Energy-Efficient Local Wake-up Scheduling in Wireless Sensor Networks

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Abstract-scheduling sensor activities is an effective way to prolong the lifetime of wireless sensor networks (WSNs). In this paper, we explore the problem of wake-up scheduling in WSNs where sensors have different lifetime. A novel local wake-up scheduling (LWS) strategy is proposed to prolong the network lifetime with full coverage constraint. In the LWS strategy, sensors are divided into a first layer set and a successor set. The first layer set which satisfies the coverage constraint is activated at the beginning. Once an active sensor runs out of energy, some sensors in the successor set will be activated to satisfy the coverage constraint. Based on the LWS strategy, this paper presents an ant colony optimization based method, namely mc-ACO, to maximize the network lifetime. The mc-ACO is validated by performing simulations on WSNs with different characteristics. A recently published genetic algorithm based wake-up scheduling method and a greedy based method are used for comparison. Simulation results reveal that mc-ACO yields better performance than the two algorithms.

Keywords-wireless sensor network; lifetime maximize; ant colony optimization; wake-up scheduling; disjoint cover set

I. INTRODUCTION

A wireless sensor network (WSN) consists of a large number of sensors which are densely deployed over a large area. Each sensor monitors a physical environment and communicates via wireless signals. With the advancements in hardware miniaturization and wireless communication technologies, WSNs have been used in various applications such as education, warfare, and traffic monitoring [1]-[3]. Regardless of the applications, extending the network lifetime is a critical issue in WSNs. This is because the sensors are battery-powered and generally difficult to be recharged.

One effective way to prolong the network lifetime is to schedule sensors' wake-up activities. In WSNs, sensors commonly have two operation modes, i.e., active mode and sleep mode. A sensor in active mode can perform its monitoring task and therefore needs to consume a relatively large amount of energy. On the contrary, a sensor in sleep mode does not perform the sensing task and consumes little energy [4][5]. Therefore, by appropriately scheduling sensors to be in low-energy sleep mode and waking them up when necessary, the network lifetime can be prolonged. In the literature, various efforts have been made on optimizing the wake-up scheduling in WSNs. These methods generally can be classified into two classes.

The first class is the disjoint cover set scheduling which prolongs the network lifetime by finding a maximum number of disjoint cover sets (see [4]-[12]). In these methods, all the cover sets found can satisfy the network requirements and are activated successively. When an active sensor runs out of energy, a new cover set will be activated to accomplish the monitoring task. The disjoint cover set scheduling discards the whole cover set when a sensor in the cover set runs out of energy. This will waste a lot of energy in applications where sensors have different lifetimes. The reason is that when a sensor runs out of energy, other sensors in the same cover set may still have residual energy. The network lifetime can be prolonged by keeping these sensors at work. As an example, Fig. 1 shows a network containing four sensors s_1 , s_2 , s_3 , and s_4 . There are four possible cover sets, i.e., $\{s_1, s_3\}, \{s_1, s_4\}, \{s_2, s_3\}, \{s_2, s_3\}, \{s_3, s_4\}, \{s_2, s_3\}, \{s_3, s_4\}, \{s_4, s_4\}$ and $\{s_2, s_4\}$. Suppose the lifetimes of these four sensors are 0.9, 1, 0.8, and 1.1 respectively. By using the disjoint cover set scheduling strategy, the maximum network lifetime is 1.0 + 0.8= 1.8 (e.g., first activate $\{s_1, s_3\}$ and then activate $\{s_2, s_4\}$). However, the network lifetime can reach 1.9(1.0+0.1+0.8 =1.9), by using the following wake-up scheduling strategy. First activate $\{s_2, s_4\}$; then activate s_1 when s_2 runs out of energy; finally activate s_3 when s_4 run out of energy. The above example demonstrates that the network lifetime can be further prolonged by efficiently utilizing the residual energy of sensors.





