


## Weyl gravity extension of Higgs inflation

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 (Received 15 June 2023; accepted 11 August 2023; published 28 August 2023)

We consider the embedding of Higgs inflation with a nonminimal coupling into the Weyl gravity. In this model, the effective current-current interactions from the heavy Weyl gauge field cancel the noncanonical Higgs kinetic term in Einstein frame, so the unitarity problem of the original Higgs inflation becomes less severe. For a simple case where the couplings of the heavy Weyl gauge field appears from the nonminimal couplings to the Ricci curvature scalar in Weyl gravity, we find that the resultant model for Higgs inflation is the same as in the Palatini formulation for Higgs inflation. The crucial difference of our model from the Palatini formulation for Higgs inflation is that there is a light Weyl gauge field coupled to the Higgs fields. We also generalize the unitarization of Higgs inflation with general covariant kinetic terms for the dilaton and the Higgs fields and realize a successful Higgs inflation, interpolating between the Palatini formulation for Higgs inflation and a Higgs-like inflation. We also discuss the Higgs mechanism for the light Weyl gauge field with an extra singlet scalar and show some interesting signatures for Higgs physics, such as the overall suppression of Higgs couplings and the direct couplings of the light Weyl gauge field to the Higgs boson.

DOI: [10.1103/PhysRevD.108.035045](https://doi.org/10.1103/PhysRevD.108.035045)

### I. INTRODUCTION

Effective field theories have been important tools for particle physics and cosmology due to the fact that it is enough to keep a finite number of parameters consistent with symmetries at low energies for accurate calculations of observable quantities. The Standard Model (SM) has been shown to be a consistent effective field theory, which is an efficient and accurate way of describing interactions of fundamental particles, probably up to very high energies, due to the null results for new physics at the Large Hadron Collider (LHC). Nonetheless, there are hints for new physics solving the empirical problems such as neutrino masses, baryon number asymmetry, dark matter, and the theoretical problems such as the hierarchy problem, the cosmological constant problem, the flavor hierarchy problem, etc.

Cosmic inflation requires a slowly rolling extra scalar field, the so-called inflaton, in the early era of cosmology, solving the problems for homogeneity, isotropy, relics, etc. Quantum fluctuations of the inflaton set the initial conditions for the standard big bang cosmology by generating

the seeds for inhomogeneities of the cosmic microwave background (CMB) and the large-scale structures. In usual cases, we need to go beyond the SM with a singlet inflaton and introduce extra couplings of the inflaton appropriately for reheating process. Although it is a challenging task to find a consistent framework for inflation models, the power spectrum of the observed CMB anisotropies such as Planck data [1] have put meaningful constraints on the inflation models through the spectral index, the tensor-to-scalar ratio, etc.

The Higgs boson is the only fundamental scalar field in the SM, so it is a good candidate for the inflaton. Then, it is an interesting possibility to regard the Higgs boson as the inflaton, instead of introducing an extrascalar field beyond the SM. Higgs inflation [2] is the minimal extension of the SM with a nonminimal coupling  $\xi_H$  of the Higgs fields to gravity, resulting in the consistent predictions for inflation. However, a large nonminimal coupling gives rise to a premature violation of unitarity from the noncanonical Higgs kinetic terms in Einstein frame, so the Higgs inflation is still an effective field theory (the validity of which is limited below the unitarity scale,  $M_P/\xi_H$  [3]). Thus, there are several proposals to unitarize the Higgs inflation in linear sigma models with an extra singlet scalar field [4–6] or with  $R^2$  or higher curvature terms [7–13].

In this article, we make an attempt to embed the Higgs inflation in the context of Weyl gravity where the conformal symmetry is gauged by a Weyl gauge field [14]. In this framework, the conformal symmetry is spontaneously broken by the vacuum expectation value (VEV) of the

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dilaton scalar. As a result, the Planck scale and the Higgs mass parameter are generated dynamically, and the Weyl gauge field coupled to the dilaton becomes heavy [15]. We study the conditions that the effective current-current interactions coming from the Weyl gauge field cancel the noncanonical Higgs kinetic terms in Higgs inflation such that the unitarity scale gets higher than in Higgs inflation. In the cases with one or two Weyl gauge fields, we show how a solution to unitarity problem in Higgs inflation and a successful inflation are achieved at the same time. The couplings of one Weyl gauge field is always absorbed into the nonminimal couplings to the Ricci curvature scalar in Weyl gravity, but those of the other Weyl gauge field is not. We show the interplay of inflationary predictions with the unitarity scale in the case with two Weyl gauge fields.

The paper is organized as follows. We first review the unitarity problem in Higgs inflation and identify the noncanonical Higgs kinetic terms in Einstein frame as the current-current interactions for the heavy Weyl gauge field. Then, we propose the Weyl invariant Lagrangian with nonminimal couplings for the dilaton and the Higgs fields, and discuss the roles of the second Weyl gauge field for unitarization and inflation, in comparison to the case with a single Weyl gauge field. Next we generalize the Weyl invariant Lagrangian with extra kinetic terms for the dilaton and the Higgs fields, which are not absorbed into the nonminimal couplings to the Ricci curvature scalar in Weyl gravity. We continue to discuss the implications of the light Weyl gauge field for Higgs physics and collider experiments. Finally, conclusions are drawn. There are two appendices for the comparison to the unitarization of Higgs inflation with a singlet scalar and the details on the generalized Weyl-invariant Lagrangian.

## II. HIGGS INFLATION AND UNITARIZATION

We consider the Higgs inflation with a nonminimal coupling in relation to the unitarity problem and sketch a new way of unitarizing the Higgs inflation in the presence of the heavy Weyl gauge coupled to the Higgs fields.

### A. Unitarity problem in Higgs inflation

The Lagrangian for Higgs inflation [2] is given by

$$\mathcal{L} = \sqrt{-g} \left( -\frac{1}{2} (M_P^2 + 2\xi_H |H|^2) R + |D_\mu H|^2 - V(H) \right), \quad (1)$$

where  $V(H)$  is the Higgs potential,

$$V(H) = m_H^2 |H|^2 + \lambda_H |H|^4. \quad (2)$$

Making a Weyl transformation by  $g_{E,\mu\nu} = \Omega g_{\mu\nu}$  with  $\Omega = 1 + 2\xi_H |H|^2/M_P^2$ , we get the Einstein frame Lagrangian as

$$\mathcal{L}_E = \sqrt{-g_E} \left( -\frac{M_P^2}{2} R + \frac{1}{\Omega} |D_\mu H|^2 + \frac{3\xi_H^2}{M_P^2} \frac{(\partial_\mu |H|^2)^2}{\Omega^2} - \frac{V(H)}{\Omega^2} \right). \quad (3)$$

Then, for  $2\xi_H |H|^2 \gg M_P^2$ , a slow-roll inflation takes place, because the Einstein frame is given by  $V_E \simeq \frac{\lambda_H}{4\xi_H^2} M_P^4 (1 + M_P^2/(2\xi_H |H|^2))^{-2}$ , and the canonical Higgs field identified during inflation is given by  $\chi/M_P = \sqrt{\frac{3}{2}} \ln(2\xi_H |H|^2/M_P^2)$ , leading to  $V_E \simeq \frac{\lambda_H}{4\xi_H^2} M_P^4 (1 + e^{-\sqrt{\frac{3}{2}}\chi/M_P})^{-2}$ . Thus, the inflationary predictions are consistent with Planck data under the condition that  $\xi_H \sim 10^4 \sqrt{\lambda_H}$  is fixed by the CMB normalization. Therefore, we need a large nonminimal coupling  $\xi_H \gg 1$  for a sizable  $\lambda_H$  during inflation.

The noncanonical Higgs kinetic term with  $\xi_H \gg 1$  in Eq. (3) contains a dimension-six operator,  $(\partial_\mu |H|^2)^2$ , with the cutoff scale given by  $\Lambda \sim M_P/\xi_H$ , so it leads to a premature violation of unitarity at  $\Lambda = M_P/\xi_H$ , which is much below the inflation scale,  $V_I \sim M_P/\sqrt{\xi_H}$  [3]. Such a low cutoff scale in the vacuum and during reheating casts doubt on the validity of the semiclassical description of Higgs inflation at large Higgs field values. For unitarizing the Higgs inflation beyond the unitarity scale, as reviewed in Appendix A, several extensions of the Higgs inflation, such as linear sigma models [4,5], singlet scalars [6],  $R^2$  term [7–12], and general higher curvature terms [10,13], have been proposed.

### B. Weyl current interactions for unitarization

For the nonminimal coupling for the Higgs in the Jordan frame, we take the Noether current for the conformal transformation, given by  $K_\mu = \partial_\mu K_H$  with  $K_H = 12\xi_H |H|^2$ . Then, we can rewrite the non-canonical Higgs kinetic term by the current-current interaction,

$$\frac{\mathcal{L}_{H,\text{eff}}}{\sqrt{-g_E}} = \frac{3\xi_H^2}{M_P^2} \frac{(\partial_\mu |H|^2)^2}{\Omega^2} = \frac{1}{48M_P^2} \frac{K_\mu K^\mu}{\Omega^2}. \quad (4)$$

The purpose of the following discussion is to propose the necessary Weyl couplings to cancel the above dangerous interactions in Weyl gravity.

Now we consider a Weyl gauge field with mass  $m_w^2 = 6g_w^2 M_P^2$ , with the following Jordan frame Lagrangian,

$$\frac{\mathcal{L}_J}{\sqrt{-g}} = -\frac{1}{4} w_{\mu\nu} w^{\mu\nu} + \frac{1}{2} m_w^2 w_\mu w^\mu - \frac{1}{2} g_w w_\mu K^\mu + \frac{1}{2} g_w^2 w_\mu w^\mu K_H. \quad (5)$$

The above form of the Lagrangian can be obtained from a nonminimal coupling of the Higgs to Weyl gravity, as will be discussed in Sec. III A and later sections. Then, after

integrating out the Weyl gauge field with its equation of motion,

$$w_\mu = \frac{g_w}{2} \frac{K_\mu}{m_w^2 + g_w^2 K_H}, \quad (6)$$

we get the effective interactions for the Higgs as

$$\begin{aligned} \frac{\mathcal{L}_{J,\text{eff}}}{\sqrt{-g}} &= -\frac{g_w^2}{8} \frac{K_\mu K^\mu}{m_w^2 + g_w^2 K_H}, \\ &= -\frac{1}{48M_P^2} \frac{K_\mu K^\mu}{\Omega}. \end{aligned} \quad (7)$$

As a result, the corresponding Einstein frame Lagrangian is

$$\frac{\mathcal{L}_{E,\text{eff}}}{\sqrt{-g_E}} = -\frac{1}{48M_P^2} \frac{K_\mu K^\mu}{\Omega^2}. \quad (8)$$

Therefore, the effective interaction coming from the Weyl gauge field in Eq. (8) cancels exactly the noncanonical Higgs kinetic term in Eq. (4), so the unitarity problem in Higgs inflation disappears. In the following, we consider a concrete realization of the effective interactions in Weyl gravity. We note that the dimension-six operator of the form  $(\partial_\mu |H|^2)^2$  as in Eq. (8) was introduced in Jordan frame for unitarity [16], but the origin of such a higher-dimensional operator was not discussed. In contrast, in our work, we will show for the first time that the counter term in Ref. [16] is the part of the Weyl invariant Lagrangian by construction.

### III. MINIMAL WEYL GRAVITY FOR HIGGS INFLATION

In this section, we introduce the minimal Weyl invariant Lagrangian with a single Weyl gauge field  $w_\mu$  and discuss the Higgs inflation and the unitarity problem in this case.

