A Novel Fuzzy Model for the Traffic Signal Control of Modern Roundabouts

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***Abstract*—Traffic signal control is a challenging task for traffic systems. As fuzzy logic is proved to be well suited to control some complex systems with uncertainties and human perception, it has been widely used to control the traffic signal in recent years. This paper proposes a novel fuzzy logic controller for signalizing modern roundabouts. Different from existing fuzzy traffic-signal controllers, the proposed controller consists of two fuzzy layers each of which has its own duty. According to the current traffic condition, one layer of the controller controls the phase sequence while the other layer determines the signal timing. By the cooperation of the two layers, the proposed controller is capable of immediately responding to the current traffic condition so as to reduce the vehicle delay or the queue length of waiting vehicles, as well as smoothing the traffic flows in order to reduce the risk of traffic jams. Simulation results prove the effectiveness of the proposed controller, for it can improve the traffic efficiency of the roundabout when compared with several existing controllers.**

Keywords-fuzzy logic; traffic signal control; roundabout

I. INTRODUCTION

The rapid development of urban population and automobile industry has caused serious traffic congestion problem in modern cities. Because the existing traffic infrastructures are hard or costly to be modified and extended, developing appropriate traffic signal control technique is the most effective way to deal with the traffic congestion problem.

In the literature, various traffic signal control methods for kinds of intersections have been proposed. Fixed-time method based on mathematical deduction and calculation is a good way to control the traffic signal [1]-[8]. If the mathematical models can reflect the characteristics of the actual traffic system, the corresponding fixed-time methods can provide very reliable and accurate control. However, as a too complicated model is hard or even impossible to solve, rational assumptions to simplify the model are needed. Obviously, a vital assumption of all the fixed-time control models is that the traffic condition should be steady. However, as the traffic condition of an intersection changes frequently depending on the time of a day and the season of a year, this assumption is always not valid.

Therefore, increasing attention has been paid on developing real-time traffic signal control methods in recent years. As fuzzy logic is capable of simulating human decision-making within a short computing time, a fuzzy logic controller (FLC) can intelligently adapt the traffic signal to the traffic condition in real time. FLC is very suitable for controlling dynamic traffic systems. Plenty of fuzzy traffic-signal controllers [9]-[15] for real-time traffic signal control have been proposed in the last decade. We classify these techniques into two categories according to the output of their fuzzy models. In the first category, the output is the extension time of the current green phase [9]-[11]. The controller adopts a preset phase circle and determines whether to maintain the current green phase or to active the next phase in the circle according to the output of the fuzzy model. In other words, the controller only adapts the phase timing. As the phase circle is always well-designed only permitting consistent traffic flows existing simultaneously in the circular lanes of the roundabout, it is less likely to cause traffic jams even when the traffic is heavy. But the use of preset phase sequences may lower the real-timeness of the controller and reduce the capacity of the roundabout. On the other hand, the second category outputs the urgency degrees of all phases at set intervals and always activates the most urgent phase [12]-[14]. In this way, the controller adapts the phase sequence. Capable of frequently adapting the phase sequence according to the changing environment, the controller has good real-timeness. However, the disadvantage is that frequently changing phase sequence may make the traffic system in a state of chaos and vulnerable to traffic jams.

This paper proposes a novel fuzzy traffic-signal controller. Unlike previous controllers which have only one fuzzy layer to calculate the extension time or urgency degrees, the proposed controller is composed of two fuzzy layers. The first layer computes the urgency degrees of different phase subsets (the phase set is divided into several phase subsets according to the directions) and gives the highest priority to the most urgent subset. The second layer computes the extension time of the current phase. If the extension value is bigger than a threshold value, the current phase is maintained; otherwise the next phase in the phase subset selected by the first layer is activated. By using the first layer, the controller is capable of changing phase sequences in order to respond to the current traffic condition and improve real-timeness of the signal control. Meanwhile, by

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the use of the second layer, the frequency of changing phase sequences is not too high, which reduces the risk of traffic jams. In general, the proposed controller is a mixture of traditional fuzzy traffic-signal controllers which has a good real-timeness as well as a low risk of traffic jam.