The second class is the non-disjoint cover set scheduling which prolongs the network lifetime by maximizing the total working time of non-disjoint cover sets (see [13]-[15]). In the non-disjoint cover set scheduling, sensors are divided into multiple non-disjoint cover sets. Each cover set can satisfy the full coverage requirement and is scheduled to work for a

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predefined time interval. When a cover set working for a predefined duration, the cover set will be scheduled into sleep mode and a new cover set will be activated to accomplish the monitoring task. One drawback of this class of methods is that they require many sensors working in intermittent periods. Therefore, these sensors will be scheduled into sleep or active mode frequently, which is harmful to circuits and may reduce the lifetime of sensors.

To better prolong the network lifetime, this paper propose a novel local wake-up scheduling (LWS) strategy. Instead of dividing the sensors into multiple disjoint cover sets, the LWS strategy divides sensors into a first layer set and a successor set. Sensors of the first layer set are activated when the network starts working, while other sensors are scheduled into sleep mode to conserve energy. Once an active sensor runs out of energy, some sensors in the successor set will be activated to satisfy the network requirements, while other sensors remain in active mode. The advantages of the proposed LWS strategy are as follows: 1) it can fully utilize the residual energy of sensors even when sensors have different lifetime. And 2) it does not require sensors changing working mode frequently. Based on the LWS strategy, this paper presents an ant colony optimization (ACO) based method, termed mc-ACO, to search for global optimal or near global optimal solutions.

ACO is a notable swarm intelligence algorithm inspired by the foraging behavior of ant species [16]. It involves a group of artificial ants travelling on a construction graph to search solutions for combinatorial optimization problems. With the aid of pheromones and heuristic information, ACO can find high quality solutions to a wide range of applications, such as job shop scheduling, cash flows scheduling, and project scheduling (see [16]-[19]). In the proposed mc-ACO, two different construction graphs are designed to guide the search. The first construction graph, with pheromone trails deposited on vertexes, is used for finding a first layer set. Meanwhile, the second construction graph with pheromone trails deposited on edges is used for finding successor cover sets. The proposed mc-ACO is validated by testing networks with different characteristics. The simulation results reveal that the proposed mc-ACO yields very promising performance.

The rest of the paper is organized as follows. Section II reviews the related work on sensor wake-up scheduling. Section III illustrates the detailed implementations of the proposed mc-ACO. The simulation studies are presented in Section IV. At last, Section V draws the conclusions.

II. RELATED WORK

Due to the effectiveness of wake-up scheduling in conserving energy in WSN applications, this research topic has attracted increasing amount of interest in academia. In [6], Cardei et al. improved the lifetime of WSNs by organizing the sensors into a maximal number of disjoint cover sets. They formulated the problem as a maximum-flow problem and presented a "maximum covers using mixed integer (MC-MIP)" algorithm. However, the programming computational cost of the MC-MIP would increase exponentially as the number of sensors enlarges. To tackle this problem, Slijepcevic et al. [7] proposed a fast greedy heuristic

algorithm, termed the "most constrained-minimally constraining covering (MCMCC)". The MCMCC is a deterministic approximation approach and shown to perform very fast even in large scale WSNs. However, the MCMCC cannot guarantee finding the global optima, for it only uses the local heuristic information. Some related work can also be found in [8][9], where heuristic algorithms are designed to maximize the number of disjoint cover sets. More recently, evolutionary algorithms have also been used to search maximum number of disjoint cover sets. In [10], Lai et al. proposed a genetic algorithm (GA) based method, termed GAMDSC. The GAMDSC reportedly gets near-optimal solutions, but it is only suitable for point coverage problems. In [11], Hu et al. designed an enhanced GA (STHGA) which involves a forward encoding scheme and three schedule transition operations to find maximum number of disjoint cover sets. The SHTGA is shown to perform better than the MCMCC and GAMDSC. In [12], Lin et al. proposed an ant colony optimization based method to maximize the lifetime of heterogeneous WSN by finding maximum disjoint cover sets.

The above methods maximize the network lifetime by finding disjoint cover sets. Several researches suggested using non-disjoint set scheduling to maximize the network lifetime. In [13], Cardei et al. showed that non-disjoint set scheduling can perform better than the disjoint cover set scheduling. The authors proved the NP-completeness of maximizing the network lifetime by the non-disjoint set scheduling and presented two heuristic algorithms to find approximation solutions. The scheduling strategy in [13] has also been extended to address more complicated network models which takes into account the energy consumption in communication and the network connective constraints [14][15].

