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RESEARCH ARTICLE

Quantized-State-Feedback-Based Neural Control for a Class of Switched Nonlinear Systems With Unknown Control Directions

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ABSTRACT This paper investigates the problem of unknown virtual control directions in a state-quantized adaptive recursive control design for a class of arbitrarily switched uncertain pure-feedback nonlinear systems in a band-limited network. State quantization is considered for state feedback control in a band-limited network. The primary contribution of this study is to provide a quantized state feedback adaptive control strategy to address the unknown control direction and arbitrarily switched nonaffine nonlinearities. Herein, a coupling problem between Nussbaum functions and quantization errors caused by quantized state feedback control laws is considered in the Lyapunov-based design and stability analysis. A state-quantized adaptive recursive control scheme using the function approximation is constructed without a priori knowledge of the signs of the control gain functions, where the estimated parameters and Nussbaum-type functions are adaptively updated via quantized states. Theoretical lemmas are derived to show that the adaptive parameters and quantization errors of the closed-loop signals are bounded using the proposed control scheme. The boundedness of the closed-loop signals and the convergence of tracking error to a neighborhood of the origin are proved using the common Lyapunov function approach. Two simulation examples are shown to illustrate the effectiveness of the proposed theoretical result.

INDEX TERMS State quantization, unknown control direction, switched pure-feedback nonlinear systems, arbitrary switching, adaptive tracking.

I. INTRODUCTION

During the past few years, various control methodologies have been developed for switched systems, which are a form of common hybrid systems, because of their application in the chemical industry, power control, mechanical operation, and other industrial productions ([1]–[10] and references therein). Among these methodologies, the common Lyapunov function approach has been extensively studied to control lower-triangular nonlinear systems with arbitrary switching. In [11]–[16], recursive and systematic control strategies using the backstepping technique [17] were presented for arbitrarily switched systems, where unmatched nonlinearities were assumed to be known. Adaptive techniques [18]–[20] have been employed to estimate system uncertainties in the nonlinear control field. To deal with unknown switched nonlinearities, approximation-based adaptive control problems of switched nonlinear systems were studied in [21]–[25]. In [26], an adaptive neural fault-tolerant control problem was addressed for uncertain switched nonstrict-feedback nonlinear systems with unmodeled dynamics and actuator faults. In [27], a finite-time tracing problem was investigated for switched nonlinear systems with backlash-like hysteresis and time-varying delays. To consider control design problems in a capacity-limited network, quantized control approaches have been studied for uncertain switched nonlinear systems with input quantization [28]–[34]. However, although control inputs were quantized, the reported design approaches were based on continuous state feedback, namely, the state

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quantization problem in the sensor-to-controller channel cannot be resolved in [28]–[34]. For the fully quantized control of switched nonlinear systems, it is significant to study the design and stability analysis methodologies using quantized state feedback information, which is discontinuous.

The signal quantization problem in the control design and analysis of nonlinear systems is an emerging research topic because of the increasing use of digital processors and communication networks with limited bandwidth (see [35]–[37] and the references therein). In recent years, function-approximation-based adaptive controllers have been proposed to address uncertain lower-triangular nonlinear systems with input quantization [28], [38]–[41]. In these results, input quantization was considered as one of input constraints and was compensated using the adaptive technique. Contrary to input quantization, state quantization causes a design difficulty owing to the discontinuously quantized state feedback in the recursive design steps. To overcome this difficulty, an adaptive backstepping control method was presented to stabilize systems with matched unknown parameters and known nonlinear functions [42]. By employing the command-filtered backstepping technique [43], lower-triangular nonlinear systems with state quantization were considered to design adaptive trackers for dealing with various control problems such as unmatched parametric uncertainties [44], state time delays [45], decentralization [46], and switched nonlinearities [47]. Although these results [44]–[47] provided several successful solutions to the state quantization problems of uncertain lower-triangular nonlinear systems, there is scope for further improvement in the following aspects. (i) The systems concerned in [44]–[47] have unity control coefficients. That is, the virtual and actual control gains are one and known. Thus, an unknown control direction problem remains open in the quantized state feedback control field of lower-triangular nonlinear systems. (ii) The systems discussed in [44]–[47] are in the strict feedback form. Thus, an adaptive quantized state feedback control problem for arbitrarily switched pure-feedback nonlinear systems with unknown nonaffine nonlinearities has not yet been addressed.

The major design difficulty in dealing with these two points lies in considering the coupling terms of Nussbaum functions and quantization errors caused by the quantized state feedback control laws in the Lyapunov-based design and stability. To overcome this difficulty, technical lemmas for analyzing these coupling terms and a quantized-state-based common adaptive control strategy for ensuring the boundedness of the quantization errors should be developed.

These observations motivate us to develop a state-quantized adaptive tracking control strategy for arbitrarily switched uncertain pure-feedback nonlinear systems with unknown control directions in a capacity-limited network. It is assumed that all states quantized by a uniform quantizer are only accessible for feedback, and the pure-feedback nonlinear functions and signs of the control gain functions induced from the mean value theorem are unknown. Compared with related works in the literature, the primary contributions of this study are as follows: (i) the unknown control direction problem is first considered in the quantized-state-based recursive control design field of lower-triangular nonlinear systems; and (ii) pure-feedback systems with unknown switched nonlinearities are first considered in the state-quantized control field. To this end, a state-quantized adaptive controller is designed using the command filtered backstepping technique and Nussbaum functions. The learning laws for the adaptive parameters and Nussbaum variables are derived using quantized states. Then, by developing technical lemmas, it is shown that the quantization errors of the virtual and actual control laws with Nussbaum functions are bounded. Based on the common Lyapunov function method and the boundedenss of quantization errors, we prove that the tracking errors are semi-globally asymptotically bounded and converge to a neighborhood of the origin in the sense of Lyapunov stability criterion. Simulation examples are included to demonstrate the effectiveness of the proposed state-quantized control methodology.

The rest of this paper consists of the following parts. A state-quantized adaptive control problem in the presence of unknown control directions and arbitrarily switched pure-feedback nonlinearities is formulated in Section 2. In Section 3, the state-quantized adaptive tracker design and the stability analysis strategy are derived. In Section 4, two simulations are provided to verify the effectiveness of the proposed approach. Finally, the conclusions are drawn in Section 5.

II. PROBLEM STATEMENT AND PRELIMINARIES

A. PROBLEM STATEMENT

Consider the following switched nonlinear pure-feedback systems with arbitrary switching

$$
\begin{aligned}
\dot{x}_i &= f_i^{\rho(t)}(\bar{x}_i, x_{i+1}) + d_i^{\rho(t)}(t), \quad i = 1, 2, \ldots, n-1 \\
\dot{x}_n &= f_n^{\rho(t)}(\bar{x}_n, u^{\rho(t)}) + d_n^{\rho(t)}(t) \\
y &= x_1
\end{aligned} \tag{1}
$$

where $\bar{x}_i = [x_1, \ldots, x_i]^\top \in \mathbb{R}^i$, $i = 1, \ldots, n$, are state vectors, $\rho(t) : [0, +\infty) \to P = \{1, 2, \ldots, p\}$ is the switching signal, $y \in \mathbb{R}$ is a system output, $u^l \in \mathbb{R}$, $\forall l \in P$ is a control input of the lth subsystem, $f_i^l(\bar{x}_i, x_{i+1}) : \mathbb{R}^{i+1} \mapsto \mathbb{R}$, $i = 1, \ldots, n$, $\forall l \in P$ are unknown smooth functions of the lth subsystem with $x_{n+1} = u^l$, and $d_i^l \in \mathbb{R}$, $i = 1, \ldots, n$, $\forall l \in P$, denote unknown external disturbances of the lth subsystem.

In this study, the system (1) and the controller are connected through a capacity-limited network environment. Thus, the measured state variables are quantized and transmitted to the control part. Then, the quantized discontinuous state variables are only available for the control design. For state quantization, a uniform quantizer is employed as a state quantizer as follows:

$$x_i^q \triangleq q(x_i) = \begin{cases} E_j, & E_j - \frac{\Delta}{2} \le x_i < E_j + \frac{\Delta}{2} \\ 0, & -\frac{\Delta}{2} \le x_i < \frac{\Delta}{2} \\ -E_j, & -E_j - \frac{\Delta}{2} \le x_i < -E_j + \frac{\Delta}{2} \end{cases} \tag{2}$$

where $i = 1, \ldots, n$, $j \in \mathbb{Z}^+$, $\Delta > 0$ is the quantization level, $E_1 = \Delta$, and $E_{j+1} = E_j + \Delta$. Then, the quantization error $z_{x,i} = x_i - x_i^q$ satisfies the property $|z_{x,i}| \le \Delta/2$.

For the quantized state feedback design in the presence of unknown switched pure-feedback nonlinearities, we need the following assumptions and lemmas.

Assumption 1 ([22], [48]): $\frac{\partial f_i^l(\bar{x}_i, x_{i+1})}{\partial x_{i+1}} \neq 0$, $i = 1, \ldots, n$, $\forall l \in P$, and there exist unknown real constants $\underline{h}_{i,0}^l > 0$ and $\bar{h}_{i,0}^l > 0$ such that $\underline{h}_{i,0}^l \le \left| \frac{\partial f_i^l(\bar{x}_i, x_{i+1})}{\partial x_{i+1}} \right| \le \bar{h}_{i,0}^l$. Additionally, the signs of $\frac{\partial f_i^l(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}$ are unknown.

Remark 1: Assumption 1 indicates that the control gain functions $\frac{\partial f_i^l(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}$ of each subsystem are strictly positive or negative. This is a sufficient condition for the controllability of system (1) [22], [48]. The signs of $\frac{\partial f_i^l(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}$ mean the virtual and actual control directions induced from the recursive control design. Thus, this study considers a state-quantized adaptive control design problem in the presence of unknown control directions of switched pure-feedback nonlinear systems.

Assumption 2 ([22]): There exist unknown constants $d_{i,0}^l > 0$, $i = 1, \ldots n$, $\forall l \in P$, such that $|d_i^l(t)| \le d_{i,0}^l$.

Assumption 3 ([22], [49]): The reference trajectory y_r and its time derivative $\dot{y}_r$ are bounded, and y_r is only available.

Assumption 4 ([42]): The state variables x_i are not available for feedback. Instead, the quantized state variables x_i^q are only available.

Remark 2: In this paper, the network control problem under a band-limited network is considered for switched nonlinear systems (1). Thus, the measured state variables are quantized before network transmission to the controller. Thus, Assumption 4 is given for the problem formulation of the adaptive quantized state feedback control design.

Lemma 1 ([50]): For a Hurwitz matrix $A \in \mathbb{R}^{n \times n}$, H is a symmetric positive definite matrix such that $A^\top H + HA = -2I$ with an identity matrix $I \in \mathbb{R}^{n \times n}$. Then, it is ensured that $\|e^{At}\| \le \beta_1 e^{-\beta_2 t}$ where $\beta_1 = \sqrt{\lambda_{\max}(H)/\lambda_{\min}(H)}$, and $\beta_2 = 1/\lambda_{\max}(H)$. Here, $\lambda_{\max}(H)$ and $\lambda_{\min}(H)$ are the maximum and minimum eigenvalues of the matrix H, respectively.

Lemma 2 ([51]): For any constant $\delta > 0$ and $\varphi \in \mathbb{R}$, it holds that $0 \le |\varphi| - \varphi \tanh(\varphi/\delta) \le 0.2785\delta$

The objective of this paper is to design a common state-quantized adaptive controller $u^l \triangleq u$, $\forall l \in P$ for system (1) under arbitrary switching and unknown control directions such that the output y follows the desired trajectory y_r.