#### A. Weyl invariant Lagrangian

In Weyl gravity, the Weyl gauge field can be introduced as a part of the redefined Christoffel symbols with a Weyl gauge coupling  $g_w$  [15,17] by

$$\tilde{\Gamma}_{\mu\nu}^\rho = \Gamma_{\mu\nu}^\rho + g_w \left( \delta_\nu^\rho w_\mu + \delta_\mu^\rho w_\nu - g_{\mu\nu} w^\rho \right), \quad (9)$$

which is Weyl invariant. Thus, the resulting Ricci scalar in Weyl gravity [15,17] is given in terms of the one in Einstein gravity and the Weyl gauge field contribution from  $w_\mu$ , as follows:

$$\tilde{R}(\tilde{\Gamma}) = R(\Gamma) - 6g_w D_\mu w^\mu - 6g_w^2 w^\mu w_\mu, \quad (10)$$

with  $D_\mu w^\mu = \partial_\mu w^\mu + \frac{1}{2} w^\rho g^{\lambda\beta} \partial_\rho g_{\lambda\beta}$ . Consequently, we can construct the minimal Lagrangian with one Weyl gauge field  $w_\mu$  and the Higgs, as follows:

$$\frac{\mathcal{L}_{\text{min}}}{\sqrt{-g}} = -\frac{1}{2} (\xi_\phi \phi^2 + 2\xi_H |H|^2) \tilde{R}(\tilde{\Gamma}) - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} - V(H, \phi), \quad (11)$$

which is Weyl invariant under the following transformations:

$$\begin{aligned} g_{\mu\nu} &\rightarrow e^{2\alpha} g_{\mu\nu}, & \phi &\rightarrow e^{-\alpha} \phi, & H &\rightarrow e^{-\alpha} H, \\ w_\mu &\rightarrow w_\mu - \frac{1}{g_w} \partial_\mu \alpha. \end{aligned} \quad (12)$$

Here, the scalar potential is given by

$$V(H, \phi) = \frac{1}{2} \lambda_{\phi H} \phi^2 |H|^2 + \frac{1}{4} \lambda_\phi \phi^4 + \lambda_H |H|^4. \quad (13)$$

The Lagrangian in Eq. (11) is expanded as follows:

$$\begin{aligned} \frac{\mathcal{L}_{\text{min}}}{\sqrt{-g}} &= -\frac{1}{2} \xi_\phi \left( \phi^2 R + 6(\partial_\mu \phi)^2 - 6(D_\mu \phi)^2 \right) - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} \\ &\quad - \xi_H \left( |H|^2 R + 6|\partial_\mu H|^2 - 6|D_\mu H|^2 \right) - V(H, \phi). \end{aligned} \quad (14)$$

Here, we rewrote the Weyl current interactions to the dilaton and Higgs fields through the nonminimal couplings by integration by parts and absorbed them in the Weyl covariant derivative terms,  $(D_\mu \phi)^2$  and  $|D_\mu H|^2$ . However, in this case, there is no net kinetic term for Higgs or dilaton in the Jordan frame. As a result,  $\phi^2 R + 6(\partial_\mu \phi)^2$  and  $|H|^2 R + 6|\partial_\mu H|^2$  are Weyl invariant, respectively, so the Weyl invariance is manifest in Einstein gravity.

Therefore, we need to extend the minimal case in Eq. (14) by introducing the Higgs kinetic term in the Jordan frame in the following way. Using Eq. (B1) with a single Weyl gauge field  $w_\mu$  in Appendix B, we can generalize the above Lagrangian in Eq. (14) to the general Weyl invariant Lagrangian [15],

$$\begin{aligned} \frac{\mathcal{L}_1}{\sqrt{-g}} &= -\frac{1}{2} \xi_\phi \left( \phi^2 R + 6(\partial_\mu \phi)^2 - 6r_\phi (D_\mu \phi)^2 \right) - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} \\ &\quad - \xi_H \left( |H|^2 R + 6|\partial_\mu H|^2 - 6r_H |D_\mu H|^2 \right) - V(H, \phi). \end{aligned} \quad (15)$$

Here, we note that  $r_\phi$  and  $r_H$  parametrize the relative strength of the gravitational couplings for two Weyl invariants,  $\phi_i^2 R + 6(\partial_\mu \phi_i)^2$  and  $(D_\mu \phi_i)^2$ , so they cannot be eliminated by the field redefinitions. But we need to take  $r_H = 1 + \frac{1}{6\xi_H}$  for the canonical kinetic term for the Higgs fields in Jordan frame in the following discussion.

We remark that Weyl symmetry appears anomalous at the quantum level in usual dimensional regularization or cutoff regularization where an explicit mass scale is introduced. However, Weyl symmetry can be respected at the quantum level in the presence of quantum scale invariance [15,18]. In this case, the renormalization scale  $\mu$  is replaced by the dilaton scalar  $\phi$  at the expense of nonrenormalizable dilaton interactions and the loop corrections can be included systematically while preserving the Weyl symmetry.

Fixing the gauge by  $\langle\phi^2\rangle = M_P^2/\xi_\phi$ , from Eq. (15), we obtain the gauge-fixed Lagrangian as follows:

$$\begin{aligned} \frac{\mathcal{L}_2}{\sqrt{-g}} = & -\frac{1}{2}(M_P^2 + 2\xi_H|H|^2)R + |\partial_\mu H|^2 - V(H) \\ & -\frac{1}{4}w_{\mu\nu}w^{\mu\nu} + \frac{1}{2}m_w^2 w_\mu w^\mu - \frac{1}{2}g_w w_\mu K^\mu + \frac{1}{2}g_w^2 w_\mu w^\mu K_H \end{aligned} \quad (16)$$

with

$$\begin{aligned} \frac{\mathcal{L}_E}{\sqrt{-g_E}} = & -\frac{M_P^2}{2}R + 6\xi_H(r_H - 1)\frac{|\partial_\mu H|^2}{\Omega} + \frac{3\xi_H^2}{M_P^2}\frac{(\partial_\mu|H|^2)^2}{\Omega^2} - \frac{V(H)}{\Omega^2} \\ & -\frac{1}{4}\tilde{w}_{\mu\nu}\tilde{w}^{\mu\nu} + \frac{1}{2}(m_w^2 + g_w^2 r_H K_H)\Omega^{-1}\tilde{w}_\mu\tilde{w}^\mu - \frac{g_w^2 r_H^2}{8\Omega}\frac{K_\mu K^\mu}{m_w^2 + g_w^2 r_H K_H}, \end{aligned} \quad (18)$$

where  $\tilde{w}_\mu$  is similarly redefined as in Eq. (B3) but with the modified  $K_H$ . This result gives rise to the Lagrangian relevant for inflation as follows:

$$\begin{aligned} \frac{\mathcal{L}_{E,\text{inf}}}{\sqrt{-g_E}} = & -\frac{M_P^2}{2}R + \frac{1}{\Omega}(6\xi_H(r_H - 1))|\partial_\mu H|^2 + \frac{3\xi_H^2}{M_P^2\Omega^2}\frac{r_\phi - r_H^2 + 2r_H(1 - r_H)\xi_H|H|^2/M_P^2}{r_\phi + 2r_H\xi_H|H|^2/M_P^2}(\partial_\mu|H|^2)^2 - \frac{V(H)}{\Omega^2}, \\ = & -\frac{M_P^2}{2}R + \frac{1}{\Omega}|\partial_\mu H|^2 + \frac{1}{M_P^2\Omega^2}\frac{3r_\phi\xi_H^2 - 3(\xi_H + \frac{1}{6})^2 - \xi_H(\xi_H + \frac{1}{6})|H|^2/M_P^2}{r_\phi + 2(\xi_H + \frac{1}{6})|H|^2/M_P^2}(\partial_\mu|H|^2)^2 - \frac{V(H)}{\Omega^2}, \end{aligned} \quad (19)$$

where we chose  $r_H = 1 + \frac{1}{6\xi_H}$  for the Higgs fields in Jordan frame to take a canonical form. Thus, expanding the above Lagrangian for the Higgs about zero, we obtain the leading higher dimensional terms,

$$\begin{aligned} \frac{\mathcal{L}_{E,\text{inf}}}{\sqrt{-g_E}} \supset & -\frac{2\xi_H}{M_P^2}|H|^2|\partial_\mu H|^2 + \frac{3}{M_P^2}\left(\xi_H^2 - \frac{1}{r_\phi}\left(\xi_H + \frac{1}{6}\right)^2\right)(\partial_\mu|H|^2)^2 \\ = & \frac{3}{M_P^2}\left(\xi_H^2 - \frac{1}{r_\phi}\left(\xi_H + \frac{1}{6}\right)^2 + \frac{\xi_H}{3}\right)(\partial_\mu|H|^2)^2 + \dots, \end{aligned} \quad (20)$$

where integration by parts are made, and the equation of motion for the Higgs fields, namely,  $\square H \simeq -\frac{\partial V_E}{\partial H}$ , with  $V_E = V(H)/\Omega^2$ , is used in the second line. Here, we note that there also appear extra higher-dimensional terms from the potential, but they are suppressed by powers of  $M_P/\sqrt{\xi_H}$ . Therefore, from Eq. (20), the unitarity scale is identified from the leading dimension-six operator by

$$V(H) = \frac{\lambda_\phi M_P^4}{4\xi_\phi^2} + \frac{\lambda_{\phi H} M_P^2}{2\xi_\phi}|H|^2 + \lambda_H|H|^2. \quad (17)$$

The second line in the above Lagrangian takes the same form as in Eq. (5), except that  $m_w^2 = 6r_\phi g_w^2 M_P^2$  and  $K_H$  are now modified to  $K_H = 12r_H \xi_H |H|^2 = 2(1 + 6\xi_H)|H|^2$ , instead of  $K_H = 12\xi_H |H|^2$ . Here, we can choose a very small  $\lambda_\phi$  for the observed cosmological constant, and the effective Higgs mass is determined by  $m_H^2 = \frac{\lambda_{\phi H}}{2\xi_\phi} M_P^2$ . For electroweak symmetry breaking, we need to choose  $\lambda_{\phi H} < 0$  and  $|\lambda_{\phi H}/(2\xi_\phi)| \ll 1$ .

## B. Einstein frame Lagrangian

Taking the same procedure as in Appendix B (namely, the conformal transformation to Einstein frame and the field redefinition of the Weyl gauge field), we obtain the Einstein frame Lagrangian after a gauge fixing,  $\langle\phi^2\rangle = M_P^2/\xi_\phi$ , as follows:

$$\Lambda_1 = \frac{M_P}{\left|\xi_H(3\xi_H + 1)\left(1 - \frac{1}{r_\phi}\right) - \frac{1}{12r_\phi}\right|^{1/2}}, \quad (21)$$

which becomes the order of the Planck scale for  $r_\phi = 1$ , independent of  $\xi_H$ . Indeed, for  $r_\phi = 1$ , the Ricci curvature scalar for the Higgs field metric becomes constant,  $R(H) = -2/M_P^2$ , so the Higgs fields live in a

four-dimensional hyperbolic space with the curvature along the order of the Planck scale. However, the higher-dimensional terms from the scalar potential lead to the cutoff scale,  $\Lambda = M_P/\sqrt{\xi_H}$ , which is still much larger than the one in Higgs inflation.

### C. Inflation

We consider the case with  $r_\phi = r_H = 1 + \frac{1}{6\xi_H}$  for simplicity. Then, the Weyl gauge field is decoupled from the Higgs fields, and the Lagrangian in Eq. (19) gets simplified to

$$\frac{\mathcal{L}_{E,\text{inf}}}{\sqrt{-g_E}} = -\frac{M_P^2}{2}R + \frac{1}{\Omega}|\partial_\mu H|^2 - \frac{\xi_H}{2M_P^2\Omega^2}(\partial_\mu|H|^2)^2 - \frac{V(H)}{\Omega^2}. \quad (22)$$

We note that the unitarity scale is given by  $\Lambda = M_P/\sqrt{\xi_H}$  for  $\xi_H \gtrsim 1$ . In unitary gauge for the Higgs fields, Eq. (22) becomes

$$\frac{\mathcal{L}_{E,\text{inf}}}{\sqrt{-g_E}} = -\frac{M_P^2}{2}R + \frac{(\partial_\mu h)^2}{2(1 + \xi_H h^2/M_P^2)^2} - \frac{V(h)}{(1 + \xi_H h^2/M_P^2)^2}. \quad (23)$$

Taking  $h \gg M_P/\sqrt{\xi_H}$  during inflation, we find that the coefficient of the Higgs kinetic term is dominated by  $1/h^4$ , so the canonical field  $\chi$  is approximated to  $\chi \simeq -M_P^2/(\xi_H h)$ . Thus, the inflaton potential in Einstein frame becomes

$$V_E = \frac{\lambda_H M_P^4}{4\xi_H^2} \left(1 + \frac{M_P^2}{\xi_H h^2}\right)^{-2} \simeq \frac{\lambda_H M_P^4}{4\xi_H^2} \left(1 + \frac{\xi_H \chi^2}{M_P^2}\right)^{-2}. \quad (24)$$

In this case, we need to choose a very small  $\xi_H$  for a successful inflation [15], so it belongs to a different class of inflation models where there is no unitarity problem below the Planck scale from the beginning. Similarly, for  $r_\phi \neq r_H$ , the Weyl gauge field is not decoupled from the Higgs. However, as far as the Weyl gauge field is sufficiently heavier than the Hubble scale during inflation, a slow-roll inflation can be still realized for a very small  $\xi_H$  even for  $r_\phi \neq r_H$  as in Ref. [15].

