this study.



Note(s): This figure shows the optimal asset allocation when using a 10% shortfall. The optimal asset allocation is executed at A where the Sharpe ratio is the highest or B where the expected return under shortfall constraints is the highest within the investment area of the shaded portion. Here, the slope of 1.28 is a one-sided *Z*-value with an 80% probability

Figure 1.
Optimal portfolio under shortfall constraints



Note(s): This figure shows that the range of efficient frontier that allows for building an optimal portfolio change as the shortfall risk, or allowable risk increases from 5 to 15%. The critical return, CPI, decreases from 10% to 0%

Figure 2. Optimal portfolio change by shortfall (left) and critical rate of return (right)

The most important step in asset allocation is defining the target rate of return. In the case of an underfunded Korea Government Employees Pension fund, the internal rate of return that increases the fund size to a certain level where the fund outcomes can cover the shortage between pension inflows and outflows by the end of the actuarial forecasting period (2075) is 5.48% [4]. In this study, this rate is used as the target rate of return for the asset allocation process.

Theoretically, SAA is conducted every mid-term (5 years). The transitional portfolio is set up linearly every year according to SAA, whereafter the annual TAA is undertaken within the permitted range of each asset class. Practically, however, many domestic mutual aid and public pension funds conduct SAA and TAA yearly, meaning that the actual asset allocation is the annual TAA and SAA just serves as an indication of the medium-term direction.

Originally, this paper attempts to compare dynamic asset allocation with SAA (5 years) every five years to static asset allocation with a reference or normal portfolio during the actuarial finance forecasting period (70 years). In static asset allocation, however, the assumption of holding a reference portfolio for a long time (70 years) is meaningless in terms of statistical significance because economic indicators that affect the expected returns of asset classes tend to converge to a certain figure after a while. In addition, as it is impossible to execute back-testing for 70 years in the Korean financial market, this study uses the past decade (2011–2020) just after the global financial crisis rather than the long-term period for the empirical analysis. There will be 10 times the SAA (5 years) simulations based on the MVO process using the recent decade's database. For the dynamic asset allocation performance, 10 times the SAA investment results are accumulated under the assumption that the first year's weights of asset classes in SAA are invested yearly instead of the annual TAA, which adjusts the initial weight within the permitted range of the first-year transitional SAA portfolio [5]. Meanwhile, for the SAA, two times the SAA performance are accumulated because the first SAA in 2011 and the sixth SAA in 2016 are maintained for the next five years, including the year.

Though a few pension funds refer to current market equilibrium returns or their in-house forecasts for the Black–Litterman model in setting an expected rate of return for each asset class, many funds use *ex post* data to determine input data. However, as the expected return based on *ex post* (or trailing) data does not reflect the future economic conditions, this paper intends to conduct asset allocation adopting the *ex ante* (or looking forward) expected return.

Among the basic input variables required for asset allocation to achieve the long-term target rate of return, the expected rate for each asset class is a very important factor in determining the weight of each asset class. The methods of calculating the expected rate of return are not only various but also controversial because of their suitability. Nevertheless, the majority of pension funds in Korea use the "building block" method that adds various spreads or premiums to risk-free interest rate, gross domestic product (GDP), and inflation (CPI). For example, stocks employ GDP (estimated) + CPI (estimated) + dividend yield (forecast), while foreign bonds use risk-free interest rate (estimated) + expected capital gains and losses (estimated) + foreign exchange hedge premium [6]. Therefore, major economic indicators, which are the basic elements of this method, should properly reflect economic shock situations, such as the coronavirus disease 2019 (COVID-19) pandemic and the global financial crisis as well as regime shifts to low-interest, low-growth, and low-volatility markets.

Monte Carlo simulation can be considered for long-term interest rate estimation. However, as it assumes a random distribution, the dynamic stochastic general equilibrium (DSGE) or vector autoregression (VAR) model is more commonly used. In this study, a VAR model is used to estimate major economic indicators, and the statistical significance of the model will be tested. The major economic indicators thus calculated are important for assessing the expected rate of return for each asset class in the asset allocation simulation described in Section 4.

Hwang (2013) and Ahn (2019) try to directly predict long-term or neutral interest rates utilizing the VAR model. In addition, it has been used by Kim *et al.* (2012), Kim and Han (2017) and Nam (2014) to, directly and indirectly, evaluate the relationship between interest rates and economic variables. As the VAR model is highly utilized in financial time series data, this study also uses it to make intra-sample predictions on interest rates, GDP and CPI to confirm the effectiveness of the estimation method.

Study variables include GDP and CPI growth rates, which are generally the most influential factors and have high correlations with interest rates. As the ultimate objectives of monetary policy are to maintain economic stability and growth, maintaining price stability and policy rates are the means of controlling them; GDP and CPI growth rates and interest rates are closely related (Lee, 2015). Furthermore, the impact of the US should not be overlooked due to Korea's economic environment with its small and open economy. Although China's influence is growing thanks to its membership in the WTO, the USA influence on Korea's economy remains powerful (Kim, 2015). Therefore, the variables selected include the quarterly GDP and CPI growth rates and the yield change rate of treasury bonds in the USA and Korea. Here, a 10-year maturity in the US and a 3-year maturity in Korea, which best reflect the marketability of each country, are selected as the benchmark treasury bonds. These are separated into negative (−) and positive (+) changes in interest rates considering the asymmetry of yield change [7].

The following is a brief process summary of how each variable is predicted through the variable setting above. Eight variables are included in the VAR models, including GDP, CPI, government bond yield hikes, and yield declines. The Asian currency crisis, global financial crisis, and Eurozone fiscal crisis are included in dummy variables. GDP and CPI are log-differentiated after multiplying by 100 and then converted to change rate (%). Government bond rates are calculated using %p of the simple differentiation, and they are divided into rise (+) and fall (−).

