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Enhanced DOA Estimation Using Linearly Predicted Array Expansion for Automotive Radar Systems

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ABSTRACT In this paper, we propose an enhanced direction-of-arrival (DOA) estimation method using linearly predicted array expansion to improve the angular resolution for automotive radar systems with a small number of antenna elements. The proposed method extracts the linear relation among signals received in the array antenna and uses it to generate extrapolated signals outside the array antenna. Finally, we apply the DOA estimation methods, such as the Bartlett and multiple signal classification algorithms, to both the original and extrapolated signals, and verify its performance through simulations and field experiments. From the simulation results, in terms of root-mean-square error and resolution probability, we observed that the proposed method had a higher angular resolution and estimation accuracy than conventional interpolation and extrapolation methods. Moreover, with the experiment results, we verified that the proposed array expansion method can be suitably applied to commercial automotive radar systems.

INDEX TERMS Array extrapolation, automotive radars, DOA estimation.

I. INTRODUCTION

Direction-of-arrival (DOA) estimation algorithms such as the Bartlett algorithm [1] or multiple signal classification (MUSIC) [2] are being employed in automotive radar systems to obtain the angular locations of detected targets. However, when multipath is generated in a variety of ways and coherent sources are present, the target position is not correctly detected. To solve this problem, many algorithms for estimating the DOA of the target using spatial smoothing have been proposed [3]–[5].

In general, when estimating the DOA of a target with the array antenna system, narrow main beamwidth and low side lobes are required to achieve fine angular resolution [6], [7]. When the main beamwidth is wide, the targets simultaneously located within the beamwidth cannot be distinguished. To obtain a narrow main beam, the number of antenna elements in the array must be increased, which increases the production costs. Thus, signal processing techniques are needed to improve the DOA estimation performance without increasing the number of actual antenna elements.

For instance, various studies have been conducted to enhance the angular resolution of array antenna system with a small number of antenna elements [8]–[15]. Specifically, methods to increase the number of antenna elements using array interpolation have been proposed in [8]–[13]. Most interpolation techniques assume that coherent sources exist. Therefore, interpolation was performed to convert non-uniform linear array (NLA) to uniform linear array (ULA) and to apply the spatial smoothing technique [8], [10]. These methods require the setup of an observation interval, called a sector, before estimating the DOA. In other words, such methods can be applied only when the angular location of the target is approximately known. If the sector is set incorrectly and the target to be found is located outside the sector, the target cannot be detected properly. Thus, the sector should be set wide at first because it does not know the location of the targets. Subsequently, since the angles of the targets are

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estimated while gradually narrowing the sector with respect to the region where the targets exist, the calculation complexity is increased by the number of times the sector is set. Moreover, array extrapolation has been proposed to enhance the angular resolution in [14] and [15]. However, the methods in [14] and [15] require updating the transformation vector whenever the signals are extrapolated. Thus, a more efficient and concise method has to be proposed. Furthermore, a limit to improving the angular resolution exists because the number of extrapolated antenna elements can only be increased to twice that of the physical antenna elements.

In this paper, we propose a linearly predicted array expansion method to enhance both the angular resolution and estimation accuracy of the DOA estimation algorithm. In our proposed method, we extract the transformation vector that represents the linear relation among received signals and generate extrapolated elements outside the physical array antenna elements. Moreover, the proposed array expansion method consists of forward and backward array expansion, which generates elements to the left and right sides of the array antenna. When using both the original and extrapolated signals, the angular locations of targets can be estimated with a higher angular resolution. The performance of our proposed method is evaluated through simulations and actual experimental results using a commercial automotive radar sensor to verify its applicability. In terms of the root-mean-square error (RMSE) and resolution probability, we found that the proposed method showed higher angular resolution and estimation accuracy compared to the conventional methods.

In addition to extending the antenna by extrapolation or interpolation, many methods have been proposed to achieve finer angular estimation performance using a small number of antennas. For example, there are minimum redundancy array [16], [17], nested array [18], [19], and co-prime array [20], [21]. They have the advantage of being able to detect more targets than the number of antennas, with limited antenna aperture size and number of antennas by maximizing degree of freedom. However, if the antennas are expanded using the proposed method, we can have better angular resolution than those using the conventional optimal NLA.

In [11], a disadvantage occurs in that it is necessary to newly acquire the position of an appropriate expanded antenna elements according to the positions of the targets to obtain a good angle estimation performance. However, in our proposed method, a uniform linear array antenna is used to generate the expanded antenna elements. Since ULA is used, there is an advantage that it is robust against multipath as compared with NLA-based techniques [22], [23]. Furthermore, the advantage of our method is that the positions of expanded antenna elements need not be calculated at that time. In addition, unlike the methods in [8]–[13], our method can estimate the position of targets at once without having to set the sector, such that the target position can be detected with little calculation irrespective of the target location. In [14], [15], the transformation vector is obtained using the covariance matrix of the received signal. With this method, the number of expanded antennas can only be generated as twice as many as the actual number of antennas. If the number of extrapolated antenna elements exceeds twice the number of physical antenna elements, the additional extrapolated received signals are generated using only the extrapolated signals. When the conventional extrapolation generates the extrapolated signals, the extrapolated signal values are inaccurate because the errors continue to be reflected. In this paper, since the expanded antenna elements are generated using the linear least-squares method, it is possible to generate more accurate expanded antenna elements than the conventional method by minimizing the error of the received signal. As a result, our method can obtain better angular resolution and angle estimation accuracy than the conventional method.

The rest of this paper is organized as follows. First, a signal model for a ULA antenna and the basic DOA estimation algorithm are presented in Section **2**. In Section **3**, our proposed DOA estimation method using linearly predicted array expansion is presented. Furthermore, simulation results using the proposed method and conventional methods are analyzed in Section **4**, and the performance of the proposed method is also examined with the actual measurement data in Section **5**. Finally, we conclude this paper in Section **6**.

II. DOA ESTIMATION IN ARRAY ANTENNA SYSTEM

A. SIGNAL MODEL FOR ULA ANTENNA

We assume a single-input multiple-output antenna system, which is composed of one transmitting antenna and N identical receiving antenna elements with uniform spacing d. If the transmitted signal is reflected from L targets, the received signal vector can be expressed as

$$\begin{aligned}\mathbf{x}(k) &= \mathbf{A}\mathbf{s}(k) + \mathbf{n}(k) \\ &= [x_1(k), x_2(k), \cdots, x_N(k)]^T, \end{aligned} \tag{1}$$

where k indicates the time index for the sampled signals and $[\cdot]^T$ denotes the transpose operator. In addition, $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_L)]$ is a steering matrix composed of steering vectors. The steering vector is given by

$$\mathbf{a}(\theta_l) = [e^{j\frac{2\pi}{\lambda}d_1 \sin\theta_l}, e^{j\frac{2\pi}{\lambda}d_2 \sin\theta_l}, \cdots, e^{j\frac{2\pi}{\lambda}d_N \sin\theta_l}]^T \quad (l = 1, 2, \cdots, L), \tag{2}$$

where λ is the wavelength of the transmitted signal, $d_i\,(i = 1, 2, \cdots, N)$ denotes the distance from the first antenna to the i-th antenna element (i.e., $d_i = (i-1)d$). θ_l denotes the angle of the l-th target, which is defined as the angle from the boresight direction of the array antenna. Moreover, $\mathbf{s}(k) = [s_1(k), s_2(k), \cdots, s_L(k)]^T$ and $\mathbf{n}(k) = [n_1(k), n_2(k), \cdots, n_N(k)]^T$ denote the incident signal and the zero-mean white Gaussian noise vectors, respectively. Here, we assume that the incident signal and noise components are uncorrelated and the power of $\mathbf{n}(k)$ is $E[\mathbf{n}(k)\mathbf{n}^H(k)] = \sigma_n^2\mathbf{I}$, where $E[\cdot]$ and $(\cdot)^H$ denote the expectation and conjugate transpose operators, respectively.