We conclude this section by saying that the minimal Weyl gravity with a very small  $\xi_H$  is an interesting possibility without a unitarity problem below the Planck scale. However, we would need a very small  $\lambda_H$  to get the correct CMB normalization, which requires a severe fine-tuning of the low-energy parameters for a small running Higgs quartic coupling at the inflation scale. Moreover, in our work, we are interested in the

dynamical mechanism for unitarizing the original Higgs inflation with a large nonminimal coupling. So, we consider a possibility of extending the Weyl symmetry in the next sections.

## IV. HIGGS INFLATION WITH EXTENDED WEYL SYMMETRY

We consider the Weyl invariant Lagrangian with the dilaton and the Higgs fields, in the presence of non-minimal couplings for them. We introduce the nonminimal couplings to the modified Ricci curvature scalar in Weyl gravity that contains the couplings of the Weyl gauge fields and add an extra kinetic term for the Higgs fields with an extra Weyl gauge field. In this case, we discuss the unitarity scale and the inflationary predictions.

### A. Lagrangian with two Weyl gauge fields

For a concrete realization of unitarization and successful Higgs inflation with a large nonminimal coupling, we consider the Lagrangian including the dilaton  $\phi$  and two Weyl gauge fields,  $w_\mu$  and  $X_\mu$ , in the Jordan frame. Another Weyl gauge field  $X_\mu$  does not appear in the Ricci scalar but it gives rise to an extra Weyl-invariant kinetic term for the Higgs field.

We first discuss the origin of two Weyl gauge fields. Since the Weyl gauge fields are originated from the local scale symmetries, we need to consider the doubled coordinates,  $x_1^\mu$  and  $x_2^\mu$ , leading to the extended diffeomorphism invariance in Weyl gravity,  $\text{Diff}_1 \times \text{Diff}_2$ , with two metric tensors,  $g_{1,\mu\nu}$  and  $g_{2,\mu\nu}$ , and the corresponding Christoffel symbols,  $\tilde{\Gamma}_{1,\mu\nu}^\rho$  and  $\tilde{\Gamma}_{2,\mu\nu}^\rho$ , Weyl gauge fields,  $w_{1,\mu}$  and  $w_{2,\mu}$ , as well as the dilaton scalars,  $\phi_1$  and  $\phi_2$ . Then, including a pair of Higgs fields,  $H_1$  and  $H_2$ , with the nonminimal couplings in the extended Weyl gravity, we can generalize the action for the conformal sector in Eq. (11) to

$$S = \sum_{i=1,2} \int dv_i \left[ -\frac{1}{2}(\xi_i \phi_i^2 + 2\zeta_i |H_i|^2) \tilde{R}(\tilde{\Gamma}_i) - \frac{1}{4} w_{i,\mu\nu} w_i^{\mu\nu} \right] + \Delta S, \quad (25)$$

where  $dv_i \equiv d^4 x_i \sqrt{-g_i}$  are the infinitesimal spacetime volumes,  $\xi_i, \zeta_i$  ( $i = 1, 2$ ) are the nonminimal couplings for the dilaton and Higgs fields, the Ricci scalar in Weyl gravity is decomposed into  $\tilde{R}(\tilde{\Gamma}_i) = R(\Gamma_i) - 6g_{w_i} D_\mu w_i^\mu - 6g_{w_i}^2 w_i^\mu w_{i,\mu}$  with  $g_{w_i}$  ( $i = 1, 2$ ) being the Weyl gauge couplings, and  $\Delta S$  are the interaction terms between the Weyl gauge fields and the general covariant derivative terms for the Higgs fields given by

$$\begin{aligned}
\Delta S = & \sum_{i=1,2} \int dv_i (-3a_i \xi_i \phi_i^2) (g_{w_1} w_{1,\mu} + \kappa_i g_{w_2} w_{2,\mu}) (g_{w_1} w_{1,\nu} + \kappa_i g_{w_2} w_{2,\nu}) g_i^{\mu\nu} \\
& + \sum_{i=1,2} \int dv_i (-6\hat{a}_i \zeta_i |H_i|^2) (g_{w_1} w_{1,\mu} + \hat{\kappa}_i g_{w_2} w_{2,\mu}) (g_{w_1} w_{1,\nu} + \hat{\kappa}_i g_{w_2} w_{2,\nu}) g_i^{\mu\nu} \\
& + \sum_{i=1,2} \int dv_i (\partial_\mu - b_i g_{w_1} w_{1,\mu} - c_i g_{w_2} w_{2,\mu}) H_i^\dagger (\partial_\nu - b_i g_{w_1} w_{1,\nu} - c_i g_{w_2} w_{2,\nu}) H_i g_i^{\mu\nu}, \quad (26)
\end{aligned}$$

with  $a_i, \hat{a}_i, \kappa_i, \hat{\kappa}_i, b_i, c_i (i = 1, 2)$  being extra couplings. Then, the action  $S$  is Weyl invariant under

$$\begin{aligned}
g_{i,\mu\nu} & \rightarrow e^{2\alpha_i(x_i)} g_{i,\mu\nu}, & \phi_i & \rightarrow e^{-\alpha_i(x_i)} \phi_i, & H_i & \rightarrow e^{-\alpha_i(x_i)} H_i, \\
w_{i,\mu} & \rightarrow w_{i,\mu} - \frac{1}{g_{w_i}} \frac{\partial \alpha_i(x_i)}{\partial x_i^\mu}, \quad (27)
\end{aligned}$$

with  $\alpha_i (i = 1, 2)$  being two independent Weyl transformation parameters. On the other hand, the extra action  $\Delta S$  with generic parameters breaks the extended diffeomorphism invariance as well as the Weyl invariance explicitly. However, after the doubled coordinates are identified by  $x_1^\mu = x_2^\mu$ ,  $\Delta S$  becomes diffeomorphism invariant, as well as Weyl invariant, provided that

$$\alpha_1 + \kappa_i \alpha_2 = 0, \quad \alpha_1 + \hat{\kappa}_i \alpha_2 = 0, \quad i = 1, 2 \quad (28)$$

and

$$b_i \alpha_1 + c_i \alpha_2 = \alpha_i, \quad i = 1, 2. \quad (29)$$

Suppose that the full diffeomorphism invariance is broken by  $\text{Diff}_1 \times \text{Diff}_2 \rightarrow \text{Diff}_0$ , such that two coordinates, two metric tensors and the dilaton scalars are identified by  $x_1^\mu = x_2^\mu$ ,  $g_{1,\mu\nu} = g_{2,\mu\nu} \equiv g_{\mu\nu}$  and  $\phi_1 = \phi_2 \equiv \phi/\sqrt{2}$ , respectively. Moreover, we also identify two Higgs fields by  $H_1 = H_2 \equiv H/\sqrt{2}$ . Furthermore, taking  $\xi_1 = \xi_2 \equiv \xi_\phi$  and  $\zeta_1 = \zeta_2 \equiv \xi_H$ , and imposing the following conditions for the parameters in the extra action  $\Delta S$  in Eq. (26),

$$a_1 + a_2 = a_1 \kappa_1^2 + a_2 \kappa_2^2 = -a_1 \kappa_1 - a_2 \kappa_2 = \frac{1}{2}, \quad (30)$$

with  $\hat{a}_i = a_i$  and  $\hat{\kappa}_i = \kappa_i (i = 1, 2)$ , and

$$b_1 = b_2 = \frac{g_{w_2} g_X}{2g_{w_1} g_w}, \quad c_1 = c_2 = -\frac{g_{w_1} g_X}{2g_{w_2} g_w}, \quad (31)$$

we obtain the effective Weyl gravity Lagrangian in terms of two redefined Weyl gauge fields,  $w_\mu$  and  $X_\mu$ , as follows:

$$\begin{aligned}
\mathcal{L}_{\text{eff}} = & \sqrt{-g} \left[ -\frac{1}{2} (\xi_\phi \phi^2 + 2\xi_H |H|^2) \tilde{R}(\tilde{\Gamma}) \right. \\
& \left. - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + |D'_\mu H|^2 \right], \quad (32)
\end{aligned}$$

where the Ricci scalar in the effective Weyl gravity is given by  $\tilde{R}(\tilde{\Gamma}) = R(\Gamma) - 6g_w D_\mu w^\mu - 6g_w^2 w^\mu w_\mu$  in terms of the redefined Christoffel symbols:

$$\begin{aligned}
\tilde{\Gamma}_{\mu\nu}^\rho & = \frac{1}{2} (\tilde{\Gamma}_{1,\mu\nu}^\rho + \tilde{\Gamma}_{2,\mu\nu}^\rho), \\
& = \Gamma_{\mu\nu}^\rho + g_w (\delta_\mu^\rho w_\nu + \delta_\nu^\rho w_\mu - g_{\mu\nu} w^\rho). \quad (33)
\end{aligned}$$

Here,  $g_w = \frac{1}{2} \sqrt{g_{w_1}^2 + g_{w_2}^2}$ , and the redefined Weyl gauge fields are given by  $w_\mu = (g_{w_1} w_{1,\mu} + g_{w_2} w_{2,\mu}) / \sqrt{g_{w_1}^2 + g_{w_2}^2}$  and  $X_\mu = (g_{w_2} w_{1,\mu} - g_{w_1} w_{2,\mu}) / \sqrt{g_{w_1}^2 + g_{w_2}^2}$ . We note that there are three equations for four unknown parameters,  $a_i, \kappa_i (i = 1, 2)$ , in Eq. (30), so generically there is one parameter family of the solutions realizing the effective Weyl gravity. Moreover, the covariant derivative with  $X_\mu$  for the Higgs fields is given  $D'_\mu H = (\partial_\mu - g_X X_\mu) H$  with an independent gauge coupling  $g_X$ .

We remark that the Weyl transformations of the redefined Weyl gauge fields,  $w_\mu$  and  $X_\mu$ , become

$$w_\mu \rightarrow w_\mu - \frac{1}{\sqrt{g_{w_1}^2 + g_{w_2}^2}} \partial_\mu (\alpha_1 + \alpha_2), \quad (34)$$

$$X_\mu \rightarrow X_\mu - \frac{1}{\sqrt{g_{w_1}^2 + g_{w_2}^2}} \partial_\mu \left( \frac{g_{w_2}}{g_{w_1}} \alpha_1 - \frac{g_{w_1}}{g_{w_2}} \alpha_2 \right). \quad (35)$$

As a result, we can identify the remaining Weyl symmetry in the effective Lagrangian in Eq. (32) by

$$w_\mu \rightarrow w_\mu - \frac{1}{g_w} \partial_\mu \alpha, \quad X_\mu \rightarrow X_\mu - \frac{1}{g_X} \partial_\mu \alpha, \quad (36)$$

with the Weyl transformation parameter being identified as

$$\alpha = \frac{1}{2} (\alpha_1 + \alpha_2) \quad (37)$$

and

$$\alpha_2 = \frac{g_{w_2}/g_{w_1} - g_X/g_w}{g_{w_1}/g_{w_2} + g_X/g_w} \alpha_1. \quad (38)$$

Thus, both of the two Weyl transformations are nontrivial, as far as  $|g_X/g_w| \neq |g_{w_2}/g_{w_1}|$ .