Simulation is conducted on a roundabout with four approaches and two circular lanes, where three different traffic conditions are tested. The proposed controller is compared with the previous fuzzy controllers which have one layer and output only extension time or urgency degrees. Simulation results and the corresponding comparisons show that the proposed controller outperforms the other controllers in terms of the vehicle delay time and average queue length. The controller is very promising for improving the traffic efficiency of modern roundabouts.

The rest of this paper is organized as follows. Section II describes the model of the roundabout. Section III presents the general method as well as the implementations of the proposed fuzzy traffic-signal controller. In Section IV, simulation is conducted and the results are analyzed. At last, a conclusion is drawn in Section V.

II. ROUNDABOUT

Roundabout is one of the most commonly employed traffic infrastructures in urban cities. According to the environmental issues and traffic conditions, there are various roundabouts with different geometric designs and signal control modules. Roundabout with two circular lanes and four approaches is widely used in both real world and academic world [6][7][13]. In this paper, we simulate this kind of roundabout.

As shown in Fig. 1, the roundabout has four approaches, each of which has three entrance lanes for each direction. From left to right, the first lane labeled “a” is the left-turn lane occupied by vehicles to turn left, e.g., vehicles from approach 0 to approach 3. The second lane labeled “b” is the go-through lane used for go-through movement, e.g., vehicles from approach 0 to approach 2. The third lane labeled “c” is the right-turn lane which permits vehicles turning right, e.g., vehicles from approach 0 to approach 1. Meanwhile, there are two circular lanes for vehicles moving around the roundabout. The inner lane is only used for left-turn traffic flows, while the outer lane is used for both the go-through traffic flows and the left-turn vehicle which is entering or leaving the inner lane. The right-turn vehicle has its unique roadway to pass the roundabout without entering any circular lanes.

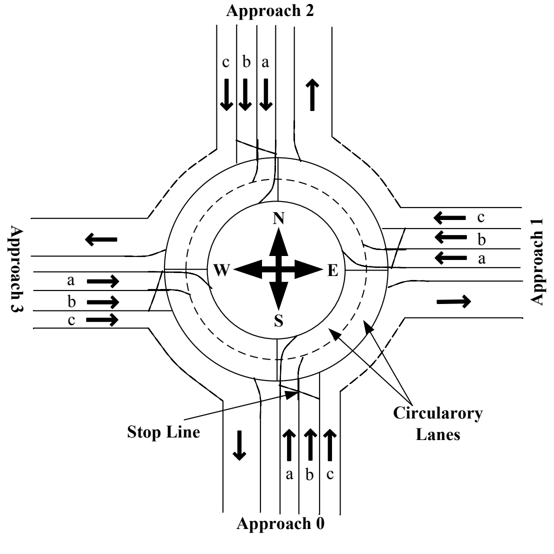


Figure. 1. Illustration of the roundabout.

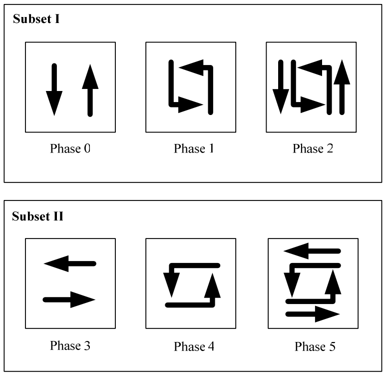


Figure. 2. Phase set of an ordinary four-approach roundabout.

In a signalized roundabout, the stop lines are located on the left-turn and go-through entrance lanes of each approach. Traffic flows behind the stop lines are controlled by traffic signals. Besides, vehicles on the right-turn lane can turn right at any time because they have their unique roadways and are independent with other traffic flows. Thus, for the roundabout, eight traffic flows consisting of the left-turn and go-through flows on each of the four approaches should be taken into consideration. Fig. 2 shows six possible phases which are commonly used for the roundabout. The phase set can be divided into two subsets according to the direction. Subset I permits north-south traffic flows (traffic flows from approaches 0 and 2) and stops the west-east traffic flows (traffic flows from approaches 1 and 3), whereas subset II does just the opposite.