III. LOCAL WAKE-UP SCHEDULING PROBLEM

A. Local Wake-up Scheduling Problem

We consider a set of sensors $S = \{s_1, s_2, ..., s_N\}$ that are randomly deployed in a $L \times W$ monitoring region. All sensors have limited power supply and can monitor a circular area of radius R. Due to the difference of initial energy and that of the energy consumption in communication, individual sensor have different lifetime. We assume the positions and the lifetime of sensors are known in advance. The local wake-up scheduling works in the following manner: Activate a subset of sensors when the network starts working. These active sensors are referred to as the first layer set, by which the whole region can be observed. Other sensors are scheduled to be in sleep mode for conserving energy. Once an active sensor runs out of energy, certain sensors in sleep mode will be activated to ensure the whole monitoring region is still observed. This process continues until the network requirement cannot be satisfied.

The general local wake-up scheduling problem can be defined as follows:

"Given a set of sensors, determine a first layer set and successors of active sensors, so that the total network lifetime can be maximized with full coverage constraint." To check whether a network is working properly, we need to compute the coverage rate of the network. First, the whole monitor region is divided into $U \times V$ small grids. Then the approximation coverage rate of the network can be computed by

$$v = \frac{C}{U \times V} \tag{1}$$

where C is the number of grids covered by at least one active sensor. According to (1), v=1 means the whole region is covered by the active sensors and the full coverage requirement is satisfied. Otherwise, the network full coverage constraint is not satisfied.

Given a set of sensors, the upper bound of the network lifetime can be estimated by

$$\tilde{T} = \min\{T_1, T_2, ..., T_{U \times V}\}$$
 (2)

where T_i is the maximum duration of the *i*-th grid be covered by the sensors, as can be computed by

$$T_i = \sum_{s_i \in N(i)} t_j, \quad i = 1, 2, ... U \times V$$
 (3)

where N(i) is the set of sensors that can cover the *i*-th grid, and t_j is the lifetime of sensor s_j . It should be noted that the true maximum network lifetime is shorter than or equal to \tilde{T} .

IV. IMPLEMENTATION OF MC-ACO

A. Solution Construction Behavior in mc-ACO

In this paper, we propose to use multiple construction graphs in ACO to guide the search. As shown in Fig. 2, the artificial ant travels the first construction graph to search for a first layer set, and then it travels the second construction graph to find a series of successor cover sets. Specifically, an artificial ant constructs a solution by the following two steps.



(a) For finding first layer SNs
 (B) For finding successor SNs
 Figure 2. Construction graphs of mc-ACO.

Step1 – Finding a first layer set: In the first construction graph, o, s_i and N respectively represent the starting point, the *i*-th sensor and the number of sensors. Pheromone is deposited on the vertexes. Starting from vertex o, artificial ants move to other vertexes one by one, and gradually find a first layer of sensors. Supposing the *k*-th ant is located at s_i , the next vertex to be visited is chosen by

$$j = \begin{cases} \max_{l \in F} \{\tau_l \eta_l^\beta\}, & \text{if } q \le q_0 \\ \text{proportion-selection-rule, otherwise} \end{cases}$$
(4)

where *F* is the set of feasible vertexes, β is a parameter, τ_i is the pheromone value on s_i , and η_i is a heuristic value which can be computed by

 η_i = number of uncovered grids that can be covered by $s_i(5)$

The proportion-selection-rule returns a vertex in a stochastic manner, where the probability of returning s_i is

$$p_{j}^{k} = \begin{cases} \frac{\tau_{j}\eta_{j}^{\beta}}{\sum_{l \in F} \tau_{l}\eta_{l}^{\beta}}, \text{if } j \in F\\ 0, & \text{otherwise} \end{cases}$$
(6)

The ant selects sensors according to (4)-(6) until the full coverage constraint is satisfied. These selected sensors form a first layer set and are activated when the network starts working.