Remark 3: Contrary to existing studies on quantized feedback adaptive control [44]–[47], a quantized state feedback control design problem in the presence of unknown signs of control gain functions and unknown switched pure-feedback nonlinearities is addressed in this paper. The main difficulty of our study is to derive a state-quantized control design and stability methodology for ensuring that the quantization errors of Nussbaum-functions-based virtual and actual control laws are bounded regardless of unknown switched pure-feedback nonlinearities. This difficulty has not been solved in the existing control works related to switched lower-triangular nonlinear systems with input quantization [28]–[34] or quantized-state-based recursive designs [44]–[47].

B. NUSSBAUM-TYPE GAIN

A continuous function $N(\xi) : \mathbb{R} \to \mathbb{R}$ is called a Nussbaum-type function if it has the following properties [52]:

$$\lim_{s \to +\infty} \sup \frac{1}{s} \int_0^s N(\xi) d\xi = +\infty$$
$$\lim_{s \to +\infty} \inf \frac{1}{s} \int_0^s N(\xi) d\xi = -\infty. \tag{3}$$

In this paper, the Nussbaum-type function $N(\xi) = e^{\xi^2} \cos((\pi/2)\xi)$ is utilized to deal with the unknown control direction problem under state quantization.

Lemma 3 ([55]): Let $V(t)$ and $\xi(t)$ be smooth functions defined on $[0, t_f)$ with $V(t) \ge 0$, $\forall t \in [0, t_f)$, and $N(\xi)$ be an even Nussbaum-type function. If the following inequality holds

$$V(t) \le c_0 + e^{-c_1 t} \int_0^t [g(x(\tau))N(\xi) + 1]\dot{\xi} e^{c_1 \tau} d\tau \tag{4}$$

for $\forall t \in [0, t_f)$ where $c_0 > 0$ and $c_1 > 0$ are constants and $g(x(t))$ is a time-varying parameter that takes values in unknown closed intervals $L := [l^-, l^+]$ with $0 \notin L$, then $V(t)$, $\xi(t)$, and $\int_0^t g(x(\tau))N(\xi)\dot{\xi} d\tau$ are bounded on $[0, t_f)$.

C. RADIAL BASIS FUNCTION NEURAL NETWORKS

Radial basis function neural networks (RBFNNs) have been widely used to approximate unknown nonlinear functions derived from controller design procedures (see [53]–[58]). Let us consider unknown continuous functions $F_i(\chi_i) \in \Omega_i \to \mathbb{R}$ with compact sets Ω_i and $i = 1, \ldots, n$. According to the universal approximation property of RBFNN [53], if the number of neural nodes is sufficiently large and the basis functions are chosen appropriately, there exist ideal bounded weight vectors $W_i^* \in \mathbb{R}^{N_i}$ such that

$$F_i(\chi_i) = W_i^{*\top} B_i(\chi_i) + \varepsilon_i(\chi_i), \quad \chi_i \in \Omega_i \tag{5}$$

where $\chi_i = [\chi_{i,1}, \ldots, \chi_{i,M_i}]^\top$ are the input vectors, $B_i(\chi_i) \in \mathbb{R}^{N_i}$ are the Gaussian basis function vectors with network nodes N_i, ε_i are the network reconstruction errors bounded by $|\varepsilon_i| \le \varepsilon_i^*$ with constants ε_i^*, and $\|W_i^*\| \le \bar{W}_i$ are satisfied with constants $\bar{W}_i$. Due to the inherent property of Gaussion functions, there exist constants B_i^* such that $\|B_i(\chi_i)\| \leq B_i^*$ [54], [56].

III. MAIN RESULTS

A. SYSTEM TRANSFORMATION

For recursive tracker design using the command-filtered backstepping technique, system (1) is transformed into an affine form. Using Assumption 1 and the implicit function theorem [56], for $\forall \bar{x}_i \in \mathbb{R}^i$, there exists a smooth ideal input $x_{i+1} = v_i^{*l}(\bar{x}_i)$ such that $f_i^l(\bar{x}_i, v_i^{*l}) = 0$. From the mean value theorem, it is obtained that [59]

$$f_i^l(\bar{x}_i, x_{i+1}) = f_i^l(\bar{x}_i, v_i^{*l}) + \int_0^1 \frac{\partial f_i^l(\bar{x}_i, x_{i+1,\lambda})}{\partial x_{i+1,\lambda}} d\lambda (x_{i+1} - v_i^{*l}) \quad (6)$$

where $i = 1, \ldots, n$, $x_{i+1,\lambda} = \lambda x_{i+1} + (1-\lambda)v_i^{*l}$; $\lambda \in [0, 1]$, $\forall l \in P$, and $x_{n+1} = u$. By defining $h_i^l = \int_0^1 \frac{\partial f_i^l(\bar{x}_i, x_{i+1,\lambda})}{\partial x_{i+1,\lambda}} d\lambda$, system (1) is represented by [22]

$$\begin{aligned} \dot{x}_i &= h_i^l x_{i+1} - h_i^l v_i^{*l} + d_i^l(t), \quad i = 1, 2, \ldots, n-1 \\ \dot{x}_n &= h_n^l u - h_n^l v_n^{*l} + d_n^l(t). \end{aligned} \quad (7)$$

B. DESIGN OF STATE-QUANTIZED ADAPTIVE TRACKER

For the command filtered backstepping design of the proposed state-quantized adaptive controller, the error signals are defined as

$$s_1 = x_1 - y_r \quad (8)$$

$$s_{i+1} = x_{i+1} - \hat{\alpha}_{i,1} \quad (9)$$

$$\tilde{\alpha}_{i,1} = \hat{\alpha}_{i,1} - \alpha_i \quad (10)$$

where $i = 1, \ldots, n-1$, s_1 and s_{i+1} are error surfaces, α_i are intermediate signals, and $\hat{\alpha}_{i,1}$ and $\tilde{\alpha}_{i,1}$ are filtered signals and filtering errors of the intermediate signals, respectively. The command filtered signals are given by the following second-order low-pass filters

$$\begin{aligned} \dot{\hat{\alpha}}_{i,1} &= \hat{\alpha}_{i,2} \\ \dot{\hat{\alpha}}_{i,2} &= -2\zeta_i\omega_i\hat{\alpha}_{i,2} - \omega_i^2(\hat{\alpha}_{i,1} - \alpha_i) \end{aligned} \quad (11)$$

with $\hat{\alpha}_{i,1}(0) = \alpha_i(0)$ and $\hat{\alpha}_{i,2}(0) = 0$, and positive constants ζ_i and ω_i.

Remark 4: Because the quantized state variables that are non-differentiable cannot be directly utilized in the recursive design using the common Lyapunov function, we present the state-quantized control design steps of switched nonlinear systems with unknown control directions. In this subsection, we first derive intermediate signals α_i using unquantized state signals. Then, a state-quantized adaptive tracker u is designed using the structure of the intermediate signals. Thus, the errors between the unquantized and quantized signals (i.e., quantization errors) are analyzed for guaranteeing closed-loop stability in the next subsection. In the proposed state-quantized adaptive tracker, the dynamics of the Nussbaum variables using quantized states are designed to ensure that the quantization errors are bounded. It is shown that the unknown control direction problem can be overcome by quantized state feedback.

Step 1: Consider the first error surface $s_1 = x_1 - y_r$. From (7), (9) and (10), we have

$$\dot{s}_1 = h_1^l s_2 + h_1^l \tilde{\alpha}_{1,1} + h_1^l \alpha_1 - h_1^l v_1^{*l} + d_1^l - \dot{y}_r. \quad (12)$$

A common Lyapunov function is defined as $V_1 = s_1^2/(2h_{1,m})$ with $h_{1,m} = \max_{l\in P}\{\bar{h}_{1,0}^l\}$. For any $l \in P$, the time derivative of V_1 is obtained as

$$\dot{V}_1 = \frac{1}{h_{1,m}} s_1(h_1^l s_2 + h_1^l \tilde{\alpha}_{1,1} + h_1^l \alpha_1 - h_1^l v_1^{*l} + d_1^l - \dot{y}_r). \quad (13)$$

Adding and subtracting s_1^4 into (13) yields

$$\dot{V}_1 = \frac{1}{h_{1,m}} s_1(h_1^l s_2 + h_1^l \tilde{\alpha}_{1,1} + h_1^l \alpha_1 + h_{1,m} g_1^l + d_1^l) - s_1^4 \quad (14)$$

where $g_1^l = -(h_1^l/h_{1,m})v_1^{*l}(x_1) + s_1^3 - \dot{y}_r/h_{1,m}$.

Using Assumptions 1 and 3, and Young's inequalities, there exist a continuous function $F_1(\chi_1)$ and a constant $\iota_1 > 0$ such that [22]

$$s_1 g_1^l \leq s_1 F_1(\chi_1) + \iota_1, \quad \forall l \in P \quad (15)$$

where $\chi_1 = [x_1, s_1]^\top$ and $F_1(\chi_1) = (1/(2\iota_1))[(\bar{v}_1^*(x_1))^2 + (M/h_{1,m})^2]s_1 + s_1^3$; $\bar{v}_1^*(x_1) = \max_{l\in P}\{v_1^{*l}(x_1)\}$ and a constant M satisfying $|\dot{y}_r| \leq M$. Using (5), it holds that $F_1(\chi_1) = W_1^{*\top}B_1(\chi_1) + \varepsilon_1(\chi_1)$ where W_1^*, B_1, and ε_1 are the ideal weighting vector, the Gaussian basis function vector, and the reconstruction error of the RBFNN, respectively. Using Young's inequality

$$s_1 W_1^{*\top} B_1(\chi_1) \leq s_1^2\theta_1 + \frac{\kappa_1}{4} \quad (16)$$

where $\theta_1 = (\bar{W}_1 B_1^*)^2/\kappa_1$ with a constant $\kappa_1 > 0$, the following inequality holds

$$\begin{aligned} \dot{V}_1 \leq{} & \frac{h_1^l}{h_{1,m}} s_1(s_2 + \tilde{\alpha}_{1,1}) - s_1^4 + \frac{\kappa_1}{4} + \iota_1 \\ & + \frac{1}{h_{1,m}} s_1(h_1^l\alpha_1 + h_{1,m}s_1\theta_1 + h_{1,m}\varepsilon_1 + d_1^l). \end{aligned} \quad (17)$$

The first intermediate signal α_1 is given by

$$\alpha_1 = N(\xi_1)\left(k_1 s_1 + s_1\hat{\theta}_1 + s_1\hat{\psi}_1 \tanh\left(\frac{s_1^2}{\delta_1}\right)\right) \quad (18)$$

where $N(\xi_1)$ is a Nussbaum function, $\hat{\theta}_1$ and $\hat{\psi}_1$ are the estimates of θ_1 and a constant ψ_1^* to be determined later, respectively, and $k_1 > 0$ and $\delta_1 > 0$ are design parameters. The tuning laws of ξ_1, $\hat{\theta}_1$, and $\hat{\psi}_1$ are designed using quantized state feedback signals at the last step. Then, from (18), the inequality (17) becomes

$$\begin{aligned} \dot{V}_1 \leq{} & -k_1 s_1^2 + \frac{h_1^l}{h_{1,m}} s_1(s_2 + \tilde{\alpha}_{1,1}) \\ & + \frac{1}{h_{1,m}} s_1(h_{1,m}\varepsilon_1 + d_1^l) - s_1^4 + \frac{\kappa_1}{4} + \iota_1 \end{aligned}$$

$$+\left(\frac{h_1^l}{h_{1,m}}N(\xi_1)+1\right)\left[k_1s_1^2+s_1^2\hat{\theta}_1+s_1^2\hat{\psi}_1\tanh\left(\frac{s_1^2}{\delta_1}\right)\right] -s_1^2\tilde{\theta}_1-s_1^2\tilde{\psi}_1\tanh\left(\frac{s_1^2}{\delta_1}\right)-s_1^2\psi_1^*\tanh\left(\frac{s_1^2}{\delta_1}\right) \quad (19)$$

where $\tilde{\theta}_1=\hat{\theta}_1-\theta_1$ and $\tilde{\psi}_1=\hat{\psi}_1-\psi_1^*$.