The sample size used in the initial model is 67 from the second quarter of 1995 to the fourth quarter of 2011. However, the sample period used for the first prediction in rolling regression is from the second quarter of 1995 to the fourth quarter of 2010, while unit root and cointegration test is conducted on the data. Though the order of variables is unimportant because the goal of utilizing VAR model is to predict interest rates, GDP, and CPI, the same order is determined and asymmetric effects on interest rates are applied, referring to Lee (2011) and Lee and Kim (2016).

Each variable's unit root test is executed before the VAR model is accepted. In general, time-series data are assumed to be stable, but most economic indicators are unstable. If these data are used in the analysis, there is a high risk of spurious regression; thus, it is very important to examine the stability of the time series through unit root tests. Table 2 shows the result of a unit root test for determining whether time series variables of government bond yields, GDP, and CPI in the US and Korea exhibit normality, as well as a Johansen cointegration test for level variables with unit roots.

Through the augmented Dickey–Fuller (ADF) test, the presence of unit roots is assessed based on the Schwartz information criterion. The results reveal that in the level variable, the null hypothesis that there is no unit root in both the cases of considering only the constant as well as a constant and a trend together could not be rejected. That is, in the level variables, all the time series variables are identified to be unstable with unit roots. Conversely, the unit root test with differentiated variables confirms that they are stable without a unit root, so the VAR model can be composed of only differenced variables.

In the above-mentioned unit root test, the level variables are found to have unit roots, so Johansen's cointegration test is conducted to determine whether there is a stable equilibrium relationship among the variables. The spurious regression issue can only be solved when there is no cointegration relationship. As shown in the test results at the bottom of Table 2, the null hypothesis that the "cointegral vector does not exist" is statistically significant at a 5% level. Therefore, these time series variables are not found to have a stable long-term equilibrium relationship.

As described above, a VAR(p) model composed of an autoregression process, wherein Y_t ($= [Y_{US,t}\ Y_{Kor,t}]$) consists of a multivariate time series of eight variables having time lag p, can be summarized as follows. Here, the variables included in Y_t are government bond rates, GDP, and CPI, considering the asymmetric effects of interest rates in the US and Korea.

$$Y_t = \delta + \sum_{i=1}^{p} \Phi_i Y_{t-i} + \varepsilon_t \tag{6}$$

	Variable			Test model: Constant	Test model: Constant + trend
Unit root test	US	Government bonds (10-year)	Level	−1.408	−2.849
			Differenced	−9.068***	−9.037***
		GDP	Level	−2.097	−2.555
			Differenced	−11.882***	−12.192***
		CPI	Level	−1.559	1.364
			Differenced	−10.263***	−10.433***
	South Korea	Government bonds (3-year)	Level	−2.176	−1.821
			Differenced	−7.452***	−6.231***
		GDP	Level	−1.978	−0.904
			Differenced	−6.977***	−7.216***
		CPI	Level	−2.05	0.325
			Differenced	−5.054***	−5.081***

	H0	Trend included	Trace	CV (95%)	Max-Eigen	CV (95%)
Johansen Test	$r = 0$	X	95.261	95.754	33.831	40.078
		O	110.302	117.708	35.757	44.497

Note(s): This table shows the results of the unit root and cointegration tests of each variable from the second quarter of 1995 to the fourth quarter of 2020 data. The unit root test was performed based on the augmented Dickey–Fuller (ADF) test. The null hypothesis for the ADF test is "the unit root exists in the data," and *** indicates a statistical significance level of 1%. The constant variable includes only the constant figure in the unit root test, and the "constant + trend" represents the constant and the time trend. The null hypothesis in the cointegration test is "no cointegration vector exists"

Table 2. Unit root and cointegration tests

Here, δ is the constant vector, while Φ_i is the estimated coefficient for the time lag variable of the interest rate, GDP and CPI growth rate, and interest rate described above.

$$Y_{US,t} = [TB10Y(+)_{US,t},\ TB10Y(-)_{US,t,}\ GDP_{US,t},\ CPI_{US,t}] \tag{7}$$

$$Y_{KOR,t} = [TB3Y(+)_{KOR,t},\ TB3Y(-)_{KOR,t,}\ GDP_{KOR,t},\ CPI_{KOR,t}] \tag{8}$$

According to analysis of optimal time lag, both AIC and BIC show that optimal p is "1" for the VAR (p) model.

In addition, the level variables are unstable with a unit root in the previous unit root test, while the differentiated variables are able to secure stability. The cointegration test shows that there is no long-term stable equilibrium relationship. Accordingly, the VAR model composed of only differentiated variables can be used by considering the characteristics of each variable. The results are shown in Table 3.

The statistical significance of each variable is found to be relatively low, and this is attributed to the small number of observations since the time series consists of quarterly data. As the goal of this study is to conduct a five-year asset allocation that requires yearly rolling analysis, the VAR model is used even though the statistical significance of each variable estimated by the VAR model is low.

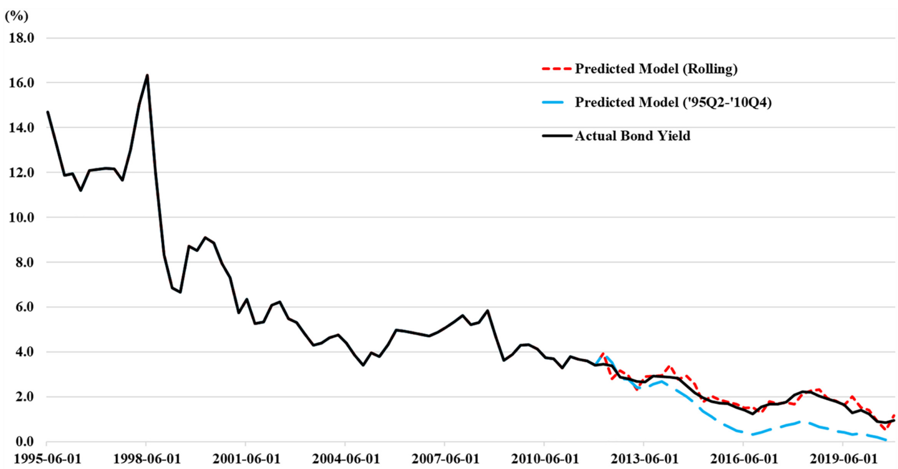
As previously mentioned, Korea is highly correlated with the US based on economic indicators given that Korea is a small and open economy. As shown in Table 3, Korea's GDP growth rate has a positive (+) relationship with US interest rates, and GDP and CPI growth rates. However, Korea's CPI growth rates are positively (+) correlated with the US interest rate when it is in an upward phase, but negatively (−) related when it is in a downward phase.