We remark on the extra Weyl-invariant terms with two Weyl gauge fields,  $w_\mu$  and  $X_\mu$ , in the effective Weyl gravity. First, Weyl gauge self-interactions,  $(g_w w_\mu - g_X X_\mu)^n$  with  $n = 2$  and  $n \geq 6$ , are Weyl-gauge invariant under the Weyl transformations in Eq. (36), but they violate the scale symmetry of the gravitational Lagrangian. On the other hand, there is a Weyl invariant self-interaction,  $(g_w w_\mu - g_X X_\mu)^4$ , which could lead to new interactions between  $X_\mu$  and the SM Higgs after the heavy Weyl gauge field  $w_\mu$  is integrated out. But they are suppressed by the mass of the heavy Weyl photon mass, so we do not include it in the following discussion. Furthermore, we could construct the Weyl-invariant terms with dilaton and Higgs fields by  $\phi^2(g_w w_\mu - g_X X_\mu)^2$  and  $|H|^2(g_w w_\mu - g_X X_\mu)^2$ . However, those terms are absent by construction in the extended Weyl gravity, for appropriate choices of the parameters in the original Lagrangian in Eq. (26) with Eqs. (30) and (31). Thus, it is sufficient to take the extra Higgs kinetic term respecting the remnant of the Weyl symmetry under  $X_\mu$ , as introduced in Eq. (32). More general cases with covariant derivative terms for the dilaton and the Higgs fields will be discussed in the next section.

Consequently, for the later discussion, we consider a Weyl invariant Lagrangian with two Weyl gauge fields,  $w_\mu$  and  $X_\mu$ , in the effective Weyl gravity with a single metric tensor, in the following simple form:

$$\begin{aligned} \frac{\mathcal{L}_2}{\sqrt{-g}} = & -\frac{1}{2}(\xi_\phi \phi^2 + 2\xi_H |H|^2) \tilde{R}(\tilde{\Gamma}) - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} \\ & - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + |D'_\mu H|^2 - V(H, \phi). \end{aligned} \quad (39)$$

The above Lagrangian is invariant under the local conformal transformation with

$$g_{\mu\nu} \rightarrow e^{2\alpha} g_{\mu\nu}, \quad \phi \rightarrow e^{-\alpha} \phi, \quad H \rightarrow e^{-\alpha} H, \quad (40)$$

and Eq. (36). We note that both Weyl gauge fields,  $w_\mu$  and  $X_\mu$ , transform with only one gauge transformation parameter  $\alpha$ . Since the Weyl gauge field  $X_\mu$  couples to gravity minimally, it can survive at low energy and it receives mass from the VEVs of Higgs and extra light singlet scalars, as will be discussed in Sec. V.

After expanding the Weyl-invariant Lagrangian in Eq. (39), we get

$$\begin{aligned} \frac{\mathcal{L}_2}{\sqrt{-g}} = & -\frac{1}{2} \xi_\phi \left( \phi^2 R + 6(\partial_\mu \phi)^2 - 6(D_\mu \phi)^2 \right) - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} \\ & - \xi_H \left( |H|^2 R + 6|\partial_\mu H|^2 - 6|D_\mu H|^2 \right) \\ & - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + |D'_\mu H|^2 - V(H, \phi), \end{aligned} \quad (41)$$

where the Weyl covariant derivatives are defined as  $D_\mu \phi = (\partial_\mu - g_w w_\mu) \phi$ ,  $D_\mu H = (\partial_\mu - g_w w_\mu) H$ . Fixing the gauge to  $\langle \phi^2 \rangle = M_P^2 / \xi_\phi$ . Then, from Eq. (41), we obtain the gauge-fixed Lagrangian as follows:

$$\begin{aligned} \frac{\mathcal{L}_2}{\sqrt{-g}} = & -\frac{1}{2} (M_P^2 + 2\xi_H |H|^2) R + |D'_\mu H|^2 - V(H) - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} \\ & - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} + \frac{1}{2} m_w^2 w_\mu w^\mu - \frac{1}{2} g_w w_\mu K^\mu + \frac{1}{2} g_w^2 w_\mu w^\mu K_H. \end{aligned} \quad (42)$$

Thus, in this case, we realize the exactly same Jordan-frame Lagrangian for the Weyl gauge field as in Eq. (5), with  $m_w^2 = 6g_w^2 M_P^2$  and  $K_H = 12\xi_H |H|^2$ .

Making a field redefinition for the Weyl gauge field by

$$\tilde{w}_\mu = w_\mu - \frac{1}{2g_w} \partial_\mu \ln(m_w^2 + g_w^2 K_H), \quad (43)$$

we can rewrite the gauge-fixed Lagrangian as

$$\begin{aligned} \frac{\mathcal{L}_2}{\sqrt{-g}} = & -\frac{1}{2} (M_P^2 + 2\xi_H |H|^2) R + |D'_\mu H|^2 - V(H) - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} \\ & - \frac{1}{4} \tilde{w}_{\mu\nu} \tilde{w}^{\mu\nu} + \frac{1}{2} (m_w^2 + g_w^2 K_H) \tilde{w}_\mu \tilde{w}^\mu - \frac{g_w^2}{8} \frac{K_\mu K^\mu}{m_w^2 + g_w^2 K_H}. \end{aligned} \quad (44)$$

## B. Einstein-frame Lagrangian and unitarity

Making a Weyl transformation of the metric,  $g_{E,\mu\nu} = \Omega g_{\mu\nu}$ , with the frame function,

$$\Omega = 1 + 2\xi_H |H|^2 / M_P^2, \quad (45)$$

and using  $m_w^2 + g_w^2 K_H = 6g_w^2 M_P^2 \Omega$  for  $m_w^2 = 6g_w^2 M_P^2$ , we get the Einstein-frame Lagrangian from Eq. (44) as

$$\begin{aligned} \frac{\mathcal{L}_E}{\sqrt{-g_E}} = & -\frac{M_P^2}{2} R + \frac{|D'_\mu H|^2}{\Omega} + \frac{3\xi_H^2 (\partial_\mu |H|^2)^2}{M_P^2 \Omega^2} - \frac{V(H)}{\Omega^2} \\ & - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{4} \tilde{w}_{\mu\nu} \tilde{w}^{\mu\nu} + \frac{1}{2} m_w^2 \tilde{w}_\mu \tilde{w}^\mu - \frac{1}{48 M_P^2} \frac{K_\mu K^\mu}{\Omega^2}. \end{aligned} \quad (46)$$

Consequently, from  $K_\mu = 12\xi_H \partial_\mu |H|^2$ , the above Einstein-frame Lagrangian becomes

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{M_P^2}{2}R + \frac{|D'_\mu H|^2}{\Omega} - \frac{V(H)}{\Omega^2} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{4}\tilde{w}_{\mu\nu}\tilde{w}^{\mu\nu} + \frac{1}{2}m_w^2\tilde{w}_\mu\tilde{w}^\mu. \quad (47)$$

As a result, the noncanonical kinetic terms for the Higgs field containing  $\xi_H^2$  are canceled out, taking the same form as in the Palatini formulation for Higgs inflation [19–21]. Moreover, the redefined Weyl gauge field  $\tilde{w}_\mu$  is decoupled from the Higgs field. Therefore, from the remaining Higgs kinetic term and the scalar potential, we can identify the unitarity cutoff as  $\Lambda = M_P/\sqrt{\xi_H}$  in the vacuum, which is much larger than the one in the original Higgs inflation,  $\Lambda = M_P/\xi_H$ , for  $\xi_H \gg 1$ .

We remark that the crucial difference from the Palatini formulation is that there exists a light Weyl gauge field  $X_\mu$ , which gets mass only from the VEVs of the Higgs and extra singlet scalars and has interesting implications for Higgs physics, as will be discussed in the later section.

### C. Inflation

Taking the unitary gauge for the Higgs fields by  $H = \frac{1}{\sqrt{2}}(0, h)^T$  in Eq. (47), the part of the Einstein-frame Lagrangian relevant for inflation is given by

$$\frac{\mathcal{L}_{E,\text{inf}}}{\sqrt{-g_E}} = -\frac{M_P^2}{2}R + \frac{1}{2} \frac{(\partial_\mu h)^2}{(1 + \xi_H h^2/M_P^2)} - \frac{V(h)}{(1 + \xi_H h^2/M_P^2)^2}. \quad (48)$$

We assume that the Higgs quartic term is dominant during inflation and  $\xi_H > 0$ . Then, we first make a field redefinition for the canonical field  $\chi$  by

$$\frac{dh}{\sqrt{1 + \xi_H h^2/M_P^2}} = d\chi, \quad (49)$$

which gives rise to

$$\chi/M_P = \frac{1}{\sqrt{\xi_H}} \ln \left[ \sqrt{\xi_H} h/M_P + \sqrt{1 + \xi_H h^2/M_P^2} \right] \quad (50)$$

or

$$h/M_P = \frac{1}{\sqrt{\xi_H}} \sinh \left[ \sqrt{\xi_H} \chi/M_P \right]. \quad (51)$$

Then, the Einstein frame potential becomes

$$V_E = \frac{\lambda_H}{4\xi_H^2} M_P^4 \tanh^4 \left[ \sqrt{\xi_H} \chi/M_P \right]. \quad (52)$$

The slow-roll parameters,  $\epsilon$  and  $\eta$ , and the number of  $e$ -foldings  $N$ , are defined as

$$\epsilon = \frac{M_P^2}{2} \left( \frac{dV_E/d\chi}{V_E} \right)^2, \quad (53)$$

$$\eta = M_P^2 \frac{d^2 V_E/d\chi^2}{V_E}, \quad (54)$$

$$N = \frac{1}{M_P} \int_{\chi_e}^{\chi} \frac{d\chi}{\sqrt{2\epsilon}}, \quad (55)$$

where  $\chi_e$  is the field value at the end of inflation, and the CMB normalization, the spectral index and the tensor-to-scalar ratio are, respectively, given by

$$A_s = \frac{1}{24\pi^2} \frac{V_E/M_P^4}{\epsilon}, \quad n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon. \quad (56)$$

Then, we take  $\xi_H h^2 \gg M_P^2$  for which we get  $h/M_P \simeq \frac{1}{\sqrt{\xi_H}} e^{\sqrt{\xi_H} \chi/M_P}$  and the inflationary parameters become

$$N \simeq \frac{1}{32\xi_H} e^{2\sqrt{\xi_H} \chi/M_P}, \quad (57)$$

$$\epsilon \simeq 128\xi_H e^{-4\sqrt{\xi_H} \chi/M_P} \simeq \frac{1}{8N^2\xi_H}, \quad (58)$$

$$\eta \simeq -32\xi_H e^{-2\sqrt{\xi_H} \chi/M_P} \simeq -\frac{1}{N}. \quad (59)$$

Thus, we obtain the approximate results for the inflationary observables,

$$A_s \simeq \frac{N^2}{12\pi^2} \frac{\lambda_H}{\xi_H}, \quad n_s \simeq 1 - \frac{2}{N}, \quad r \simeq \frac{2}{N^2\xi_H}. \quad (60)$$

Taking the observed value  $A_s = 2.1 \times 10^{-9}$  with  $N = 50\text{--}60$ , we have the following relation

$$\xi_H = 1.0 \times 10^{10} \lambda_H \quad (61)$$

in comparison to the case in the metric formulation for Higgs inflation, which requires  $\xi_H \sim 10^4 \sqrt{\lambda_H}$ .

We also get the spectral index,  $n_s = 0.960\text{--}0.967$ , for  $N = 50\text{--}60$ , which is consistent with the Planck data,  $n_s = 0.967 \pm 0.0037$  [1]. Thus, the inflationary predictions in our model coincide with those in the Palatini formulation for Higgs inflation [19–21]. Namely, the nonminimal coupling  $\xi_H$  is larger than that of the original Higgs inflation for a sizable  $\lambda_H$ , resulting in quite a tiny tensor-to-scalar ratio  $r$ . However, even for a large  $\xi_H$ , the cutoff scale does not fall below the typical energy scales of inflation and reheating [22].



