Pole Inflation from Broken Noncompact Isometry in Weyl Gravity

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We propose the microscopic origin of the pole inflation from the scalar fields of broken noncompact isometry in Weyl gravity. We show that the $SO(1, N)$ isometry in the field space in combination with the Weyl symmetry relates the form of the nonminimal couplings to the one of the potential in the Jordan frame. In the presence of an explicit breaking of the $SO(1, N)$ symmetry in the coefficient of the potential, we realize the pole inflation near the pole of the inflaton kinetic term. Applying our results to the Higgs or Peccei-Quinn (PQ) inflation models, we find that there is one parameter family of the solutions for the pole inflation, depending on the overall coefficient of the Weyl covariant derivatives for scalar fields. The same coefficient not only makes the predictions of the pole inflation varying, being compatible with the Planck data, but also determines the mass of the Weyl gauge field. We also show that the isocurvature perturbations of the axion can be suppressed sufficiently during the PQ pole inflation, and the massive Weyl gauge field produced during reheating serves as a dark matter candidate.

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Introduction—Inflation has been a main paradigm for the early universe by which various problems in the standard Big Bang cosmology are solved and the initial conditions for the flat, homogeneous, and isotropic universe can be explained. A slowly rolling scalar field, the so called inflaton, is required to derive inflation, and its quantum fluctuations generate necessary inhomogeneities observed in the cosmic microwave background (CMB) and large scale structures. Higgs inflation with a nonminimal coupling [1] has drawn a lot of attention because it shows a possibility that inflation is realized by the Higgs field within the standard model (SM), but a consistent picture beyond Higgs inflation should emerge due to the problem with large nonminimal coupling [2–4].

There is another class of inflation models where the inflaton has a conformal coupling to gravity [4,5], so that inflation takes place close to the pole of the inflaton kinetic term in the Einstein frame. This is dubbed pole inflation. The concept of the inflation for α-attractor models was also introduced in Refs. [6–9]. Global conformal symmetry can be gauged by a local conformal symmetry or Weyl symmetry. As a result, the Planck mass can be generated dynamically from the vacuum expectation value of the dilaton field or one of the scalar fields of a nonlinear sigma model type [10]. Furthermore, multifield models with Weyl symmetry including the SM Higgs were considered for inflation [11–13].

The SM is based on the gauge principle explaining the forces in nature after the gauge symmetries are broken spontaneously, and there is an approximate custodial symmetry for SM Higgs fields, which is broken only by the $U(1)_Y$ gauge coupling and Yukawa couplings. A similar gauge principle is applied to the theory of gravitation such that the conformal symmetry is gauged and it is broken spontaneously. As a result, Einstein gravity is reproduced, up to a massive Weyl gauge field, which couples to gravity minimally. The goal of this article is to make the Weyl gauge symmetry manifest in the extension with extra scalar multiplets beyond the dilaton and the metric tensor, so it is suitable for a unified description of the gravity-Higgs system based on both the gauge symmetry principle and the extended custodial symmetry for the dilaton and the extra multiple fields. The scalar sector contains an extended custodial symmetry for the SM Higgs or the Peccei-Quinn (PQ) fields, which is the inflaton candidate with an appropriate form of the scalar potential in the context of the pole inflation. The full content of the SM or its nongravitational extensions can be easily accommodated in this setup.

In this Letter, we propose the multifield models for inflation respecting both the Weyl symmetry and the broken noncompact isometry in the field space such as $SO(1, N)$, which is the extension of the isometry or custodial symmetry of nondilaton scalar fields. After the Weyl symmetry is broken spontaneously due to the vacuum expectation value (VEV) of the dilaton, the $SO(1, N)$ symmetry is spontaneously broken to $SO(N)$, and the Planck scale is generated. In this scenario, we pursue a concrete realization of the pole inflation in Weyl gravity, which is applicable to the cases with

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the SM Higgs fields [14] and the PQ singlet scalar field [15,16] transforming under electroweak symmetry or a global $U(1)$ PQ symmetry, respectively. For inflation, we introduce an explicit breaking for the $SO(1,N)$ symmetry only in the effective quartic coupling, but the $SO(N)$ symmetry remains unbroken in the Lagrangian. We discuss the roles of the Weyl covariant derivatives for scalar fields for the mass of the Weyl gauge field and the solutions for pole inflation [17].