Figure 3 shows the two forecasting models for Korea's interest rates; one is rolling regression with fixed sample data and the other is rolling regression with rolling sample data. The first model estimates economic indicators yearly from 2011 to 2020 for the subsequent

Dependent variable	△KTB3Y(+)_Kor,t	△KTB3Y(−)_Kor,t	△GDP_Kor,t	△CPI_Kor,t
△TB10Y(+)_US,t−1	0.137	0.104	0.612	0.276
	0.110	0.172	0.396	0.259
△TB10Y(−)_US,t−1	0.025	0.145	0.368	−0.082
	0.081	0.126	0.290	0.190
△GDP_US,t−1	0.008	−0.016	0.004	−0.013
	0.019	0.030	0.070	0.046
△CPI_US,t−1	0.010	−0.002	0.002	−0.051
	0.044	0.068	0.156	0.102
△KB3Y(+)_Kor,t−1	0.123*	−0.195*	−0.167	0.085
	0.072	0.111	0.256	0.168
△KB3Y(−)_Kor,t−1	−0.003	0.275***	−0.287*	0.048
	0.043	0.067	0.153	0.101
△GDP_Kor,t−1	−0.009	0.013	0.146**	0.014
	0.020	0.031	0.072	0.047
△CPI_Kor,t−1	0.034	0.053	−0.136	−0.159*
	0.030	0.047	0.109	0.071
Constant term	0.101*	−0.195**	0.841***	0.698*
	0.058	0.091	0.209	0.137
R-squared	0.105	0.269	0.217	0.104

Note(s): (1) *, ** and *** indicate the 10%, 5% and 1% statistical significance levels, respectively; (2) In the interest rate parentheses, a (+) represents the phase of interest rates hike, and a (−) represents the phase of decreasing interest rates; This table illustrates the correlation coefficients between differenced variables through the VAR model and the standard deviation of the Korean economic indicators

Table 3. Estimation results for the VAR model (only Korean results reported)



Note(s): The difference between the two prediction models is 0.92% in Q3 of 2015. It began to expand, peaking at 1.7% in Q3 of 2019, and then narrowed slightly. This figure shows the comparison of the predicted value using rolling sample with the value using fixed sample against actual Korean government bond yield. The first one is observed to be closer to the actual economic figures than the second one

Figure 3.
Comparing actual interest rates with prediction model result

five years based on the fixed samples from the second quarter of 1995 to the fourth quarter of 2010. The second model, however, forecasts economic indicators through rolling regression [8], which moves samples quarterly from the third quarter of 1995 to the first quarter of 2011, and from the fourth quarter of 1995 to the second quarter of 2011, and so on.

The predicted value of rolling regression with a rolling sample, which updates recent data and can improve prediction capability, is observed to be closer to the actual economic figures than the one estimated by the fixed sample. Predicting the long-term future through a fixed sample does not properly reflect the changing economic environments afterward. Put differently, rolling regression appears to be a useful tool for predicting interest rates.

The major economic indicator figures estimated through the VAR model are used to calculate the expected return for each asset class via the "building block" method. For example, using samples from the second quarter of 1995 to the fourth quarter of 2010, the VAR model at the end of 2010 estimates each economic indicator for each year from 2011 to 2015, and then we calculate the average value of each economic indicator. Dividend yield, credit spread, and foreign exchange hedge premium are added to these economic indicator numbers to determine the expected rate of return by asset class [9]. Through these processes, the SAA for five years, from 2011 to 2015, is derived using the MVO method.

4. Empirical analysis of two asset allocation methods

The result of the average interest rate, GDP, and CPI estimations over the five-year period from 2011 to 2020 is shown in Table 4. Based on the VAR model, which passes statistical verification, the average five-year values of economic indicators, including the given year, are calculated in the table.

By appending past average spread or premium to the estimated economic indicators, the annual expected returns of different asset classes are computed over the next 5 years as presented in Table 5.

(Unit: %)

Year	Korean GDP	Korean CPI	Korean interest rate	US GDP	US CPI	US interest rate
2011	3.45	2.60	2.84	2.25	1.89	1.80
2012	1.05	0.74	2.53	0.52	0.63	1.51
2013	1.55	1.18	2.76	1.10	1.31	2.75
2014	1.39	0.62	1.68	0.87	0.99	2.17
2015	1.14	0.93	1.66	0.91	0.87	2.17
2016	2.17	1.33	1.44	1.15	1.41	2.11
2017	1.17	0.70	1.62	0.78	0.60	2.30
2018	1.43	0.99	1.58	0.98	0.94	2.58
2019	2.05	1.29	1.14	1.21	1.28	1.69
2020	0.22	0.35	2.13	−0.13	−0.02	1.14

Note(s): This table shows the average for five years of Korea and US economic indicators estimated through the VAR model in Table 3. For example, 3.45% of Korea's GDP in 2011 is the average value for 2011–2015 estimated at the end of 2010. The rest of the values are estimated in the same manner

Table 4. Estimation of basic economic variables

(Unit: %)