B. BARTLETT ALGORITHM

In this paper, we evaluate the performance of the proposed array expansion method mainly using the Bartlett DOA estimation algorithm. In the Bartlett algorithm, the weight vector $\mathbf{w}^*$ that maximizes the output signal power of the array antenna has to be determined [24] as

$$\mathbf{w}^* = \arg\max_{\mathbf{w}} E\left[\left|\mathbf{w}^H\mathbf{x}(k)\right|^2\right] = \arg\max_{\mathbf{w}} \left\{E\left[|s(k)|^2\right]\left|\mathbf{w}^H\mathbf{a}(\theta)\right|^2 + \sigma_n^2\|\mathbf{w}\|_2^2\right\}, \quad (3)$$

where $|\cdot|$ denotes the absolute value and $\|\cdot\|_2$ denotes the l_2-norm. This assumes that the power of the weighted noise component is constant, i.e., $\|\mathbf{w}\|_2^2$ has to be set as unity. Thus, the optimal solution of (3) is given by

$$\mathbf{w}^* = \frac{\mathbf{a}(\theta)}{\sqrt{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}}, \quad (4)$$

and the power of the weighted output can be expressed as

$$P(\theta) = E\left[\left|\mathbf{w}^{*H}\mathbf{x}(k)\right|^2\right] = \frac{\mathbf{a}^H(\theta)\mathbf{R}_{xx}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}, \quad (5)$$

where $\mathbf{R}_{xx} = E[\mathbf{x}(k)\mathbf{x}^H(k)]$ is the autocorrelation matrix of the received signal vector given by (1). Because power $P(\theta)$ has the highest value at the angle where the target is located, the angle of the target can be estimated by searching θ [24].

In practice, the exact statistics for $\mathbf{s}(k)$ and $\mathbf{n}(k)$ are unknown. If processes $\mathbf{s}(k)$ and $\mathbf{n}(k)$ are considered as ergodic, the ensemble average can be replaced by the time average. Thus, we can estimate $\mathbf{R}_{xx}$ using time-averaged autocorrelation matrix as

$$\hat{\mathbf{R}}_{xx} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}(k)\mathbf{x}^H(k), \quad (6)$$

where K is the number of time samples used to calculate the matrix. The number of time samples is related to the range resolution of the radar system.

III. PROPOSED LINEARLY PREDICTED ARRAY EXPANSION

In this section, an advanced DOA estimation method is proposed to enhance the angular resolution of the Bartlett algorithm. The method extracts the linear relation among received signals, and the extrapolated signals are generated outside the regions covered by the actual array antenna. However, because the proposed method is based on the linearity of the phase of the received signals in uniform linear array system, it is not applicable to nonuniform linear arrays. The extrapolation includes forward and backward linearly predicted array expansions. Then, using both the original and extrapolated signals, we estimate the angle of the target.

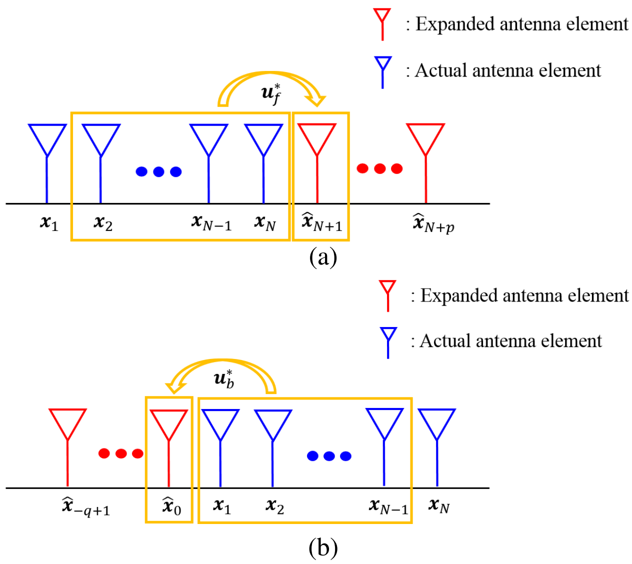

FIGURE 1. Linearly predicted expansion (a) forward (b) backward.

A. FORWARD LINEARLY PREDICTED ARRAY EXPANSION

In the forward linearly predicted array expansion, we extrapolate the received signal beyond the N-th antenna element, as illustrated in Fig. 1(a). To generate the signal at the $(N+1)$-th antenna element located at distance Nd to the right of the first antenna element, the linear relation among the N received signals is extracted. If the K time-sampled received signal vector at the i-th antenna element is expressed as

$$\mathbf{x}_i = [x_i(1),\ x_i(2),\ \cdots,\ x_i(K)]^T \quad (i = 1,\ 2,\ \cdots,\ N). \quad (7)$$

Since the uniform linear array is used, linearity exists between the phases of the received signals. Thus, $\mathbf{x}_N$ can be expressed as a linear combination of $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_{N-1}$:

$$\mathbf{x}_N \approx [\mathbf{x}_1,\ \mathbf{x}_2,\ \cdots,\ \mathbf{x}_{N-1}]\cdot\mathbf{u}_f = \mathbf{X}_f\mathbf{u}_f = \tilde{\mathbf{x}}_N. \quad (8)$$

Here, $\mathbf{u}_f$ is the vector composed of coefficients for the linear combination and $\mathbf{X}_f = [\mathbf{x}_1,\ \mathbf{x}_2,\ \cdots,\ \mathbf{x}_{N-1}]$. To find the coefficient vector, we solve the least squares problem which is defined as

$$\mathbf{u}_f^* = \arg\min_{\mathbf{u}_f}\|\mathbf{x}_N - \tilde{\mathbf{x}}_N\|_2^2, \quad (9)$$

which is the process of minimizing the error between the actual received signal and the predicted received signal. In (9), to obtain $\mathbf{u}_f^*$, which minimizes the norm of the difference between $\mathbf{x}_N$ and $\tilde{\mathbf{x}}_N$, we use the method of linear least squares (LLS) [25]. Since $\mathbf{X}_f$ is not a square matrix, there is no inverse matrix. Thus, $\mathbf{u}_f^*$ is calculated as

$$\mathbf{u}_f^* = \mathbf{X}_f^H(\mathbf{X}_f\mathbf{X}_f^H)^{-1}\mathbf{x}_N. \quad (10)$$

Using this transformation vector that is extracted from the original received signals, the forward linearly predicted array expansion can be conducted to extrapolate signals to the right side of the array. The transformation vector can be generated in the same way even when the number of targets is plural

Assuming that the antenna element is located outside and to the right of the original array antenna, its corresponding signal can be generated using transformation vector $\mathbf{u}_f^*$ and received signals $\mathbf{x}_2, \mathbf{x}_3, \cdots, \mathbf{x}_N$. In general, when assuming the $(N+p)$-th $(p \geq 1)$ antenna element is located outside and to the right of the original array antenna, its signal can be sequentially generated as

$$\hat{\mathbf{x}}_{N+p} = \mathbf{Y}_f^{(p)}\mathbf{u}_f^*, \tag{11}$$

where

$$\mathbf{Y}_f^{(p)} = \begin{cases} [\mathbf{x}_2, \mathbf{x}_3, \cdots, \mathbf{x}_N] \\ \text{for } p=1 \\ [\mathbf{x}_3, \mathbf{x}_4, \cdots, \mathbf{x}_N, \hat{\mathbf{x}}_{N+1}] \\ \text{for } p=2 \\ [\mathbf{x}_{p+1}, \cdots, \mathbf{x}_N, \hat{\mathbf{x}}_{N+1}, \cdots, \hat{\mathbf{x}}_{N+p-1}] \\ \text{for } 3 \leq p < N-1 \\ [\mathbf{x}_N, \hat{\mathbf{x}}_{N+1}, \cdots, \hat{\mathbf{x}}_{N+p-1}] \\ \text{for } p = N-1 \\ [\hat{\mathbf{x}}_{p+1}, \hat{\mathbf{x}}_{p+2}, \cdots, \hat{\mathbf{x}}_{N+p-1}] \\ \text{for } p \geq N \end{cases}. \tag{12}$$

Once $\mathbf{u}_f^*$ is calculated for K time-sampled received signal vector, it can be repeatedly used to generate extrapolated signals. Within a snapshot, the linearity between the phases of the received signals are preserved, so the same transformation vector can be used to generate the expanded signals. Since the transformation vector with the smallest error is obtained using the linear least squared method, the error between the expanded signal and the real signal is also considerably small.