The setup—We consider the dilaton χ, an N-dimensional scalar multiplet, $\Phi = (1/\sqrt{2})(\phi_1, \phi_2, \ldots, \phi_N)^T$, composed of N real scalar fields, and the Weyl gauge field w_μ in Weyl gravity. It can accommodate the SM Higgs doublet for $N=4$ or the PQ singlet scalar field for $N=2$. Then, the Jordan frame Lagrangian for bosons respecting the Weyl invariance and the $SO(1,N)$ isometry, in $\{\chi, \phi_i\}$ with $i = 1, 2, \ldots, N$, is given by

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = (1+a)\left[-\frac{1}{12}(\chi^2 - \phi_i^2)R - \frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2}(\partial_\mu \phi_i)^2\right] + \frac{1}{2}a(D_\mu \chi)^2 - \frac{1}{2}a(D_\mu \phi_i)^2 - \frac{1}{4}w_{\mu\nu}w^{\mu\nu} - V, \quad (1)$$

with

$$V(\chi, \phi_i) = \frac{1}{\langle \chi^4 \rangle} f(\phi_i^2/\chi^2)(\chi^2 - \phi_i^2)^2. \quad (2)$$

Here, we note that the Weyl gauge transformations are

$$g_{\mu\nu} \to e^{2\alpha(x)} g_{\mu\nu}, \qquad \chi \to e^{-\alpha(x)}\chi,$$
$$\phi_i \to e^{-\alpha(x)}\phi_i, \qquad w_\mu \to w_\mu - \frac{1}{g_w}\partial_\mu \alpha(x), \quad (3)$$

with $\alpha(x)$ being an arbitrary real transformation parameter. Then, the covariant derivatives for the dilaton and the Higgs fields are given by

$$D_\mu \chi = (\partial_\mu - g_w w_\mu)\chi, \qquad D_\mu \phi_i = (\partial_\mu - g_w w_\mu)\phi_i, \quad (4)$$

with g_w being the Weyl gauge coupling, and the field strength tensor for the Weyl gauge field is $w_{\mu\nu} = \partial_\mu w_\nu - \partial_\nu w_\mu$. We normalized the scalar kinetic terms in Eq. (1), up to a constant parameter a. We did not include the SM gauge interactions for the Higgs fields explicitly, but they can be also introduced easily. $f(\phi_i^2/\chi^2)$ is an arbitrary function of ϕ_i^2/χ^2, respecting the Weyl gauge symmetry, but it breaks the $SO(1,N)$ isometry down to $SO(N)$ explicitly. If $f(\phi_i^2/\chi^2)$ is a constant parameter, the full $SO(1,N)$ is respected, but it leads to a constant vacuum energy after a gauge fixing, as will be shown later. Thus, in order to consider the inflationary cosmology with a slow-roll inflaton, $f(\phi_i^2/\chi^2)$ must not be constant.

Due to the Weyl symmetry in the Jordan frame Lagrangin in Eq. (1), the form of the Lagrangian is the same in any other frames related by the Weyl transformations unless a gauge for the Weyl symmetry is fixed. Thus, we first fix the gauge for the Weyl symmetry with $\chi = \langle \chi \rangle = \sqrt{6/(1+a)}$ in the Jordan frame, so we break the Weyl symmetry and the $SO(1,N)$ symmetry spontaneously. Then, the Lagrangian in Eq. (1) becomes

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}\left(1 - \frac{1}{6}(1+a)\phi_i^2\right)R + \frac{1}{2}(\partial_\mu \phi_i)^2 + \frac{1}{2}a g_w w_\mu \partial^\mu \phi_i^2 - \frac{1}{2}a g_w^2 \phi_i^2 w_\mu w^\mu - \frac{1}{4}w_{\mu\nu}w^{\mu\nu} + \frac{1}{2}m_w^2 w_\mu w^\mu - V(\langle \chi \rangle, \phi_i), \quad (5)$$

with

$$m_w^2 = a g_w^2 \langle \chi^2 \rangle = \frac{6 a g_w^2}{1+a}, \quad (6)$$

$$V(\langle \chi \rangle, \phi_i) = f(\phi_i^2/\langle \chi^2 \rangle)\left(1 - \frac{1}{6}(1+a)\phi_i^2\right)^2. \quad (7)$$

In the presence of electroweak symmetry breaking, there is an additional contribution to the Weyl gauge field by $a g_w^2 v^2$, but it is negligible as compared to the one from the dilaton VEV.

For $a = 0$, we get the same form of the Higgs part of the Lagrangian as in the Higgs pole inflation where the Higgs is conformally coupled to gravity and both the effective Planck scale and the Higgs potential depend on the same factor, $(1 - \frac{1}{6}\phi_i^2)$ [14]. But, in this case, the Weyl gauge field would be massless, while being decoupled from the Higgs fields. However, for $a \neq 0$, the Weyl gauge field becomes massive, and we can generalize the Higgs pole inflation, as will be discussed later. As compared to the case with conformal symmetry in Ref. [5], our results rely on the spontaneously broken Weyl gauge symmetry. Thus, there are extra interaction terms between the Weyl gauge field and the Higgs fields. The same results hold for the PQ pole inflation.