Step i $(i=2,\ldots,n-1)$*:* From (7), (9), (10), and (11), we have

$$\dot{s}_i=h_i^ls_{i+1}+h_i^l\tilde{\alpha}_{i,1}+h_i^l\alpha_i-h_i^lv_i^{*l}+d_i^l-\hat{\alpha}_{i-1,2}. \quad (20)$$

For any $l\in P$, the time derivative of a common Lyapunov function candidate $V_i=s_i^2/(2h_{i,m})$ with $h_{i,m}=\max_{l\in P}\{\bar{h}_{i,0}^l\}$ is represented by

$$\dot{V}_i=\frac{1}{h_{i,m}}s_i(h_i^ls_{i+1}+h_i^l\tilde{\alpha}_{i,1}+h_i^l\alpha_i-h_i^lv_i^{*l} +d_i^l-\hat{\alpha}_{i-1,2}). \quad (21)$$

Adding and subtracting s_i^4 into (21) yields

$$\dot{V}_i=\frac{1}{h_{i,m}}s_i(h_i^ls_{i+1}+h_i^l\tilde{\alpha}_{i,1}+h_i^l\alpha_i+h_{i,m}g_i^l+d_i^l)-s_i^4 \quad (22)$$

where $g_i^l(\chi_i)=-(h_i^l/h_{i,m})v_i^{*l}+s_i^3-\hat{\alpha}_{i-1,2}/h_{i,m}$ with $\chi_i=[\bar{x}_i,s_i,\hat{\alpha}_{i-1,2}]^\top$.

Then, there exist a continuous function $F_i(\chi_i)$ and a constant $\iota_i>0$ such that

$$s_ig_i^l\le s_iF_i(\chi_i)+\iota_i,\quad \forall l\in P. \quad (23)$$

The nonlinear function $F_i(\chi_i)$ is approximated by the RBFNN as follows: $F_i(\chi_i)=W_i^{*\top}B_i(\chi_i)+\varepsilon_i(\chi_i)$ where W_i^*, B_i, and ε_i are the ideal weighting vector, the Gaussian basis function vector, and the reconstruction error of the RBFNN, respectively. Using Young's inequality, we have $s_iW_i^{*\top}B_i(\chi_i)\le s_i^2\theta_i+\kappa_i/4$ where $\theta_i=(\bar{W}_iB_i^*)^2/\kappa_i$ and $\kappa_i>0$ is a constant. Then, the following inequality holds

$$\dot{V}_i\le\frac{h_i^l}{h_{i,m}}s_i(s_{i+1}+\tilde{\alpha}_{i,1})-s_i^4+\frac{\kappa_i}{4}+\iota_i +\frac{1}{h_{i,m}}s_i(h_i^l\alpha_i+h_{i,m}s_i\theta_i+h_{i,m}\varepsilon_i+d_i^l). \quad (24)$$

The ith intermediate signal α_i is designed as

$$\alpha_i=N(\xi_i)\left(k_is_i+s_i\hat{\theta}_i+s_i\hat{\psi}_i\tanh\left(\frac{s_i^2}{\delta_i}\right)\right) \quad (25)$$

where $N(\xi_i)$ is a Nussbaum function, $\hat{\theta}_i$ and $\hat{\psi}_i$ are estimates of θ_i and a constant ψ_i^* to be determined later, respectively, and $k_i>0$ and $\delta_i>0$ are design parameters. Then, substituting (25) into (24) gives

$$\dot{V}_i\le-k_is_i^2+\frac{h_i^l}{h_{i,m}}s_i(s_{i+1}+\tilde{\alpha}_{i,1}) +\frac{1}{h_{i,m}}s_i(h_{i,m}\varepsilon_i+d_i^l)-s_i^4+\frac{\kappa_i}{4}+\iota_i +\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\left[k_is_i^2+s_i^2\hat{\theta}_i+s_i^2\hat{\psi}_i\tanh\left(\frac{s_i^2}{\delta_i}\right)\right] -s_i^2\tilde{\theta}_i-s_i^2\tilde{\psi}_i\tanh\left(\frac{s_i^2}{\delta_i}\right)-s_i^2\psi_i^*\tanh\left(\frac{s_i^2}{\delta_i}\right) \quad (26)$$

where $\tilde{\theta}_i=\hat{\theta}_i-\theta_i$ and $\tilde{\psi}_i=\hat{\psi}_i-\psi_i^*$.

Step n: Using (7) and (9), the time derivative of a common Lyapunov function candidate $V_n=s_n^2/(2h_{n,m})$ with $h_{n,m}=\max_{l\in P}\{\bar{h}_{n,0}^l\}$ is obtained as

$$\dot{V}_n=\frac{1}{h_{n,m}}s_n(h_n^l(u-\alpha_n)+h_n^l\alpha_n+h_{n,m}g_n^l+d_n^l)-s_n^4 \quad (27)$$

where $l\in P$, α_n is an intermediate signal, $g_n^l(\chi_n)=-(h_n^l/h_{n,m})v_n^{*l}+s_n^3-\hat{\alpha}_{n-1,2}/h_{n,m}$ with $\chi_n=[\bar{x}_n,s_n,\hat{\alpha}_{n-1,2}]^\top$. Then, there exist a continuous function $F_n(\chi_n)$ and a constant $\iota_n>0$ such that

$$s_ng_n^l\le s_nF_n(\chi_n)+\iota_n,\quad \forall l\in P. \quad (28)$$

The nonlinear function $F_i(\chi_i)$ is approximated by the RBFNN as follows: $F_n(\chi_n)=W_n^{*\top}B_n(\chi_n)+\varepsilon_n(\chi_n)$ where W_n^*, B_n, and ε_n are the ideal weighting vector, the Gaussian basis function vector, and the reconstruction error of the RBFNN, respectively.

Using Young's inequality $s_nW_n^{*\top}B_n(\chi_n)\le s_n^2\theta_n+\kappa_n/4$ with $\theta_n=(\bar{W}_nB_n^*)^2/\kappa_n$ and a constant κ_n, it holds that

$$\dot{V}_n\le\frac{h_n^l}{h_{n,m}}s_n(u-\alpha_n)-s_n^4+\frac{\kappa_n}{4}+\iota_n +\frac{1}{h_{n,m}}s_n(h_n^l\alpha_n+h_{n,m}s_n\theta_n+h_{n,m}\varepsilon_n+d_n^l). \quad (29)$$

The intermediate signal α_n is selected as

$$\alpha_n=N(\xi_n)\left(k_ns_n+s_n\hat{\theta}_n+s_n\hat{\psi}_n\tanh\left(\frac{s_n^2}{\delta_n}\right)\right) \quad (30)$$

where $N(\xi_n)$ is a Nussbaum function, $\hat{\theta}_n$ and $\hat{\psi}_n$ are estimates of θ_n and ψ_n^* to be determined later, respectively, and $k_n>0$ and $\delta_n>0$ are design parameters.

Substituting (30) into (29) yields

$$\dot{V}_n\le-k_ns_n^2+\frac{h_n^l}{h_{n,m}}s_n(u-\alpha_n) +\frac{1}{h_{n,m}}s_n(h_{n,m}\varepsilon_n+d_n^l)-s_n^4+\frac{\kappa_n}{4}+\iota_n +\left(\frac{h_n^l}{h_{n,m}}N(\xi_n)+1\right)\left[k_ns_n^2+s_n^2\hat{\theta}_n+s_n^2\hat{\psi}_n\tanh\left(\frac{s_n^2}{\delta_n}\right)\right] -s_n^2\tilde{\theta}_n-s_n^2\tilde{\psi}_n\tanh\left(\frac{s_n^2}{\delta_n}\right) -s_n^2\psi_n^*\tanh\left(\frac{s_n^2}{\delta_n}\right) \quad (31)$$

where $\tilde{\theta}_n=\hat{\theta}_n-\theta_n$ and $\tilde{\psi}_n=\hat{\psi}_n-\psi_n^*$.

To design the state-quantized adaptive tracker u using Nussbaum functions, we first define the quantized-state-based error surfaces s_i' as

$$s_1'=x_1^q-y_r$$
$$s_{i+1}'=x_{i+1}^q-\hat{\alpha}_{i,1}' \quad (32)$$

where $i = 1, \ldots, n-1$ and $\hat{\alpha}'_{i,1}$ are filtered signals of virtual control laws α'_i which are obtained from the following second-order command filters

$$\dot{\hat{\alpha}}'_{i,1} = \hat{\alpha}'_{i,2}$$
$$\dot{\hat{\alpha}}'_{i,2} = -2\zeta_i\omega_i\hat{\alpha}'_{i,2} - \omega_i^2(\hat{\alpha}'_{i,1} - \alpha'_j) \tag{33}$$

with $\hat{\alpha}'_{i,1}(0) = \alpha'_i(0)$ and $\hat{\alpha}'_{i,2}(0) = 0$.

The common virtual control laws α'_i and the actual control law u using quantized state feedback signals are presented as

$$\alpha'_i = N(\xi_i)\left(k_i s'_i + s'_i\hat{\theta}_i + s'_i\hat{\psi}_i \tanh\left(\frac{s'^2_i}{\delta_i}\right)\right) \tag{34}$$
$$u = \alpha'_n \tag{35}$$
$$\dot{\xi}_i = \gamma_{\xi,i}\left(k_i s'^2_i + s'^2_i\hat{\theta}_i + s'^2_i\hat{\psi}_i \tanh\left(\frac{s'^2_i}{\delta_i}\right) - \sigma_{\xi,i}s'^2_i\xi_i\right) \tag{36}$$
$$\dot{\hat{\theta}}_i = \gamma_{\theta,i}\left(s'^2_i - \sigma_{\theta,i}s'^2_i\hat{\theta}_i\right) \tag{37}$$
$$\dot{\hat{\psi}}_i = \gamma_{\psi,i}\left(s'^2_i \tanh\left(\frac{s'^2_i}{\delta_i}\right) - \sigma_{\psi,i}s'^2_i\hat{\psi}_i\right) \tag{38}$$

where $i = 1, \ldots, n$ and $\gamma_{\xi,i}$, $\gamma_{\theta,i}$, $\gamma_{\psi,i}$, $\sigma_{\xi,i}$, $\sigma_{\theta,i}$, and $\sigma_{\psi,i}$ are the positive design parameters.

Remark 5: In the existing studies on quantized state feedback adaptive control [42], [44]–[47], the signs of the control coefficient functions were assumed to be known. That is, the signs of nonlinear control gains were known in [42] and nonlinear strict-feedback systems with unity control gains were considered in [44]–[47]. However, in this paper, the control directions (i.e., the signs of the control coefficient functions h^l_i) are assumed to be unknown. To deal with this problem, the state-quantized adaptive tracking scheme using Nussbaum functions is presented in (34)-(38). In (36)-(38), the tuning laws of the adaptive parameters and Nussbaum variables are derived using the quantized states.

C. STABILITY ANALYSIS USING QUANTIZATION ERRORS

Let us define the quantization errors of the recursively induced closed-loop signals as

$$z_{s,i} = s_i - s'_i, \quad z_{\alpha,j} = \alpha_j - \alpha'_j,$$
$$z_{\hat{\alpha},j,1} = \hat{\alpha}_{j,1} - \hat{\alpha}'_{j,1}, \quad z_{\hat{\alpha},j,2} = \hat{\alpha}_{j,2} - \hat{\alpha}'_{j,2},$$
$$z_u = \alpha_n - u = \alpha_n - \alpha'_n \tag{39}$$

where $i = 1, \ldots, n$ and $j = 1, \ldots, n-1$.

Then, we derive the following lemmas to show the boundedness of quantization errors.

Lemma 4: For any constant $\delta > 0$ and any $\varphi_1, \varphi_2 \in \mathbb{R}$, it holds that

1) $|\varphi_1 \tanh(\varphi_1^2/\delta) - \varphi_2 \tanh(\varphi_2^2/\delta)| \leq 1.6017|\varphi_1 - \varphi_2|$
2) $|\varphi_1^2 \tanh(\varphi_1^2/\delta) - \varphi_2^2 \tanh(\varphi_2^2/\delta)| \leq 1.1997|\varphi_1^2 - \varphi_2^2|$

Proof: See Appendix I. ■

Lemma 5: Consider the adaptive laws (36)-(38).