Step2 – Finding successor cover sets: Once an active sensor runs out of energy, the second construction graph is utilized to search for successor sensors to satisfy the coverage requirement. In the second construction graph, os_i represents the sensor which runs out of energy, while s_i represents the candidate successor sensor. The pheromone is deposited on the edges. Supposing s_i runs out of energy, the artificial ant will move to vertex os_i , and chooses a successor sensor by (4) - (6). Here pheromones are deposited on edges, hence τ_i in (4) and (6)

should be replaced by τ_{ij} , which denotes the pheromone on the edge between os_i and s_j . If the full coverage constraint cannot be satisfied, the ant returns to os_i and selects a new sensor again until finding a new successor cover set. When the ant successfully finds a successor cover set, it will move to one vertex among $os_1, os_2, ..., os_N$, according to the shortest lifetime sensor in the newly found cover set. Then the ant continues to find a new successor cover set by the above methods. This process repetitively until no successor cover set can be found.

B. Algorithm Framework

The mc-ACO involves three main steps to search for promising solutions, as illustrated as follows.

Step 1 – Initialization: This step initializes pheromones on the construction graphs. First of all, a feasible solution is obtained by using a greedy algorithm. The greedy algorithm finds the first layer of sensors by repetitively selecting a sensor with the largest heuristic value computed by (5). Once an active sensor runs out of energy, sleeping sensors with the largest heuristic value are activated to satisfy the coverage requirement. This process is repeated until the full coverage constraint cannot be satisfied. Then the pheromone values are initialized as

$$\tau_i = \tau_{ij} = t_0$$

where τ_i is the pheromone on vertex s_i of the first construction graph, τ_{ij} is the pheromone on the edge between os_i and s_j , t_0 is the network lifetime of the solution found by the greed algorithm.

Step 2 – Solution construction: In this step, each artificial ant finds a feasible solution by the mechanism given in section III-

B. At the end of each construction process, the pheromone values are updated by

$$\begin{cases} \tau_i = (1 - \varphi) \cdot \tau_i + \varphi \cdot t_0, \text{ if } s_i \in X \\ \tau_{ij} = (1 - \varphi) \cdot \tau_{ij} + \varphi \cdot t_0, \text{ if } \overline{os_i s_j} \in X \end{cases}$$
(13)

where φ is the pheromone decay coefficient, *X* is the solution found by the current ant, $s_i \in X$ means that s_i belongs to the first layer set of the solution, while $\overline{os_i s_j} \in X$ means that when sensor s_i runs out of energy, s_j will be activated in the solution.

Step 3 –Global pheromone updating: The global pheromone updating process is performed when all ants complete their solution construction procedures. The aim of this operation is to increase the pheromones on the path of the best-so-far ant, so that subsequent ants can use the pheromone information to explore promising regions of the search space. The final pheromone is updated by

$$\begin{cases} \tau_i = (1 - \rho) \cdot \tau_i + \rho \cdot t_{best}, \text{ if } s_i \in X_{best} \\ \tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \rho \cdot t_{best}, \text{ if } \overline{\sigma s_i s_j} \in X_{best} \end{cases}$$
(14)

where ρ is the global pheromone update coefficient, X_{best} is the best-so-far solution and t_{best} is the network lifetime of the best-so-far solution.

There is a repetition from *Step2* to *Step3* until meeting the termination condition.

V. SIMULATIONS AND COMPARISONS

A. Simulations Settings

In this section, we perform simulations to validate the proposed mc-ACO. Different test cases are designed to test the algorithm performance. All sensors are randomly deployed in a 100m \times 100m rectangle region. The initial lifetime of each sensor is uniformly distributed between 0.8 and 1.2. The sensing radius of all sensors are set to be R = 30m.

In order to demonstrate the effectiveness and efficiency of the proposed strategy, the STHGA method proposed in [11] and the G-MSC in [13] are used for comparison. The STHGA is a recently published GA-based method which maximizes the network lifetime by finding the maximal number of disjoint cover sets. We compare mc-ACO with this method to demonstrate that using local wake-up scheduling strategy can work more effective than the disjoint cover set scheduling strategy. The G-MSC is a greedy algorithm which divides sensors into non-disjoint cover sets by applying a step-by-step selection policy. At each step, G-MSC greedily selects a "critical" target (e.g., the one observed by least sensors) and then selects the sensor with the greatest contribution (e.g., the one observes most uncovered targets) to the "critical" target. We set the working time of each cover set to be the shortest lifetime of sensors in the cover set.

