# Swarm Intelligence Inspired Multicast Routing: An Ant Colony Optimization Approach ${ }^{\star}$ 

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#### Abstract

The advancement of network induces great demands on a series of applications such as the multicast routing. This paper firstly makes a brief review on the algorithms in solving routing problems. Then it proposes a novel algorithm called the distance complete ant colony system (DCACS), which is aimed at solving the multicast routing problem by utilizing the ants to search for the best routes to send data packets from a source node to a group of destinations. The algorithm bases on the framework of the ant colony system (ACS) and adopts the Prim's algorithm to probabilistically construct a tree. Both the pheromone and heuristics influence the selection of the nodes. The destination nodes in the multicast network are given priority in the selection by the heuristics and a proper reinforcement proportion to the destination nodes is studied in the case experiments. Three types of heuristics are tested, and the results show that a modest heuristic reinforcement to the destination nodes can accelerate the convergence of the algorithm and achieve better results.


## 1 Introduction

With the development of network technologies, application demands and quality requests are getting higher and higher. As the topologies of networks are complex, efficient packet routing methods are quite meaningful to the real world [1].

The complexity of the network makes deterministic algorithms incapable to react in time. So a variety of heuristic algorithms for network routing emerged in recent years. Di Caro and Dorigo [2] introduced an approach called AntNet to the adaptive learning of routing tables in a communication network. Gelenbe et al. 3] recently proposed an experimental investigation of path discovery using genetic algorithms (GAs). There are also some algorithms for network routing, such as some agent-based algorithms (eg., the ant-based control (ABC) system [4), neural heuristics [5. These algorithms are promising but still need enhancements.

Multicast routing [6]-11 first appeared in ARPANET in packet-switched store-and-forward communication networks. The packets in multicast routing

[^0]are sent from a source node to several destinations without or with certain constraints such as the requirements of the quality-of-service (QoS) and real-time communications. The unconstrained multicast routing problem, which is known to be NP-complete [10], is equivalent to the Steiner tree problem in graphs.

This paper aims at solving the unconstrained multicast routing problem using an ant algorithm. The proposed algorithm, which is called the distance complete ant colony system (DCACS), bases on an ant colony system framework [12] and takes advantage of the distance complete graph (DCG) to find the multicast tree with the lowest cost in the network. Each ant starts from a randomly chosen destination nodes (including the source node) and follows the structure of the Prim's minimum spanning tree (MST) [13] algorithm to construct the multicast tree. The ants select a next node in a deterministic way, or explore one by a probabilistic selection. After an ant has finished its tour, the solution is checked further for a potentially better tree by the classical Prim's MST algorithm and the redundancy trimming strategy. The pheromone in the network is updated by the local and global pheromone update mechanism.

The genetic algorithm presented in [10] and the ant algorithm described in 15 are used to compare the performance of the proposed DCACS algorithm. The ant algorithm in [15] has been applied to solve Steiner tree problems. However, their algorithm differs from ours in which: 1) Their algorithm bases on the ant system (AS), but ours on the ant colony system (ACS). 2) The ants in their algorithm start from all the destination nodes and merge together when an ant steps on the route that another ant has passed. Our ants start from a randomly chosen destination node, and each ant builds their own tree. 3) Their algorithm has conflict detection and avoidance mechanism as their ants may conflict with each other resulting in erroneous results. But ours do not need to do so. 4) The pheromone update methods are different between the two algorithms.

The other contribution of this paper is that it analyzes the results of giving priority in the selection of destination nodes by the heuristics. Three types of heuristic methods are tested. Proper heuristic methods and parameters settings to reinforce pheromone values to the destination nodes and the other nodes are concluded by experiments on the Steiner cases from the OR-Library [14.

The paper is organized as follows. Section 2 introduces the description of multicast routing problems and their mathematical definition. The construction method of the distance complete graph (DCG) is also introduced. Section 3 describes the implementation and main techniques of the proposed DCACS algorithm. In Section 4, the proposed algorithm is tested by some cases to analyze its performance. The conclusion of the paper is made in Section 5.