(i) For any initial conditions such that $\tilde{\theta}_i(0) \in \Omega_{\tilde{\theta},i}$, it holds that $\tilde{\theta}_i(t) \in \Omega_{\tilde{\theta},i}$, $\forall t \geq 0$ where $i = 1, \ldots, n$ and $\Omega_{\tilde{\theta},i} = \{\tilde{\theta}_i| \, |\tilde{\theta}_i| \leq \Lambda_{\tilde{\theta},i}\}$ are compact sets with unknown constants $\Lambda_{\tilde{\theta},i}$.

(ii) For any initial conditions such that $\tilde{\psi}_i(0) \in \Omega_{\tilde{\psi},i}$, it holds that $\tilde{\psi}_i(t) \in \Omega_{\tilde{\psi},i}$, $\forall t \geq 0$ where $i = 1, \ldots, n$ and $\Omega_{\tilde{\psi},i} = \{\tilde{\psi}_i| \, |\tilde{\psi}_i| \leq \Lambda_{\tilde{\psi},i}\}$ are compact sets with unknown constants $\Lambda_{\tilde{\psi},i}$.

(iii) For any initial conditions such that $\xi_i(0) \in \Omega_{\xi,i}$, it holds that $\xi_i(t) \in \Omega_{\xi,i}$, $\forall t \geq 0$ where $i = 1, \ldots, n$ and $\Omega_{\xi,i} = \{\xi_i| \, |\xi_i| \leq \Lambda_{\xi,i}\}$ are compact sets with unknown constants $\Lambda_{\xi,i}$.

Proof: See Appendix II. ■

Remark 6: To analyze the closed-loop stability using the state-quantized adaptive tracking scheme (34)-(38), we need to prove that the quantization error term $u - \alpha_n$ in (31) is bounded by the proposed state-quantized control scheme. To this end, we derive Lemma 5. Lemma 5 indicates that the proposed adaptive laws (36)-(38) ensure the boundedness of the parameter estimation errors $\tilde{\theta}_i(t)$ and $\tilde{\psi}_i(t)$, and the tuning parameter $\xi_i(t)$. Lemma 5 is used to prove the closed-loop stability using the state-quantized adaptive tracking scheme (34)-(38) in Theorem 1.

Lemma 6: Consider the quantization errors $z_{s,i}$, $z_{\alpha,j}$, $z_{\hat{\alpha},j,1}$, $z_{\hat{\alpha},j,2}$, z_u, and the uniform quantizer (2). There exist positive constants $Z_{s,i}$, $Z_{\alpha,j}$, $Z_{\hat{\alpha},j}$, and Z_u such that $|z_{s,i}| \leq Z_{s,i}$, $|z_{\alpha,j}| \leq Z_{\alpha,j}$, $\|z_{\hat{\alpha},j}\| \leq Z_{\hat{\alpha},j}$, and $|z_u| \leq Z_u$ where $i = 1, \ldots, n$, $j = 1, \ldots, n-1$, and $z_{\hat{\alpha},j} = [z_{\hat{\alpha},j,1}, z_{\hat{\alpha},j,2}]^\top$.

Proof: The recursive proof steps are derived to show the boundedness of the quantization errors.

i) Since $|z_{x,i}| \leq \Delta/2$ is ensured from the definition of the quantizer (2), $z_{s,1}$ is bounded as

$$|z_{s,1}| = |z_{x,1}| \leq \Delta/2 \triangleq Z_{s,1}. \tag{40}$$

From (18) and (34), $z_{\alpha,1}$ is obtained as

$$z_{\alpha,1} = N(\xi_1)\Big[k_1(s_1 - s'_1) + \hat{\theta}_1(s_1 - s'_1) + \hat{\psi}_1\left\{s_1 \tanh\left(\frac{s_1^2}{\delta_1}\right) - s'_1 \tanh\left(\frac{s'^2_1}{\delta_1}\right)\right\}\Big]. \tag{41}$$

From Lemma 5, it is satisfied that

$$|\xi_i| \leq \Lambda^*_{\xi,i}, \quad |\tilde{\theta}_i| \leq \Lambda^*_{\tilde{\theta},i}, \quad |\tilde{\psi}_i| \leq \Lambda^*_{\tilde{\psi},i} \tag{42}$$

where $i = 1, \ldots, n$, $\Lambda^*_{\xi,i} = \max\{|\xi_i(0)|, \Lambda_{\xi,i}\}$, $\Lambda^*_{\tilde{\theta},i} = \max\{|\tilde{\theta}_i(0)|, \Lambda_{\tilde{\theta},i}\}$, and $\Lambda^*_{\tilde{\psi},i} = \max\{|\tilde{\psi}_i(0)|, \Lambda_{\tilde{\psi},i}\}$ are constants. From the boundedness of ξ_i, there exists a positive constant $\Lambda_{N,i}$ such that

$$|N(\xi_i)| \leq \Lambda_{N,i}, \quad i = 1, \ldots, n. \tag{43}$$

In addition, using Lemma 4-(i) and $|z_{s,1}| \leq Z_{s,1}$, we have the inequality

$$\left|s_1 \tanh\left(s_1^2/\delta_1\right) - s'_1 \tanh\left(s'^2_1/\delta_1\right)\right| \leq 1.6017\, Z_{s,1} \triangleq Z_{\tanh,1}. \tag{44}$$

By substituting $\hat{\theta}_1 = \tilde{\theta}_1 + \theta_1$ and $\hat{\psi}_1 = \tilde{\psi}_1 + \psi_1^*$ into (41), it is obtained that

$$|z_{\alpha,1}| \leq \Lambda_{N,1}\big[k_1 Z_{s,1} + (\Lambda^*_{\tilde{\theta},1} + \theta_1)Z_{s,1} + (\Lambda^*_{\tilde{\psi},1} + \psi_1^*)Z_{\tanh,1}\big] \triangleq Z_{\alpha,1}. \quad (45)$$

From (11) and (33), the dynamics of the quantization error $z_{\hat{\alpha},1} = [z_{\hat{\alpha},1,1}, z_{\hat{\alpha},1,2}]^\top$ is given by

$$\dot{z}_{\hat{\alpha},1} = A_1 z_{\hat{\alpha},1} + D_1 z_{\alpha,1} \quad (46)$$

where $A_1 = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & -2\zeta_1\omega_1 \end{bmatrix}$ is a stable matrix and $D_1 = [0, \omega_1^2]^\top$. The solution of the differential equation (46) is represented by

$$z_{\hat{\alpha},1}(t) = e^{A_1 t} z_{\hat{\alpha},1}(0) + \int_0^t e^{A_1(t-\tau)} D_1 z_{\alpha,1}(\tau)d\tau. \quad (47)$$

Since the matrix A_1 is invertible and $|z_{\alpha,1}| \leq Z_{\alpha,1}$, it is satisfied that for all $t \geq 0$,

$$\|z_{\hat{\alpha},1}(t)\| \leq \|e^{A_1 t}\|\|z_{\hat{\alpha},1}(0)\| + Z_{\alpha,1}\|D_1\|\|A_1^{-1}(I - e^{A_1 t})\|. \quad (48)$$

According to Lemma 1, $\|e^{A_1 t}\| \leq \beta_{1,1}e^{-\beta_{1,2}t}$ is satisfied with constants $\beta_{1,1} > 0$ and $\beta_{1,2} > 0$. Owing to $z_{\hat{\alpha},1,1}(0) = z_{\alpha,1}(0)$ and $z_{\hat{\alpha},1,2}(0) = 0$, we have $\|z_{\hat{\alpha},1}(0)\| = |z_{\alpha,1}(0)|$. Then, it holds that

$$\|z_{\hat{\alpha},1}(t)\| \leq \beta_{1,1}|z_{\alpha,1}(0)| + Z_{\alpha,1}\|D_1\|\|A_1^{-1}\|(1 + \beta_{1,1}) \triangleq Z_{\hat{\alpha},1} \quad (49)$$

for all $t \geq 0$. Thus, the inequalities $|z_{\hat{\alpha},1,1}| \leq Z_{\hat{\alpha},1}$ and $|z_{\hat{\alpha},1,2}| \leq Z_{\hat{\alpha},1}$ hold.

ii) Using $|z_{x,i}| \leq \Delta/2$, $|z_{\hat{\alpha},i-1,1}| \leq Z_{\hat{\alpha},i-1}$, (42), (43), and Lemma 4-(i), $z_{s,i}$ and $z_{\alpha,i}$ are bounded as

$$|z_{s,i}| \leq |z_{x,i}| + |z_{\hat{\alpha},i-1,1}| \leq \Delta/2 + Z_{\hat{\alpha},i-1} \triangleq Z_{s,i} \quad (50)$$

$$|z_{\alpha,i}| \leq \Lambda_{N,i}\big[k_i Z_{s,i} + (\Lambda^*_{\tilde{\theta},i} + \theta_i)Z_{s,i} + (\Lambda^*_{\tilde{\psi},i} + \psi_i^*)Z_{\tanh,i}\big] \triangleq Z_{\alpha,i} \quad (51)$$

where $i = 2, \ldots, n-1$, and $Z_{s,i}$ and $Z_{\alpha,i}$ are positive constants.
From (11) and (33), the dynamics of $z_{\hat{\alpha},i} = [z_{\hat{\alpha},i,1}, z_{\hat{\alpha},i,2}]^\top$ is given by

$$\dot{z}_{\hat{\alpha},i} = A_i z_{\hat{\alpha},i} + D_i z_{\alpha,i} \quad (52)$$

where $i = 2, \ldots, n-1$, $A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix}$, and $D_i = [0, \omega_i^2]^\top$. Using the similar procedures to (47)-(49), $z_{\hat{\alpha},i}$ is bounded as

$$\|z_{\hat{\alpha},i}(t)\| \leq Z_{\hat{\alpha},i}, \quad \forall t \geq 0 \quad (53)$$

where $i = 2, \ldots, n-1$ and $Z_{\hat{\alpha},i} = \beta_{1,i}|z_{\hat{\alpha},i}(0)| + Z_{\alpha,i}\|D_i\|\|A_i^{-1}\|(1 + \beta_{1,i})$ with a constant $\beta_{1,i} > 0$. Thus, the inequalities $|z_{\hat{\alpha},i,1}| \leq Z_{\hat{\alpha},i}$ and $|z_{\hat{\alpha},i,2}| \leq Z_{\hat{\alpha},i}$ hold.

iii) Using $z_u = \alpha_n - \alpha_n'$, $|z_{s,n}| \leq Z_{s,n}$, $|z_{\hat{\alpha},n-1,1}| \leq Z_{\hat{\alpha},n-1}$, (42), (43), and Lemma 4-(i) yields

$$|z_u| \leq \Lambda_{N,n}\big[k_n Z_{s,n} + (\Lambda^*_{\tilde{\theta},n} + \theta_n)Z_{s,n} + (\Lambda^*_{\tilde{\psi},n} + \psi_n^*)Z_{\tanh,n}\big] \triangleq Z_u \quad (54)$$

with a constant Z_u.

This completes the proof of Lemma 6. ■

Consider an overall Lyapunov candidate V as

$$V = \sum_{i=1}^{n} V_i + \sum_{i=1}^{n-1} \tilde{\alpha}_i^\top G_i \tilde{\alpha}_i \quad (55)$$

where $\tilde{\alpha}_i = [\tilde{\alpha}_{i,1}, \tilde{\alpha}_{i,2}]^\top$; $\tilde{\alpha}_{i,2} = \hat{\alpha}_{i,2}$.