The parameter settings of mc-ACO are listed in Table I, while the parameter settings of STHGA are set according to [11]. As both STHGA and mc-ACO are stochastic algorithms that may obtain different results in different runs, we run STHGA and mc-ACO respectively for 30 independent times on each test case and use the average results for comparison.

TABLE I. SUMMARY OF PARAMETER SETTINGS IN MC-ACO

paramete	value	summary	
т	10	Number of artificial ants	
q_0	0.9	Exploitation rate	
φ	0.5	Pheromone decay coefficient	
ρ	0.5	Global pheromone update coefficient	
β	2	Determine the importance of heuristic information	
MAXFES	1000	The maximum number of fitness evaluations	

B. Comparison Results

 TABLE II.
 COMPARISON RESULTS ON 300-SENSOR NETWORKS

Test case			STHGA		G-MSC	mc-ACO
id	Ĉ	Ĩ	\overline{C}	\overline{T}	Т	\overline{T}
1	19	18.8	19	15.4367	18.8	18.8
2	20	19.8	20	16.3667	19.7	19.8
3	21	20	21	17.14	19.8	20
4	20	19.8	20	16.3067	19.4	19.8
5	20	19.5	20	16.2833	19.2	19.5
6	17	16.6	17	13.6767	16.3	16.6
7	21	20.3	21	17.0733	19.9	20.3
8	17	16.6	17	13.7267	16.6	16.6
9	19	19.4	19	15.5367	19.3	19.4
10	22	21.3	22	17.92	20.2	21.3

Table II lists the results computed by STHGA, g-LWS, and mc-ACO on ten 300-sensor networks. In Table II, \tilde{C} and \tilde{T} respectively represent the upper bound of disjoint cover sets and the upper bound of network lifetime; \overline{C} means the average number of cover sets found by the STHGA in 30 independent runs, while the \overline{T} in the fifth and seventh column respectively represent the average network lifetime found by the STHGA and mc-ACO. As G-MSC is a deterministic algorithm that only provides a fix solution for a problem, we use T to represent the network lifetime found by the algorithm. It can be observed that, although STHGA can find the maximum number of cover sets on all cases, the total network lifetimes are shorter than the upper bound of network lifetimes. This is because the lifetime of a cover set is dependent on the shortest lifetime sensors. When the sensor with the shortest lifetime runs out of energy, the resident energy of other sensors in the same cover set cannot be used any more. This results in a lot of waste. The G-MSC finds the upper bound of network lifetime on some test cases, such as case 1 and 8. However, the G-MSC cannot achieve the global optima on other cases. This is because the G-MSC only utilizes local heuristic information to construct solutions, which makes it easily get trapped into local optima. By making use of the global search ability of ACS and effective local heuristic information, the mc-ACO performs the best on all test cases. It finds the upper bound of network lifetimes on all test cases.

Fig. 3 shows the trends of network lifetime with the number of sensors. For each setting, ten different networks are generated for testing and the average results are used for comparison. It can be observed that the average network lifetime increases when the number of sensor enlarges. This is because more sensors can be assigned to cover a target when the sensor number enlarges. The curves on the figure also demonstrate that mc-ACO outperforms G-MSC and STHGA in terms of network lifetime.



Figure 3. Comparison results on networks with different number of sensors.

VI. CONCLUSIONS

The wake-up scheduling of sensors has significant impact on the lifetime and coverage of a WSN. In this paper, we present an ant colony based local wake-up scheduling method to prolong the network lifetime with full coverage constraint. In the proposed mc-ACO, the artificial ants search solutions in a two-phased manner. The first phase finds a set of sensors that satisfy fully coverage constraint, while the second phase finds the successors of sensors which run out of energy. Two construction graphs are accordingly designed to guide the artificial ants to search partial solutions in the two phases. Simulation results on different networks demonstrate that the proposed mc-ACO yields better performance than the STHGA and the G-MSC. Future work is to extend the algorithm framework to other network models such as those take into account the network connectivity constraints and the routing strategy.

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