Fuzzy-Based Pareto Optimality for Many-Objective Evolutionary Algorithms

Zhenan He, Gary G. Yen, *Fellow, IEEE,* and Jun Zhang, *Senior Member, IEEE*

***Abstract*—Evolutionary algorithms have been effectively used to solve multiobjective optimization problems with a small number of objectives, two or three in general. However, when problems with many objectives are encountered, nearly all algorithms perform poorly due to loss of selection pressure in fitness evaluation solely based upon the Pareto optimality principle. In this paper, we introduce a new fitness evaluation mechanism to continuously differentiate individuals into different degrees of optimality beyond the classification of the original Pareto dominance. The concept of fuzzy logic is adopted to define a fuzzy Pareto domination relation. As a case study, the fuzzy concept is incorporated into the designs of NSGA-II and SPEA2. Experimental results show that the proposed methods exhibit better performance in both convergence and diversity than the original ones for solving many-objective optimization problems.**

***Index Terms*—Fuzzy logic, multiobjective evolutionary algorithm, NSGA-II, Pareto optimality, SPEA2.**

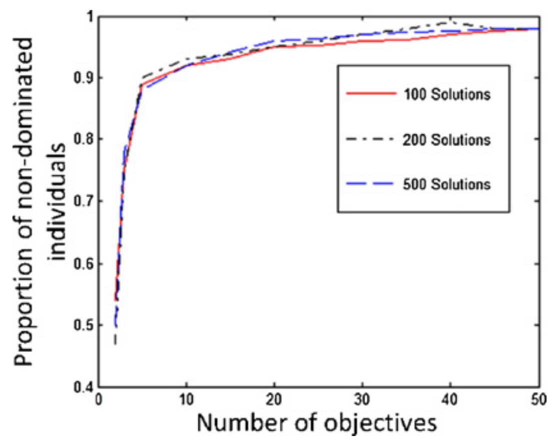

Fig. 1. Proportion of non-dominated individuals within a population.

I. Introduction

Evolutionary algorithms have been effectively used to explore the Pareto-optimal fronts in multiobjective optimization problems (MOPs). In literature, most of these multiobjective evolutionary algorithms (MOEAs) and their variants based on fundamentals in evolutionary strategies, particle swarm optimization, differential evolution, or artificial immune systems work well only in problems with a small number of objectives, mainly in two or three objectives. However, many real-world MOPs involve more than five conflicting objectives, which are commonly referred to as many-objective optimization problems and the performance of most MOEAs deteriorates severely in the face of problems with such a large number of objectives [1]. The main reason that MOEAs lose exploring capability in solving many-objective optimization problems is largely due to the ineffective definition of Pareto optimality. Consider the definition of the Pareto dominance relation.

Pareto Dominance Relation: For a minimization problem, a vector $\overrightarrow{u} = f(\overrightarrow{x_u}) = (u_1, \ldots, u_M)$ is said to dominate $\overrightarrow{v} = f(\overrightarrow{x_v}) = (v_1, \ldots, v_M)$, denoted by $\vec{u} < \vec{v}$, if and only if $\forall i \in \{, \ldots, M\}$, $u_i \le v_i$ and $\exists i \in \{1, \ldots, M\}$, $u_i < v_i$. When any two given vectors $\vec{u}$ and $\vec{v}$ are compared, there are only two possible conclusions according to the Pareto dominance relation:

1) either $\vec{u}$ dominates $\vec{v}$ or $\vec{u}$ is dominated by $\vec{v}$, or
2) $\vec{u}$ and $\vec{v}$ are non-dominated with respect to each other.