Gauge-fixed Lagrangian in Einstein frame—Using Eq. (5) and the redefined Weyl gauge field, we can rewrite the Lagrangian in the Jordan frame, as follows:

$$\frac{\mathcal{L}_{J,\mathrm{eff}}}{\sqrt{-g_J}} = -\frac{1}{2}\left(1 - \frac{1}{6}(1+a)\phi_i^2\right)R + \frac{1}{2}(\partial_\mu \phi_i)^2 - \frac{1}{48}a(1+a)\cdot\frac{(\partial_\mu \phi_i^2)^2}{1 - \frac{1}{6}(1+a)\phi_i^2} - f(\phi_i^2/\langle \chi^2 \rangle)\left(1 - \frac{1}{6}(1+a)\phi_i^2\right)^2 - \frac{1}{4}\tilde{w}_{\mu\nu}\tilde{w}^{\mu\nu} + \frac{1}{2}m_w^2\left(1 - \frac{1}{6}(1+a)\phi_i^2\right)\tilde{w}_\mu \tilde{w}^\mu, \quad (8)$$

where

$$\tilde{w}_\mu \equiv w_\mu - \frac{1}{2g_w}\partial_\mu \ln(m_w^2 - a g_w^2 \phi_i^2). \quad (9)$$

Here, we used Eq. (6) for m_w^2.

Now making a rescaling of the metric by $g_{\mu\nu,J} = g_{\mu\nu,E}/\Omega$, with $\Omega = 1-\frac{1}{6}(1+a)\phi_i^2$, we obtain the Einstein frame Lagrangian from Eq. (8) as

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}R + \frac{3}{4\Omega^2}(\partial_\mu\Omega)^2 + \frac{1}{2}\frac{(\partial_\mu\phi_i)^2}{\Omega} - \frac{1}{48}a(1+a)\cdot\frac{(\partial_\mu\phi_i^2)^2}{\Omega^2} - f(\phi_i^2/\langle\chi^2\rangle) - \frac{1}{4}\tilde{w}_{\mu\nu}\tilde{w}^{\mu\nu} + \frac{1}{2}m_w^2\tilde{w}_\mu\tilde{w}^\mu. \quad (10)$$

Thus, the redefined Weyl gauge field $\tilde{w}_\mu$ is decoupled from the Higgs fields, and it couples to gravity minimally. We note that the Weyl gauge field has an arbitrary mass depending the Weyl gauge coupling g_w and a, and there is a Z_2 symmetry for $\tilde{w}_\mu$ in the Lagrangian. So, the Weyl gauge field could be a good candidate for dark matter, which is gravitationally produced during inflation or reheating.

From $\partial_\mu\Omega = -\frac{1}{6}(1+a)\partial_\mu\phi_i^2$, we can recast the Einstein frame Lagrangian without the Weyl gauge field in a simpler form,

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}R + \frac{1}{2}\frac{(\partial_\mu\phi_i)^2}{\left(1-\frac{1}{6}(1+a)\phi_i^2\right)^2} + \frac{1}{12}(1+a)\cdot\frac{\frac{1}{4}(\partial_\mu\phi_i^2)^2 - \phi_j^2(\partial_\mu\phi_i)^2}{\left(1-\frac{1}{6}(1+a)\phi_i^2\right)^2} - V_E(\phi_i), \quad (11)$$

with $V_E(\phi_i) = f(\phi_i^2/\langle\chi^2\rangle)$. Therefore, the Higgs kinetic terms in the above Lagrangian are of the same form as in the Higgs pole inflation [14], except with an arbitrary parameter a.

For the pole inflation, we take the coefficient of the Jordan frame potential as

$$f(\phi_i^2/\chi^2) = V_0 + \frac{1}{2}m_\phi^2\langle\chi^2\rangle\cdot\frac{\phi_i^2}{\chi^2} + \frac{1}{4}\lambda_\phi\langle\chi^4\rangle\cdot\frac{(\phi_i^2)^2}{\chi^4}. \quad (12)$$

Here, V_0 corresponds to the vacuum energy, which respects the $SO(1,N)$ symmetry, but m_ϕ^2, λ_ϕ terms break the $SO(1,N)$ symmetry into $SO(N)$. Then, under the gauge condition, $\chi = \langle\chi\rangle = \sqrt{6/(1+a)}$, it leads to the standard form of the Higgs-like potential in Eq. (11) as

$$V_E(\phi_i) = \frac{1}{2}m_\phi^2\phi_i^2 + \frac{1}{4}\lambda_\phi(\phi_i^2)^2. \quad (13)$$

Pole inflation models in Weyl gravity—From the general Einstein frame Lagrangian obtained in the previous section, we discuss the generalization of the pole inflation scenarios with the SM Higgs doublet or the PQ singlet scalar field in Weyl gravity.