B. BACKWARD LINEARLY PREDICTED ARRAY EXPANSION

Besides the forward array expansion, we extrapolate the received signal beyond the first antenna element, as illustrated in Fig. 1(b). Similar to the forward array expansion, we extract the linear relation among the N received signals. For this expansion, the relation between $\mathbf{x}_1$ and $\mathbf{x}_N, \mathbf{x}_{N-1}, \cdots, \mathbf{x}_2$ can be expressed as the linear combination:

$$\mathbf{x}_1 \approx [\mathbf{x}_N, \mathbf{x}_{N-1}, \cdots, \mathbf{x}_2] \cdot \mathbf{u}_b = \mathbf{X}_b\mathbf{u}_b = \tilde{\mathbf{x}}_1, \tag{13}$$

where $\mathbf{u}_b$ is the vector composed of coefficients for the linear combination. Then, the optimal backward transformation vector $\mathbf{u}_b^*$ can be obtained by solving the least squares problem, which is calculated as

$$\mathbf{u}_b^* = \arg\min_{\mathbf{u}_b} \left\| \mathbf{x}_1 - \tilde{\mathbf{x}}_1 \right\|_2^2. \tag{14}$$

Similar to (10), we can extract the transformation vector $\mathbf{u}_b^*$ by using the LLS method, which minimizes the norm of the difference between $\mathbf{x}_1$ and $\tilde{\mathbf{x}}_1$. Thus, the solution of (14) is given by

$$\mathbf{u}_b^* = \mathbf{X}_b^H(\mathbf{X}_b\mathbf{X}_b^H)^{-1}\mathbf{x}_1. \tag{15}$$

As in the case of $\mathbf{u}_f^*$, $\mathbf{u}_b^*$ is used for the backward linearly predicted array expansion to extrapolate signals to the left side of the array.

As shown in Fig. 1(b), assuming that the antenna element is located outside and to the left of the original array antenna, its corresponding signal can be generated using transformation vector $\mathbf{u}_b^*$ and $\mathbf{x}_{N-1}, \mathbf{x}_{N-2}, \cdots, \mathbf{x}_1$. In general, when assuming the $(-q+1)$-th $(q \geq 1)$ antenna element is located outside and to the the left of the original array antenna, its signal can be sequentially generated as

$$\hat{\mathbf{x}}_{-q+1} = \mathbf{Y}_b^{(q)}\mathbf{u}_b^*, \tag{16}$$

where

$$\mathbf{Y}_b^{(q)} = \begin{cases} [\mathbf{x}_{N-1}, \mathbf{x}_{N-2}, \cdots, \mathbf{x}_1] \\ \text{for } q=1 \\ [\mathbf{x}_{N-2}, \mathbf{x}_{N-3}, \cdots, \mathbf{x}_1, \hat{\mathbf{x}}_0] \\ \text{for } q=2 \\ [\mathbf{x}_{N-q}, \cdots, \mathbf{x}_1, \hat{\mathbf{x}}_0, \cdots, \hat{\mathbf{x}}_{-q+2}] \\ \text{for } 3 \leq q < N-1 \\ [\mathbf{x}_1, \hat{\mathbf{x}}_0, \cdots, \hat{\mathbf{x}}_{-q+2}] \\ \text{for } q = N-1 \\ [\hat{\mathbf{x}}_{N-q}, \hat{\mathbf{x}}_{N-q-1}, \cdots, \hat{\mathbf{x}}_{-q+2}] \\ \text{for } q \geq N \end{cases}. \tag{17}$$

In (17), $\hat{\mathbf{x}}_0$ and $\hat{\mathbf{x}}_{-q+1}$ denote the signals generated from the antenna elements located at distances d and qd to the left of the first antenna element, respectively. Similar to $\mathbf{u}_f^*$, once $\mathbf{u}_b^*$ is calculated for K time-sampled received signal vector, it can be repeatedly used to generate extrapolated signals.

Finally, by using the proposed forward and backward linearly predicted array expansion, we can generate extrapolated signals $\hat{\mathbf{x}}_{-q+1}, \cdots, \hat{\mathbf{x}}_0, \hat{\mathbf{x}}_{N+1}, \cdots, \hat{\mathbf{x}}_{N+p}$. Then, the Bartlett algorithm is conducted using both the received signals by the N elements of the actual array antenna and the $(p+q)$ generated signals, and we can achieve an improved angular resolution.

IV. SIMULATION RESULTS

In this section, we verify the performance of the proposed method through simulations. In our simulation, the number of antenna elements in the receiving array antenna is set as four, which is typical in automotive radar systems [11], [26], [27]; the signal-to-noise ratio (SNR) of the received signal is set to 10 dB; the number of time sample K is set to 1361. The number of time samples is set as above to configure as the same to the actual radar system. In addition, two types of antenna spacing are used, i.e., 0.8λ and 1.8λ. To prove that the proposed algorithm works well regardless of the position of the targets, the simulations were performed with varying target positions.

First, we compare the DOA estimation performance using forward, backward, and the combination of both array expansion methods to determine the direction to expand

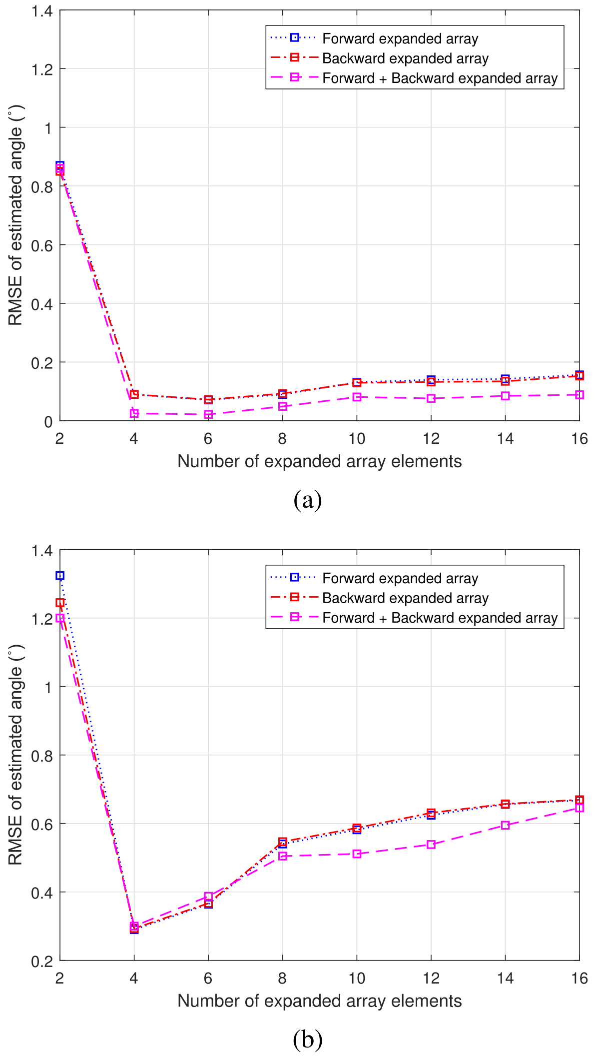

FIGURE 2. **RMSE according to the number of expanded antenna elements (a) without normalization and (b) with normalization.**

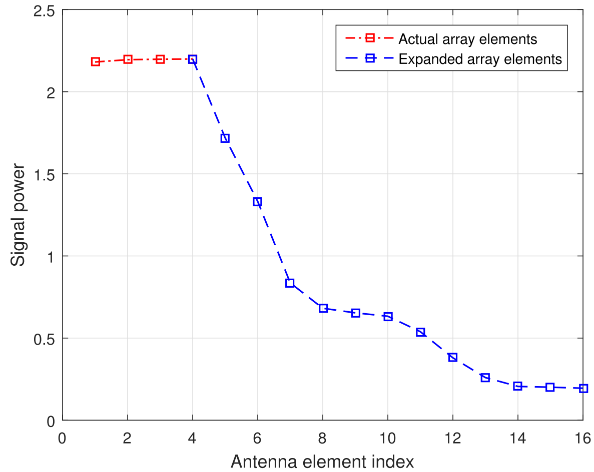

FIGURE 3. **Power of the actual (1 to 4) and extrapolated (5 to 16) signals.**

the array. By increasing the number of antenna elements in the three array expansion methods, we calculate the RMSE as

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{l=1}^{L}\sum_{m=1}^{M}\left\{(\theta_l - \hat{\theta}_l^{(m)})^2\right\}}{LM}}\ (^\circ), \tag{18}$$

where M denotes the number of simulation runs and $\hat{\theta}_l^{(m)}$ denotes the estimated angular location of the l-th target in the m-th ($m = 1, 2, \cdots, M$) simulation. We executed $M = 1000$ simulation runs under the same conditions. In addition, we disregarded the RMSE for the case when the number of targets was incorrectly determined.