We remark that, from the Einstein-frame Lagrangian in Eq. (47), the Weyl gauge field  $X_\mu$  gets mass during inflation as follows:

$$m_X^2 = \frac{g_X^2 h^2}{1 + \xi_H h^2 / M_P^2},$$

$$\simeq \frac{g_X^2 M_P^2}{\xi_H} \left(1 + e^{-\sqrt{\xi_H} X / M_P}\right)^{-1}, \quad (62)$$

which is sufficiently larger than the Hubble scale,  $H_I^2 \simeq \lambda_H M_P^2 / (4\xi_H^2)$ , for  $g_X^2 \gg \lambda_H / (4\xi_H)$ . Therefore, we can safely ignore the dynamics of the Weyl gauge field  $X_\mu$  during inflation.

## V. GENERAL COUPLINGS FOR TWO WEYL GAUGE FIELDS

We generalize the discussion in the previous section with extra Weyl-covariant kinetic terms for the dilaton and the Higgs fields and discuss the unitarity scale and the inflationary predictions.

### A. General Lagrangian in Weyl gravity

Introducing the extra parameters,  $r_\phi$  and  $r_H$ , and using the results in Appendix B, we consider the most general Lagrangian in Weyl gravity with two Weyl gauge fields and the Ricci scalar given by Eq. (10) as follows:

$$\frac{\mathcal{L}_G}{\sqrt{-g}} = -\frac{1}{2}(\xi_\phi \phi^2 + \xi_H |H|^2) \tilde{R} + 3\xi_\phi (r_\phi - 1) (D_\mu \phi)^2 + 6\xi_H (r_H - 1) |D_\mu H|^2$$

$$- \frac{1}{4} w_{\mu\nu} w^{\mu\nu} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + (1 - 6\xi_H (r_H - 1)) |D'_\mu H|^2 - V(H, \phi). \quad (63)$$

Then, we expand the above Lagrangian by using Eq. (10) as follows:

$$\frac{\mathcal{L}_G}{\sqrt{-g}} = -\frac{1}{2} \xi_\phi \left( \phi^2 R + 6(\partial_\mu \phi)^2 - 6r_\phi (D_\mu \phi)^2 \right) - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} - \xi_H \left( |H|^2 R + 6|\partial_\mu H|^2 - 6r_H |D_\mu H|^2 \right)$$

$$- \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + (1 - 6\xi_H (r_H - 1)) |D'_\mu H|^2 - V(H, \phi). \quad (64)$$

Here, we note that the net Higgs kinetic term is normalized to be canonical in a Jordan frame.

Following a similar step in Appendix B as in the case with  $r_\phi = r_H = 1$ , we obtain the Einstein-frame Lagrangian with gauge fixing,  $\langle \phi^2 \rangle = M_P^2 / \xi_\phi$ , as follows:

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{M_P^2}{2} R + 6\xi_H (r_H - 1) \frac{|\partial_\mu H|^2}{\Omega} + (1 - 6\xi_H (r_H - 1)) \frac{|D'_\mu H|^2}{\Omega}$$

$$+ \frac{3\xi_H^2}{M_P^2 \Omega^2} \frac{r_\phi - r_H^2 + 2r_H(1 - r_H)\xi_H |H|^2 / M_P^2}{r_\phi + 2r_H \xi_H |H|^2 / M_P^2} (\partial_\mu |H|^2)^2$$

$$- \frac{V(H)}{\Omega^2} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{4} \tilde{w}_{\mu\nu} \tilde{w}^{\mu\nu} + \frac{1}{2} (m_w^2 + g_w^2 r_H K_H) \Omega^{-1} \tilde{w}_\mu \tilde{w}^\mu, \quad (65)$$

where  $\tilde{w}_\mu$  is the redefined heavy Weyl gauge field, given by

$$\tilde{w}_\mu = w_\mu - \frac{1}{2g_w} \partial_\mu \ln(m_w^2 + g_w^2 r_H K_H), \quad (66)$$

with  $K_H = 12\xi_H |H|^2$ , and the mass of the heavy Weyl gauge field becomes  $m_w^2 = 6r_\phi g_w^2 M_P^2$ . Then the heavy Weyl gauge field  $\tilde{w}_\mu$  is generically not decoupled from the Higgs field, except for  $r_\phi = r_H$ .

We now discuss the unitarity scale in the presence of the general gravitational couplings. To this, we focus on the noncanonical Higgs kinetic terms as follows:

$$\frac{\mathcal{L}_{\text{kin}}}{\sqrt{-g_E}} = \frac{|\partial_\mu H|^2}{1 + 2\xi_H |H|^2 / M_P^2}$$

$$+ \frac{3\xi_H^2}{M_P^2} \frac{r_\phi - r_H^2 + 2r_H(1 - r_H)\xi_H |H|^2 / M_P^2}{(1 + 2\xi_H |H|^2 / M_P^2)^2 (r_\phi + 2r_H \xi_H |H|^2 / M_P^2)}$$

$$\cdot (\partial_\mu |H|^2)^2. \quad (67)$$

Therefore, the cutoff scale depends on various combinations of  $\xi_H$ ,  $r_\phi$ , and  $r_H$ , but it can be generically much larger than the one in Higgs inflation. For instance, we obtain the leading dimension-six operator for the Higgs,

$$\begin{aligned} \frac{\mathcal{L}_{\text{kin}}}{\sqrt{-g_E}} &\supset -\frac{2\xi_H}{M_P^2}|H|^2|\partial_\mu H|^2 + \frac{3\xi_H^2}{M_P^2}\left(1 - \frac{r_H^2}{r_\phi}\right)(\partial_\mu|H|^2)^2 \\ &= \frac{1}{M_P^2}\left(3\xi_H^2\left(1 - \frac{r_H^2}{r_\phi}\right) + \xi_H\right)(\partial_\mu|H|^2)^2 + \dots, \end{aligned} \quad (68)$$

where integration by parts are made and the equation of motion for the Higgs fields is used in the second line. Thus, the unitarity scale identified from the leading dimension-six operator is

$$\Lambda_1 = \frac{M_P}{\left|3\xi_H^2\left(1 - \frac{r_H^2}{r_\phi}\right) + \xi_H\right|^{1/2}}, \quad (69)$$

which can be much larger than the one in Higgs inflation, depending on  $\xi_H$ ,  $r_\phi$ , and  $r_H$ . On the other hand, even higher-dimensional derivative terms and the higher-dimensional terms in the scalar potential lead to the cutoff scale,  $\Lambda = M_P/\sqrt{\xi_H}$ .

For  $r_\phi = r_H$ , the Einstein-frame Lagrangian becomes

$$\begin{aligned} \frac{\mathcal{L}_E}{\sqrt{-g_E}} &= -\frac{M_P^2}{2}R + 6\xi_H(r_H - 1)\frac{|\partial_\mu H|^2}{\Omega} \\ &+ (1 - 6\xi_H(r_H - 1))\frac{|D'_\mu H|^2}{\Omega} \\ &+ \frac{3\xi_H^2(1 - r_H)}{\Omega^2 M_P^2}(\partial_\mu|H|^2)^2 - \frac{V(H)}{\Omega^2} \\ &- \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{4}\tilde{w}_{\mu\nu}\tilde{w}^{\mu\nu} + \frac{1}{2}m_w^2\tilde{w}_\mu\tilde{w}^\mu, \end{aligned} \quad (70)$$

so the heavy Weyl gauge field  $\tilde{w}_\mu$  is decoupled from the Higgs fields in Einstein frame. In this case, we find that the unitarity scale is given by  $\Lambda_1 = M_P/\sqrt{3\xi_H^2(1 - r_H) + \xi_H}$  from the leading dimension-six operator and  $\Lambda = M_P/\sqrt{\xi_H}$  from the other higher-dimensional terms.

## B. Inflation with $r_\phi = r_H$

For  $r_\phi = r_H$ , the heavy Weyl gauge field is decoupled from the Higgs field and the Higgs kinetic term in Eq. (67) becomes simplified. For  $r_\phi \neq r_H$ , the Weyl gauge field is not decoupled from the Higgs in Eq. (65), but a slow-roll inflation can be maintained even in this case, as far as the Weyl gauge field is sufficiently heavier than the Hubble scale during inflation. For simplicity in the following discussion, we choose  $r_\phi = r_H$  for the inflation with a varying unitarity scale depending on  $r_H$ .

In the following, we take  $r_\phi = r_H$ , for which the relevant Einstein-frame Lagrangian for inflation is simplified to

$$\begin{aligned} \frac{\mathcal{L}_{E,\text{inf}}}{\sqrt{-g_E}} &= -\frac{M_P^2}{2}R + \frac{1}{2\Omega^2}\left(1 + \xi_H(1 + 3\xi_H(1 - r_H))\right)h^2/M_P^2 \\ &\times (\partial_\mu h)^2 - \frac{V(h)}{\Omega^2}, \end{aligned} \quad (71)$$

with  $\Omega = 1 + \xi_H h^2/M_P^2$ . The parameter  $r_H$  must satisfy  $r_H > 0$ , for the Weyl gauge field mass to be positive. Also, to avoid a ghost mode in the large field region with  $\xi_H^2 h^2 \gg M_P^2$ , we consider a case with  $0 < r_H < 1$ . In the following discussion, we set  $M_P = 1$  for notational simplicity.

In order to see the modification for  $r_H \neq 1$ , we first introduce the field redefinition by

$$\xi_H h^2 = e^{2\sqrt{\xi_H}\hat{\chi}}. \quad (72)$$

Then, we can rewrite Eq. (71) in terms of  $\hat{\chi}$  as

$$\begin{aligned} \frac{\mathcal{L}_{E,\text{inf}}}{\sqrt{-g_E}} &= -\frac{M_P^2}{2}R + \frac{1}{2}\frac{(1 + 3\xi_H(1 - r_H) + e^{-2\sqrt{\xi_H}\hat{\chi}})}{(1 + e^{-2\sqrt{\xi_H}\hat{\chi}})^2} \\ &\times (\partial_\mu \hat{\chi})^2 - \frac{\lambda_H}{4\xi_H^2}\left(1 + e^{-2\sqrt{\xi_H}\hat{\chi}}\right)^{-2}. \end{aligned} \quad (73)$$

We can make a further redefinition of the inflaton by

$$\chi = \sqrt{1/\xi_H + 3(1 - r_H)}\text{Arctanh}\sqrt{\frac{1 + 3\xi_H(1 - r_H) + e^{-2\sqrt{\xi_H}\hat{\chi}}}{1 + 3\xi_H(1 - r_H)}} - \sqrt{3(1 - r_H)}\text{Arccoth}\sqrt{\frac{3\xi_H(1 - r_H)}{1 + 3\xi_H(1 - r_H) + e^{-2\sqrt{\xi_H}\hat{\chi}}}}. \quad (74)$$

But, this is not of invertible form for the canonical inflaton  $\chi$ . Thus, instead we compute the slow-roll parameters and the number of  $e$ -foldings in terms of  $\hat{\chi}$  as below.