## 2 Multicast Routing Problem and Distance Complete Graph

In this section, the formal description and definition of the multicast routing problem will be made and the distance complete graph topology on which the proposed algorithm based will also be described.

### 2.1 Definition of the Multicast Routing Problem

A network graph is denoted as $G=\left(V_{G}, E_{G}, \Omega\right)$, where $V_{G}$ is the set of nodes in the network, $E_{G}$ is the set of edges that connect two nodes in $V_{G}$, and $\Omega$ is the set of constraints. If the problem is constraint-free, then $\Omega=\emptyset$. In a multicast routing problem, the $V_{G}$ is divided into three subsets as $V_{S}, V_{T}, V_{I}$, where $V_{S}$ is the set of source nodes, $V_{T}$ is the set of destinations, and $V_{I}$ is the set of intermediate nodes. Each edge $e$ which belongs to $V_{G}$ has a positive cost $c(e)$ for passing the edge. If there is no edge connecting two nodes in $V_{G}$, the corresponding cost will be set as $\infty$, which indicates that the edge does not exist. The multicast routing problem is to find a tree with the minimum cost connecting the nodes in $V_{S}$ and $V_{T}$, and the resulting tree must comply with the constraints in $\Omega$. The formal definition of a multicast routing problem is to find a tree $T=\left(V_{S}+V_{T}+V_{\theta}, E_{\theta}\right)$, where $V_{\theta} \subseteq V_{I}$ and $E_{\theta} \subseteq E_{G}$, with

$$
\min \sum_{e \in T} c(e), T \text { satisfies constraints in } \Omega
$$

Fig. 1 shows an example of the multicast routing. There are twelve nodes and some edges. The gray solid node 1 is the source node, and the black solid nodes 2,6 , and 11 are the destination nodes. Now the problem is to find a tree which has all the destination nodes connected to the source node with the minimum total cost within the constraints. The black edges between nodes $2,1,5,6,7$, 11 construct the result tree in the example via the intermediate nodes 5 and 7 . As the problem considered in this paper is the one without constraints, which is an equivalence of the Steiner tree in graphs, we do not differentiate the source nodes and the destination nodes. We call both of the nodes as destination nodes. The sets $V_{S}$ and $V_{T}$ are combined to be denoted as $V_{D}$.


Fig. 1. An example of a multicast routing network

### 2.2 Distance Complete Graph

The network in multicast routing is not a fully-connected graph, which means that some nodes are not directly connected by a single edge, though they are attainable via other nodes and the corresponding edges. A distance complete graph (DCG) of a network is the one that each pair of nodes is logically connected and the cost (or distance) on that edge is the cost of the shortest path between the nodes in the actual network.

One of the classical algorithms for generating an undirected DCG is the Floyd algorithm [16]. Fig. 2 shows an example of a topology with six nodes. The solid edges in Fig. 2 are the actual edges existing between the two nodes, and the numbers beside the edges are the corresponding cost values. We can see that


Fig. 2. An example of transforming edges (a,b) and (a,d) in the DCG
nodes a and $b$ are not adjacent, but they can reach each other via node c or d . The algorithm is to change the original graph in Fig. 2(a) to the DCG in Fig. 2(b), where a logical edge (shown as a dashed line) is added with the minimum cost from a to b . The shortest route from a to b is via node c , so the cost $d_{\mathrm{ab}}$ of the logical edge in the DCG is 2.5 . Because the graph is undirected, we have $d_{\mathrm{ba}}=d_{\mathrm{ab}}$. To nodes a and d, although they are connected directly, there is a shorter route via nodes e and f. So $d_{\mathrm{ad}}=d_{\mathrm{da}}=1.5$.

The time complexity of the Floyd algorithm is $O\left(\left|V_{G}\right|^{3}\right)$. The ant algorithm proposed in this paper is based on the DCG, so the construction of the DCG must be preprocessed. After constructing the DCG, the ants can be dispatched to construct multicast trees.