III. THE NOVEL FUZZY MODEL

A. General Idea

The general idea of traffic signal control based on fuzzy logic is that the fuzzy controller receives the current traffic condition of the roundabout as input and outputs the active phase to the traffic signal module. According to the selected phase, the traffic signal module allows some traffic flows to move and stops the others behind the stop lines. Then the traffic condition of the roundabout changes, and after a time period the controller accordingly generates the next active phase.

The proposed traffic-signal controller is composed of two fuzzy layers. The first layer computes the urgency degrees (*UD*) of the two phase subsets by the end of each phase. If the current running subset has a larger *UD* than the other subset,

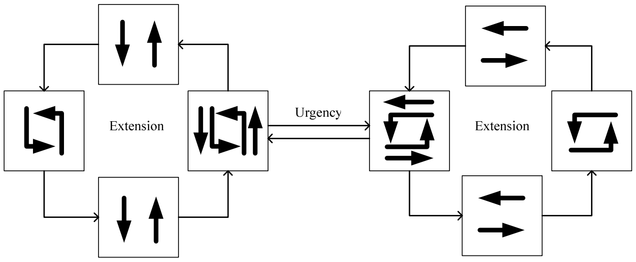


Figure. 3. Phase sequences of FUZZY-MIX controller.

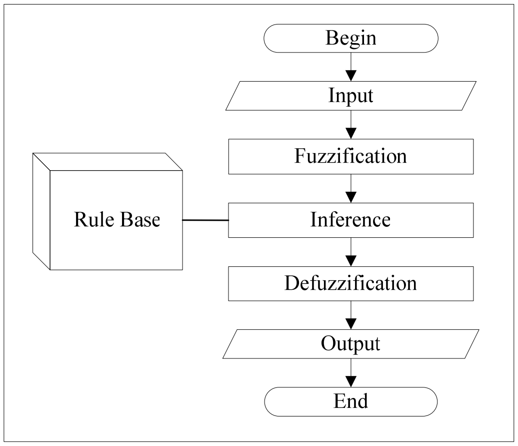


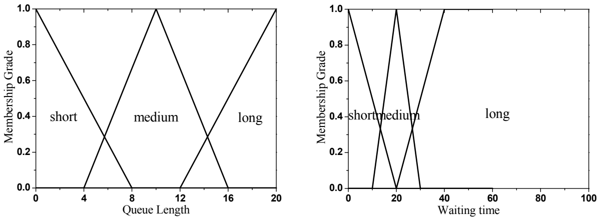
Figure. 4. Flowchart of a Fuzzy .

the second layer is activated. The second layer computes the extension time (*ET*) of the current green phase. If *ET* is larger than a predefined threshold value (Φ time units), the phase is kept unchanged; otherwise the next phase in the current subset is activated and allocated with an initial duration (Θ time units). The phase sequences of the two subsets are shown in Fig. 3. On the other hand, if the current running subset has a smaller *UD* than the other subset, the system will execute an all-red time (θ time units) and then jumps to the entrance phase of the other subset. As shown in Fig. 3, the entrance phases of the two subsets are the phases with maximum active flows, i.e., phase 1 in subset I and phase 4 in subset II.

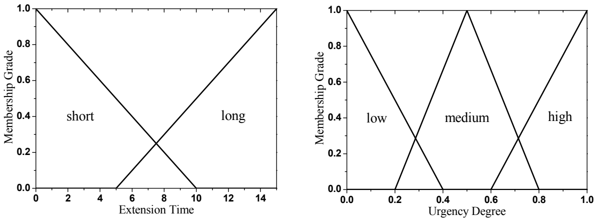
As shown in Fig. 4, a fuzzy logic controller **[16][17]** consists of a fuzzification module, a fuzzy rule base, an inference engine, and a defuzzification module. The fuzzification module performs membership functions that convert the crisp input values into linguistic values with fuzzy membership grades. The inference engine receives the fuzzification results and employs the rules in the fuzzy rule base to infer fuzzy (linguistic) outputs. It is capable of simulating human decision-making and is the kernel of the fuzzy logic controller. At last, the defuzzification module converts the fuzzy outputs into a crisp value. In the following, the implementations of the proposed controller are described.