First, for  $e^{-2\sqrt{\xi_H}\hat{\chi}} \ll 1$ , we obtain the slow-roll parameters as

$$\epsilon = \frac{1}{2V}\left(\frac{d\hat{\chi}}{d\chi}\right)^2\left(\frac{dV}{d\hat{\chi}}\right)^2 \simeq \frac{8\xi_H e^{-4\sqrt{\xi_H}\hat{\chi}}}{1 + 3\xi_H(1 - r_H)}, \quad (75)$$

$$\eta = \frac{d\hat{\chi}}{d\chi}\frac{d}{d\hat{\chi}}\left(\frac{dV}{d\hat{\chi}}\right) \simeq -\frac{8\xi_H e^{-2\sqrt{\xi_H}\hat{\chi}}}{1 + 3\xi_H(1 - r_H)}, \quad (76)$$

and the number of  $e$ -foldings is

$$N = \int_{\chi_e}^{\chi} \frac{d\hat{\chi}}{d \ln V / d\hat{\chi}} \left( \frac{d\chi}{d\hat{\chi}} \right)^2, \quad (77)$$

$$\simeq \frac{e^{2\sqrt{\xi_H \hat{\chi}}}}{8\xi_H} \left( 1 + 3\xi_H(1-r_H) - 6\xi_H^{3/2}(1-r_H)\hat{\chi}e^{-2\sqrt{\xi_H \hat{\chi}}} \right).$$

Then, we can get an approximate solution for  $\hat{\chi}$  to the above equation, as follows:

$$e^{-2\sqrt{\xi_H \hat{\chi}}} \simeq \frac{1 + 3\xi_H(1-r_H)}{8\xi_H N} \times \left[ 1 - \frac{3}{8N}(1-r_H) \ln \frac{8\xi_H N}{1 + 3\xi_H(1-r_H)} \right]. \quad (78)$$

As a result, we can rewrite the slow-roll parameters in terms of the number  $e$ -foldings as follows:

$$\epsilon \simeq \frac{1 + 3\xi_H(1-r_H)}{8\xi_H N^2} \left[ 1 - \frac{3}{8N}(1-r_H) \ln \frac{8\xi_H N}{1 + 3\xi_H(1-r_H)} \right]^2, \quad (79)$$

$$\eta \simeq -\frac{1}{N} \left[ 1 - \frac{3}{8N}(1-r_H) \ln \frac{8\xi_H N}{1 + 3\xi_H(1-r_H)} \right]. \quad (80)$$

Consequently, we get the general expressions for the inflationary observables as

$$A_s \simeq \frac{N^2}{12\pi^2} \frac{\lambda_H}{\xi_H} \frac{1}{1 + 3\xi_H(1-r_H)} \times \left[ 1 - \frac{3}{8N}(1-r_H) \ln \frac{8\xi_H N}{1 + 3\xi_H(1-r_H)} \right]^{-2}, \quad (81)$$

$$n_s \simeq 1 - \frac{2}{N} \left[ 1 - \frac{3}{8N}(1-r_H) \ln \frac{8\xi_H N}{1 + 3\xi_H(1-r_H)} \right], \quad (82)$$

$$r \simeq \frac{2(1 + 3\xi_H(1-r_H))}{\xi_H N^2} \times \left[ 1 - \frac{3}{8N}(1-r_H) \ln \frac{8\xi_H N}{1 + 3\xi_H(1-r_H)} \right]^2. \quad (83)$$

Then, for  $r_H = 1$ , we recover the results in Eq. (60). Otherwise, all the inflationary observables are different from the case with  $r_H = 1$ . In particular, the spectral index can be corrected sizably. For instance, from  $\xi_H = 10^{10}\lambda_H$ ,  $\lambda_H = 0.01$ , and  $N = 50$ , we get  $\frac{3(r_H-1)}{4N} \ln(8\xi_H N) = 0.4(r_H - 1)$ , so the deviation in the spectral index can be significant.

Moreover, for  $r_H < 1$ , the CMB normalization determines the nonminimal coupling approximately by

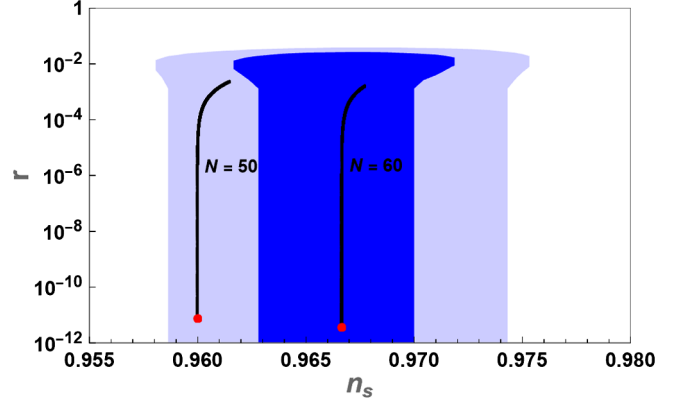


FIG. 1. The spectral index  $n_s$  and the tensor-to-scalar ratio  $r$  as a function of  $r_H$ . The bounds on  $n_s$  and  $r$ , from the combined results of Planck, WMAP, BICEP, and Keck data, are shown in dark and light blue regions, within  $1\sigma$  and  $2\sigma$  errors, respectively. The red points correspond to  $r_H = 1$  for  $N = 50, 60$ , respectively.

$$\xi_H \simeq \frac{1}{6(1-r_H)} \left( \sqrt{1 + N^2(1-r_H)\lambda_H/(\pi^2 A_s)} - 1 \right). \quad (84)$$

For  $N^2(1-r_H)\lambda_H/(\pi^2 A_s) \ll 1$ , we recover the value of  $\xi_H$  in Eq. (61) with  $r_H = 1$ ; for  $N^2(1-r_H)\lambda_H/(\pi^2 A_s) \gg 1$ , we get  $\xi_H \simeq (N/6\pi)\sqrt{\lambda_H/(A_s(1-r_H))} \simeq 6 \times 10^3 / \sqrt{1-r_H}$  for  $\lambda_H = 0.01$  and  $N = 50$ .

In Fig. 1, we show the predicted values for the spectral index  $n_s$  and the tensor-to-scalar ratio  $r$  by fixing  $N = 50, 60$  and  $\lambda_H = 0.01$  but varying  $r_H$  in the range of  $0 < r_H < 1$ . We set  $\xi_H$  from the CMB normalization, and imposed the bounds on  $n_s$  and  $r$ , for Planck data combined with WMAP, BICEP, and Keck data [23]. The dark blue and light blue regions are within  $1\sigma$  and  $2\sigma$  errors. The red dots are for  $r_H = 1$  with  $N = 50, 60$ , respectively, which is equivalent to the minimal Weyl gravity or the Palatini formulation for Higgs inflation. But, for  $r_H \neq 1$ , our model connects between the Palatini formulation for Higgs inflation and a Higgs-like inflation continuously. There was a similar construction of interpolating between the original Higgs inflation and its Palatini formulation in the context of Einstein-Cartan gravity [24].

We also remark that from the Einstein-frame Lagrangian in Eq. (70), the Weyl gauge field  $X_\mu$  gets mass during inflation as follows:

$$m_X^2 = \frac{a_H g_X^2 h^2}{1 + \xi_H h^2 / M_P^2}, \quad (85)$$

$$\simeq \frac{a_H g_X^2 M_P^2}{\xi_H} \left( 1 + e^{-\sqrt{\xi_H \hat{\chi}} / M_P} \right)^{-1},$$

with  $a_H = 1 - 6\xi_H(r_H - 1)$ . Thus, the Weyl gauge field mass is sufficiently larger than the Hubble scale for  $a_H g_X^2 \gg \lambda_H / \xi_H$  as in the case with  $r_H = 1$  in the previous subsection, so we can safely ignore the dynamics of the

Weyl gauge field  $X_\mu$  in the case with general Weyl invariant terms.

## VI. LIGHT WEYL GAUGE FIELD AND HIGGS PHYSICS

The Weyl gauge field  $w_\mu$  coupled to the dilaton  $\phi$  gets a large mass after the Weyl symmetry is broken spontaneously. On the other hand, another Weyl gauge field  $X_\mu$  couples only to the Higgs fields and extra light scalars, so it receives a mass at low energy. As the Higgs fields couple to the light Weyl gauge field directly via the covariant kinetic term, we discuss some interesting implications for Higgs physics and new resonance searches at colliders.

### A. Higgs mechanism for light Weyl gauge field

In our model, the extra Weyl gauge field  $X_\mu$  can be light, as far as it couples to an extra light scalar  $s$ , with the covariant kinetic term in Jordan frame as follows:

$$\frac{\Delta\mathcal{L}}{\sqrt{-g}} = \frac{1}{2} (D'_\mu s)^2, \quad (86)$$

with  $D'_\mu s = (\partial_\mu - g_X X_\mu)s$ . We note that the extra light scalar  $s$  also couples to the heavy Weyl gauge field  $w_\mu$ , being consistent with the Weyl symmetry, for instance, through  $D_\mu s = (\partial_\mu - c g_w w_\mu - (1-c)g_X X_\mu)s$ , with  $c$  being constant. Then, there appears a mixing between  $w_\mu$  and  $X_\mu$  in this case. But, for  $g_X \langle s \rangle \ll g_w \langle \phi \rangle = g_w M_P / \sqrt{\xi_\phi}$ , we can safely decouple the heavy Weyl gauge field  $w_\mu$  without changing our conclusions below.

Then, from Eq. (86), we get the leading interactions between the light Weyl gauge field and the Higgs and singlet fields in Einstein frame as

$$\begin{aligned} \mathcal{L}_{X,\text{int}} &= a_H \left( -g_X X_\mu \partial^\mu |H|^2 + g_X^2 X_\mu X^\mu |H|^2 \right) \\ &\quad - \frac{1}{2} g_X X_\mu \partial^\mu s^2 + \frac{1}{2} g_X^2 X_\mu X^\mu s^2, \end{aligned} \quad (87)$$

with  $a_H \equiv 1 - 6\xi_H(r_H - 1)$ . Here, we assume that the extra singlet scalar  $s$  does not change the previous discussion on inflation for  $s = 0$  during inflation, but it is crucial for the Higgs mechanism for the light Weyl gauge field.

After the Higgs and the singlet get VEVs, we expand them by  $H = (0, v + h)^T / \sqrt{2}$  and  $s = v_s + \tilde{s}$ . Then, the light Weyl gauge field  $X_\mu$  gets a nonzero mass by

$$m_X^2 = g_X^2 (a_H v^2 + v_s^2). \quad (88)$$

Moreover, there appears a mixing between the Weyl gauge field and the Higgs fields in Eq. (87), which can be

eliminated by a gauge fixing term for the Weyl gauge field,

$$\mathcal{L}_{gf} = -\frac{1}{2\zeta} \left( \partial_\mu X^\mu + \zeta g_X (a_H v h + v_s \tilde{s}) \right)^2, \quad (89)$$

with  $\zeta$  being the gauge fixing parameter. Thus, the wouldbe Goldstone,  $G_X \sim a_H v h + v_s \tilde{s}$ , is eaten by the Weyl gauge field  $X_\mu$ , and the orthogonal combination,  $h_{\text{SM}} \sim v_s h - a_H v \tilde{s}$ , is identified as the SM Higgs boson. From the gauge fixing term in Eq. (89), the wouldbe Goldstone has mass,  $m_{G_X}^2 = \zeta g_X^2 ((a_H v)^2 + v_s^2)$ , so the Goldstone  $G_X$  would be decoupled in unitary gauge with  $\zeta \rightarrow \infty$  as usual in spontaneously broken gauge theories.

We remark that in order to keep the wouldbe Goldstone boson massless in the scalar potential, we need to take the alignment limit for the mass mixing between Higgs and singlet scalars. To this, we need to set the quartic potential for them to  $\lambda_H (|H|^2 + \frac{1}{2a_H} s^2)^2$ . Then, there would be a generic fine-tuning for the mixing quartic coupling between the singlet and the Higgs, but the aforementioned quartic coupling is protected by the  $SO(5)$  symmetry in the limit of  $a_H = 1$  or  $r_H = 1$ , for which the unitarity scale becomes maximized to  $M_P / \sqrt{\xi_H}$ . Then, during inflation, the effective mass for the singlet scalar becomes  $m_s^2 \simeq \lambda_H M_P^2 / (2\xi_H a_H)$  for  $|H|^2 \gtrsim M_P^2 / (2\xi_H)$ , which is much larger than the Hubble scale during inflation,  $H_I^2 \sim \lambda_H M_P^2 / (12\xi_H^2)$ , for a large  $\xi_H$ . Therefore, we can safely ignore the dynamics of the singlet scalar  $s$  during inflation.