Figure 2 shows the RMSE for the three variations of our proposed array expansion methods according to the number of expanded array antenna elements when the targets are located at -1° and 4° with the antenna spacing of 1.8λ. In this simulation, because the number of antenna elements is expanded to both sides when the combined forward and backward expansion is used, we calculated the RMSE at increments of two for all the variations of the proposed method. Because the angular displacement between the two targets was within the half-power beamwidth, it is difficult to resolve the two targets with the conventional Bartlett algorithm [28]. However, when the array elements are only expanded by two, the targets were distinguished. The resolution probability increases even if only two expanded antennas are added; however, because the RMSE is relatively high, it is necessary to increase the number of expanded antennas to four or more for a more accurate angle estimation. Because the signal generation is similar in the forward only and the backward only expansions, both methods show almost the equivalent DOA estimation performance. When the number of expanded antenna elements is less than four, little performance difference exist in the three expansion methods. However, for array expansions of or above four elements, the combined forward and backward array expansion method clearly outperforms the individual expansions. After this initial evaluation, we used the combined expansion for the remainder of the simulations.

Figure 2(a) also shows that the RMSE values does not decrease when the number of expanded elements is more than a certain number. This phenomenon can be related to the power of the extrapolated signal as shown in Fig. 3, where the power of the 4 received signals is higher than that of the extrapolated signals using forward expansion. Furthermore, the power of the extrapolated signals converges to zero as the number of expanded antenna elements increases. This can be explained as follows. Let transformation vector $\mathbf{u}_f^*$ be expressed as

$$\mathbf{u}_f^* = [u_{f_1},\ u_{f_2},\ u_{f_3}]^T, \tag{19}$$

where u_{f_j} ($j = 1, 2, 3$) is the j-th element in $\mathbf{u}_f^*$. In this simulation, because the sum of the magnitude of u_{f_j} is smaller than one, the power of the extrapolated signals gradually decreases [29]. Thus, the newly generated signals with small power cannot considerably improve the angle estimation performance, and the RMSE does not further decrease, as shown in Fig. 2(a). To solve this problem, we considered the normalization of the extrapolated signals as shown in Fig. 2(b). However, because the transformation vector $\mathbf{u}_f^*$ is a vector that minimizes the error of the extrapolated signals using the linear least-squares method, if we normalize the extrapolated signals, the weight of the extrapolated signals becomes large. Because the extrapolated signals are inaccurate relative to the actual signals, the angular estimation error increases as the weight multiplied by the extrapolated signals increases.

In general, the side lobes increase when the antenna spacing is above 0.5λ. Furthermore, when the antenna spacing is larger than 1.0λ, undesired grating lobes are generated [7], which cannot be distinguished from the main lobe. Thus, it is difficult to estimate the angular location of the target through conventional spectral-based methods such as the Bartlett algorithm. If the antenna spacing is large, the angular resolution of targets is better. However, larger antenna spacing has the disadvantage that grating lobes are generated near the boresight. However, we can mitigate the side lobes and prevent the generation of grating lobes in the field of view of the array antenna, using the proposed array expansion method. Figure 4 shows the DOA estimation of the Bartlett algorithm with and without the proposed array expansion method for antenna spacings of 0.8λ and 1.8λ and 8 expanded antenna elements. Figure 4(a) shows that unlike the conventional Bartlett algorithm, the algorithm including the proposed expansion distinguishes two adjacent targets. Moreover, we compare the DOA estimation performance when applying the Bartlett with the proposed method to 0.8λ ULA antenna to when applying the conventional Bartlett to 2.4λ ULA antenna. The angles estimated in both cases are $(-1.9°, 5.4°)$ and $(-2.2°, 5.7°)$, respectively. For the ULA antenna with spacing 2.4λ, because the half-power beamwidth is approximately 7.4°,the two targets can be separated; however, grating lobes appear in the range from $-30°$ to $-15°$ and from 15° to 30°. Due to these grating lobes, the angles of the targets cannot be exactly estimated. On the other hand, because the grating lobes do not appear in the field of view when using the proposed method, grating lobes cannot be confused with main lobes. Thus, these results suggest that DOA estimation and its angular resolution can be improved using the proposed method, even when the antenna spacing is smaller than 1.0λ. Similar trends can be appreciated when considering a spacing of 1.8λ, and grating lobes appear when using a spacing of 5.4λ, as shown in Fig. 4(b).

Next, we compare the performance of the proposed method with that of the conventional interpolation using LLS method [11]. Figure 5(a) shows the DOA estimation of the Bartlett algorithm with and without the proposed method, and that of the Bartlett algorithm using the conventional LLS method. For the conventional LLS method [11], sector $\Theta = [\theta_L, \theta_R] = \{\theta \mid \theta_L \leq \theta \leq \theta_R\}$ must be set, which indicates the observation interval in the field of view of the array antenna.

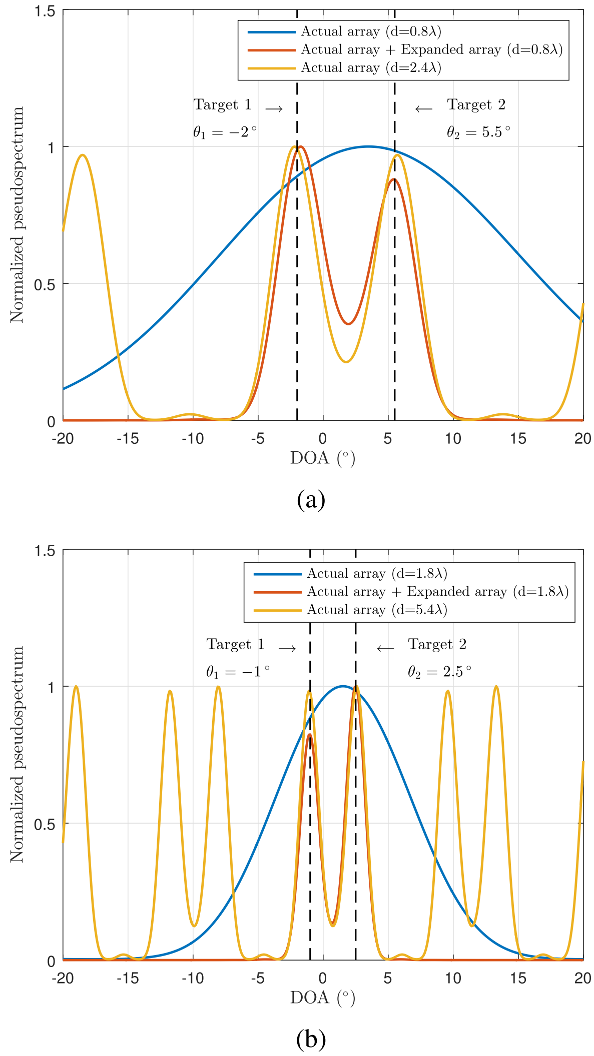

FIGURE 4. DOA estimation of the Bartlett algorithm with and without the proposed array expansion method for spacing of (a) 0.8λ and (b) 1.8λ.