Year	Domestic stock	Foreign stock	Domestic bond	Foreign bond	Alternative investment
2011	6.40	6.31	3.96	5.15	5.57
2012	7.60	7.08	3.91	4.73	5.89
2013	2.97	3.94	3.36	4.36	4.36
2014	3.85	4.87	3.19	5.21	4.90
2015	3.29	4.33	2.53	3.77	4.32
2016	3.64	4.39	1.95	3.32	4.30
2017	5.24	5.06	1.97	2.99	4.70
2018	3.44	3.72	2.57	1.90	4.00
2019	4.73	4.74	2.14	2.42	4.34
2020	5.41	4.86	1.69	1.66	4.47

Note(s): This table shows the expected rate of return for each asset class determined by adding various spreads or risk premiums to economic indicators projected by the VAR model

Table 5. Estimation of expected return by asset classes

The probability of portfolio returns from domestic CPI is set at 15%, as Korean national pension and public employee pension services use a shortfall risk of 15% to implement SAA over the mid-term (5 years). To consider the stability of asset allocation, it is constrained to give more weight on bonds than stocks. The minimum and maximum weights of domestic and foreign stocks, domestic bonds, foreign stocks, and alternative investments are given as 5–25%, 10–60% and 10–30%, respectively. In particular, domestic bond weight is set higher because the Japanese Government Pension Investment Fund (GPIF) and the majority of Korean pension funds use pay-as-you-go [10] systems, which take a higher portion of the domestic bond. Additionally, there is no change in constraints during the empirical analysis period based on the comparisons between both asset allocation methodologies.

There are two methodologies in selecting optimal asset allocation strategies, which include choosing portfolios with the highest Sharpe ratio and highest return within the shortfall risk boundary. The former method has a disadvantage of high sensitivity to the weights of asset classes based on the efficient frontier slope. Bonds and alternative investments that have low volatility are often given more weight (corner solution). In this study, the latter method is applied because the size of the fund under the underfunded status needs to be quickly expanded to reach the long-term ALM.

Table 6 illustrates the optimal asset allocation plan and corresponding expected portfolio return, volatility, and probability of shortfall that reflect constraints, such as min-max allocation weights and allowed risk limits based on correlation and expected return and risk of asset classes. Rebalancing occurs yearly in static asset allocations with a total of 10 times annually, whereas it occurs every five years for dynamic asset allocations in 2011 and 2016. In the first simulation, the average expected portfolio return falls far below the long-term target rate, while the optimal asset allocation is selected based on the maximum rate of return, not on the risk-adjusted rate. Specifically, the case when the expected return reaches above the mid- and long-term target rate of 5.48% only occurs once, while above the target rate of 5% occurs twice.

The rate of returns illustrated in Table 6 is calculated as the first-year mid-term return of the SAA over the five years. Dynamic asset allocation uses the annually accumulated return, whereas static asset allocation uses returns accumulated in 2011 and 2016, under the assumption that the allocation plan continues over the next five years. By back-testing the actual return on the calculated asset allocation weights, dynamic asset allocation performs better than static asset allocation not only in annual returns but also in risk-adjusted returns as shown in Simulation I of Table 7.

(Unit: %)

Year	Domestic stock	Foreign stock	Domestic bond	Foreign bond	Alternative investment	$E(r_p)$	σ_p	SF
2011	20.3%	25.0%	30.0%	10.0%	14.7%	5.40%	6.87%	9.44%
2012	16.6%	25.0%	30.0%	10.0%	18.4%	5.76%	6.32%	9.35%
2013	5.0%	12.8%	30.0%	30.0%	30.0%	4.28%	2.61%	0.04%
2014	5.0%	11.9%	30.0%	30.0%	30.0%	4.76%	1.85%	0.00%
2015	5.0%	23.0%	30.0%	12.0%	30.0%	3.67%	2.62%	0.23%
2016	8.1%	25.0%	30.0%	10.0%	30.0%	3.60%	3.05%	1.62%
2017	25.0%	24.1%	30.0%	10.0%	10.9%	3.93%	3.98%	5.25%
2018	11.2%	18.8%	30.0%	10.0%	30.0%	3.24%	2.26%	0.32%
2019	21.9%	25.0%	30.0%	10.0%	13.1%	3.67%	3.88%	4.49%
2020	25.0%	22.7%	30.0%	10.0%	12.3%	3.68%	4.46%	9.34%

Note(s): In this table, the dynamic asset allocation plan is shown every year a total of 10 times. The static asset allocation is in 2011 and 2016 with a total of two times. The expected rate of return, standard deviation and shortfall risk are estimated figures of the asset allocation simulation for each year

Table 6. Yearly dynamic and static asset allocation in Simulation I

		Annual rate of return	Volatility	Sharpe ratio
Simulation I	Dynamic asset allocation	6.37%	4.53%	0.86
	Static asset allocation	5.71%	4.04%	0.79
Simulation II	Dynamic asset allocation	6.30%	4.54%	0.95
	Static asset allocation	6.16%	4.92%	0.85
Simulation III	Dynamic asset allocation	6.81%	6.25%	0.77
	Static asset allocation	6.64%	6.13%	0.76

Note(s): This table shows the annual rate of returns and Sharpe ratio of the two asset allocation methods. Simulation I is an asset allocation based on the original forecasting data by the VAR model, and Simulation II is carried out by adding filtering to the economic indicators, setting new upper and lower bands of economic indicators and raising the lower and upper limits of stock weights by 10%. Simulation III is conducted by increasing the lower and upper bounds of stocks and alternative investments to 20 and 40%, respectively, as well as lowering the domestic bonds to 10 and 30% in addition to filtering

Table 7. Asset allocation efficiency from dynamic and static strategies

Over a 10-year period of empirical analysis, there is no significant difference between both methods in the first-half period. However, dynamic asset allocation performs better in the second-half period because it reflects constantly changing economic and financial environments by rebalancing yearly. Furthermore, in the first year (2016) of the second-half period, static asset allocation assigns only 8.1% to domestic Korean stock, which has a high realized rate of return and carries the weight over the five years, whereas dynamic asset allocation actively rebalances the weight up to 25%. Comparing the cumulative effect between the two methods, Figure 4 shows similar results that dynamic asset allocation is superior to static asset allocation in the second half.