### B. Weyl gauge field interactions

As we discussed with the gauge-fixing Lagrangian in the previous subsection, we introduce a mixing angle between the Higgs and singlet scalars by

$$\begin{pmatrix} h \\ \tilde{s} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h_{\text{SM}} \\ G_X \end{pmatrix}, \quad (90)$$

with  $\tan \theta = (a_H v) / v_s$ . Since the SM particles couple only to  $h$ , the SM Higgs couplings for the physical scalar,  $h_{\text{SM}}$ , are modified as compared to the SM. Thus, the decays and production channels for the SM Higgs are modified by the mixing angle. Therefore, in order to be consistent with Higgs data, we need to take the mixing angle sufficiently small, for instance,  $\sin \theta \simeq (a_H v) / v_s \lesssim 0.3(0.03)$  at the 10 (1)% level.

Taking the general covariant gauge with the gauge fixing term for the Weyl gauge field in Eq. (89), we can reduce the interaction terms of the Weyl gauge field to the wouldbe Goldstone  $G_X$  and the SM Higgs boson  $h_{\text{SM}}$  in the following:

$$\begin{aligned}
\mathcal{L}_{X,\text{int}} &= -a_H g_X X_\mu h \partial^\mu h + \frac{1}{2} a_H g_X^2 X_\mu X^\mu (h^2 + 2vh) - g_X X_\mu \tilde{s} \partial^\mu \tilde{s} + \frac{1}{2} g_X^2 X_\mu X^\mu (\tilde{s}^2 + 2v_s \tilde{s}), \\
&= -g_X (a_H \cos^2 \theta + \sin^2 \theta) X_\mu h_{\text{SM}} \partial^\mu h_{\text{SM}} - g_X (a_H - 1) \cos \theta \sin \theta X_\mu (G_X \partial^\mu h_{\text{SM}} + h_{\text{SM}} \partial^\mu G_X) \\
&\quad - g_X (a_H \sin^2 \theta + \cos^2 \theta) X_\mu G_X \partial^\mu G_X + g_X^2 \sqrt{(a_H v)^2 + v_s^2} X_\mu X^\mu G_X + \frac{1}{2} g_X^2 (a_H \cos^2 \theta + \sin^2 \theta) X_\mu X^\mu h_{\text{SM}}^2 \\
&\quad + g_X^2 (a_H - 1) \cos \theta \sin \theta X_\mu X^\mu G_X h_{\text{SM}} + \frac{1}{2} g_X^2 (a_H \sin^2 \theta + \cos^2 \theta) X_\mu X^\mu G_X^2.
\end{aligned} \tag{91}$$

Here, we note that the perturbativity conditions on the Weyl gauge field couplings are given by

$$g_X < 1, \quad a_H g_X^2 < 1. \tag{92}$$

In unitary gauge, the wouldbe Goldstone is decoupled, so we only have to consider the Weyl gauge field couplings for the SM Higgs,  $X_\mu h_{\text{SM}} \partial^\mu h_{\text{SM}}$  and  $X_\mu X^\mu h_{\text{SM}}^2$ . But the derivative couplings of the Weyl gauge field can be written as  $X_\mu h_{\text{SM}} \partial^\mu h_{\text{SM}} = -\frac{1}{2} \partial_\mu X^\mu h_{\text{SM}}^2$ , etc., up to total derivative terms, so there is no decay of the on-shell Weyl gauge field because of  $\partial_\mu X^\mu = 0$ . Moreover, we note that there is no linear Higgs coupling to the Weyl gauge field, so there is no additional decay of the SM Higgs boson. Since there is no direct coupling between the Weyl gauge field and the other SM particles (fermions and gauge bosons), it is challenging to test the Weyl gauge field at current collider experiments.

We remark the effects of a gauge kinetic mixing for the Weyl gauge field for testing the Weyl gauge field models. In the presence of a gauge kinetic mixing between the Weyl gauge field and the hypercharge gauge boson  $B_\mu$  in the following form,

$$\mathcal{L}_{\text{gmix}} = -\frac{1}{2} \sin \xi X_{\mu\nu} B^{\mu\nu}, \tag{93}$$

with  $B_{\mu\nu}$  being the gauge field strength for the hypercharge gauge boson, we need to diagonalize the gauge kinetic terms and the mass matrix for neutral gauge bosons simultaneously. As a result, the electroweak neutral gauge bosons and the Weyl gauge field are mixed [25,26] by

$$\begin{pmatrix} B_\mu \\ W_{3\mu} \\ X_\mu \end{pmatrix} = \begin{pmatrix} c_W & t_\xi s_\zeta - s_W c_\zeta & -s_W s_\zeta - t_\xi c_\zeta \\ s_W & c_W c_\zeta & c_W s_\zeta \\ 0 & -s_\zeta / c_\xi & c_\zeta / c_\xi \end{pmatrix} \begin{pmatrix} \tilde{A}_\mu \\ \tilde{Z}_\mu \\ \tilde{X}_\mu \end{pmatrix}. \tag{94}$$

Here,  $c_\xi \equiv \cos \xi$ ,  $t_\xi \equiv \tan \xi$ ,  $c_W \equiv \cos \theta_W$ ,  $s_W \equiv \sin \theta_W$ , and  $\zeta$  is the mixing angle between  $Z$  and  $X$  bosons, given [25,26] by

$$\tan(2\zeta) = \frac{m_Z^2 s_W \sin(2\xi)}{m_X^2 - m_Z^2 (c_\xi^2 - s_W^2 s_\xi^2)}. \tag{95}$$

In the limit of  $m_X^2 \ll m_Z^2$  (or  $m_X^2 \gg m_Z^2$ ) and  $|\xi| \ll 1$ , we obtain  $\zeta \approx -s_W \xi$  (or  $\zeta \approx \frac{m_Z^2}{m_X^2} s_W \xi$ ). Moreover, the mass eigenvalues for  $Z$ -like and  $X$ -like gauge bosons are given [25,26] by

$$m_{1,2}^2 = \frac{1}{2} \left[ m_Z^2 (1 + s_W^2 t_\xi^2) + m_X^2 / c_\xi^2 \pm \sqrt{(m_Z^2 (1 + s_W^2 t_\xi^2) + m_X^2 / c_\xi^2)^2 - 4m_Z^2 m_X^2 / c_\xi^2} \right]. \tag{96}$$

We first recall that the current interactions in the interaction basis are given by

$$\begin{aligned}
\mathcal{L}_{\text{EM/NC}} &= g_X X_\mu J_X^\mu + e (s_W W_{3\mu} + c_W B_\mu) J_{\text{EM}}^\mu \\
&\quad + \frac{e}{2s_W c_W} (c_W W_{3\mu} - s_W B_\mu) J_Z^\mu,
\end{aligned} \tag{97}$$

where  $J_X^\mu$  are the Weyl current given by  $J_X^\mu = -a_H \partial_\mu |H|^2 + \frac{1}{2} \partial_\mu s^2$ , and  $J_{\text{EM}}^\mu$  and  $J_Z^\mu$  are electromagnetic and neutral currents. Then, using Eq. (94), we can approximate the above current interactions in the basis of mass eigenstates [25,26], for  $\varepsilon \equiv c_W t_\xi \approx c_W \xi \ll 1$ , as

$$\begin{aligned}
\mathcal{L}_{\text{EM/NC}} &\simeq e \tilde{A}_\mu J_{\text{EM}}^\mu + \tilde{Z}_\mu \left[ \frac{e}{2s_W c_W} J_Z^\mu + \varepsilon g_X t_W J_X^\mu \right] \\
&\quad + \tilde{X}_\mu \left[ g_X J_X^\mu - e \varepsilon J_{\text{EM}}^\mu \right].
\end{aligned} \tag{98}$$

Therefore, we find that the redefined Weyl gauge field  $\tilde{X}_\mu$  couples to the electromagnetic current  $J_{\text{EM}}^\mu$  too, so it is possible to produce the Weyl gauge field directly at the LHC and other collider experiments. Consequently, we can test the Weyl gauge field models by the interplay of standard decay modes into a pair of SM particles (apart from Higgs) appearing in non-Weyl  $Z'$  models and the Weyl current interaction.

## VII. CONCLUSIONS

We considered the embedding of Higgs inflation with a nonminimal coupling into the Weyl gravity where the unitarity problem of the original Higgs inflation is less severe, thanks to the heavy Weyl gauge field coupled to the Higgs fields. When the couplings of the heavy Weyl gauge field are absorbed into the nonminimal couplings to the Ricci curvature scalar in Weyl gravity, we found that the resultant model for Higgs inflation is the same as in the Palatini formulation for Higgs inflation. The covariant kinetic term for the Higgs fields with the second light Weyl gauge field is necessary for a successful Higgs inflation with a large nonminimal coupling. Thus, the crucial difference of our model from the Palatini formulation for Higgs inflation is that there is a light Weyl gauge field coupled to the Higgs fields.

We also generalized the unitarization of Higgs inflation with Weyl gauge fields in the presence of general covariant kinetic terms for the dilaton and the Higgs fields. In this case, we realized a successful Higgs inflation, interpolating between the Palatini formulation for Higgs inflation and a Higgs-like inflation. However, due to the unitarity problem, the region of the parameter space close to the Palatini formulation for Higgs inflation is favored.

We also showed that the light Weyl gauge field gets a small mass due to the VEV of an extra singlet scalar and the Goldstone boson associated with the light Weyl gauge field is a mixture of the Higgs and extra singlet scalars. Therefore, we found that the mixing between the Higgs and extra singlet scalars can be constrained by Higgs data. Furthermore, in the presence of a gauge kinetic mixing for the light Weyl gauge field, there are interesting signatures for  $Z'$ -like resonances at the LHC, with the direct couplings of the light Weyl gauge field to the SM Higgs bosons, unlike in usual  $Z'$  models.

## ACKNOWLEDGMENTS

The work is supported in part by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (Grants No. NRF-2022R1A2C2003567 and No. NRF-2021R1A4A2001897).

## APPENDIX A: COMPARISON TO UNITARIZATION WITH SINGLET SCALARS

### 1. Linear sigma models

We consider an induced gravity model for the sigma field  $\sigma$ , with the following Jordan frame Lagrangian [4,5],

$$\frac{\mathcal{L}_\sigma}{\sqrt{-g}} = -\frac{1}{2}\sigma^2 R + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{4}\lambda_\sigma\left(\sigma^2 - M_P^2 - 2\xi_H|H|^2\right)^2. \quad (\text{A1})$$

Then, after integrating out the sigma field with its equation of motion,

$$\sigma^2 = M_P^2 + 2\xi_H|H|^2, \quad (\text{A2})$$

we obtain the effective nonminimal coupling for the Higgs from the one for the sigma field in Eq. (A1).

From Eq. (A1), the effective Higgs quartic coupling is related to the running quartic couplings by

$$\lambda_{\text{eff}} = \lambda_H - \lambda_\sigma\xi_H^2, \quad (\text{A3})$$

where  $\lambda_\sigma\xi_H^2 < 1$  should be taken from perturbativity.