B. Variables and Membership Functions

Fuzzy variables used in the proposed controller are: queue length (*QL*) and waiting time (*WT*) as inputs to reflect the current traffic condition, and *ET* and *UD* as outputs to control the traffic signal. *QL* of a phase is the average number of vehicles detected behind the stop lines which have green signal in the phase, and *WT* is the average duration of these vehicles since they have arrived the roundabout. Moreover, *QL* and *WT* of a phase subset are represented by those of its entrance phase.



(a) Membership functions of *QL* (b) Membership functions of *WT*



(c) Membership functions of *ET* (d) Membership functions of *UD*

Figure. 5. Membership functions of fuzzy variables.

Membership functions of the four input/output variables are shown in Fig. 5, in which the *x*-axis is the quantity of a variable while the *y*-axis is the corresponding membership grade. It can be observed in Fig. 5(a) that *QL* has three membership functions including *short_QL*(*x*), *medium_QL*(*x*), and *long_QL*(*x*) as defined in (1)-(3).

$$short\_QL(x)=\begin{cases}(8-x)/8, & \text{if } x<8\\ 0, & \text{if } x\geq 8\end{cases} \tag{1}$$

$$medium\_QL(x)=\begin{cases}0, & \text{if } x\leq 4 \text{ or } x\geq 16\\ (x-4)/6, & \text{if } 4<x<10\\ (16-x)/6, & \text{if } 10\leq x<16\end{cases} \tag{2}$$

$$high\_QL(x)=\begin{cases}0, & \text{if } x\leq 12\\ (x-12)/8, & \text{if } x>12\end{cases} \tag{3}$$

In the same way, *WT* has three membership functions including *short_WT*(*x*), *medium_WT*(*x*), and *long_WT*(*x*); *ET* has two membership functions including *short_ET*(*x*) and *long_ET*(*x*); and *UD* has three membership functions including *low_UD*(*x*), *medium_UD*(*x*), and *high_UD*(*x*). These membership functions are illustrated in Fig. 5(b), (c), and (d) respectively. In the fuzzification module, the membership functions of input variables *QL* and *WT* are performed to fuzzify the crisp inputs in order to obtain the corresponding membership grades.

Table I Fuzzy Rule Base

Rule	Inputs		Output 1	Output 2
	QL	*WT*	*ET*	*UD*
$R_{1\text{-}1}/R_{2\text{-}1}$	short	short	short	low
$R_{1\text{-}2}/R_{2\text{-}2}$	short	medium	short	low
$R_{1\text{-}3}/R_{2\text{-}3}$	short	long	short	medium
$R_{1\text{-}4}/R_{2\text{-}4}$	medium	short	short	low
$R_{1\text{-}5}/R_{2\text{-}5}$	medium	medium	long	medium
$R_{1\text{-}6}/R_{2\text{-}6}$	medium	long	long	high
$R_{1\text{-}7}/R_{2\text{-}7}$	long	short	long	medium
$R_{1\text{-}8}/R_{2\text{-}8}$	long	medium	long	high
$R_{1\text{-}9}/R_{2\text{-}9}$	long	long	long	high

C. Fuzzy Rule Base and Fuzzy Inference

Table I shows the fuzzy rule base. It is composed of eighteen rules including nine rules ($R_{1\text{-}1}$-$R_{1\text{-}9}$) to infer *ET* and nine rules ($R_{2\text{-}1}$-$R_{2\text{-}9}$) to infer *UD*. For example, $R_{1\text{-}1}$ means that `if` *QL* `is` short `and` *WT* `is` short `then` *ET* `is` short; $R_{2\text{-}1}$ means that `if` *QL* `is` short `and` *WT* `is` short `then` *UD* `is` low; $R_{1\text{-}7}$ stands for that `if` *QL* `is` long `and` *WT* `is` short `then` *ET* `is` long; $R_{2\text{-}7}$ stands for that `if` *QL* `is` long `and` *WT* `is` short `then` *UD* `is` medium.