Several issues arise when economic indicators computed from the VAR model are directly applied to derive expected returns by asset classes. While the five-year average GDP and CPI are used to derive the expected return, changing economic conditions results in the over- or under-estimation problems, leading to a larger expected return volatility. Moreover, as stocks are given minimum weights during the period when GDP or CPI forecasts are low, expected portfolio returns reach far below the long-term target rate of 5.48% even if the realized stock return is high. Put differently, an adequate regulation on time series such as GDP and CPI is necessary to reduce the volatility of forecasts and estimations, especially when there is an external shock such as the global financial crisis and COVID-19 pandemic. Furthermore, as the mid-term trend of economic indicators is crucial in asset allocation, filtering methods can be considered. Hence, in this study, we apply a band-pass filter [11] before estimating economic indicators. The filtering generates a smoothing effect on the expected return volatility and reduces a sudden change in allocation weights.

The second simulation adds the filtering method and sets the lower and upper boundaries to the existing economic indicator estimations to smooth the expected return volatility of each asset class. The boundaries of GDP, CPI, and interest rate are set at 1.5–5%, 1–3%, and "current interest rate ±1%," respectively [12]. Additionally, the lower and upper stock weight boundaries are increased by 10% because the case when the expected portfolio return is above the long-term target rate of 5.48% occurs only once.

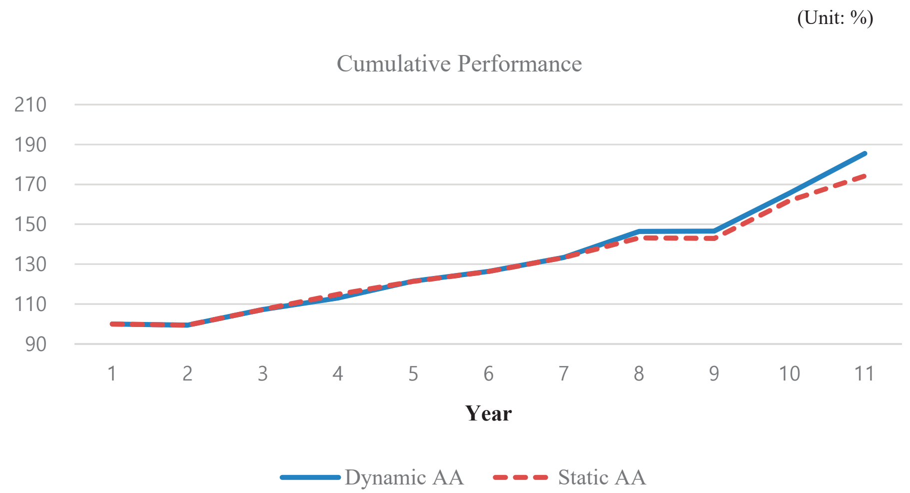


Figure 4.
Simulation I: Performance of dynamic vs. static asset allocation

Note(s): This figure compares the cumulative effect between the two asset allocation methods over a ten-year period. Simulation I is an asset allocation based on the original data estimates derived from the VAR model

As a result of asset allocation under these conditions, there are only two observed cases of expected portfolio return above the mid- and long-term rate of return of 5.48% and six cases for the expected portfolio return above 5%, resulting in the average expected return over 10 years at 5%, which is still below the long-term target rate. The result can be explained by the initial setting of lower minimum and maximum weights on stock and higher weights on domestic bonds for stable asset management through regulation on asset return volatility. For instance, in 2013, while the expected return of the domestic bond is 3.48%, a minimum weight of 30% is forced to be allocated on domestic bond. Consequently, while the expected returns of domestic and foreign stock are 6% and 6.17%, respectively, with a small difference, because the domestic stock is assigned a minimum weight of 15% only, the overall expected portfolio return becomes 5.07%.

Notwithstanding the regulation of asset allocation weights, the cumulative realized return over the back-testing period as in Simulation II of Table 7 exceeds the target rate for both dynamic and static asset allocations due to recent domestic and foreign stock market boom. Moreover, in terms of the annual and risk-adjusted return, static asset allocation shows a higher performance than dynamic asset allocation.

Meanwhile, dynamic asset allocation performs better in the first five years, while static asset allocation performs better in the second five. This is because, in the first half, static asset allocation maintains a minimum value of 15%, while dynamic asset allocation actively increases the proportion of foreign stocks close to 35%. In the second half, static asset allocation maintains the share of foreign stocks at 35%, while dynamic asset allocation adjusts its weight down to 25% over time.

The third simulation adjusts the minimum and maximum weights of asset classes in a direction that increases the weight of risky assets and decreases the weight of riskless assets. The minimum and maximum allocation weights of stocks and alternative investment increase to 20–40%, and domestic bond decrease to 10–30%. The purpose of changing weights is to increase the chance of expected portfolio return to be the higher long-term target rate such that the 10-year average expected return would exceed the long-term target rate. Furthermore, to set the lower and upper bounds, the quarterly window sliding [13] method over the past five years is used.

In Simulation III, there are four cases of expected portfolio return exceeding the long-term target return of 5.48% and seven cases exceeding 5%. The average expected return is 5.53% over the 10 cases, which exceeds the long-term target return and demonstrates an improvement compared with the first two simulations.