## 3 Distance Complete ACS for Multicast Routing Problem

The structure of the proposed distance complete ACS (DCACS) is based on the procedure of constructing a minimum spanning tree (MST) by the Prim's algorithm [13. The following section will present how to solve a multicast routing problem.

### 3.1 The Prim's Algorithm and the Pheromone and Heuristics Mechanism

Suppose $S$ is a set with one node and $V$ is the set of nodes in an undirected connected graph excluding the node in S . The classical Prim's algorithm [13] is to add the node $j$ to $S$, provided that the edge $(i, j)$ has the minimum cost, $i \in S$, and $j \in V-S$. The algorithm terminates when $S=V$. In the proposed algorithm, the selection criteria for the next node are not simply based on the cost of the edges, but the product of the pheromone value and the heuristic value.

The product of pheromone and heuristics for an edge $(i, j)$ is denoted as $\tau(i, j) \eta(i, j)$, where $\tau(i, j)$ is the pheromone value and $\eta(i, j)$ is the heuristic value. The initial pheromone value $\tau_{0}$ for every edge is discussed in 3.2. Three types of heuristic values are tested in this paper.

$$
H_{a}: \eta(i, j)=\left\{\begin{array}{l}
{\left[\mu_{D} /\left(1 / d_{i j}\right)\right]^{\beta}, \text { if } j \in V_{D}}  \tag{1}\\
{\left[\mu_{I} /\left(1 / d_{i j}\right)\right]^{\beta}, \text { otherwise }}
\end{array}\right.
$$

$$
\begin{gather*}
H_{b}: \eta(i, j)=\left\{\begin{array}{l}
\mu_{D} /\left(1 / d_{i j}\right)^{\beta}, \text { if } j \in V_{D} \\
\mu_{I} /\left(1 / d_{i j}\right)^{\beta}, \text { otherwise }
\end{array}\right.  \tag{2}\\
H_{c}: \eta(i, j)=\left\{\begin{array}{l}
\left(\mu_{D}\right)^{\beta} /\left(1 / d_{i j}\right), \text { if } j \in V_{D} \\
\left(\mu_{I}\right)^{\beta} /\left(1 / d_{i j}\right), \text { otherwise }
\end{array}\right. \tag{3}
\end{gather*}
$$

where $\mu_{D}$ and $\mu_{I}$ are the reinforcement proportion to the destinations and intermediate nodes respectively. $\mu_{D}=\max \left(\left|V_{I}\right| /\left|V_{G}\right|,\left|V_{D}\right| /\left|V_{G}\right|\right) . \mu_{I}=\min \left(\left|V_{I}\right| /\left|V_{G}\right|\right.$, $\left.\left|V_{D}\right| /\left|V_{G}\right|\right) . d_{i j}$ is the cost of the logical edge in the DCG between nodes $i$ and $j$. The parameter $\beta$ is a positive constant.

The destination nodes are more likely to be selected by using the above heuristic methods. With the same cost by the edges, the greedy operation prefers the nodes in $V_{D}$. The performance of the algorithm with different heuristic methods will be tested in Section 4 with a discussion on the design of the heuristics.

### 3.2 The Ant's Search Behavior

In this part, the process on how an ant searches for routes to build a tree is described step by step. It is a probabilistic Prim's algorithm in the framework of ACS [12] and the algorithm operates in the DCG.

Step 1: Initialization
At first, an ant $k$ is placed on a randomly chosen destination node $i$. Then it begins its search.