Generalized Higgs pole inflation: We realize pole inflation with the SM Higgs inflation [14] as an example for $N = 4$. In unitary gauge, the Higgs fields composing an $SU(2)_L$ doublet take $\phi_1 = h$ and $\phi_2 = \phi_3 = \phi_4 = 0$, so the second kinetic term in Eq. (11) vanishes. Then, the Einstein frame Lagrangian in Eq. (11) becomes

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}R + \frac{1}{2}\frac{(\partial_\mu h)^2}{\left(1-\frac{1}{6}(1+a)h^2\right)^2} - V_E(h), \quad (14)$$

with $V_E(h) = \frac{1}{2}m_H^2h^2 + \frac{1}{4}\lambda_H h^2$ after m_ϕ^2, λ_ϕ in Eq. (13) is replaced by the Higgs parameters, m_H^2, λ_H, respectively. This takes precisely the same form as in the Higgs pole inflation [14], again except the parameter a. For $m_H^2 < 0$ and $\lambda_H > 0$, the VEV of the Higgs is determined by $\langle h\rangle = v = \sqrt{-m_H^2/\lambda_H}$.

As a result, making the Higgs kinetic term canonical for

$$h = \langle\chi\rangle\tanh\left(\frac{\psi}{\langle\chi\rangle}\right), \quad (15)$$

we obtain the inflaton Lagrangian in Eq. (14) as

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}R + \frac{1}{2}(\partial_\mu\psi)^2 - V_E(\psi), \quad (16)$$

where the inflaton potential with the Higgs quartic coupling only becomes

$$V_E(\psi) = 9\lambda_H\tanh^4\left(\frac{\psi}{\langle\chi\rangle}\right). \quad (17)$$

Then, the Higgs pole inflation corresponds to $a = 0$ or $\langle\chi\rangle = \sqrt{6}$. As compared to T α-attractor models with the potential $V_E \sim \tanh^{2n}(\psi/\sqrt{6\alpha})$ [6], we can identify the model parameters by $1 + a = (1/\alpha)$ and $n = 2$. Thus, for $a = [0, 1]$, the T α-attractor models vary by $\alpha = [1, \frac{1}{2}]$.

We note that the interaction Lagrangian of the Higgs boson to the electroweak bosons in unitary gauge in the Einstein frame contains

$$\mathcal{L}_{h,\mathrm{gauge}} = F(h)\left(2g^2W_\mu W^\mu + (g'B_\mu - gW_\mu^3)^2\right), \quad (18)$$

with

$$F(h) \equiv \frac{h^2}{8[1-\frac{1}{6}(1+a)h^2]} = \frac{1}{8}\langle\chi\rangle^2\sinh^2\left(\frac{\psi}{\langle\chi\rangle}\right), \quad (19)$$

where we used Eq. (15) in the second line. Then, during inflation with $\psi \gg \langle\chi\rangle$, the effective masses for the electroweak gauge bosons are given by $M_W^2 \gg \frac{1}{4}g^2M_P^2$ and $M_Z^2 \gg \frac{1}{4}(g^2 + g'^2)M_P^2$, so they are safely decoupled from the inflaton. On the other hand, after inflation, $\psi \ll \langle\chi\rangle$, for which $\langle\chi\rangle^2\tanh^2(\psi/\langle\chi\rangle) \simeq \psi^2$, so we can recover the standard interactions of the Higgs boson to the electroweak gauge bosons, so reheating can proceed.

From the slow-roll parameters with Eq. (16), we get the spectral index and the tensor-to-scalar ratio at horizon exit in terms of the number of e-foldings, as follows:

$$n_s = 1 - \frac{64(8N + \langle\chi^2\rangle)}{256N^2 - \langle\chi^4\rangle}, \quad (20)$$

$$r = \frac{512\langle\chi^2\rangle}{256N^2 - \langle\chi^4\rangle}. \quad (21)$$