Then, the fuzzy inference is applied to combine these "`if-then`" rules into a mapping from fuzzy input set to fuzzy output set. In the inference engine of FUZZY-MIX, rule matching is conducted by using fuzzy intersection (the min operator), while rule merging is done by employing fuzzy union (the max operator). For example, suppose that the membership grade of "*QL* `is` short" is u_1 and the membership grade of "*WT* `is` short" is u_2, the inference strength of $R_{1\text{-}1}$ is computed by $r_{11} = \min\{u_1, u_2\}$. Moreover, as "*ET* `is` short" is the consequent of $R_{1\text{-}1}$, $R_{1\text{-}2}$, $R_{1\text{-}3}$, and $R_{1\text{-}4}$, the membership grade of "*ET* `is` short" is computed by $o_1 = \max\{r_{11}, r_{12}, r_{13}, r_{14}\}$.

D. Defuzzification

Finally, the defuzzification module is executed. It is necessary because the traffic system needs crisp signal timings. A height defuzzification method **[18]** is adopted, in which the resultant output is sensitive to all the corresponding rules executed. At first, the centroid of each membership functions is computed. Then the final crisp output is the average of these centroids, weighted by the membership grades (heights). The output of the fuzzyfication is defined as

$$O = \frac{\sum_{i=1}^{k} o_i \times c_i}{\sum_{i=1}^{k} o_i} \qquad (4)$$

where k is the number of membership functions of the output variable; o_i is the membership grade of the ith fuzzy output, c_i is the centroid of the ith membership function. For example, suppose that the membership grades of "*ET* `is` short" and "*ET* `is` long" are 0.408 and 0.083 respectively, as the centroid of *short_ET*(*x*) and *long_ET*(*x*) are 2.5 and 12.5 respectively, the final crisp output of *ET* is computed as $O_{ET} = (0.408 \times 2.5 + 0.083 \times 12.5)/(0.408 + 0.083) = 4.19$.

Table II Tested Traffic Conditions

Lane	Condition A	Condition B	Condition C
0-L	0.15→0.3	0.1→0.2	0.05→0.15
0-S	0.15→0.3	0.1→0.2	0.16→0.29
1-L	0→0.05	0.1→0.2	0.1→0.18
1-S	0→0.05	0.1→0.2	0.2→0.31
2-L	0.15→0.3	0.1→0.2	0.07→0.17
2-S	0.15→0.3	0.1→0.2	0.14→0.29
3-L	0→0.05	0.1→0.2	0.13→0.18
3-S	0→0.05	0.1→0.2	0.24→0.34

IV. SIMULATION

A. Simulation Settings

The simulation program is developed with Visual C++, run on a machine with Intel Pentium Dual CPU, 1.99 GHz/500 MB of RAM. The geometric design and phase composing of the roundabout are shown in Fig. 1 and Fig. 2 respectively and described in Section II. The arrivals of vehicles on different lanes are independent and subject to Poisson distribution [15]. We perform the simulations on three different traffic conditions. The Poisson arrival rates of vehicles on each entrance lanes of these conditions are shown in Table II, where 0-L, 1-L, 2-L, and 3-L stand for the left-turn entrance lanes of approaches 0, 1, 2, and 3 respectively; 0-S, 1-S, 2-S, and 3-S represent the go-straight entrance lanes of approaches 0, 1, 2, and 3 respectively; a time unit is equal to 0.5 seconds; the Poisson arrival rate of each entrance lane starts at a beginning value and smoothly increase to an ending value over time. For example, if Condition A is tested, the probability of a vehicle being generated on the left-turn lane of approach 0 per time unit starts at 0.15 and increases to 0.3. The three different conditions have their own characteristics. In Condition A, the differences between the north-south flow rates (the arrival rates of vehicles from approaches 0 and 2) and the west-east flow rates (the arrival rates of vehicles from approaches 1 and 3) are conspicuous, whereas in Condition B, all the entrance lanes have same vehicle arrival rates. Condition C is a heavy traffic condition in which the flow rates increase to a relatively high value after a period of time.