As shown in Simulation III of Table 7, dynamic asset allocation performs relatively well in terms of annual return and Sharpe ratio. While dynamic asset allocations outperform static asset allocation in the first five years, the latter surpasses it in the second five years. This is because, in the first half, static asset allocation maintains a minimum value of 20% for domestic and foreign stocks, while dynamic asset allocation increases overseas stock up to 40% actively, which ensues high realized returns. However, in the second half, static asset allocation continues to maintain a maximum value of 40% of the overseas stock, while dynamic asset allocation gradually reduces the proportion of overseas stock to the minimum value of 20%, which also realizes a high return.

As Simulations I–III are short periods of back-testing, the performance of both asset allocation methods differs depending on the stock market phase between the first and second half periods. In Simulation I, dynamic asset allocation is superior to static asset allocation in the second half, but in Simulations II and III, dynamic asset allocation outperforms static asset allocation in the first half (Figures 5 and 6, and Appendix 3). However, if a 30-year long-term reference portfolio is constructed, such as the Japanese GPIF or the Canadian CPP, the performance of dynamic asset allocation actively responding to market conditions might inevitably be much higher than static asset allocation.

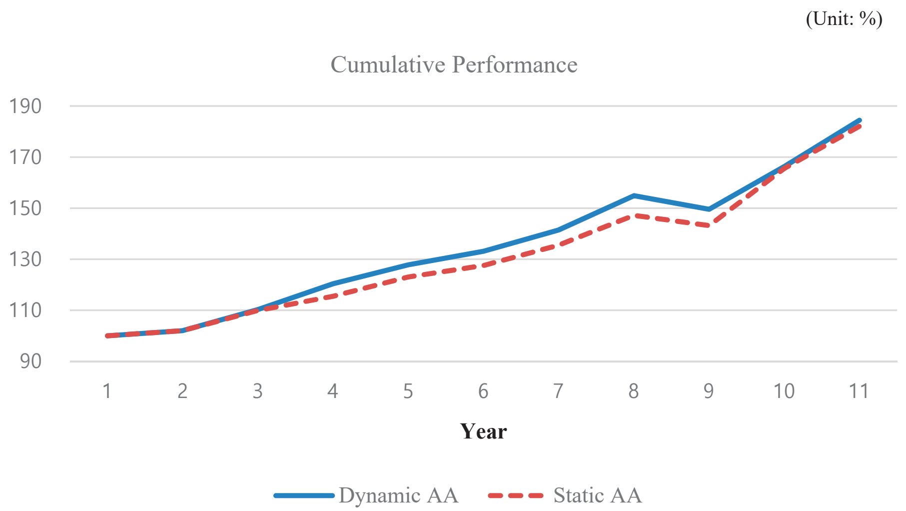


Figure 5.
Simulation II:
Performance of
dynamic vs. static
asset allocation

Note(s): This figure compares the cumulative performance between the two asset allocation methods over a ten-year period. Simulation II is conducted by adding filtering to the estimation of economic indicators, setting new upper and lower limits of economic indicators, and raising both the minimum and maximum ratio of stocks by 10%

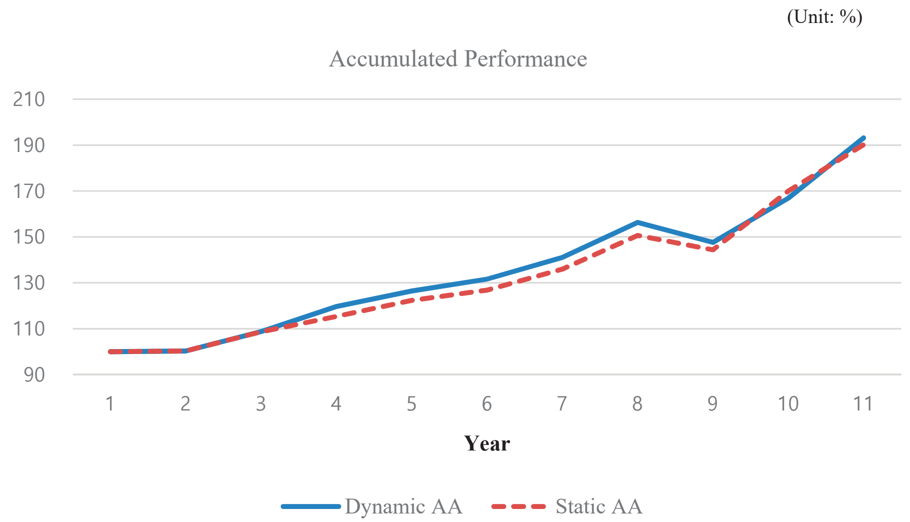


Figure 6.
Simulation III:
Performance of
dynamic vs. static
asset allocation

Note(s): This figure compares the cumulative performance between the two asset allocation methods over a ten-year period. Simulation III is an asset allocation conducted by increasing the minimum and maximum ratio of domestic/foreign stocks and alternative investments to 20% and 40%, respectively, and lowering the proportion of domestic bonds to 10% and 30%, respectively

5. Conclusion

The MVO model is a commonly used practical asset allocation method that conducts asset allocation by using the expected return and risk of each asset and correlation among different asset classes. The most important factor in this process is setting the expected rate of return for each asset class, but there is much controversy over its suitability. Alternative investments, in particular, suffer from a lack of appropriate integrated benchmarks in addition to the difficulty of calculating the expected return.

Most domestic pension funds adopt the "building block" method, which adds spreads or premiums to the risk-free, GDP growth, or inflation rates for setting expected returns; thus, GDP, CPI, and risk-free rates are used as their basic variables. Generally, past economic indicators or realized rates of returns, referred to as *ex post* or trailing, are used as input data for the expected rate of returns. This, however, does not fully reflect the future economic situation; therefore, we attempt to conduct asset allocation using the *ex ante* (or looking forward) expected rate of return through the outlook of future economic indicators. Estimating major economic indicators should reflect economic shocks, such as the COVID-19 pandemic and the global financial crisis, as well as regime shifts to low-interest, low-growth, and low-volatility markets. VAR is used to estimate and forecast long-term interest rates in this study and is also applied to GDP and CPI estimation because it is widely used in financial time series data.