Theorem 1: Consider uncertain switched nonlinear pure-feedback systems (1) with the state quantizer (2) and unknown control directions. For initial conditions satisfying $V(0) \leq \mu$ with a positive constant μ, the state-quantized adaptive controller (i.e., (34)-(38)) guarantees that all signals of the closed-loop system are semi-globally uniformly ultimately bounded and the tracking error s_1 converges to an adjustable compact set including the origin.

Proof: Let us define $\bar{\tilde{\alpha}}_{j,1} = [\tilde{\alpha}_{1,1}, \ldots, \tilde{\alpha}_{j,1}]$ and $\bar{s}_i = [s_1, \ldots, s_i]^\top$ for $j = 1, \ldots, n-1$ and $i = 1, \ldots, n$. The time derivative of $\tilde{\alpha}_i$ is obtained as

$$\dot{\tilde{\alpha}}_i = A_i\tilde{\alpha}_i + D\Xi_i \quad i = 1, \ldots, n-1 \quad (56)$$

where $A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix}$ are Hurwitz matrices, $D = [1, 0]^\top$, and

$$\begin{aligned}
&\Xi_1(\xi_1, \bar{s}_2, \tilde{\alpha}_{1,1}, y_r, \dot{y}_r, \hat{\theta}_1, \hat{\psi}_1) \\
&= \frac{\partial N(\xi_1)}{\partial \xi_1}\dot{\xi}_1\left(k_1 s_1 + s_1\hat{\theta}_1 + s_1\hat{\psi}_1 \tanh\left(\frac{s_1^2}{\delta_1}\right)\right) \\
&\quad + N(\xi_1)\Bigg[k_1\dot{s}_1 + \dot{s}_1\hat{\theta}_1 + s_1\dot{\hat{\theta}}_1 + \dot{s}_1\hat{\psi}_1 \tanh\left(\frac{s_1^2}{\delta_1}\right) \\
&\quad + s_1\dot{\hat{\psi}}_1 \tanh\left(\frac{s_1^2}{\delta_1}\right) + 2\frac{s_1^2}{\delta_1}\hat{\psi}_1 \operatorname{sech}^2\left(\frac{s_1^2}{\delta_1}\right)\Bigg] \\
&\Xi_j(\xi_j, \bar{s}_{j+1}, \bar{\tilde{\alpha}}_{j,1}, y_r, \tilde{\alpha}_{j-1,2}, \hat{\theta}_j, \hat{\psi}_j) \\
&= \frac{\partial N(\xi_j)}{\partial \xi_j}\dot{\xi}_j\left(k_j s_j + s_j\hat{\theta}_j + s_j\hat{\psi}_j \tanh\left(\frac{s_j^2}{\delta_j}\right)\right) \\
&\quad + N(\xi_j)\Bigg[k_j\dot{s}_j + \dot{s}_j\hat{\theta}_j + s_j\dot{\hat{\theta}}_j + \dot{s}_j\hat{\psi}_j \tanh\left(\frac{s_j^2}{\delta_j}\right) \\
&\quad + s_j\dot{\hat{\psi}}_j \tanh\left(\frac{s_j^2}{\delta_j}\right) + 2\frac{s_j^2}{\delta_j}\hat{\psi}_j \operatorname{sech}^2\left(\frac{s_j^2}{\delta_j}\right)\Bigg]
\end{aligned}$$

for $j = 2, \ldots, n-1$. For any positive definite matrix Q_i, the equation $A_i^\top G_i + G_i A_i = -Q_i$ is satisfied with a positive definite symmetric matrix G_i.

Substituting (19), (26), (31), and (56) into the time derivative of V gives

$$\begin{aligned}\dot{V} \leq \sum_{i=1}^{n}\Bigg[&-k_i s_i^2+\frac{1}{\gamma_{\xi,i}}\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\dot{\xi}_i\\&+\frac{1}{h_{i,m}}s_i(h_{i,m}\varepsilon_i+d_i^l)-s_i^4+\frac{\kappa_i}{4}+\iota_i\\&-s_i^2\tilde{\theta}_i-s_i^2\tilde{\psi}_i\tanh\left(\frac{s_i^2}{\delta_i}\right)-s_i^2\psi_i^*\tanh\left(\frac{s_i^2}{\delta_i}\right)\\&+\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\Bigg\{\Bigg(k_i s_i^2+s_i^2\hat{\theta}_i\\&+s_i^2\hat{\psi}_i\tanh\left(\frac{s_i^2}{\delta_i}\right)\Bigg)-\frac{\dot{\xi}_i}{\gamma_{\xi,i}}\Bigg\}\Bigg]\\&+\sum_{i=1}^{n-1}\left[\frac{h_i^l}{h_{i,m}}s_i(s_{i+1}+\tilde{\alpha}_{i,1})\right]+\frac{h_n^l}{h_{n,m}}s_n(u-\alpha_n)\\&+\sum_{i=1}^{n-1}[-\tilde{\alpha}_i^\top Q_i\tilde{\alpha}_i+2\tilde{\alpha}_i^\top G_i D\Xi_i]. \end{aligned} \tag{57}$$

Then, by applying (36), we have

$$\begin{aligned}&\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\left\{\left(k_i s_i^2+s_i^2\hat{\theta}_i+s_i^2\hat{\psi}_i\tanh\left(\frac{s_i^2}{\delta_i}\right)\right)-\frac{\dot{\xi}_i}{\gamma_{\xi,i}}\right\}\\&=\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\Bigg[(k_i+\hat{\theta}_i-\sigma_{\xi,i}\xi_i)(s_i^2-s_i'^2)\\&\quad+\hat{\psi}_i\left\{s_i^2\tanh\left(\frac{s_i^2}{\delta_i}\right)-s_i'^2\tanh\left(\frac{s_i'^2}{\delta_i}\right)\right\}\Bigg]\\&\quad+\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\sigma_{\xi,i}s_i^2\xi_i. \end{aligned} \tag{58}$$

Lemma 4-(ii) leads to $|s_i^2\tanh(s_i^2/\delta_i)-s_i'^2\tanh(s_i'^2/\delta_i)| \leq 1.1997|s_i^2-s_i'^2|$. From $\hat{\theta}_i=\tilde{\theta}_i+\theta_i$, $\hat{\psi}_i=\tilde{\psi}_i+\psi_i^*$, Lemmas 5 and 6, and $s_i^2-s_i'^2=2(s_i-s_i')s_i-(s_i-s_i')^2$, there exist positive constants $\Lambda_{1,i}$ and $\Lambda_{2,i}$ such that

$$\begin{aligned}&\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\Bigg[(k_i+\hat{\theta}_i-\sigma_{\xi,i}\xi_i)(s_i^2-s_i'^2)\\&\quad+\hat{\psi}_i\left\{s_i^2\tanh\left(\frac{s_i^2}{\delta_i}\right)-s_i'^2\tanh\left(\frac{s_i'^2}{\delta_i}\right)\right\}\Bigg]\\&\leq\left|\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right||s_i^2-s_i'^2|(k_i+|\hat{\theta}_i|+\sigma_{\xi,i}|\xi_i|\\&\quad+1.1997|\hat{\psi}_i|)\\&\leq\Lambda_{1,i}|s_i|+\Lambda_{2,i} \end{aligned} \tag{59}$$

where $\Lambda_{1,i} \triangleq 2Z_{s,i}(\Lambda_{N,i}+1)(k_i+(\Lambda_{\tilde{\theta},i}^*+\theta_i)+\sigma_{\xi,i}\Lambda_{\xi,i}^*+1.1997(\Lambda_{\tilde{\psi},i}^*+\psi_i^*))$ and $\Lambda_{2,i} \triangleq Z_{s,i}^2(\Lambda_{N,i}+1)(k_i+(\Lambda_{\tilde{\theta},i}^*+\theta_i)+\sigma_{\xi,i}\Lambda_{\xi,i}^*+1.1997(\Lambda_{\tilde{\psi},i}^*+\psi_i^*))$.

Using (58) and (59), (57) becomes

$$\begin{aligned}\dot{V} \leq \sum_{i=1}^{n}\Bigg[&-k_i s_i^2+\frac{1}{\gamma_{\xi,i}}\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\dot{\xi}_i\\&+\frac{1}{h_{i,m}}s_i(h_{i,m}\varepsilon_i+d_i^l)-s_i^4+\frac{\kappa_i}{4}+\iota_i\\&-s_i^2\tilde{\theta}_i-s_i^2\tilde{\psi}_i\tanh\left(\frac{s_i^2}{\delta_i}\right)-s_i^2\psi_i^*\tanh\left(\frac{s_i^2}{\delta_i}\right)\\&+\Lambda_{1,i}|s_i|+\Lambda_{2,i}+\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\sigma_{\xi,i}s_i^2\xi_i\Bigg]\\&+\sum_{i=1}^{n-1}\left[\frac{h_i^l}{h_{i,m}}s_i(s_{i+1}+\tilde{\alpha}_{i,1})\right]+\frac{h_n^l}{h_{n,m}}s_n(u-\alpha_n)\\&+\sum_{i=1}^{n-1}\left[-\tilde{\alpha}_i^\top Q_i\tilde{\alpha}_i+2\tilde{\alpha}_i^\top G_i D\Xi_i\right]. \end{aligned} \tag{60}$$

Since $\tilde{\theta}_i$, $\tilde{\psi}_i$, and ξ_i are bounded from Lemma 5, there exist positive functions Ξ_1^* and Ξ_j^* such that

$$\begin{aligned}&|\Xi_1(\xi_1,\bar{s}_2,\tilde{\alpha}_{1,1},y_r,\dot{y}_r,\hat{\theta}_1,\hat{\psi}_1)|\\&\quad\leq\Xi_1^*(\bar{s}_2,\tilde{\alpha}_{1,1},y_r,\dot{y}_r)\\&|\Xi_j(\xi_j,\bar{s}_{j+1},\bar{\tilde{\alpha}}_{j,1},y_r,\tilde{\alpha}_{j-1,2},\hat{\theta}_j,\hat{\psi}_j)|\\&\quad\leq\Xi_j^*(\bar{s}_{j+1},\bar{\tilde{\alpha}}_{j,1},y_r,\tilde{\alpha}_{j-1,2}) \end{aligned} \tag{61}$$

for $j=2,\ldots,n-1$. Then, using Lemmas 5 and 6, we have

$$\frac{h_i^l}{h_{i,m}}s_i(s_{i+1}+\tilde{\alpha}_{i,1}) \leq s_i^2+\frac{1}{2}s_{i+1}^2+\frac{1}{2}\|\tilde{\alpha}_i\|^2 \tag{62}$$

$$\frac{h_n^l}{h_{n,m}}s_n(u-\alpha_n) \leq \frac{s_n^2 Z_u^2}{\bar{\epsilon}_1}+\frac{\bar{\epsilon}_1}{4} \tag{63}$$

$$|2\tilde{\alpha}_i^\top G_i D\Xi_i| \leq \frac{\Xi_i^{*2}\|G_i\|^2\|\tilde{\alpha}_i\|^2}{\epsilon_{i,1}}+\epsilon_{i,1} \tag{64}$$

$$\Lambda_{1,i}|s_i| \leq \frac{\Lambda_{1,i}^2 s_i^2}{\epsilon_{i,2}}+\frac{\epsilon_{i,2}}{4} \tag{65}$$

$$\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\sigma_{\xi,i}s_i^2\xi_i \leq \frac{1}{2}s_i^4+\frac{1}{2}(\Lambda_{N,i}+1)^2\sigma_{\xi,i}^2\Lambda_{\xi,i}^{*2} \tag{66}$$

$$-s_i^2\tilde{\psi}_i\tanh\left(\frac{s_i^2}{\delta_i}\right) \leq \frac{1}{2}s_i^4+\frac{1}{2}\Lambda_{\tilde{\psi},i}^{*2} \tag{67}$$

$$\frac{1}{h_{i,m}}s_i(h_{i,m}\varepsilon_i+d_i^l) \leq \frac{s_i^2\varepsilon_i^2}{\bar{\epsilon}_{i,2}}+\frac{1}{\bar{\epsilon}_{i,2}h_{i,m}^2}s_i^2 d_i^{l2}+\frac{\bar{\epsilon}_{i,2}}{2} \tag{68}$$

where $\epsilon_{i,1}$, $\epsilon_{i,2}$, $\bar{\epsilon}_1$ and $\bar{\epsilon}_{i,2}$ are positive constants.