As a result, from Eq. (20), we obtain the spectral index as $n_s = 0.9662$–0.9666 for $N = 60$ and $a = [0, 1]$, which agrees with the observed spectral index from Planck, $n_s = 0.967 \pm 0.0037$ [18]. Moreover, we also predict the tensor-to-scalar ratio as $r = 0.000\,83 - 0.0033$ for $N = 60$ and $a = [0, 1]$, which is compatible with the bound from the combined Planck and Keck data [19], $r < 0.036$. We also find that the CMB normalization, $A_s = (1/24\pi^2)(V_I/\epsilon_*) = 2.1 \times 10^{-9}$, sets the inflation energy scale by

$$\lambda_H = (3.4 \times 10^{-9})r. \quad (22)$$

Thus, for a given r, we need the Higgs quartic coupling during inflation to be $\lambda_H = 2.8 \times 10^{-12}$–$1.1 \times 10^{-11}$. Such a tiny quartic coupling for the SM Higgs could be achieved when the corresponding beta function is sufficiently small in the presence of the couplings of singlet scalar fields to the SM Higgs [14].

In Fig. 1, we depict the inflationary predictions of the pole inflation in Weyl gravity in the spectral index n_s vs the tensor-to-scalar ration r. We show the results for $a = 0$ and $a = 1$ in blue and red lines, respectively, while the number of e-foldings is bounded between $N = 50$ and 60 at the pairs of blue or red bullets. We overlay the bounds from Planck on the spectral index within 1σ and 2σ errors in the yellow and green regions, respectively.

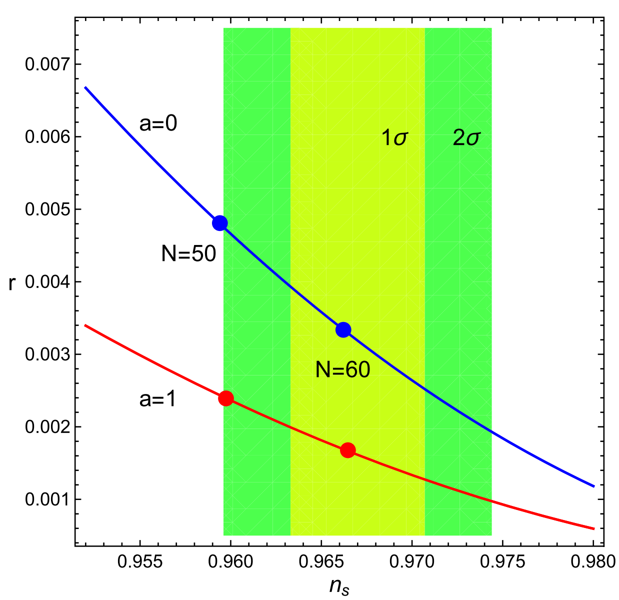

FIG. 1. Spectral index n_s vs tensor-to-scalar ratio r for the pole inflation in Weyl gravity. Blue and red solid lines are the cases with $a = 0, 1$, respectively, and the blue or red bullets indicate the boundaries where the number of e-foldings is given by $N = 50$, 60. The Planck bounds on the spectral index within 1σ and 2σ errors are shown in the yellow and green regions, respectively.

Generalized PQ pole inflation—We now realize the pole inflation with the PQ singlet scalar field [15,16] as an example with a complex scalar field, that is, $N = 2$. In this case, a PQ complex scalar field, $\Phi = (1/\sqrt{2})(\phi_1 + i\phi_2)$, transforms under the global $U(1)$ PQ symmetry. Taking the PQ field in the polar representation, $\Phi = (1/\sqrt{2})\rho e^{i\theta}$, the Einstein frame Lagrangian in Eq. (11) becomes

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}R + \frac{1}{2}\frac{(\partial_\mu\rho)^2}{\left(1 - \frac{1}{6}(1+a)\rho^2\right)^2} + \frac{1}{2}(1+a)\cdot\frac{\rho^2(\partial_\mu\theta)^2}{\left(1 - \frac{1}{6}(1+a)\rho^2\right)} - V_E(\rho), \quad (23)$$

where $V_E(\rho) = \frac{1}{2}m_\Phi^2\rho^2 + \frac{1}{4}\lambda_\Phi\rho^4$ after m_ϕ^2, λ_ϕ in Eq. (13) is replaced by the PQ parameters, m_Φ^2, λ_Φ, respectively. This takes precisely the same form as in the PQ pole inflation [15,16], again except for the parameter a. For $m_\Phi^2 < 0$ and $\lambda_\Phi > 0$, the VEV of the PQ field is determined by $\langle\Phi\rangle = f_a = \sqrt{-m_\Phi^2/\lambda_\Phi}$.