Three fuzzy controllers are applied to control the signal of the roundabout. Despite the proposed controller, another two fuzzy controllers adopting fuzzy models which are widely used in the literature [9]-[14] are also implemented. In the following statement, the fuzzy controller which outputs the extension time of the current phase is termed FUZZY-TIM for it actually determines the phase timing. Meanwhile, the controller which adjusts the phase sequence according to the urgency degrees is termed FUZZY-SEQ. The proposed controller with two fuzzy layers hybridizes FUZZY-TIM and FUZZY-SEQ is termed FUZZY-MIX.

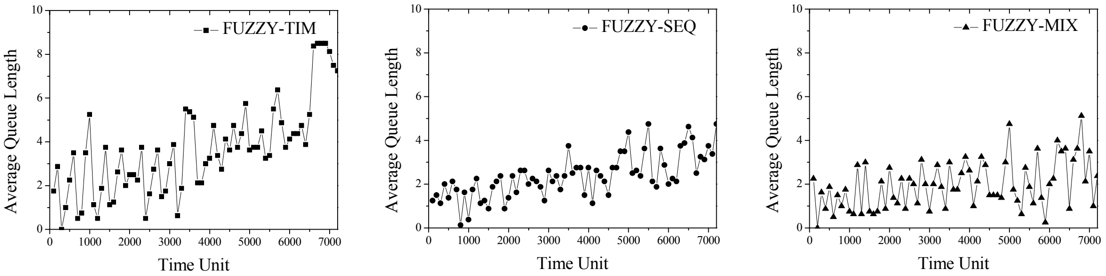
Each simulation lasts 7200 time units (equals to 1 h). In the controllers, the initial duration Θ for each phase is set as 10, the threshold value Φ for extending the green phase is set as 5, the interval time Δ for recomputing *UD* (used in FUZZY-SEQ) is set as 5, and the length θ of all-red time is set as 2. The membership functions of FUZZY-TURN, FUZZY-JUMP, and FUZZY-MIX are shown in Fig. 5.

B. Simulation Results

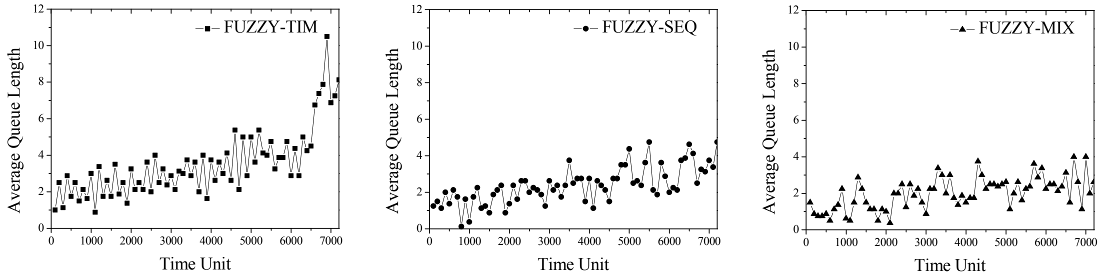
Table III Average Vehicle Delay Obtained by the Three Controllers

Controller	Condition A	Condition B	Condition C
FUZZY-TIM	29.93406	23.6764	30.55463
FUZZY-SEQ	19.55003	21.72118	--
FUZZY-MIX	15.60689	13.57572	14.34392