In this simulation, the sector of the conventional LLS method is given as $[-15°, 15°]$ with 8 expanded antenna elements. As shown in Fig. 5(a), regardless of the range of the sector, the proposed method has better angle estimation accuracy than the conventional LLS method. In Fig 5(b), when the sector is $[-15°, 15°]$, the angular location of the targets using conventional LLS method cannot be distinguished, and only one target is estimated at 1.1°. In this case, no apparent performance difference exists compared to the conventional Bartlett algorithm. However, when using the proposed method, the angular locations of the targets are accurately estimated at $-0.9°$ and 2.9°. In the conventional LLS method, when the range of the sector is close to the range of target angles, the accuracy of the estimation is improved. Thus, we use more narrow sector as $[-5°, 5°]$, and the estimation results are also given in Fig. 5(b). When the range of the sector gets narrow, the angles of the targets are estimated as $-1.7°$ and 1.8°, which are close to actual angles; however, even when the narrow range of the sector is used, the estima-

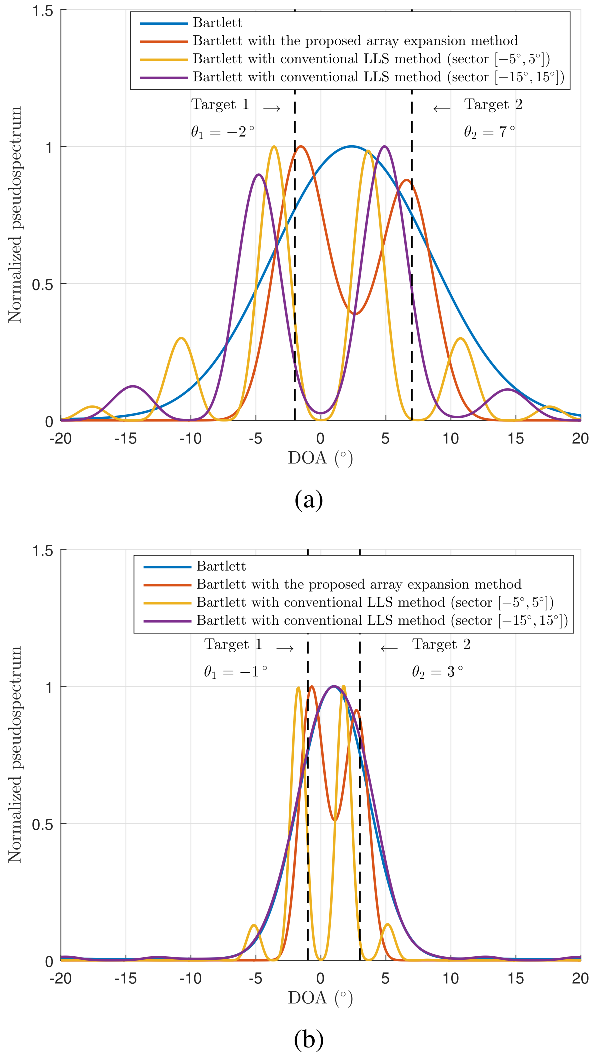

FIGURE 5. DOA estimation of the Bartlett algorithm with and without the proposed array expansion method, and it using the conventional LLS method for spacing of (a) 0.8λ and (b) 1.8λ.

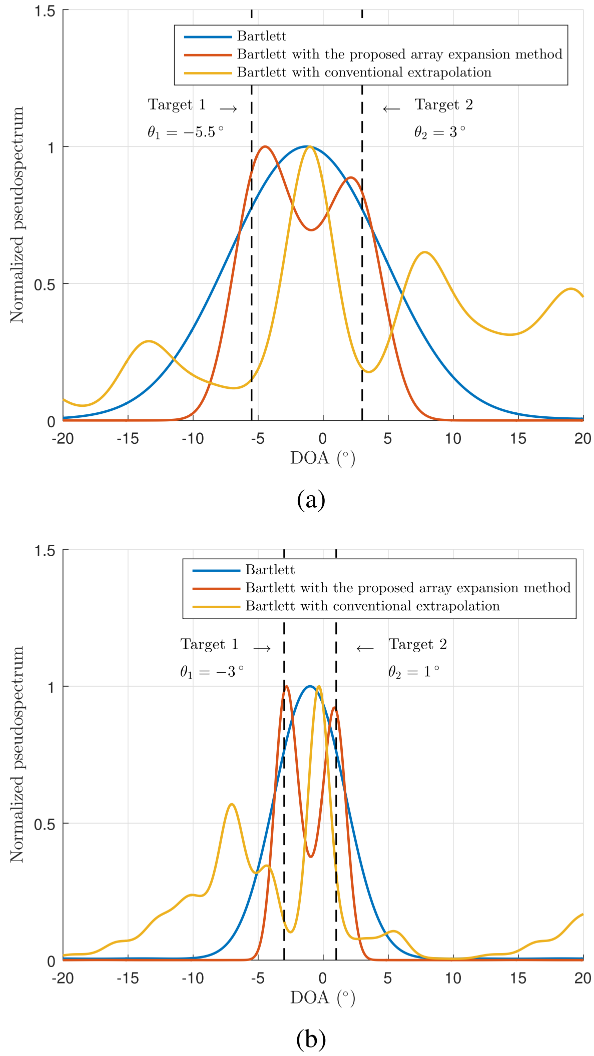

FIGURE 6. DOA estimation of the Bartlett algorithm with and without the proposed array expansion method, and it using conventional extrapolation for spacing of (a) 0.8λ (b) 1.8λ.

tion error is larger than that of using the proposed method. In addition, if the target exists outside the sector, the conventional LLS method cannot accurately estimate the position of the target [10]- [12]. Overall, the proposed method outperforms conventional LLS method in estimation accuracy and angular resolution without requiring to define a sector and regardless of antenna spacing.

Then, we compare the DOA estimation using the proposed method with that of using conventional extrapolation [14], [15]. In the conventional extrapolation, a transformation vector is obtained by using a correlation matrix of the received signal, and the number of antenna elements can be expanded at each side only by the same number of the actual antenna elements [14]. Therefore, we considered 8 expanded antenna elements. Figures 6(a) and 6(b) show the DOA estimation of the Bartlett algorithm using the proposed array expansion and conventional extrapolation for spacing of 0.8λ and 1.8λ, respectively. When the proposed method is applied, the two targets are completely separated, and the estimated angles are −4.8° and 2.2° for 0.8λ. On the other hand, when using conventional extrapolation, the exact angular locations of the targets cannot be estimated because four peak values appear in the pseudospectrum. Even when the antenna spacing is larger, as shown in Fig. 6(b), the proposed method clearly outperforms the conventional extrapolation.

We also evaluate the performance of the proposed method and conventional extrapolation by varying the number of expanded antenna elements. Figures 7 and 8 show the RMSE and resolution probability according to the number of expanded antenna elements, respectively. The resolution probability is defined as the rate of successfully separated targets over the simulations, where targets were located at −1° and 3° with antenna spacing of 1.8λ. For both methods, when the number of expanded antenna elements is below 4, the two targets are mostly indistinguishable as shown in Fig. 8. However, for increased number of expanded

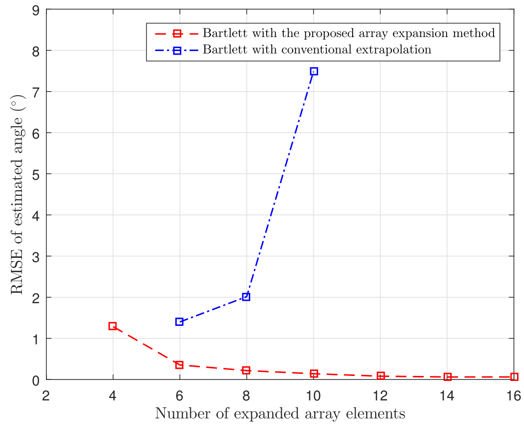

FIGURE 7. RMSE according to the number of antenna elements using the proposed method and conventional extrapolation.