Making the kinetic term for the radial mode canonically normalized by

$$\rho = \langle\chi\rangle \tanh\left(\frac{\psi}{\langle\chi\rangle}\right), \quad (24)$$

we obtain Eq. (23) as

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}R + \frac{1}{2}(\partial_\mu\psi)^2 + 3\sinh^2\left(\frac{\psi}{\langle\chi\rangle}\right)(\partial_\mu\theta)^2 - V_E(\psi), \quad (25)$$

where the inflaton potential with the PQ quartic coupling only is given by

$$V_E(\psi) = 9\lambda_\Phi \tanh^4\left(\frac{\psi}{\langle\chi\rangle}\right). \quad (26)$$

Therefore, the same inflationary predictions as in the Higgs pole inflation are maintained, as far as the Higgs quartic coupling in Eq. (22) is replaced by the PQ quartic coupling. In this case, a similarly tiny quartic coupling for the PQ field can be stable under the renormalization group running for small Yukawa couplings and mixing quartic couplings of the PQ field [15,16].

On the other hand, if the PQ symmetry is not broken explicitly, the angular mode or the axion would be massless before the QCD phase transition, so there exists a nonzero isocurvature perturbation from the angular mode. The Planck satellite [18] sets a bound on the isocurvature perturbations by

$$\beta_{\rm iso} \equiv \frac{P_{\rm iso}(k_*)}{P_\zeta(k_*) + P_{\rm iso}(k_*)} < 0.038, \tag{27}$$

at 95% C.L., with $P_\zeta(k_*) = 2.1 \times 10^{-9}$ at $k_* = 0.05\ {\rm Mpc}^{-1}$, leading to

$$\left(\frac{\Omega_a}{\Omega_{\rm DM}}\right)^2 \frac{H_I^2}{\pi^2 \theta_*^2 f_{a,{\rm eff}}^2} < 8.3 \times 10^{-11}. \tag{28}$$

The effective decay constant of the axion is large at the horizon exit, namely, $f_{a,{\rm eff}} \simeq 22 M_P$ for $|a| \lesssim 1$ [16]. Thus, for $\Omega_a = \Omega_{\rm DM}$ and $\theta_* = \pi$, we find that $H_I < 1.1 \times 10^{15}$ GeV. Therefore, the current bound from the isocurvature perturbation is consistent with our predicted value, $H_I = (2.48 \times 10^{14}\ {\rm GeV})\sqrt{r}$, with $r \lesssim 0.0033$ for $|a| \lesssim 1$ in our model.

Gravitational production of Weyl photon dark matter—In the postinflationary period of the pole inflation, the coherent oscillation of the inflaton starts, reheating the universe. Assuming that a quartic term in the Einstein frame potential in Eq. (13) is dominant as in Higgs or PQ pole inflation models, the potential for the canonical inflaton becomes $V_E(\psi) \simeq \lambda_\phi \psi^4$, for which the equation of state for the inflaton during reheating becomes radiationlike [20], i.e., $\omega_\psi = \frac{1}{3}$. Then, in the presence of the decay and scattering of the inflaton, the postinflationary dynamics in the perturbative regime is governed by the Boltzmann equations [14],

$$\dot{\rho}_\psi + 3(1 + w_\psi) H \rho_\psi = -(1 + w_\psi)\Gamma_\psi \rho_\psi, \tag{29}$$

$$\dot{\rho}_R + 4H\rho_R = (1 + w_\psi)\Gamma_\psi \rho_\psi, \tag{30}$$

with $H^2 = (1/3M_P^2)(\rho_\psi + \rho_R)$. Here, ρ_ψ, ρ_R are the energy densities for the inflaton and the SM radiation bath, and Γ_ψ contains the decay and scattering rates for the inflaton. Then, the evolution of the inflaton energy density is approximately given by $\rho_\psi \simeq \rho_{\psi,{\rm end}}(a/a_{\rm end})^{-3(1+w_\psi)}$, and the SM radiation energy density is also approximated to

$$\rho_R(a) \simeq \frac{8 M_P \Gamma_\psi \sqrt{\rho_{\psi,{\rm end}}}}{\sqrt{3}(5 - 3w_\psi)} \left(\frac{a}{a_{\rm end}}\right)^{-\frac{3}{2}(1+w_\psi)} \times \left(1 - \left(\frac{a}{a_{\rm end}}\right)^{-\frac{1}{2}(5-3w_\psi)}\right). \tag{31}$$

Thus, the reheating temperature $T_{\rm RH}$ is determined by $\rho_\psi = \rho_R = (\pi^2 g_{\rm RH}/30) T_{\rm RH}^4$.