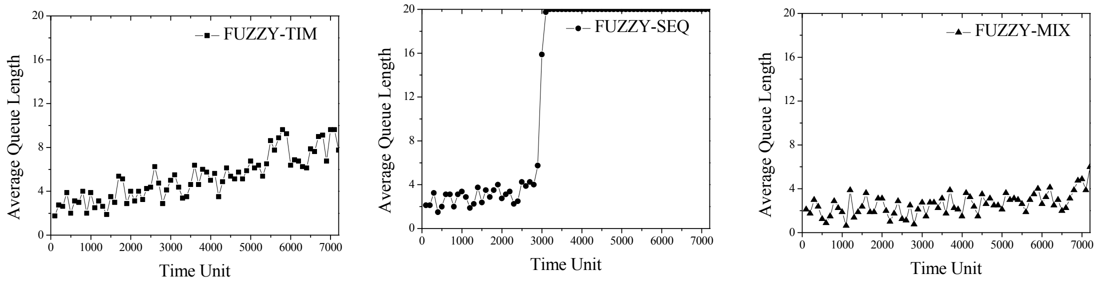
As an index of performance, the average delay time of vehicles during the simulation is computed and presented in Table III. First, compare the previous one-layer FUZZY-TIM and FUZZY-SEQ, it can be observed that FUZZY-SEQ obtains less vehicle delay time than FUZZY-TIM in controlling Condition A and Condition B. Especially when the differences between the north-south flow rates and the west-east flow rates are conspicuous (Condition A), FUZZY-SEQ can obtain a much less vehicle delay time than FUZZY-TIM. This is because that FUZZY-SEQ always changes its green phase to the currently most urgent phase. The controller is much easier to switch the phase from one direction to the other direction than FUZZY-TIM (which complies with predefined phase circle). Therefore, FUZZY-SEQ is more readily adapt to changing needs and circumstances and outperforms the FUZZY-TIM. However, if the roundabout has a relatively heavy condition such as Condition C, FUZZY-SEQ



(a) Condition A



(b) Condition B



(c) Condition C

Figure. 6. Average Queue Length obtained by FUZZY-TIM, FUZZY-SEQ, and FUZZY-MIX.

encounters some troubles. In Table III, the average delay time of using FUZZY-SEQ to control the roundabout with Condition C is not listed. Because big traffic jam happens in this simulation. The FUZZY-SEQ controller frequently changes the phase sequences in order to improve real-timeness, but the cost is that the risk of traffic jams is increased. In contrast, FUZZY-TIM adopts well-designed phase circle which does not cause traffic jams even when the traffic condition is very heavy.

As a mixture of FUZZY-TIM and FUZZY-SEQ, the proposed FUZZY-MIX controller outperforms the other two controllers for all the tested conditions. The reasons are summarized as follows. First of all, the second layer of FUZZY-MIX uses consistent phase sequences, which helps to smooth the traffic flows and reduce congestion. Second, by the use of the first layer, FUZZY-MIX is capable of breaking the phase sequences if the other direction of the roundabout is in more urgent than the current direction. By this way, the proposed controller can immediately respond to the current need of the roundabout. Third, compared to FUZZY-SEQ which considers the urgency degrees among the phases, the FUZZY-MIX concerns the urgency degrees among the phase subsets. The coarse-grained FUZZY-MIX controller does not change the phase sequences as frequent as the FUZZY-SEQ controller, and therefore induces less all-red time than the FUZZY-JUMP. The decrease in all-red time helps to improve the traffic efficiency. Last, when switching the phase from one phase subset to the other, the entrance phase of the subset is set as the phase which has maximum active flows. As the all-red time (before the entrance phase starts up) clears the vehicles in the circular lanes of the roundabout, the vehicles of the urgent direction can occupy the circular lanes as soon as possible. This mechanism helps to maximally relieve the traffic burden of the urgent direction.

In addition, the average length of the eight waiting-vehicle queues of the roundabout during the simulation is shown in Fig. 6. It can be observed that the average queue length increases along with the increase of vehicle arrival rates. FUZZY-SEQ outperforms FUZZY-TIM when controlling Condition A and B, but encounters a traffic jam when the traffic condition is heavy. FUZZY-MIX has the shortest average queue length during the simulation, which further demonstrates the effectiveness and efficiency of the proposed fuzzy traffic-signal controller.

V. CONCLUSION

In this paper, a novel fuzzy traffic-signal controller is proposed. The controller has two fuzzy layers to control phase sequences and timings. The first layer helps to immediately respond to the changing environment in order to improve real-timeness, while the second layer smoothes the traffic flows and contributes to a low risk of traffic jams. The proposed controller is used to control the traffic signal of a modern roundabout. Simulation results show the effectiveness and efficiency of the proposed controller.

As the membership functions adopted in the fuzzy controller is given by human perception, they cannot guarantee the best performance of the controller. Future work is that we can develop some intelligent algorithm to optimize the membership functions of the controller in order to further improve the performance of the proposed controller.

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