Applying these inequalities to (57) yields

$$\begin{aligned}\dot{V} \leq &-(k_1-1)s_1^2-\sum_{i=2}^{n-1}\left(k_i-\frac{3}{2}\right)s_i^2-\left(k_n-\frac{1}{2}\right)s_n^2\\&-\sum_{i=1}^{n-1}\left(q_i-\frac{1}{2}\right)\|\tilde{\alpha}_i\|^2+\sum_{i=1}^{n-1}\frac{\Xi_i^{*2}\|G_i\|^2\|\tilde{\alpha}_i\|^2}{\epsilon_{i,1}}\\&+\sum_{i=1}^{n}\frac{1}{\gamma_{\xi,i}}\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\dot{\xi}_i\\&+\sum_{i=1}^{n}\left[s_i^2\psi_i-s_i^2\psi_i^*\tanh\left(\frac{s_i^2}{\delta_i}\right)\right]+C_1 \end{aligned} \tag{69}$$

where $q_i = \lambda_{\min}(Q_i)$, $\psi_j = \Lambda_{1,j}^2/\epsilon_{j,2} + \varepsilon_j^2/\bar{\epsilon}_{j,2} + d_j^{l2}/(\bar{\epsilon}_{j,2}h_{j,m}^2) - \tilde{\theta}_j$, $j = 1,\ldots,n-1$, $\psi_n = \Lambda_{1,n}^2/\epsilon_{n,2} + \varepsilon_n^2/\bar{\epsilon}_{n,2} + d_n^{l2}/(\bar{\epsilon}_{n,2}h_{n,m}^2) - \tilde{\theta}_n + Z_u^2$, and $C_1 = \sum_{i=1}^n [(1/2)(\Lambda_{N,i}+1)^2\sigma_{\xi,i}^2\Lambda_{\xi,i}^2+(1/2)\Lambda_{\tilde{\psi},i}^{*2}+\Lambda_{2,i}+(\kappa_i+\epsilon_{i,2})/4+\iota_i] + \sum_{i=1}^{n-1}(\epsilon_{i,1}+\bar{\epsilon}_{i,2}/2)+\bar{\epsilon}_1/4$.

From Assumption 2 and the boundedness of $\tilde{\theta}_i$, $\tilde{\psi}_i$, and ε_i, it holds that

$$\begin{aligned}|\psi_j| &\leq \Lambda_{1,j}^2/\epsilon_{j,2} + \varepsilon_j^{*2}/\bar{\epsilon}_{j,2} + d_j^{*2}/(\bar{\epsilon}_{j,2}h_{j,m}^2) + \Lambda_{\tilde{\theta},j}^* \triangleq \psi_j^* \\ |\psi_n| &\leq \Lambda_{1,n}^2/\epsilon_{n,2} + \varepsilon_n^{*2}/\bar{\epsilon}_{n,2} + d_n^{*2}/(\bar{\epsilon}_{n,2}h_{n,m}^2) + \Lambda_{\tilde{\theta},n}^* \\ &\quad + Z_u^2 \triangleq \psi_n^* \end{aligned} \tag{70}$$

where $j = 1,\ldots,n-1$ and $d_i^* = \max_{l\in P}\{d_{i,0}^l\}$, $i = 1,\ldots,n$. Then, from Lemma 2, we have the following inequality

$$|s_i^2\psi_i| \leq s_i^2\psi_i^* \leq s_i^2\psi_i^* \tanh\left(\frac{s_i^2}{\delta_i}\right) + 0.2785\psi_i^*\delta_i \tag{71}$$

for $i = 1,\ldots,n$.

Consider the compact sets $\Omega_i \in \mathbb{R}^{2(i-1)+1}$, $i = 1,\ldots,n$ and $\Omega_R \in \mathbb{R}^2$ as $\Omega_i = \{s_1^2/(2h_{1,m}) + \sum_{j=1}^{i-1}[s_{j+1}^2/(2h_{j+1,m}) + \tilde{\alpha}_j^\top G_j\tilde{\alpha}_j] \leq \mu\}$ and $\Omega_R = \{y_r^2 + \dot{y}_r^2 \leq R\}$ with a positive constant R. Since the sets Ω_i and Ω_R are compact, $\Omega_i \times \Omega_R$ are also compact sets. Then, there exist constants $\breve{\Xi}_i$ such that $|\Xi_i^*| \leq \breve{\Xi}_i$ on $\Omega_i\times\Omega_R$ for $i = 1,\ldots,n-1$. Setting the design parameters $k_1 = \bar{\eta}_1 + 1$, $k_j = \bar{\eta}_j + (3/2)$, $j = 2,\ldots,n-1$, $k_n = \bar{\eta}_n + (1/2)$, and $q_i = (1/2) + \breve{\Xi}_i^2\|G_i\|^2/\epsilon_{i,1} + \bar{q}_i$, $i = 1,\ldots,n-1$, with positive constants $\bar{\eta}_1$, $\bar{\eta}_j$, $\bar{\eta}_n$, and $\bar{q}_i$, it is satisfied that

$$\begin{aligned}\dot{V} \leq &-\sum_{i=1}^n \bar{\eta}_i s_i^2 - \sum_{i=1}^{n-1}\bar{q}_i\|\tilde{\alpha}_i\|^2 \\ &-\sum_{i=1}^{n-1}\left(1-\frac{\Xi_i^{*2}}{\breve{\Xi}_i^2}\right)\frac{\breve{\Xi}_i^2\|G_i\|^2\|\tilde{\alpha}_i\|^2}{\epsilon_{i,1}} \\ &+\sum_{i=1}^n\frac{1}{\gamma_{\xi,i}}\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\dot{\xi}_i + C_2\end{aligned} \tag{72}$$

where $C_2 = C_1 + \sum_{i=1}^n 0.2785\psi_i^*\delta_i$. Since $|\Xi_i^*| \leq \breve{\Xi}_i$ on $\Omega_i \times \Omega_R$, for $i = 1,\ldots,n-1$, the inequality (72) becomes

$$\dot{V} \leq -\eta V + \sum_{i=1}^n\frac{1}{\gamma_{\xi,i}}\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\dot{\xi}_i + C_2 \tag{73}$$

where $\eta = \min_{i=1,\ldots,n,j=1,\ldots,n-1}\{2h_{i,m}\bar{\eta}_i, \bar{q}_j/\lambda_{\max}(G_j)\}$.

Solving the above inequality, we have

$$V(t) \leq e^{-\eta t}\int_0^t\sum_{i=1}^n\frac{1}{\gamma_{\xi,i}}\left(\frac{h_i^l}{h_{i,m}}N(\xi_i)+1\right)\dot{\xi}_i e^{\eta\varsigma}d\varsigma + C_3 \tag{74}$$

where $C_3 = V(0) + C_2/\eta$. By Assumption 1, we have $|h_i^l/h_{i,m}| \leq 1$. Then, from Lemma 3, it is ensured that $V(t)$, $\xi_i(t)$, $i = 1,\ldots,n$, and $\int_0^t\sum_{i=1}^n\frac{1}{\gamma_{\xi,i}}\frac{h_i^l}{h_{i,m}}N(\xi_i)\dot{\xi}_i d\varsigma$ are bounded for $\forall t \in [0, t_f)$. According to Proposition 2 in [60], it also holds that $t_f = \infty$. Thus, all the closed-loop signals are bounded. This implies that s_i, α_i, $\hat{\alpha}_{j,1}$, $\hat{\alpha}_{j,2}$, and α_n are bounded for $i = 1,\ldots,n$ and $j = 1,\ldots,n-1$. From Lemma 6, s_i', α_i', $\hat{\alpha}_{j,1}'$, $\hat{\alpha}_{j,2}'$, and u are also bounded. Thus, there exist constants $\bar{\Lambda}$ such that $|\sum_{i=1}^n((h_i^l/h_{i,m})N(\xi_i)+1)\dot{\xi}_i/\gamma_{\xi,i}| \leq \bar{\Lambda}$ on $\Omega_n \times \Omega_R$ where $i = 1,\ldots,n$.

Thus, the inequality (73) becomes

$$\dot{V} \leq -\eta V + C_4 \tag{75}$$

where $C_4 = C_2+\bar{\Lambda}$. Thus, the time derivative of V is negative for $V = \mu$ when $\eta > C_4/\mu$. This implies that $V \leq \mu$ is an invariant set. Moreover, the inequality (74) can be rewritten as

$$V(t) \leq (V(0) - C_4/\eta)e^{-\eta t} + C_4/\eta. \tag{76}$$

Owing to $(1/(2h_{1,m}))s_1^2 \leq V$, the tracking error s_1 converges to a compact set $\Omega_O = \{s_1| \, |s_1| \leq \sqrt{2h_{1,m}C_4/\eta}\}$. By adjusting η, the compact set Ω_O can be made arbitrarily small. ■

Remark 7: The design parameters presented in the proof of Theorem 1 are sufficient conditions. The guidelines for setting design parameters are provided as follows.

(i) Increasing k_i helps to increase η. Thus, the convergence bound $\sqrt{2h_{1,m}C_4/\eta}$ of the tracking error can be reduced.
(ii) Setting $\sigma_{\psi,i}$, $\sigma_{\theta,i}$, and $\sigma_{\xi,i}$ to be small constants and increasing $\gamma_{\psi,i}$, $\gamma_{\theta,i}$, and $\gamma_{\xi,i}$ results in the faster learning speed of $\hat{\psi}_i$, $\hat{\theta}_i$, and ξ_i respectively.
(iii) As δ_i decreases, C_4 also decreases. Thus, the convergence bound of the tracking error can be reduced.

IV. SIMULATION EXAMPLE

A. EXAMPLE 1

Consider the following switched second-order nonlinear system:

$$\begin{aligned}\dot{x}_1 &= f_1^{\rho(t)}(\bar{x}_2) + d_1^{\rho(t)}(t) \\ \dot{x}_2 &= f_2^{\rho(t)}(\bar{x}_2, u^{\rho(t)}) + d_2^{\rho(t)}(t) \\ y &= x_1\end{aligned} \tag{77}$$

where $\rho(t) : [0,+\infty) \to P = \{1,2\}$, $f_1^1 = (2 + 0.2\cos(x_1))x_2 + 0.2x_1^2$, $f_1^2 = (2 + 0.1e^{x_1})x_2 + 0.5\sin(x_1)$, $f_2^1 = (2 + 0.2\cos(x_1x_2))u$, $f_2^2 = (2 + 0.1e^{x_1x_2})u + 0.5x_1x_2$, $d_1^1 = d_1^2 = 0.1\sin(\pi t)$, and $d_2^1 = d_2^2 = 0.1\cos(\pi t)$. The nonlinear functions f_1^1, f_1^2, f_2^1, and f_2^2, and the disturbances d_1^1, d_1^2, d_2^1, and d_2^2 are assumed to be unknown. For the simulation, the initial conditions are set to $\bar{x}_2 = [0, 0]^\top$, the quantization interval is given as $\Delta = 0.01$, and the reference signal is defined as $y_r(t) = 0.5\cos(0.5t)+0.5\cos(1.1t)$. The proposed common quantized state feedback controller for this simulation is given by

$$\begin{aligned}\alpha_1' &= N(\xi_1)\left(k_1s_1' + s_1'\hat{\theta}_1 + s_1'\hat{\psi}_1\tanh\left(\frac{s_1'^2}{\delta_1}\right)\right) \\ u &= N(\xi_2)\left(k_2s_2' + s_2'\hat{\theta}_2 + s_2'\hat{\psi}_2\tanh\left(\frac{s_2'^2}{\delta_2}\right)\right)\end{aligned}$$