Step 2: Exploration or Exploitation
The ant probabilistically chooses to do exploration or exploitation to select the next node $j$ based on the state transition rule [12], which is controlled by (4)

$$
j=\left\{\begin{array}{c}
\arg \max _{r \in \Theta_{k}}\{\tau(i, r) \eta(i, r)\}, \text { if } q \leq q_{0} \text { (exploitation) }  \tag{4}\\
J, \text { otherwise (exploration) }
\end{array}\right.
$$

where $i$ is the node that the ant has visited, $\Theta_{k}$ is the set of nodes that haven't been visited by the current ant $k, q_{0}$ is a predefined parameter in $[0,1]$ controlling the proportion of doing exploitation and exploration, $J$ is a random node chosen by (5). $\tau(i, r)$ stands for the pheromone value on the edge connecting nodes $i$ and $r . \eta(i, r)$ is the heuristic value for choosing node $r$ from node $i$. Note that the DCG is constructed, so any pairs of nodes are logically connected. If a random number $q$ is smaller than or equal to $q_{0}$, the ant will choose the next unvisited node with the maximum product of pheromone and heuristic value. This is called the exploitation step. Otherwise, the next node $j$ is chosen by (5) with a probability distribution, which is called the exploration step.

$$
p_{k}(i, j)=\left\{\begin{array}{c}
\frac{\tau(i, j) \eta(i, j)}{\sum_{r \in \Theta_{k} \tau(i, r) \eta(i, r)},}, \text { if } j \in \Theta_{k}  \tag{5}\\
0, \text { otherwise }
\end{array}\right.
$$

Step 3: Edge Extension and Local Pheromone Update
When an ant is building a solution, the pheromone value on the visited edges is decreased. It is done for the avoiding too many ants that tail after the same route
and letting the other ants search the network diversely. The local pheromone update is done in two sub-steps, one is the update of pheromone on logical edges, and the other is on actual edges. The pheromone value on an edge $(i, j)$ is updated as (6).

$$
\begin{equation*}
\tau(i, j)=(1-\rho) \tau(i, j)+\rho \tau_{\min } \tag{6}
\end{equation*}
$$

where $\rho \in(0,1]$ is the pheromone evaporation rate. $\tau_{\text {min }}$ is the lower boundary value of the pheromone on every edge. Since $\tau(i, j) \geq \tau_{\text {min }}$, the updated pheromone value is smaller than or equal to the original one.

After choosing the next node $j$, an ant $k$ moves from node $i$ to node $j$. Since the ant moves on the DCG, the logical edge $(i, j)$ can be extended to the actual route. The logical and actual edges in the route have their pheromone updated by (6). Fig. 3) shows an example of updating an edge ( 1,6 ), which means that the ant just moved from node 1 to node 6 . The logical edge $(1,6)$ can be extended to be a route $1,2,3,4,5,6$. Then the pheromone values on the corresponding edges $(1,2),(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,4),(3$, $5),(3,6),(4,5),(4,6),(5,6)$ and their symmetric edges $($ e.g. $(2,1))$ are also updated. After the local pheromone update, the pheromone density on the edges corresponding to the logical edges decreases.


Fig. 3. Example of extending a logical edge (1, 6)

Step 4: Has the ant finished the job?
The nodes walked by the ant and the ones that are extended from the logical edges are regarded as having been visited. The termination condition for an ant is whether it has visited all the destination nodes. If the termination condition is unsatisfied, go to Step 2 for a further search. Otherwise, the ant has finished searching for a multicast tree.

### 3.3 Redundancy Trimming

After an ant has finished building a multicast tree connecting the source node and all the destination nodes, the tree must be checked for redundancy.

Firstly, we apply the classical Prim's algorithm to the visited nodes. If the cost of the generated MST is smaller than that of the tree built by the ant, the ant's solution is replaced by the MST.

Secondly, we check for useless intermediate nodes. The intermediate nodes that have only one degree of connectivity are deleted (e.g. node 10 will be deleted in Fig. (1).

The above two processes are repeated until the tree cannot be optimized any more. The redundancy trimming can also be used as a deterministic algorithm
for multicast routing problems. Take the whole set of nodes in $V_{G}$ as the input, the tree can be gradually trimmed and reduced. The resulting tree is a MST, and it is a feasible solution for the problem. However, the experiments in Section 4 show that this method can not generate the best tree. It is a sub-optimal result, which can be used to initialize the pheromone value. Hence, the initial pheromone value on every edge is set as

$$
\begin{equation*}
\tau_{0}=1 /\left(\left|V_{G}\right| T_{s}\right) \tag{7}
\end{equation*}
$$

where $T_{s}$ is the cost of the tree generated by the deterministic redundancy trimming method.