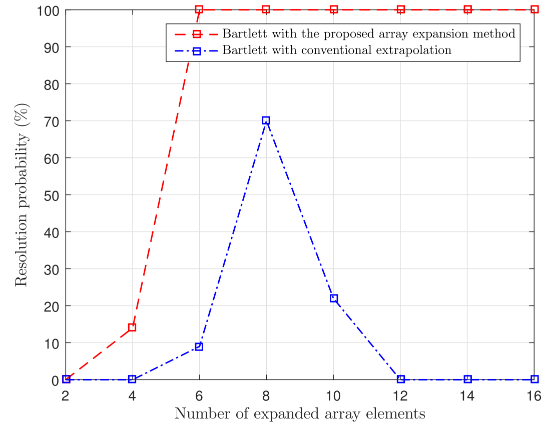

FIGURE 8. Resolution probability according to the number of antenna elements using the proposed method and conventional extrapolation.

antenna elements, the angle estimation accuracy tends to improve. The best performance of conventional extrapolation is achieved when the number of expanded antenna elements is approximately twice the number of the actual antenna elements. However, the performance of the proposed method has more improvement when the number of antenna elements is more than two times of the number of actual antenna elements. Moreover, the proposed method has a lower computational complexity than the conventional extrapolation, which requires the calculation of the transformation vector whenever signals are generated [14], [15]. In the process of creating an expanded array, an expanded signal is first generated using the actual received signal. However, if more than $2N$ expanded antennas are generated, only the expanded signal will be used to generate the new expanded signal, so the accuracy of the expanded signal will be lowered. However, since the proposed method generates an expanded signal with fairly high accuracy, the angle estimation performance is improved even if the number of antennas is increased to $3N$. Although the optimal number of the expanded antennas varies depending on the position of the target, basically, $2N$ is most appropriate to generate, and in some cases, performance increases even when $3N$ expanded signals are generated.

Fig. 9 shows how RMSE varies with the number of targets. Even though the number of targets is the same, the accuracy varies depending on the position of the target. As the number of targets increases, the angle estimation accuracy decreases. Furthermore, the conventional extrapolation method cannot accurately estimate the number of targets when the number of targets is more than 3, but the proposed method is quite accurate even if the number of targets increases. Fig. 10 shows how RMSE varies with the number of snapshots. The SNR was 0 dB, the DOAs of targets were set to $-2°$ and $4°$, respectively, and 10000 simulations were performed. When multiple snapshots are used, the accuracy of the used data to find DOA of targets can be increased, so the angle estimation performance is improved. In Fig. 10, as the number of snapshots used increases, the RMSE of estimated angle is decreased

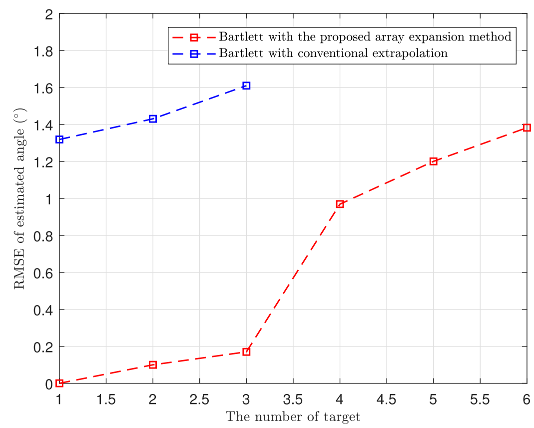

FIGURE 9. Resolution probability according to the number of targets using the proposed method and conventional extrapolation.

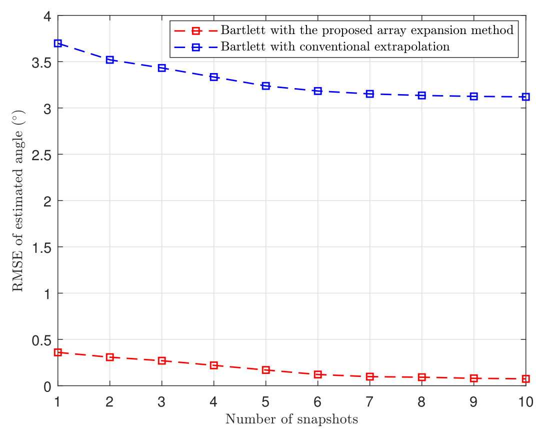

FIGURE 10. RMSE according to the number of snapshot using the proposed method and conventional extrapolation.

for both algorithms. Fig. 11 and 12 show how the angular resolution and RMSE vary with SNR. The performance of the proposed method and the conventional extrapolation method are compared. To compare the performance under the same conditions as the conventional method, the number of

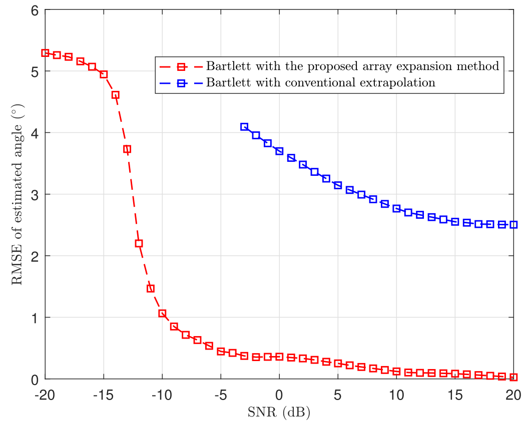

FIGURE 11. **RMSE according to SNR using the proposed method and conventional extrapolation.**

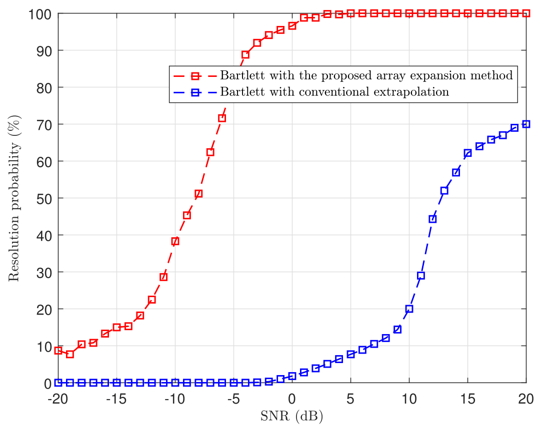

FIGURE 12. **Resolution probability according to SNR using the proposed method and conventional extrapolation.**

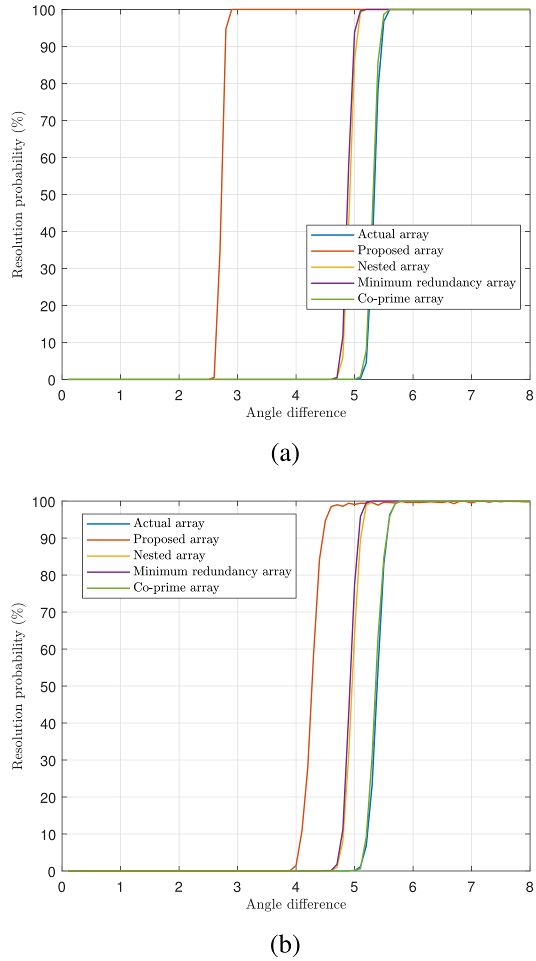

FIGURE 13. **Resolution probability according to the DOA difference using the actual array, proposed array, minimum redundancy array, nested array, co-prime array, when SNR is (a) 10dB (b) −5dB.**

extrapolated antennas was set to 8. The SNR was increased by 1 dB, and 10000 simulations were performed for each. The DOAs of targets were set to $-2°$ and $4°$, respectively. In Fig. 12, the conventional method distinguishes two targets from when the SNR is above -3dB. Thus, we calculate RMSE only when the SNR is above -3dB. The performance of the proposed method is slightly degraded when the SNR is very low, but otherwise, the performance is much improved than that of the conventional extrapolation method.