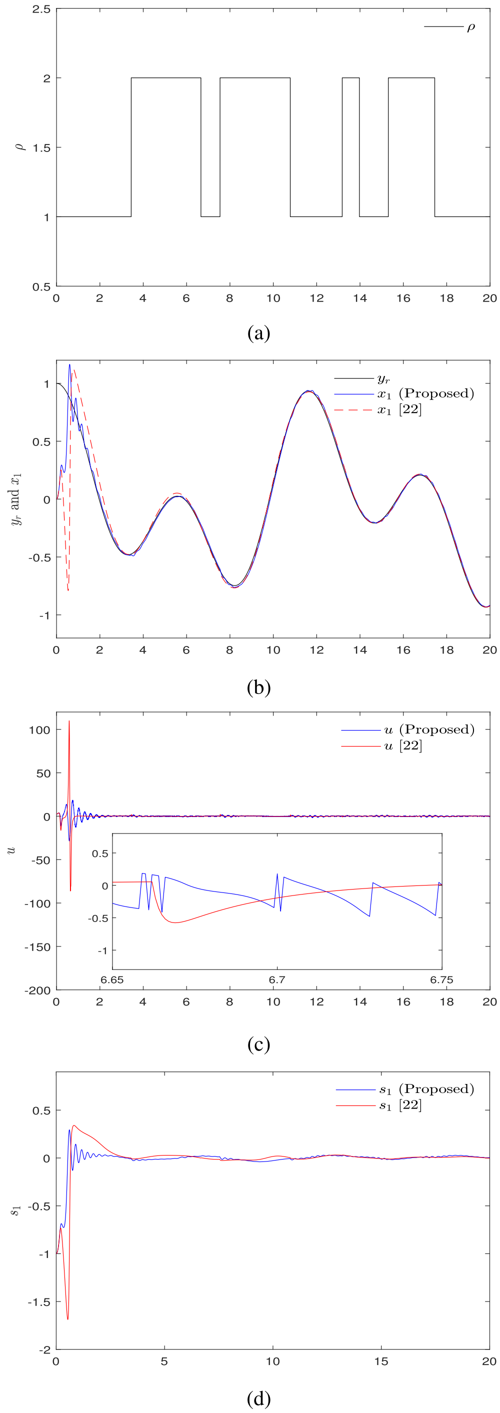

FIGURE 1. Comparison of the simulation results for Example 1 (a) Switching signal ρ (b) Tracking result (i.e., x_1 and y_r) (c) Control input u (d) Tracking error s_1.

$$\dot{\xi}_i = \gamma_{\xi,i}\left(k_i s_i'^2 + s_i'^2\hat{\theta}_i + s_i'^2\hat{\psi}_i \tanh\left(\frac{s_i'^2}{\delta_i}\right) - \sigma_{\xi,i}s_i'^2\xi_i\right)$$
$$\dot{\hat{\theta}}_i = \gamma_{\theta,i}\left(s_i'^2 - \sigma_{\theta,i}s_i'^2\hat{\theta}_i\right)$$
$$\dot{\hat{\psi}}_i = \gamma_{\psi,i}\left(s_i'^2 \tanh\left(\frac{s_i'^2}{\delta_i}\right) - \sigma_{\psi,i}s_i'^2\hat{\psi}_i\right)$$

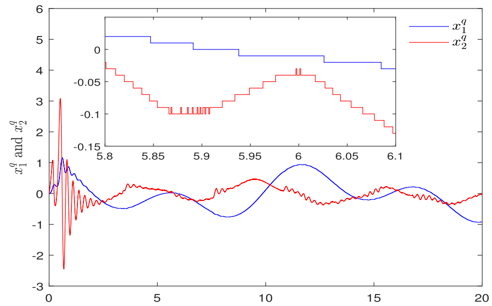

FIGURE 2. Quantized states of the proposed approach for Example 1.

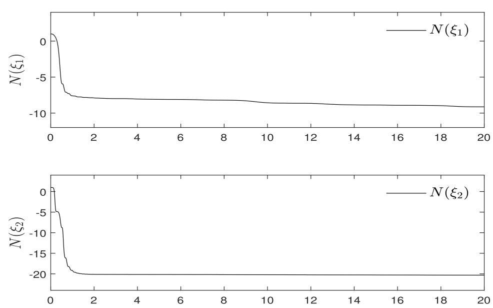

FIGURE 3. Outputs of Nussbaum functions $N(\xi_1)$ and $N(\xi_2)$ of the proposed approach for Example 1.

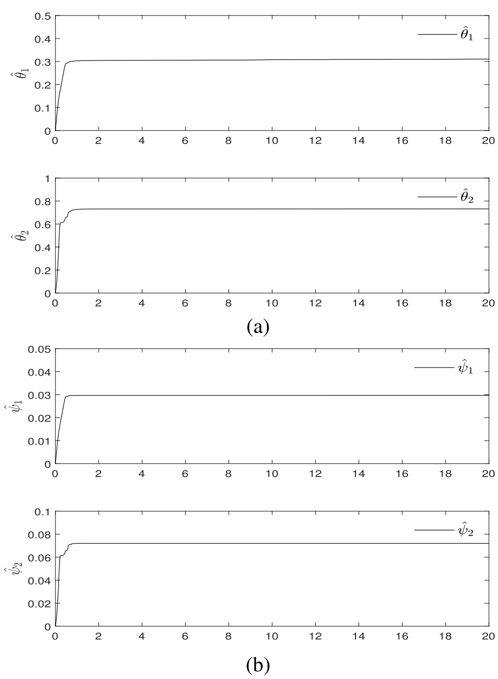

FIGURE 4. Estimates of θ_i and ψ_i of the proposed approach for Example 1 (a) $\hat{\theta}_1$ and $\hat{\theta}_2$ (c) $\hat{\psi}_1$ and $\hat{\psi}_2$.

$$\dot{\hat{\alpha}}'_{1,1} = \hat{\alpha}'_{1,2}$$
$$\dot{\hat{\alpha}}'_{1,2} = -2\zeta_1\omega_1\hat{\alpha}'_{1,2} - \omega_1^2(\hat{\alpha}'_{1,1} - \alpha'_1) \tag{78}$$

where $i = 1, 2$, $k_1 = 1.1$, $k_2 = 2$, $\delta_1 = \delta_2 = 0.1$, $\gamma_{\xi,1} = 4$, $\gamma_{\xi,2} = 1$, $\gamma_{\theta,1} = \gamma_{\theta,2} = 1$, $\gamma_{\psi,1} = \gamma_{\psi,2} = 0.1$,

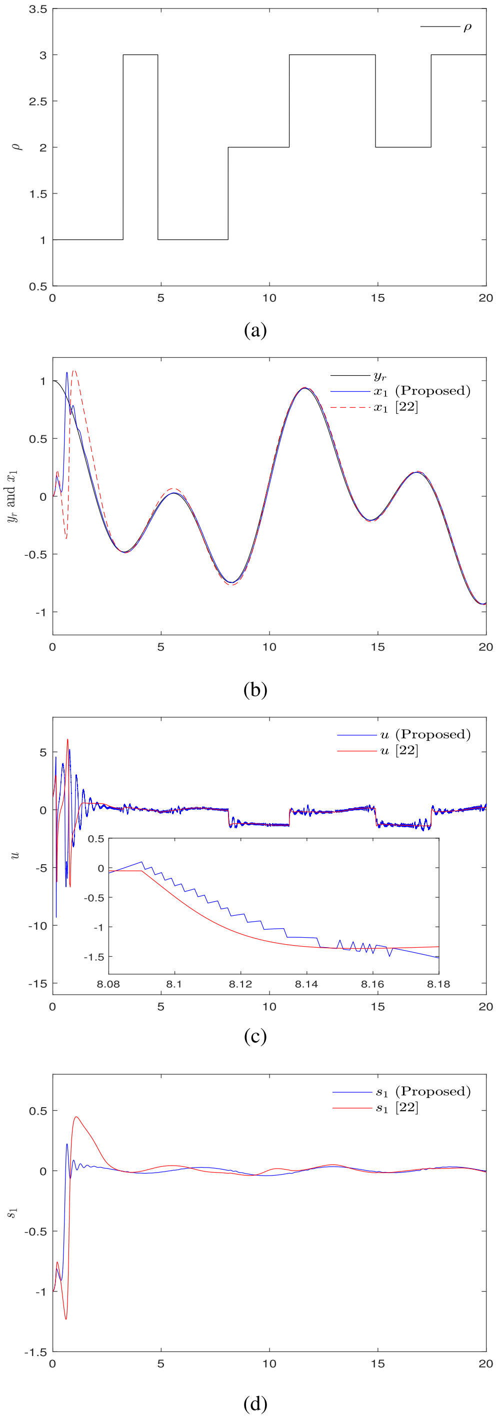

FIGURE 5. Comparison of the simulation results for Example 2 (a) Switching signal ρ (b) Tracking result (i.e., x_1 and y_r) (c) Control input u (d) Tracking error s_1.

$\sigma_{\xi,1} = \sigma_{\xi,2} = \sigma_{\theta,1} = \sigma_{\theta,2} = \sigma_{\psi,1} = \sigma_{\psi,2} = 0.01$, $\zeta_1 = 0.707$, and $\omega_1 = 30$.

The simulation results using the proposed approach are compared with the results obtained by the neural network-based adaptive tracker designed without signal quantization in [22]. The design parameters of the previous controller [22] are set to be the same as those for the proposed controller. The switching signal is given in Fig. 1(a). The tracking results, control inputs, and tracking errors are compared in Fig. 1(b)-(d). Although the proposed tracker is designed based on quantized state feedback, the tracking performance of the proposed approach is comparable to that of the unquantized state feedback controller [22] for switched nonlinear systems. Fig. 2 displays the time responses of the quantized states of the proposed approach. The outputs of the Nussbaum functions for the proposed approach are plotted in Fig. 3. The estimates of θ_1, θ_2, ψ_1, and ψ_2 for the proposed approach are shown in Fig. 4. Based on these simulation results, we can see that the tracking error converges to the neighborhood of the origin. This means that the proposed quantized feedback control strategy ensures good tracking performance even if all the states are quantized for feedback and the control directions and pure-feedback nonlinearities are unknown under arbitrary switching.

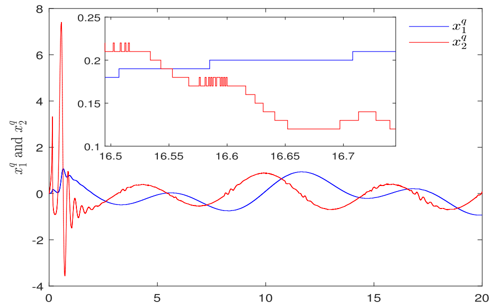

FIGURE 6. Quantized states of the proposed approach for Example 2.

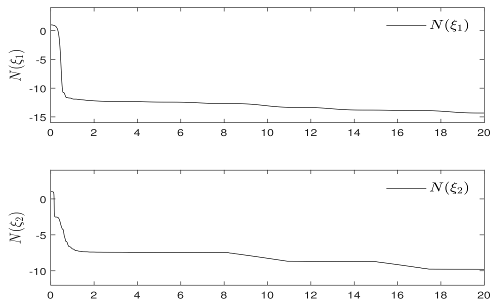

FIGURE 7. Outputs of Nussbaum functions $N(\xi_1)$ and $N(\xi_2)$ of the proposed approach for Example 2.