### 3.4 Global Pheromone Update

After the $m$ ants have performed the tree construction, the global pheromone update is applied to the best tree ever been found in order to reinforce the pheromone values. The pheromone values on the actual edges of the best-so-far tree are updated as (8).

$$
\begin{equation*}
\tau(i, j)=(1-\rho) \tau(i, j)+\rho \Delta \tau \tag{8}
\end{equation*}
$$

where $\Delta \tau=1 / T_{\text {best }}, T_{\text {best }}$ is the total cost of the best-so-far tree.
The pheromone values on the logical edges in the best-so-far tree are also updated. Suppose an actual route between a pair of nodes $i$ and $j$ in the best tree is $\left(a_{0}, a_{1}, a_{2}, \ldots, a_{\psi}\right)$, where $a_{0}=i$ and $a_{\psi}=j$. $\psi$ is the number of edges in the route, then the new pheromone value on the edge $(i, j)$ equals to

$$
\begin{equation*}
\tau(i, j)=\sum_{l=0}^{\psi-1} \tau\left(a_{l}, a_{l+1}\right) / \psi \tag{9}
\end{equation*}
$$

## 4 Experiment and Discussions

The test cases used in this paper are the Steiner problems in group b from the OR-Library [14. The eighteen problems are tabulated in Table 1 with the graph size from 50 to 100 . In the table, $\left|V_{G}\right|$ stands for the number of nodes, and $\left|V_{D}\right|$ is the number of destination nodes. $\left|E_{G}\right|$ is the number of actual edges in the graph. $O P T$ is the optimum of the problem. $T_{s}$ is the cost of the tree generated by using only the redundancy trimming method. It can be seen that the value of $T_{s}$ is not good compared to the best value.

The values of the parameters are set empirically as $q_{0}=0.9, \rho=0.1, \tau_{\text {min }}=$ $\tau_{0}$. The maximum iteration number is set as 500 , but once the ants have found the optimum, the algorithm terminates. The same combination of parameters values are tested for ten times independently.

Table 1. Eighteen Steiner b test cases

| No. | $\left\|V_{G}\right\|$ | $E_{G}$ | $V_{D}$ | $O P T$ | $T_{s}$ | No. | $\mid V_{G}$ | $E_{G}$ | $V_{D}$ | $O P T$ | $T_{s}$ | No. | $\left\|V_{G}\right\|$ | $E_{G}$ | $V_{D}$ | $O P T$ | $T_{s}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 50 | 63 | 9 | 82 | 89 | 7 | 75 | 94 | 13 | 111 | 112 | 13 | 100 | 125 | 17 | 165 | 189 |
| 2 | 50 | 63 | 13 | 83 | 96 | 8 | 75 | 94 | 19 | 104 | 111 | 14 | 100 | 125 | 25 | 235 | 243 |
| 3 | 50 | 63 | 25 | 138 | 144 | 9 | 75 | 94 | 38 | 220 | 227 | 15 | 100 | 125 | 50 | 318 | 333 |
| 4 | 50 | 100 | 9 | 59 | 63 | 10 | 75 | 150 | 13 | 86 | 95 | 16 | 100 | 200 | 17 | 127 | 171 |
| 5 | 50 | 100 | 13 | 61 | 68 | 11 | 75 | 150 | 19 | 88 | 103 | 17 | 100 | 200 | 25 | 131 | 144 |
| 6 | 50 | 100 | 25 | 122 | 130 | 12 | 75 | 150 | 38 | 174 | 186 | 18 | 100 | 200 | 50 | 218 | 227 |