The application of the basic economic indicators calculated by a VAR model to determine the expected rate of return for each asset class leads to several problems. When volatility in time series such as GDP and CPI increases due to external shocks, the figures for a specific year are over- or under-estimated, even when using the five-year average for GDP and CPI to calculate the expected return. Thus, the expected return of asset classes also becomes volatile. To overcome these problems and derive long-term trends, filtering is introduced. This smooths out the volatility of the expected rate of return for asset classes, alleviating the possibility of significantly rebalancing the portfolio. Additionally, the upper and lower economic indicator limits are set to reduce such volatility. Finally, to achieve a high target return for the fully or partly underfunded pension funds, additional simulations are conducted by adjusting the maximum and minimum proportions of each asset class to increase the proportion of risky assets and lower the weights of safe assets. The purpose of this process is to increase the opportunity that the average expected return on asset allocations for a total of 10 times exceeds the long-term target return.

Based on three asset allocation simulations, the following insights are derived. First, economic indicator filtering and upper-lower bound computation from recent five-year economic indicator filtering is necessary to secure data stability and upper-lower bound computation is also needed to reduce the expected return volatility by using the bounds as the model constraint. Second, to reach the ALM goal, the expected portfolio return must be higher than the long-term target rate. Hence, despite the higher volatility, more stocks should be allocated than low-yielding assets, such as bonds. Following the above-mentioned steps, the back-testing results from the third and fourth simulations show that dynamic asset allocation has a higher annual yield and risk-adjusted return (Sharpe ratio) than static asset allocation. Put differently, a dynamic asset allocation approach that reflects changing economic and financial environments is more effective than static asset allocation (Constant mix strategy) that keeps allocation weights constant over five or more years.

This study is limited by the fact that the dynamic asset allocation method using the MVO model fails to account for the rebalancing costs of adjusting the portfolio, unlike multi-term stochastic linear programming. Therefore, further research must be conducted through a reasonable and sophisticated cost estimation. In addition, while this study includes the aftermath period of the global financial crisis, the backtesting period is relatively short. It is also necessary to consider the past 30 years, including the Asian currency crisis and the information technology bubble burst. Furthermore, in this study, we calculate the expected rate of return for each asset class using the *ex ante* (or forward-looking) method; however, it would be meaningful to compare the efficiency of dynamic and static asset allocation using the *ex post* (or trailing) method. Finally, in this study, we use VAR to estimate interest rates, but the performance of both asset allocation methods can also be compared and analyzed using Monte Carlo simulation or the DSGE model.

Notes

1. Japan GPIF name it as a reference portfolio (Horie, 2017); Canada CPPIB refers to a normal portfolio (Office of the Chief Actuary, 2007)
2. Most Korean public pensions are prohibited from borrowing except in short-term liquidity shortage situations.
3. See Appendix 1.
4. According to the Korea Government Employees Pension Service's IPS, the fund had financial assets of 8.89tn won as of the end of 2019. This means that the IRR covering 16.43tn won which is the government support per year in 2075 is 5.48%.
5. The NPS, etc., conduct TAA yearly within the permitted range of each asset class based on the SAA plan (5 years), and transition portfolios are organized linearly over 5 years as an annual TAA.
6. See Appendix Tables A1 and A2 for details.
7. A negative (–) change rate variable means actual rate of negative (–) change, and 0 if it is positive (+); the positive (+) change rate variable is the actual rate of positive (+) change, and 0 if it is negative (–). Lee (2011) and Lee and Kim (2016) reflect the asymmetric effect using this method.
8. The initial sample composition includes those from the second quarter of 1995 to the fourth quarter of 2010, followed by the third quarter of 1995 to the first quarter of 2011 → from the fourth quarter of 1995 to the second quarter of 2011 → . . . → From the second quarter of 2005 to the fourth quarter of 2020 (final), it involves a repeated removal and addition of one-quarter.
9. Refer to Tables A1 and A2 in Appendix 2 for details.
10. The "pay-as-you-go" system is a pension funding method. In the event when the fund runs out or becomes exhausted, complete reserves are given up and the future generation supports the current generation. The current fund thus has a "payment reserve" characteristic.
11. Band-pass filter is a method of extracting cyclical patterns within a certain period (short-term 2–8 years, medium-term 8–20 years) from financial time series data. Band-pass filter has an advantage in easily identifying circulation phases but has a disadvantage in having a high uncertainty in the analysis result whenever new information is added (end-point problem).
12. The upper and lower bounds of economic indicators have been calculated quarterly from 2000 to 2020 through a rolling method, with an upper bound at 85% of the band and a lower bound at the band's minimum value. Details can be provided upon request.
13. The window sliding method calculates annual data by carrying forward data every quarter, increasing the data size (i.e. 2006.Q1–2010.Q4, 2006.Q2–2010.Q2, . . ., 2019.Q1–2019.Q4).

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Appendix 1

Multistage stochastic linear programming

Multistage stochastic linear programming, one of the dynamic asset allocation models, is a method of solving the problem of future uncertainty in static asset allocation. While linear programming assumes information that is already known, the actual input variable figures in asset allocation continue to change. Accordingly, stochastic linear programming tends to resolve the flaws facing future uncertainties. The method is first introduced by Dantzig (1995) and Beale (1995), and subsequently, Kusy and Ziemba (1986) verify a simple two-stage “recourse” model.