### 2. Starobinsky model

We can add an  $R^2$  term in Higgs inflation by

$$\mathcal{L}_{R2} = \sqrt{-g}\alpha R^2, \quad (\text{A4})$$

which is dual to the scalaron Lagrangian,

$$\mathcal{L}_{R2} = -2\alpha\chi R - \alpha\chi^2. \quad (\text{A5})$$

Then, after the field redefinitions,

$$g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}, \quad \chi \rightarrow \Omega^2\chi, \quad H \rightarrow \Omega H, \quad (\text{A6})$$

with  $\Omega^{-2} = (1 + \frac{\sigma}{\sqrt{6}})^2$ , and

$$\Omega^{-2} + 2\xi_H|H|^2 + 4\alpha\chi = 1 - \frac{1}{3}|H|^2 - \frac{1}{6}\sigma^2, \quad (\text{A7})$$

it was shown that the Higgs- $R^2$  Lagrangian can be recast into a linear sigma model type [9–12],

$$\begin{aligned} \frac{\mathcal{L}_{H+R2}}{\sqrt{-g}} &= -\frac{1}{2}\left(1 - \frac{1}{6}\sigma^2 - \frac{1}{3}|H|^2\right)R + \frac{1}{2}(\partial_\mu\sigma)^2 + |\partial_\mu H|^2 \\ &\quad - \frac{1}{144\alpha}\left[\left(\sigma + \frac{\sqrt{6}}{2}\right)^2 + 6\left(\xi_H + \frac{1}{6}\right)|H|^2 - \frac{3}{2}\right]^2. \end{aligned} \quad (\text{A8})$$

As a result, there is no unitarity problem below the Planck scale, as far as the following perturbativity conditions are fulfilled,

$$\frac{1}{\alpha} \left( \xi_H + \frac{1}{6} \right) < 1, \quad \frac{1}{\alpha} \left( \xi_H + \frac{1}{6} \right)^2 < 1. \quad (\text{A9})$$

In this case, there is a similar shift in the effective Higgs quartic coupling by

$$\lambda_{\text{eff}} = \lambda_H - \frac{1}{4\alpha} \left( \xi_H + \frac{1}{6} \right)^2. \quad (\text{A10})$$

### 3. Singlet scalar with a triple coupling

We consider a real singlet scalar  $S$ , with the following Lagrangian [6]:

$$\mathcal{L} = \frac{1}{2} (\partial_\mu S)^2 - \frac{1}{2} m_S^2 S^2 - \mu S |H|^2. \quad (\text{A11})$$

Then, after integrating out the singlet scalar by

$$S = -\frac{\mu}{m_S^2} |H|^2, \quad (\text{A12})$$

we get the effective interactions for the Higgs doublet as

$$\mathcal{L}_{\text{eff}} = \frac{\mu^2}{2m_S^4} (\partial_\mu |H|^2)^2 + \frac{\mu^2}{2m_S^2} |H|^4. \quad (\text{A13})$$

Therefore, we obtain the desired noncanonical kinetic term for the Higgs with

$$\begin{aligned} \frac{\mathcal{L}}{\sqrt{-g}} = & -\frac{1}{2} (M_P^2 + 2\xi_H |H|^2) R + 6\xi_H (r_H - 1) |\partial_\mu H|^2 + (1 - 6\xi_H (r_H - 1)) |D'_\mu H|^2 - V(H) - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} \\ & + \frac{1}{2} m_w^2 w_\mu w^\mu - \frac{1}{2} r_H g_w w_\mu K^\mu + \frac{1}{2} r_H g_w^2 w_\mu w^\mu K_H, \end{aligned} \quad (\text{B2})$$

where  $m_w^2 = 6r_\phi g_w^2 M_P^2$ . For  $r_\phi = r_H = 1$ , we recover the simple Lagrangian introduced in Eq. (41) in the text.

In this case, making a field redefinition of the Weyl gauge field by

$$\begin{aligned} \tilde{w}_\mu &= w_\mu - \frac{g_w r_H}{2} \cdot \frac{1}{m_w^2 + g_w^2 r_H K_H} K_\mu, \\ &= w_\mu - \frac{1}{2g_w} \partial_\mu \ln(m_w^2 + g_w^2 r_H K_H), \end{aligned} \quad (\text{B3})$$

we obtain the general gauge-fixed Lagrangian as

$$\begin{aligned} \frac{\mathcal{L}}{\sqrt{-g}} = & -\frac{1}{2} (M_P^2 + 2\xi_H |H|^2) R + 6\xi_H (r_H - 1) |\partial_\mu H|^2 + (1 - 6\xi_H (r_H - 1)) |D'_\mu H|^2 - V(H) - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{4} \tilde{w}_{\mu\nu} \tilde{w}^{\mu\nu} \\ & + \frac{1}{2} (m_w^2 + g_w^2 r_H K_H) \tilde{w}_\mu \tilde{w}^\mu - \frac{g_w^2 r_H^2}{8} \frac{K_\mu K^\mu}{m_w^2 + g_w^2 r_H K_H}. \end{aligned} \quad (\text{B4})$$

Here, we find that  $m_w^2 + g_w^2 r_H K_H = 6r_\phi g_w^2 M_P^2 (1 + 2(r_H/r_\phi) \xi_H |H|^2/M_P^2)$ . As a result, we also get the general Einstein-frame Lagrangian as

$$\frac{\mu^2}{m_S^4} = \frac{3\xi_H^2}{M_P^2}, \quad (\text{A14})$$

and there is a tree-level shift in the Higgs quartic coupling by

$$\lambda_{\text{eff}} = \lambda_H - \frac{\mu^2}{2m_S^2}. \quad (\text{A15})$$

## APPENDIX B: GENERAL WEYL-INVARIANT LAGRANGIAN

Introducing extra parameters,  $r_\phi$  and  $r_H$ , we consider the most general Lagrangian with Weyl invariance as follows:

$$\begin{aligned} \frac{\mathcal{L}_G}{\sqrt{-g}} = & -\frac{1}{2} \xi_\phi \left( \phi^2 R + 6(\partial_\mu \phi)^2 - 6r_\phi (D_\mu \phi)^2 \right) - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} \\ & - \xi_H \left( |H|^2 R + 6|\partial_\mu H|^2 - 6r_H |D_\mu H|^2 \right) \\ & - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + (1 - 6\xi_H (r_H - 1)) |D'_\mu H|^2 - V(H, \phi). \end{aligned} \quad (\text{B1})$$

Here, we chose the net Higgs kinetic term to be canonical in the Jordan frame without loss of generality. Then, fixing the gauge to  $\langle \phi^2 \rangle = M_P^2/\xi_\phi$ , the gauge-fixed Lagrangian becomes

$$\begin{aligned} \frac{\mathcal{L}_E}{\sqrt{-g_E}} = & -\frac{M_P^2}{2}R + 6\xi_H(r_H - 1)\frac{|\partial_\mu H|^2}{\Omega} + (1 - 6\xi_H(r_H - 1))\frac{|D'_\mu H|^2}{\Omega} + \frac{3\xi_H^2(\partial_\mu|H|^2)^2}{M_P^2\Omega^2} - \frac{V(H)}{\Omega^2} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{4}\tilde{w}_{\mu\nu}\tilde{w}^{\mu\nu} \\ & + \frac{1}{2}(m_w^2 + g_w^2 r_H K_H)\Omega^{-1}\tilde{w}_\mu\tilde{w}^\mu - \frac{g_w^2 r_H^2}{8\Omega}\frac{K_\mu K^\mu}{m_w^2 + g_w^2 r_H K_H}. \end{aligned} \quad (\text{B5})$$

So, for the general Weyl-invariant Lagrangian, the noncanonical Higgs kinetic terms are not canceled completely, and the Weyl gauge field  $\tilde{w}_\mu$  is not decoupled from the Higgs field.

For simplicity, we take the case with  $r_H = r_\phi$ . Then, from  $m_w^2 + g_w^2 r_H K_H = 6r_\phi g_w^2 M_P^2 \Omega$ , a simplification arises as follows:

$$\begin{aligned} \frac{\mathcal{L}_E}{\sqrt{-g_E}} = & -\frac{M_P^2}{2}R + 6\xi_H(r_H - 1)\frac{|\partial_\mu H|^2}{\Omega} + (1 - 6\xi_H(r_H - 1))\frac{|D'_\mu H|^2}{\Omega} \\ & + \frac{3\xi_H^2(1 - r_H)(\partial_\mu|H|^2)^2}{M_P^2\Omega^2} - \frac{V(H)}{\Omega^2} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{4}\tilde{w}_{\mu\nu}\tilde{w}^{\mu\nu} + \frac{1}{2}m_w^2\tilde{w}_\mu\tilde{w}^\mu. \end{aligned} \quad (\text{B6})$$

In this case, we find that the noncanonical Higgs kinetic terms are not canceled out, but the Weyl gauge field  $\tilde{w}_\mu$  is decoupled from the Higgs field.

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- [1] Y. Akrami *et al.* (Planck Collaboration), *Astron. Astrophys.* **641**, A10 (2020).
- [2] F. L. Bezrukov and M. Shaposhnikov, *Phys. Lett. B* **659**, 703 (2008).
- [3] C. P. Burgess, H. M. Lee, and M. Trott, *J. High Energy Phys.* **09** (2009) 103; J. L. F. Barbon and J. R. Espinosa, *Phys. Rev. D* **79**, 081302 (2009); C. P. Burgess, H. M. Lee, and M. Trott, *J. High Energy Phys.* **07** (2010) 007; M. P. Hertzberg, *J. High Energy Phys.* **11** (2010) 023.
- [4] G. F. Giudice and H. M. Lee, *Phys. Lett. B* **694**, 294 (2011); H. M. Lee, *Phys. Lett. B* **722**, 198 (2013).
- [5] H. M. Lee, *Phys. Rev. D* **98**, 015020 (2018); S. M. Choi, Y. J. Kang, H. M. Lee, and K. Yamashita, *J. High Energy Phys.* **05** (2019) 060.
- [6] J. L. F. Barbon, J. A. Casas, J. Elias-Miro, and J. R. Espinosa, *J. High Energy Phys.* **09** (2015) 027.
- [7] Y. Ema, *Phys. Lett. B* **770**, 403 (2017).
- [8] D. Gorbunov and A. Tokareva, *Phys. Lett. B* **788**, 37 (2019).
- [9] Y. Ema, K. Mukaida, and J. van de Vis, *J. High Energy Phys.* **11** (2020) 011.
- [10] H. M. Lee and A. G. Menkara, *J. High Energy Phys.* **09** (2021) 018.
- [11] S. Aoki, H. M. Lee, and A. G. Menkara, *J. High Energy Phys.* **10** (2021) 178.
- [12] S. Aoki, H. M. Lee, A. G. Menkara, and K. Yamashita, *J. High Energy Phys.* **05** (2022) 121.
- [13] D. Y. Cheong, H. M. Lee, and S. C. Park, *Phys. Lett. B* **805**, 135453 (2020).
- [14] H. Weyl, *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* **1918**, 465 (1918); P. A. M. Dirac, *Proc. R. Soc. A* **333**, 403 (1973); E. Scholz, arXiv:1111.3220; E. Scholz, *Einstein Stud.* **14**, 261 (2018); E. Scholz, *Einstein Stud.* **13**, 171 (2017).
- [15] D. M. Ghilencea and H. M. Lee, *Phys. Rev. D* **99**, 115007 (2019).
- [16] R. N. Lerner and J. McDonald, *Phys. Rev. D* **82**, 103525 (2010).
- [17] D. M. Ghilencea, *Phys. Rev. D* **101**, 045010 (2020).
- [18] M. Shaposhnikov and D. Zenhausern, *Phys. Lett. B* **671**, 162 (2009); R. Armillis, A. Monin, and M. Shaposhnikov, *J. High Energy Phys.* **10** (2013) 030; D. M. Ghilencea, *Phys. Rev. D* **93**, 105006 (2016); D. M. Ghilencea, Z. Lalak, and P. Olszewski, *Eur. Phys. J. C* **76**, 656 (2016).
- [19] F. Bauer and D. A. Demir, *Phys. Lett. B* **665**, 222 (2008).
- [20] T. Takahashi and T. Tenkanen, *J. Cosmol. Astropart. Phys.* **04** (2019) 035.
- [21] R. Jinno, M. Kubota, K. y. Oda, and S. C. Park, *J. Cosmol. Astropart. Phys.* **03** (2020) 063.
- [22] Y. Mikura and Y. Tada, *J. Cosmol. Astropart. Phys.* **05** (2022) 035.
- [23] P. A. R. Ade *et al.* (BICEP and Keck Collaboration), *Phys. Rev. Lett.* **127**, 151301 (2021).
- [24] M. Shaposhnikov, A. Shkerin, I. Timiryasov, and S. Zell, *J. Cosmol. Astropart. Phys.* **02** (2021) 008; **10** (2021) E01.
- [25] S. M. Choi and H. M. Lee, *J. High Energy Phys.* **09** (2015) 063.
- [26] S. M. Choi, H. M. Lee, Y. Mambrini, and M. Pierre, *J. High Energy Phys.* **07** (2019) 049.