On the other hand, the Weyl photon can be produced from the gravitational scattering of the inflaton as well as the gravitational scattering of the radiation during or after reheating. Thus, solving the Boltzmann equations for the number density of the Weyl photon during and after reheating [23], we obtain the relic abundance of the Weyl photon as

$$\Omega_w h^2 = 1.6 \times 10^8 m_w \left(\frac{g_{*,0}}{g_{\rm RH}}\right) \left(Y_{w,{\rm inflaton}} + Y_{w,{\rm thermal}} + Y_{w,{\rm reheating}}\right), \tag{32}$$

with

$$Y_{w,{\rm inflaton}} \simeq \frac{2.53 g_{RH}^{3/4} \lambda_\phi^{3/4} \psi_{\rm end}^3}{M_P^3}, \tag{33}$$

$$Y_{w,{\rm thermal}} \simeq \frac{56\,469 T_{\rm RH}^3}{128\pi^6 \sqrt{10 g_{\rm RH}} M_P^3}, \tag{34}$$

$$Y_{w,{\rm reheating}} \simeq \begin{cases} Y_{w,{\rm thermal}}, & \text{PQ inflation}, \\ \frac{18\,759}{18\,823} Y_{w,{\rm thermal}}, & \text{Higgs inflation}. \end{cases} \tag{35}$$

Here, $g_{*,0} = 3.91$ and $g_{\rm RH} = 106.75$ are taken, and $\psi_{\rm end}$ is the inflation field value at the end of inflation, set to $\psi_{\rm end} \simeq 1.5 M_P$, and $\lambda_\phi = 10^{-11}$ at reheating from CMB normalization. In the case of the Higgs pole inflation, we kept only the SM fermions and gauge bosons for thermal scattering during reheating ($Y_{w,{\rm reheating}}$) as the Higgs fields have large field-dependent masses. We also note that the contribution from the inflaton scattering is independent of the reheating temperature [24], unlike the case with a matterlike inflaton during reheating [25,26].

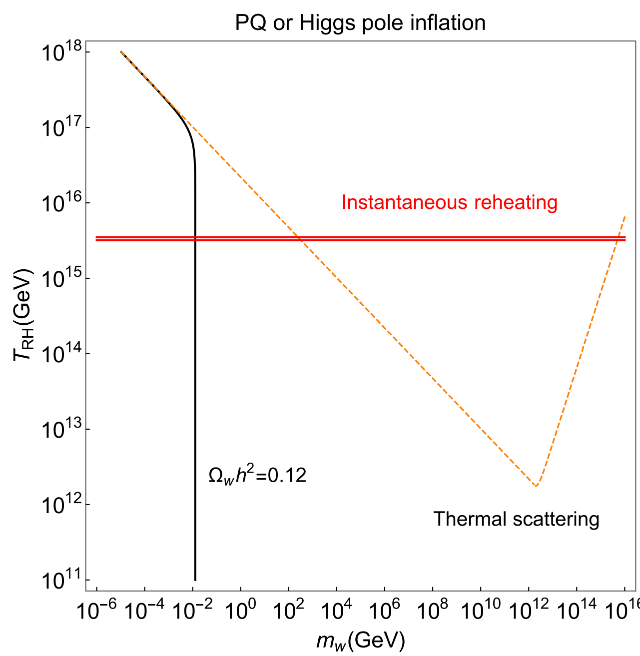

FIG. 2. Reheating temperature $T_{\rm RH}$ vs Weyl photon mass m_w in the pole inflation with a quartic potential. The orange dashed line corresponds to the contour satisfying the correct relic density with thermal scattering only, while the black line is the case after both inflaton scattering and thermal scattering are included. A small discrepancy between the cases for PQ and Higgs pole inflation models is not shown.

In Fig. 2, the black lines show the contours for the correct relic density for the Weyl photon dark matter in T_{RH} vs m_w for the PQ or Higgs pole inflations. The orange dashed lines correspond to the correct relic density when only the thermal scattering processes during and after reheating are taken into account. The temperature for instantaneous reheating becomes maximal, as shown in the red lines. We find that the contribution from the inflaton scattering is independent of the reheating temperature and dominant as compared to the thermal scattering [27]. The observed dark matter abundance is accounted for by the Weyl photon of about 10 MeV mass. A small difference between the relic abundances in the PQ and Higgs pole inflations is not visible in Fig. 2.

Conclusions—We presented the microscopic origin of pole inflation with scalar fields in the N-dimensional multiplet in Weyl gravity. We showed that the broken $SO(1,N)$ isometry in the field space in combination with Weyl symmetry restricts the form of the Lagrangian in the Jordan frame such that the vacuum energy is dominant during inflation near the pole of the kinetic term for the inflaton in the Einstein frame. An explicit breaking of the $SO(1,N)$ symmetry to $SO(N)$ is necessary for a slow-roll inflation near the pole.