Table 2. Comparisons of DCACS with the GA in 10 and the ant algorithm in 15

| No. | GA $[10]$ | ant [15] | DCACS | No. | GA[10] | ant [15] | DCACS | No. | GA [10] | ant [15] | DCACS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 82 | 82 | 82 | 7 | 111 | - | 111 | 13 | 165 | 167.3 | 168 |
| 2 | 83 | 83 | 83 | 8 | 104 | 104 | 104 | 14 | 235 | 235.3 | 235 |
| 3 | 138 | 138 | 138 | 9 | 220 | 220 | 220 | 15 | 318 | 318 | 318 |
| 4 | 59 | 59 | 59 | 10 | 86 | 87.4 | 86 | 16 | 127 | 133 | 127 |
| 5 | 61 | 61 | 61 | 11 | 88 | 89 | 88 | 17 | 131 | - | 131 |
| 6 | 122 | - | 122 | 12 | 174 | - | 174 | 18 | 218 | 225.5 | 218 |

- The values are not available in (15.


### 4.1 Analysis on the Heuristic Method

The basic structures of $\mathrm{H}_{a}, \mathrm{H}_{b}$, and $\mathrm{H}_{c}$ are the same, except for the role of the parameter $\beta$. The reinforcement proportion to the destination nodes and the intermediate nodes is normalized to be in $[0,1]$ and $\mu_{D}+\mu_{I}=1$. In the traditional ACS for the traveling salesman problem by Dorigo and Gambardella [12], the heuristics is defined as $\left(1 / c_{i j}\right)^{\beta}$, and the $\beta(\beta>0)$ is used to determine the relative importance of pheromone versus cost. In our algorithms in solving multicast routing problems, a series of tests is made to judge in which way the parameter $\beta$ should be used and what its value should be.

The results by the three $\mathrm{H}_{a}, \mathrm{H}_{b}$, and $\mathrm{H}_{c}$ are compared in Fig. 4. Each subfigure is composed of three parts with the corresponding heuristic methods, and each method has been tested using 10,50 and 100 ants. The figures illustrate the success rates of the algorithm to the corresponding Steiner tree problems. It can be seen that $\mathrm{H}_{a}$ and $\mathrm{H}_{c}$ are more robust than $\mathrm{H}_{b}$, which means that the emphasis by $\beta$ to $\mu_{D}$ and $\mu_{I}$ is more significant than to the cost of the edges. The best $\beta$ for $\mathrm{H} 2_{a}$ is $1,2,3$ and for $\mathrm{H} 2_{c}$ is $1,2,3,4$ in most cases. The best value of $\mu^{\beta}$ is shown to be from 8 to 16 in the test cases.

### 4.2 Comparison with Other Algorithms

We use the genetic algorithm in [10] and the ant algorithm in [15] to compare the performance of the proposed algorithm. The parameters used in the proposed DCACS with $\mathrm{H} 2_{c}$ are set as $m=10 \sim 100, \beta=3, \rho=0.1$, and $q_{0}=0.9$.

The obtained results are tabulated and compared in Table 2. The genetic algorithm presented in [10] can find the optimal tree with $100 \%$ success to all


Fig. 4. Comparison between $\mathrm{H} 2_{a}, \mathrm{H} 2_{b}$ and $\mathrm{H} 2_{c}$
the eighteen test problems. Our proposed DCACS also can achieve $100 \%$ success to the test cases except for Stein b13. In [15, only the Stein b1~b5, b8~b11, b13~b16, and b18 are tested. However, the results obtained by our algorithm are much better and faster according to the descriptions in [15.

## 5 Conclusion

This paper proposed an algorithm DCACS according to the ant colony system on the distance complete graph to solve the multicast routing problems without constraints. The ants base on the structure of Prim's algorithm and probabilistically select the nodes to construct a multicast tree. After the tree has been generated, the redundant nodes are deleted. Pheromone update and heuristic reinforcement to the destination nodes efficiently guide the algorithm to the optimum. The proposed algorithm is tested by eighteen Steiner cases and three kinds of heuristic methods are evaluated. By comparing the results with other algorithms, the proposed algorithm is very promising in solving multicast routing problems.

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