The core of this method, whether in the second or multistage level, is to take corrective measures after an event occurs under the concept of “recourse.” Multistage stochastic linear programming can be expressed as follows in a general model.

$$min_{\ x_0 \in X} \left[g_0(x_0) + \sum_{\tau}^{K} E_{\xi_1, \cdots \xi_\tau} Q_\tau \left(x_0,\ \widehat{x_1}, \cdots\ \widehat{x_{\tau-1}}\ , \widetilde{\xi_1}, \cdots, \widetilde{\xi_\tau} \right) \right]$$

stage 0 = current time point; $\tau = 1, \ldots, K$ (= Estimated expiration time); Q = recourse function, x_0 = current asset allocation, $\widehat{x_i}$ = future asset allocation, $\xi = \widehat{x_i}$ conditional probability that the two preceding allocations will occur.

The basic concept of this model is to specifically reflect the decision-making variables following the first period in the model. At this time, an important premise is “non-expectancy,” which means that the determinant x_1 in the first step is independent of the second step of the scenario, suggesting that the determinants are based only on current or past information. If c_0 is used instead of g_0 in the general model above, the objective function is expressed as cost. A stochastic linear model that minimizes this cost is a way of deriving the optimal asset allocation at the current time by minimizing the sum of the decision-making costs in the first period and the expected costs for the scenario-based revision in the next period, that is, the rebalancing costs incurred in the process of modifying future asset allocations based on conditional probabilities (Cho *et al.*, 2001; Dantzig and Infanger, 1993).

In this study, the demonstration period is over 10 years. However, this model has an estimation period of only 3–5 years, which makes it impractical to use for dynamic asset allocation simulation.

Appendix 2

Classification	Factor decomposition of expected rate of return
Domestic bonds	Risk-free interest rate (estimation) + term spread + credit spread + expected capital gain or loss
Overseas bonds	Risk-free interest rate (estimation) + credit spread + expected capital gain or loss + foreign exchange hedge premium
Stock	GDP (estimation) + CPI (estimation) + dividend yield (prediction)
Alternative investment	Weighted average of expected return by sector (Assumptions: Real Estate: Private Equity Fund: Infrastructure ratio of 4:4:2 & Domestic: Foreign ratio of 5:5)

Table A1. Components of the expected rate of return

Asset classes	Components	Detailed basis for calculation
Domestic bonds	Risk-free interest rate	Estimates of 3-year treasury bond interest rate
	Credit spread	Average of five-year spread between three-year treasury bonds and corporate bonds (A—AA+)
	Term spread	Average of three-year spread between three-year and five-year treasury bonds
	Expected capital gains and losses	Estimates of interest rate model and reflection of market duration
Foreign bonds	Risk-free interest rate	Estimates of 10-year US bond interest rate
	Credit spread	Average of five-year spread between 10-year US bonds and corporate bonds (A—AA+)
	Foreign Exchange Hedge Premium	Annualize the weighted average premium of USD, EUR, JPY, and GBP per month
Domestic stocks	GDP, CPI	Estimates of Korean real GDP and CPI
	Dividend yield	Bloomberg survey forecast
Foreign stock	GDP, CPI	Estimates of the US's real GDP and CPI
Alternative investment	Domestic real estate	Estimates of Korean CPI + 3%p
	Foreign real estate	Estimates of US CPI + 3%p
	Domestic private equity	(Expected return on domestic bond benchmarks + 2%) × 0.7 + (Expected return on domestic stock benchmarks + 1%) × 0.3
	Foreign private equity	(Expected return on foreign bond benchmarks + 2%) × 0.7 + (Expected return on foreign stock benchmarks + 2%) × 0.3
	Domestic SOC	Estimates of Korean CPI + 3%p
	Foreign SOC	Estimates of US CPI + 4%p

Note(s): (1) The spread and premium of the previous five years are calculated in an annual rolling manner; (2) Refer to the Public Employee Pension Park 2020 Asset Management Guidelines (IPS) for alternative investment spreads

Table A2. Detailed basis for each component of the expected rate of return

Appendix 3
Simulations II and III: asset allocation (proposal)

Table A3. Simulation II: Asset allocation (proposal)

(Unit: %)

Year	Domestic stock	Foreign stock	Domestic bond	Foreign bond	Alternative investment	$E(r_p)$	σ_p	SF
2011	15.0	15.0	30.0	28.3	11.7	5.3	4.8	2.2
2012	24.7	25.3	30.0	10.0	10.0	5.3	8.2	14.3
2013	15.0	34.9	30.0	10.0	10.1	5.1	7.3	13.5
2014	15.0	35.0	30.0	10.0	10.0	6.7	5.2	0.8
2015	15.0	35.0	30.0	10.0	10.0	5.8	4.7	1.5
2016	15.0	35.0	30.0	10.0	10.0	5.4	4.5	3.9
2017	22.5	27.4	30.0	10.0	10.1	4.4	4.0	7.4
2018	28.5	20.6	30.0	10.0	10.9	4.2	3.4	6.8
2019	33.4	16.6	30.0	10.0	10.0	4.3	4.3	11.3
2020	22.7	15.0	30.0	10.0	22.3	3.8	3.6	7.5

Table A4. Simulation III: Asset allocation (proposal)

(Unit: %)

Year	Domestic stock	Foreign stock	Domestic bond	Foreign bond	Alternative investment	$E(r_p)$	σ_p	SF
2011	20.0	20.0	10.0	29.5	20.5	5.26	6.34	6.67
2012	28.3	20.0	10.0	10.0	31.7	5.40	8.38	14.23
2013	20.0	38.2	10.0	10.0	21.8	5.51	8.79	14.64
2014	20.0	39.9	10.0	10.0	20.1	6.70	6.31	2.34
2015	20.0	39.1	10.0	10.0	20.9	6.14	5.61	2.41
2016	20.0	40.0	10.0	10.0	20.0	6.25	5.44	2.86
2017	20.0	39.3	10.0	10.0	20.7	5.14	4.93	5.61
2018	39.1	20.4	10.0	10.0	20.6	4.87	4.18	4.84
2019	39.9	20.0	10.0	10.0	20.1	5.35	5.25	6.48
2020	36.8	20.0	10.0	10.0	23.2	4.72	5.67	14.99

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