From the Higgs or PQ inflation models with Weyl symmetry, we found that one parameter family of the pole inflation exists, depending on the overall coefficient a of the Weyl covariant derivatives for scalar fields. The same coefficient not only makes the inflationary predictions for the spectral index and the tensor-to-scalar ratio varying, being compatible with the Planck data, but also determines the mass of the Weyl gauge field. As compared to the T α-attractor models [5], our results show an interesting consequence that the tensor-to-scalar ratio decreases as the Weyl photon mass, $m_w^2 = [6a\, g_w^2/(1+a)]$, increases, for $a = [0, 1]$.

A successful inflation with Higgs or PQ fields is possible while the isocurvature perturbations of the axion can be suppressed sufficiently in the latter case, due to a large effective axion decay constant during inflation. We also showed that the massive Weyl photon can be a dark matter candidate of about 10 MeV mass, which is produced dominantly from the inflaton scattering during reheating.

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[1] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659**, 703 (2008).
[2] C. P. Burgess, H. M. Lee, and M. Trott, J. High Energy Phys. 09 (2009) 103; 07 (2010) 007; J. L. F. Barbon and J. R. Espinosa, Phys. Rev. D **79**, 081302(R) (2009); M. P. Hertzberg, J. High Energy Phys. 11 (2010) 023.
[3] G. F. Giudice and H. M. Lee, Phys. Lett. B **694**, 294 (2011).
[4] Y. Ema, K. Mukaida, and J. van de Vis, J. High Energy Phys. 11 (2020) 011; H. M. Lee and A. G. Menkara, J. High Energy Phys. 09 (2021) 018.
[5] R. Kallosh and A. Linde, J. Cosmol. Astropart. Phys. 12 (2013) 006.
[6] R. Kallosh, A. Linde, and D. Roest, J. High Energy Phys. 11 (2013) 198.
[7] B. J. Broy, M. Galante, D. Roest, and A. Westphal, J. High Energy Phys. 12 (2015) 149.
[8] M. Galante, R. Kallosh, A. Linde, and D. Roest, Phys. Rev. Lett. **114**, 141302 (2015).
[9] T. Terada, Phys. Lett. B **760**, 674 (2016).
[10] L. Smolin, Nucl. Phys. **B160**, 253 (1979).
[11] D. M. Ghilencea and H. M. Lee, Phys. Rev. D **99**, 115007 (2019).
[12] Y. Tang and Y. L. Wu, J. Cosmol. Astropart. Phys. 03 (2020) 067.
[13] S. Aoki and H. M. Lee, Phys. Rev. D **108**, 035045 (2023).
[14] S. Clery, H. M. Lee, and A. G. Menkara, J. High Energy Phys. 10 (2023) 144.
[15] H. M. Lee, A. G. Menkara, M. J. Seong, and J. H. Song, J. High Energy Phys. 05 (**2024**) 295.
[16] H. M. Lee, A. G. Menkara, M. J. Seong, and J. H. Song, Eur. Phys. J. C **84**, 1260 (2024).
[17] See Supplemental Material at http://link.aps.org/supplemental/10.1103/skhx-yc43 for details on redefined Weyl gauge field, slow-roll parameters and isocurvature perturbations in the model.
[18] Y. Akrami *et al.* (Planck Collaboration), Astron. Astrophys. **641**, A10 (2020).
[19] P. A. R. Ade *et al.* (BICEP and Keck Collaborations), Phys. Rev. Lett. **127**, 151301 (2021).
[20] For the Higgs pole inflation, the detailed analysis on perturbative reheating and some discussion on the perturbation equations were given in Ref. [14], but it would be also important to study the preheating effects in the pole inflation models [21,22]. But, we postpone a detailed analysis on preheating to a future work.
[21] Y. Ema, R. Jinno, K. Mukaida, and K. Nakayama, J. Cosmol. Astropart. Phys. 02 (**2017**) 045.
[22] E. I. Sfakianakis and J. van de Vis, Phys. Rev. D **99**, 083519 (2019).
[23] M. A. G. Garcia, K. Kaneta, Y. Mambrini, and K. A. Olive, J. Cosmol. Astropart. Phys. 04 (2021) 012.
[24] S. Clery, Y. Mambrini, K. A. Olive, and S. Verner, Phys. Rev. D **105**, 075005 (2022).
[25] Y. Tang and Y. L. Wu, Phys. Lett. B **774**, 676 (2017).
[26] S. Aoki, H. M. Lee, A. G. Menkara, and K. Yamashita, J. High Energy Phys. 05 (**2022**) 121.
[27] M. Garny, A. Palessandro, M. Sandora, and M. S. Sloth, J. Cosmol. Astropart. Phys. 02 (**2018**) 027.