To investigate the improvement of the proposed array expansion method compared to the angular resolution of various NLAs proposed previously, we simulated the resolution probability by increasing the angular interval between the targets by 0.1°. The total antenna aperture size is set to 11λ and 6 antennas are used. Fig. 13 shows how the angular resolution varies with the antenna array type for the two SNRs. In Fig. 13(a) When the SNR is 10dB, the angular resolution is improved by more than 2° compared to other NLAs. However, because the proposed method is sensitive to SNR changes, the angular resolution of the proposed method approaches that of conventional NLAs under low

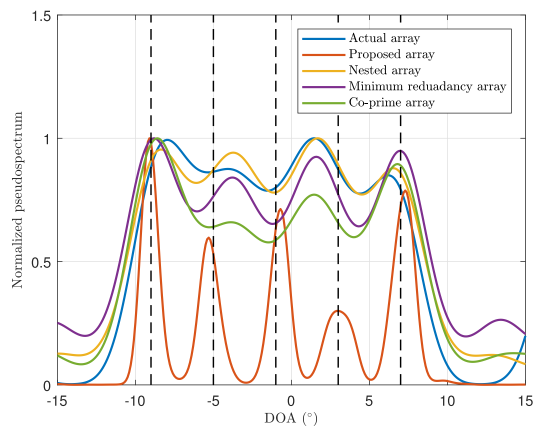

FIGURE 14. **Normalize Bartlett pseudospectra of different antenna arrays.**

SNR conditions in Fig. 13(b). Moreover, in Fig. 14, the additional simulations were performed in the presence of five targets in order to check whether the proposed method is applicable even in the presence of a large number of targets. When multiple targets are in close proximity and the targets

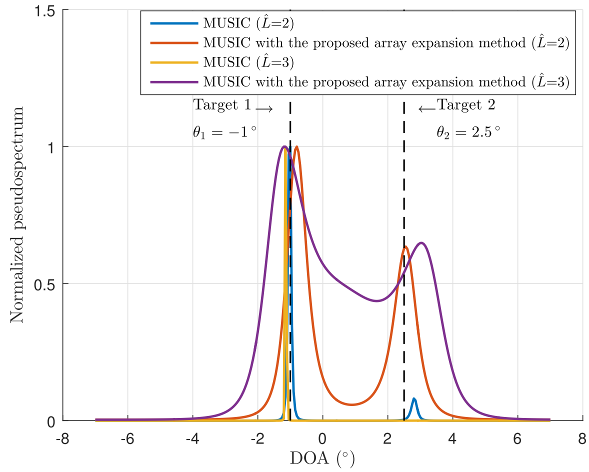

FIGURE 15. DOA estimation of the MUSIC algorithm with and without the proposed array expansion method considering correctly estimated ($\hat{L} = 2$) and incorrectly estimated ($\hat{L} = 3$) number of targets.

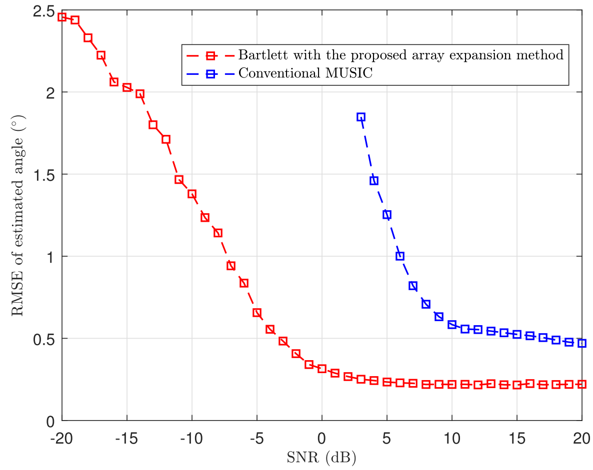

FIGURE 16. RMSE according to SNR using MUSIC with the proposed method and conventional MUSIC.

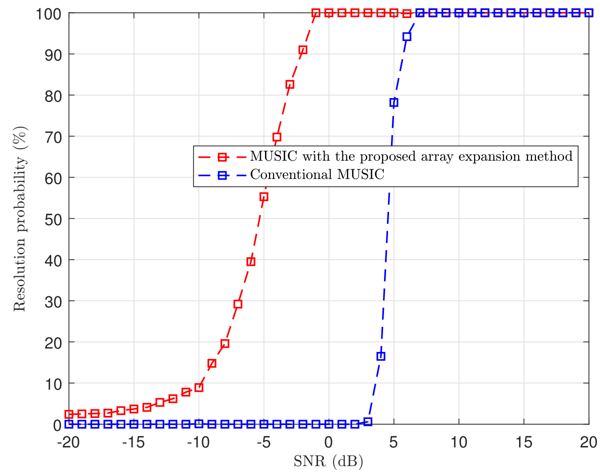

FIGURE 17. Resolution probability according to SNR using MUSIC with the proposed method and conventional MUSIC.

are detected using the conventional NLAs, it appears that there are three or four targets because not all targets can be distinguished. However, all targets are correctly detected by using the proposed array expansion method.

In addition, we apply the proposed array expansion to the MUSIC DOA estimation algorithm, which delivers a high resolution and is reported to outperform the conventional Bartlett algorithm [24]. MUSIC algorithm is one of the subspace based algorithms. MUSIC is a method of estimating the DOA of the target using the orthogonality between the received signal and the noise component. After eigenvalue decomposition of the covariance matrix of the received signal, the eigenvector corresponding to the noise component is multiplied by the steering vector to find the DOA of the target. However, to find the DOA of targets using the MUSIC algorithm, the number of targets should be estimated in advance.

The MUSIC angle estimation algorithm is performed assuming that the targets correspond to the most dominant eigenvalues obtained through the eigenvalue decomposition of the correlation matrix of the received signal. Therefore, if the number of estimated targets is more than the number of actual targets, the noise component that corresponds to a large eigenvalue can be recognized as a target. Conversely, if the number of estimated targets is less than the number of actual targets, the signal component that corresponds to a small eigenvalue can be recognized as noise. Thus, in the MUSIC algorithm, the difference between the estimated and actual number of targets undermines the algorithm performance [27]. Figure 15 shows the DOA estimation of the MUSIC algorithm with and without the proposed array expansion method for two targets located at $-1°$ and $2.5°$, respectively and the antenna spacing of 1.8λ. In addition, we tested the algorithm considering estimated number of targets $\hat{L} = 2$ and $\hat{L} = 3$, to evaluate the effect of an erroneous number of targets in the latter case (i.e., $\hat{L} = 3$). As shown in the figure, when $\hat{L} = 3$, the two targets cannot be identified by the MUSIC algorithm. However, when the proposed method is applied, the angular locations of the targets can be correctly found regardless of the estimated number of targets. Furthermore, for statistical analysis, we analyzed the RMSE and resolution probability through 10000 simulations. RMSE is calculated using the equation in (18). The resolution probability is calculated from the number of times that the targets were properly separated into two targets during a total of 10,000 tests. As shown in Table 1, when the proposed method is applied, the two targets are completely separated. In addition, although the MUSIC algorithm cannot detect the targets correctly if the number of targets is not known in advance, the resolution probability is increased by 20% or more when the proposed method is applied. This is almost the same as when the conventional MUSIC algorithm is applied when the number of targets is known. The RMSE is also lowered, which considerably improves the angle estimation accuracy.

TABLE 1. RMSE and resolution probability of MUSIC DOA estimation methods for two targets.

DOA estimation method	RMSE (°)	Resolution probability (%)
MUSIC ($\hat{L} = 2$)	0.22	92.37
MUSIC with the proposed method ($\hat{L} = 2$)	0.13	100
MUSIC ($\hat{L} = 3$)	0.34	68.23
MUSIC with the proposed method ($\hat{L} = 3$)	0.27	91.58

TABLE 2. RMSE and resolution probability of various DOA estimation methods for three targets.