B. EXAMPLE 2

In this simulation, we consider the tracking problem of the Van der Pol oscillator that represents various types of electrical circuits [61]. The following dynamics of the oscillator is given by

$$\begin{aligned} \dot{x}_1 &= f_1^{\rho(t)}(\bar{x}_2) + d_1^{\rho(t)}(t) \\ \dot{x}_2 &= f_2^{\rho(t)}(\bar{x}_2, u^{\rho(t)}) + d_2^{\rho(t)}(t) \\ y &= x_1 \end{aligned} \tag{79}$$

where $\rho(t) : [0, +\infty) \to P = \{1, 2, 3\}$, $f_1^1 = f_1^2 = f_1^3 = x_2$, $f_2^1 = -2(x_1^2 - 1)x_2 - x_1 + (2 + \sin(x_1 x_2))$

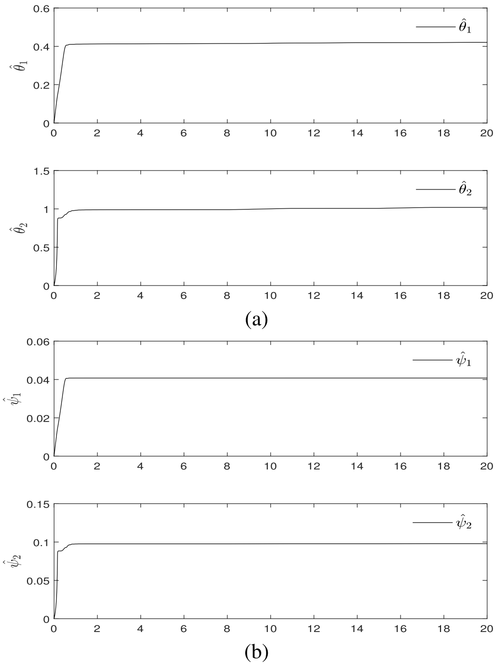

FIGURE 8. Estimates of θ_i and ψ_i of the proposed approach for Example 2 (a) $\hat{\theta}_1$ and $\hat{\theta}_2$ (b) $\hat{\psi}_1$ and $\hat{\psi}_2$.

$[u + (1/3)u^3 + \sin(u)]$, $f_2^2 = \cos(x_1x_2) + (2 + \sin(x_1x_2))[(1/3)u^3 + \cos(u)]$, and $f_2^3 = \sin(x_2^2) + (2 + \cos(x_1x_2))[3u^3 + \sin(u)]$, $d_1^1 = d_1^2 = d_1^3 = 0.1\cos(t)$ and $d_2^1 = d_2^2 = d_2^3 = 0.2\sin(t)$. Here, f_1^1 and f_2^1 represent the inherent nonlinearities of the Van der Pol oscillator. To consider the switched pure-feedback nonlinearities, it is assumed that the nonlinear function f_2^1 is switched to f_2^2 and f_2^3. For the simulation, the initial conditions are set to $\bar{x}_2 = [0, 0]^\top$, the quantization interval is given as $\Delta = 0.01$, and the reference signal is defined as $y_r(t) = 0.5\cos(0.5t) + 0.5\cos(1.1t)$. The design parameters for this simulation are chosen as $k_1 = 1.1$, $k_2 = 1$, $\delta_1 = \delta_2 = 0.1$, $\gamma_{\xi,1} = 3$, $\gamma_{\xi,2} = 1$, $\gamma_{\theta,1} = \gamma_{\theta,2} = 1$, $\gamma_{\psi,1} = \gamma_{\psi,2} = 0.1$, $\sigma_{\xi,1} = \sigma_{\xi,2} = \sigma_{\theta,1} = \sigma_{\theta,2} = \sigma_{\psi,1} = \sigma_{\psi,2} = 0.01$, $\zeta_1 = 0.707$, and $\omega_1 = 30$.

A comparison of the simulation results is presented in Fig. 5. It is observed that the performance of the proposed quantized feedback tracker is similar to that of the unquantized state feedback controller [22], regardless of state quantization. Fig. 6 displays the time responses of the quantized states of the proposed approach. The outputs of the Nussbaum functions for the proposed approach are given in Fig. 7. The boundedness of the estimated parameters $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\psi}_1$, and $\hat{\psi}_2$ for the proposed approach is illustrated in Fig. 8. These figures demonstrate the effectiveness of the proposed theoretical result.

V. CONCLUSION

We have addressed a state-quantized adaptive tracker design problem for arbitrarily switched uncertain pure-feedback nonlinear systems with unknown control directions. Contrary to the related results in literature, our major contributions lie in the use of the quantized state feedback information and the analysis of quantization errors to derive an adaptive tracking controller in the presence of unknown control directions and switched nonaffine nonlinearities. The stability of the closed-loop system has been proved by inducing bounding lemmas and using the common Lyapunov function method, Simulation results have successfully verified the proposed theoretical approach. Further studies on the quantized-state-based prescribed performance control problem [62]–[64] of switched nonlinear systems can be recommended as future work.

APPENDIX I: PROOF OF LEMMA 4

(i) Consider the function $T_1(\varphi) = \varphi\tanh\left(\varphi^2/\delta\right)$ with a constant $\delta > 0$ and any $\varphi \in \mathbb{R}$. Then, from the straightforward algebraic manipulation, the derivative of $T_1(\varphi)$ with respect to φ is bounded as

$$\begin{aligned}\frac{dT_1(\varphi)}{d\varphi} &= \tanh\left(\frac{\varphi^2}{\delta}\right) + 2\frac{\varphi^2}{\delta}\operatorname{sech}^2\left(\frac{\varphi^2}{\delta}\right)\\ &\le 1.6017. \end{aligned} \tag{80}$$

Using (80), we obtain the following inequality

$$\begin{aligned} &T_1(\varphi_1) - T_1(\varphi_2)\\ &\quad= \int_{\varphi_2}^{\varphi_1}\left[\tanh\left(\frac{\varphi^2}{\delta}\right) + 2\frac{\varphi^2}{\delta}\operatorname{sech}^2\left(\frac{\varphi^2}{\delta}\right)\right]d\varphi,\\ &|T_1(\varphi_1) - T_1(\varphi_2)|\\ &\quad\le \left|\int_{\varphi_2}^{\varphi_1} 1.6017\,d\varphi\right|\\ &\quad= 1.6017|\varphi_1 - \varphi_2|. \end{aligned} \tag{81}$$

The above inequality leads to

$$\begin{aligned}&\left|\varphi_1\tanh\left(\frac{\varphi_1^2}{\delta}\right) - \varphi_2\tanh\left(\frac{\varphi_2^2}{\delta}\right)\right|\\ &\quad\le 1.6017|\varphi_1 - \varphi_2|. \end{aligned} \tag{82}$$

(ii) Consider the function $T_2(\varphi) = \varphi\tanh(\varphi/\delta)$ with a constant $\delta > 0$ and any $\varphi \in \mathbb{R}$. Then, the derivative of $T_2(\varphi)$ with respect to φ is bounded as

$$\begin{aligned}\frac{dT_2(\varphi)}{d\varphi} &= \tanh\left(\frac{\varphi}{\delta}\right) + \frac{\varphi}{\delta}\operatorname{sech}^2\left(\frac{\varphi}{\delta}\right)\\ &\le 1.1997. \end{aligned} \tag{83}$$

Then it holds that

$$\begin{aligned} T_2(\varphi_1^2) - T_2(\varphi_2^2) &= \int_{\varphi_2^2}^{\varphi_1^2}\left[\tanh\left(\frac{\varphi}{\delta}\right)\right.\\ &\quad\left. + \frac{\varphi}{\delta}\operatorname{sech}^2\left(\frac{\varphi}{\delta}\right)\right]d\varphi,\\ |T_2(\varphi_1^2) - T_2(\varphi_2^2)| &\le \left|\int_{\varphi_2^2}^{\varphi_1^2} 1.1997\,d\varphi\right|\\ &= 1.1997|\varphi_1^2 - \varphi_2^2|. \end{aligned} \tag{84}$$

The above inequality leads to

$$\left|\varphi_1^2 \tanh\left(\frac{\varphi_1^2}{\delta}\right) - \varphi_2^2 \tanh\left(\frac{\varphi_2^2}{\delta}\right)\right| \leq 1.1997|\varphi_1^2 - \varphi_2^2|. \quad (85)$$

Then, Lemma 4 is proved.

APPENDIX II: PROOF OF LEMMA 5

(i) Let us define a Lyapunov function $V_{\theta,i} = (1/(2\gamma_{\theta,i}))\tilde{\theta}_i^\top \tilde{\theta}_i$. Then, the time derivative of $V_{\theta,i}$ is

$$\begin{aligned}\dot{V}_{\theta,i} &= \tilde{\theta}_i^\top \left(s_i'^2 - \sigma_{\theta,i} s_i'^2 \hat{\theta}_i\right) \\ &= \tilde{\theta}_i^\top \left(s_i'^2 - \sigma_{\theta,i} s_i'^2 (\theta_i + \tilde{\theta}_i)\right). \end{aligned} \quad (86)$$

Then, it holds that

$$\dot{V}_{\theta,i} \leq \|\tilde{\theta}_i\| s_i'^2 \left(1 + \sigma_{\theta,i}\theta_i - \sigma_{\theta,i}\|\tilde{\theta}_i\|\right). \quad (87)$$

$\dot{V}_{\theta,i}$ is negative when $\|\tilde{\theta}_i\| > \Lambda_{\tilde{\theta},i}$ where $\Lambda_{\tilde{\theta},i} \triangleq (1 + \sigma_{\theta,i}\theta_i)/\sigma_{\theta,i}$. Therefore, if $\tilde{\theta}_i(0) \in \Omega_{\tilde{\theta},i}$, it is ensured that $\tilde{\theta}_i(t) \in \Omega_{\tilde{\theta},i}$, $\forall t \geq 0$.

(ii) Consider $V_{\psi,i} = (1/(2\gamma_{\psi,i}))\tilde{\psi}_i^2$. From $\tanh(\cdot) \leq 1$ and $\hat{\psi}_i = \psi_i^* + \tilde{\psi}_i$, $\dot{V}_{\psi,i}$ satisfies

$$\dot{V}_{\psi,i} \leq |\tilde{\psi}_i| s_i'^2 \left(1 + \sigma_{\psi,i}\psi_i^* - \sigma_{\psi,i}|\tilde{\psi}_i|\right). \quad (88)$$

From the inequality (88), $\dot{V}_{\psi,i}$ is negative when $|\tilde{\psi}_i| > \Lambda_{\tilde{\psi},i}$ with $\Lambda_{\tilde{\psi},i} \triangleq (1 + \sigma_{\psi,i}\psi_i^*)/\sigma_{\psi,i}$. Therefore, if $\tilde{\psi}_i(0) \in \Omega_{\tilde{\psi},i}$, it is guaranteed that $\tilde{\psi}_i(t) \in \Omega_{\tilde{\psi},i}$, $\forall t \geq 0$.

(iii) Consider $V_{\xi,i} = (1/(2\gamma_{\xi,i}))\xi_i^2$. The boundedness of $\tilde{\theta}_i$ and $\tilde{\psi}_i$ are shown in i and ii. Therefore, it holds that

$$\begin{aligned}\dot{V}_{\xi,i} \leq |\xi_i| s_i'^2 \big(&k_i + (\theta_i + \Lambda_{\tilde{\theta},i}^*) \\ &+ (\psi_i^* + \Lambda_{\tilde{\psi},i}^*) - \sigma_{\xi,i}|\xi_i|\big)\end{aligned} \quad (89)$$

where $\Lambda_{\tilde{\theta},i}^* = \max\{|\tilde{\theta}_i(0)|, \Lambda_{\tilde{\theta},i}\}$ and $\Lambda_{\tilde{\psi},i}^* = \max\{|\tilde{\psi}_i(0)|, \Lambda_{\tilde{\psi},i}\}$. Then, by defining $\Lambda_{\xi,i} \triangleq (k_i + (\theta_i + \Lambda_{\tilde{\theta},i}^*) + (\psi_i^* + \Lambda_{\tilde{\psi},i}^*))/\sigma_{\xi,i}$, we have that if $\xi_i(0) \in \Omega_{\xi,i}$, then $\xi_i(t) \in \Omega_{\xi,i}$, $\forall t \geq 0$.

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