DOA estimation method	RMSE (°)	Resolution probability (%)
Bartlett algorithm	-	0
Bartlett with the proposed method	0.27	100
Bartlett with conventional LLS method	1.03	47.94
Bartlett with conventional extrapolation	-	0
MUSIC ($\hat{L} = 3$)	0.69	88.59
MUSIC with the proposed method ($\hat{L} = 3$)	0.97	93.06

Furthermore, in Fig. 16 and 17, additional simulation is performed to verify that the proposed array expansion method is well applied to MUSIC algorithm even at low SNR. The DOAs of targets were set to $-1°$ and 2.5°, and the SNR was increased from -20dB to 20dB in 1dB steps. Applying the proposed array expansion method to MUSIC can achieve good separation of both targets even under low SNR. In addition, the accuracy of the angle estimation is also improved, which means that the RMSE is lower than that of the conventional MUSIC. In addition, we verified whether the proposed array expansion method can be applied when more than two targets exist. Hence, Table 2 shows a comparison among the Bartlett and MUSIC algorithms, with and without the proposed array expansion method, the Bartlett with the conventional LLS method, and the Bartlett with the conventional extrapolation for three targets located at $-8°$, $-1°$, and 7°, and correct the estimation for $\hat{L} = 3$ of the number of targets for the MUSIC algorithm. As shown in Table 2, the three targets cannot be distinguished using both the conventional Bartlett algorithm and the Bartlett algorithm with the conventional extrapolation method. However, the three targets are successfully decomposed using the proposed array expansion method. Moreover, applying the proposed method to the MUSIC algorithm shows that the RMSE is slightly higher but the separation probability is higher than that of the conventional MUSIC algorithm. Therefore, we confirmed that the proposed method can achieve more performance improvement when applied to the Bartlett algorithm than the MUSIC algorithm. In the simulation results, the DOA estimation using the Bartlett algorithm with the proposed method obtains angles closer to the actual values. Furthermore, because the computational complexity of the Bartlett algorithm is much lower than that of the MUSIC algorithm, which relies on the eigenvalue decomposition [27], the former with the proposed array expansion method can be considered as the most suitable DOA estimation method for automotive radar systems.

V. EXPERIMENTAL RESULTS

This section presents the performance of the proposed method from the experiments with an actual automotive radar system. We conducted the experiments in a test field using a long-range radar manufactured by Mando Corporation (Republic of Korea). This system has a field of view from $-10°$ to 10° and maximum detection range of 200 m. In addition, the number of transmitting and receiving antenna elements are 1 and 4, and the antenna spacing between receiving antenna elements is 1.8λ. Moreover, the system transmits a 76.5 GHz frequency-modulated continuous wave radar signal with bandwidth of 500 MHz and 10 ms sweep time for the up- and down-chirp signals. The transmitted signal is multiplied by signals reflected from targets, and the multiplied signal passes through a low-pass filter. The proposed array expansion method is applied to the time-domain low-pass filtered output.

In the experimental scenario, we place only two target vehicles in the field of view of the radar system to mitigate the effects of other external factors and analyze only the signals reflected from them, as shown in Fig. 18. Two identical target vehicles are placed at the same distance but at different angles. Two vehicles are placed 30 m away from the radar-equipped vehicle, and 4 m apart from each other. In addition, they are located at $-5°$ and 5° in the direction that the radar-equipped vehicle is looking at. For the measured signals in this environment, the normalized pseudospectrums of the Bartlett algorithm with and without the proposed array expansion method, and the Bartlett algorithm with conventional LLS method and conventional extrapolation are shown in Fig. 19. Similar to the simulation results, the Bartlett algorithm by itself was unable to distinguish the two targets and they were recognized as a single target. By applying the proposed method, the two targets were identified, and their angular positions accurately estimated at $-5.3°$ and 5.8°. Even though a narrow sector including the two target vehicles was set when using conventional LLS method as

TABLE 3. RMSE and resolution probability of various DOA estimation methods in actual experiments.

DOA estimation method	RMSE (°)	Resolution probability (%)
Bartlett algorithm	-	0
Proposed method with 8 expanded signals	3.97	94.33
Proposed method with 12 expanded signals	3.73	99.33
Conventional LLS method with 8 expanded signals	5.74	53.67
Conventional extrapolation with 8 expanded signals	11.63	15.33

FIGURE 18. Experimental setup with two targets located in front of a radar-equipped vehicle.

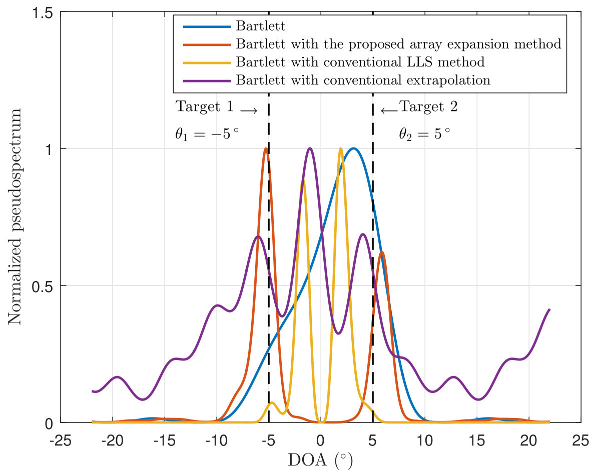

FIGURE 19. DOA estimation of the conventional Bartlett algorithm with and without the proposed array expansion method, and it using conventional LLS method and conventional extrapolation.

[−5° 5°], the estimated values were highly inaccurate. Moreover, the Bartlett algorithm with conventional extrapolation showed poor and unreliable identification and angle estimation results.

To quantify the performance of the proposed method in the statistical aspect, we evaluated the received signals from 300 measurements considering the same experimental conditions and calculated the RMSE and resolution probability for the different methods, as listed in Table 3. The comparison includes the Bartlett algorithm, as well as the algorithm with the proposed array expansion method, conventional LLS method, and conventional extrapolation. The Bartlett algorithm by itself was unable to identify the two targets from any of the 300 signals (i.e., resolution probability of 0%), and thus, the RMSE was not calculated. The Bartlett algorithm with either conventional LLS method or conventional extrapolation poorly identified the two targets, and the estimation accuracy was low. However, the Bartlett algorithm with the proposed method showed the best performance with the highest resolution probability and the lowest RMSE among the compared methods. Moreover, in the proposed method, we can increase the number of array elements further in contrast to the conventional extrapolation. The table also shows the DOA estimation performance when 12 signals are generated by our proposed method. In this case, the two targets are separated by more than 99% resolution probability, which is more improved than using 8 extrapolated signals.

VI. COMPUTATIONAL COMPLEXITY

The computational complexity of the Bartlett algorithm is about $O(P(2N-1)(N+P))$. P denotes the sampling grid of angle. The computational complexity of conventional MUSIC algorithm is about $O(N^3 + JNL)$ to proceed eigenvalue decomposition. J is the samples of the MUSIC null-spectrum function. In the proposed algorithm, the computational complexity depends on how many expanded antennas are generated. Assuming that the number of expanded antennas is $2N$, the computational complexity is approximately $O((18P + 4K)N^2)$. Furthermore, we measured the actual algorithm execution time. When the execution time of the Bartlett algorithm is 1, the execution time of the proposed method with Bartlett algorithm is 1.3 and the MUSIC algorithm is 1.8. Compared to the conventional Bartlett algorithm, the computational complexity is slightly increased, but the computational complexity is still lower than that of the MUSIC algorithm.

VII. CONCLUSION

In this paper, we proposed an improved DOA estimation method using linearly predicted array expansion for automotive radar systems having a small number of antenna elements. The performance of the proposed method was verified by simulations and actual experiments. In particular, we tested the proposed method using the Bartlett and MUSIC algorithms, which are widely applied in automotive radar systems. Unlike conventional LLS method, our proposed method does not require a predefined sector.

Furthermore, the proposed method has a low computational complexity than conventional extrapolation which requires the repeated calculation of transformation vectors to generate extrapolated signals. Overall, the proposed method outperforms the other approaches in every simulation and experiment with respect to estimation accuracy and angular resolution. Therefore, the proposed method could improve the low angular resolution caused by a small number of antenna elements. We expect that the proposed method will be an efficient and simple approach to improve DOA estimation without requiring additional hardware for automotive radar systems.

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S. Lee was with Seoul National University, Seoul, South Korea, when this research was conducted.

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