Therefore, two individuals can only be differentiated under the first condition. However, from the definition, the greater the number of objectives, the more difficult it becomes to satisfy the first condition. For example, consider that $\vec{u}$ dominates $\vec{v}$ in a 2-D MOP and a 5-D MOP. To arrive at the first condition, for the 2-D problem, $\vec{u}$ only needs to be better than $\vec{v}$ in one objective and no worse in the other objective. However, for the 5-D problem, $\vec{u}$ should be better than $\vec{v}$ in at least one objective and no worse than $\vec{v}$ in all of the remaining objectives. On the other hand, it is fairly easy to arrive at "non-dominated with respect to each other" relation when two individuals are compared based on the Pareto dominance relation in a higher-dimensional problem. Fig. 1 shows the proportion of non-dominated individuals in the initial populations (i.e., set at 100, 200, and 500) who are randomly generated for the benchmark function DTLZ2 [2], a scalable benchmark problem, under various numbers of objectives from two to 50 with a step size of one. Every data point was averaged over 50 independent runs of the same initialization. From this figure, the proportion of non-dominated individuals rises quickly with the number of objectives increasing from

Manuscript received May 3, 2012; revised September 3, 2012, December 14, 2012, and March 29, 2013; accepted April 9, 2013. Date of publication April 12, 2013; date of current version March 28, 2014.

Z. He and G. G. Yen are with the School of Electrical and Computer Engineering, Oklahoma State University, Stillwater, OK 74075 USA (e-mail: gyen@okstate.edu; zhenan@okstate.edu).

J. Zhang is with the Department of Computer Science, Sun Yat-Sen University, Guangzhou 510006, China (e-mail: junzhanghk@mail.sysu.edu.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TEVC.2013.2258025

two to five. When the number of objectives exceeds five, the proportion of non-dominated individuals in a randomly generated initial population is higher than 90%. This leads to significantly diminishing selection pressure during the evolutionary process no matter how the MOEA is designed, if the Pareto dominance relation is used. Therefore, although Pareto optimality is effective to facilitate the convergence of the population in low-dimensional problems, it is not effective in maintaining selection pressure during the evolution process in many-objective optimization problems. In addition to the deficiency in the definition of Pareto dominance stated above, other complications such as visualization of high-dimensional objective spaces [1], a large number of individuals needed to obtain a good representation of the Pareto front [1], and a very high computational cost [3], have contributed to the challenges of solving many-objective optimization problems. From the above discussions, the main difficulties of many-objective optimization problems are caused by a large number of objectives. Naturally, the efforts in addressing this issue have led to strategies to reduce the number of objectives without losing information. For instance, Brockhoff and Zitzler [3] first identify conflict and non-conflict relationships between each pair of objectives and then combine non-conflicting objectives into one objective. Deb and Saxena [4] propose a principle component analysis method to adaptively finding the correct lower-dimensional interactions of each objective by iteratively progressing from the interior of the search space toward the Pareto-optimal region. Singh *et al*. [5] identify whether or not each objective is redundant based on an approximated non-dominated front which is roughly generated beforehand. Although objective reduction works in some special conditions, serious limitations remain. Of course, for real-world environments, problems exist whose objectives cannot be further reduced. For these problems, the methods stated above can only rely upon a relative order of importance of the objectives [4]. In some problems a very small number of objectives can be eliminated, potentially making the difference. For example, for a 10-D problem, reducing only one or two objectives does not help much to solve the problem in an effective manner. Even if the number of objectives is reduced sufficiently, it is not clear how the Pareto front derived in a reduced low-dimensional space can portray the true Pareto front in the original higher dimensional space. Moreover, Said *et al*. [6] use the decision maker's preferences to set an error constant δ and incorporate these preferences into the Pareto dominance definition to guide the search toward the area of interest in the Pareto front. Because of the drawbacks of objective reduction methods and the lack of preference information in most of the real-world applications, the focus of this paper is solely on designing a new fitness measure through the definition of Pareto dominance to continuously differentiate individuals into different degrees of optimality beyond the classification of the original Pareto dominance. Here, the notion of fuzzy logic [7] is adopted. Based on this notion, a fuzzy Pareto dominance (FD) relation is defined and incorporated into the designs of NSGA-II [8] and SPEA2 [9], as a case study, instead of the original Pareto dominance principle. The resulting fuzzy dominance NSGA-II and SPEA2 are applied to search for a Pareto optimal set for many-objective optimization problems by maintaining the selection pressure toward the Pareto front throughout the entire evolutionary process. Please note that the same fuzzy dominance concept can be easily incorporated into other MOEA designs. The remaining sections complete the presentation of this paper. Section II outlines selected literature for fitness evaluations. Section III elaborates on the proposed fuzzy Pareto dominance relation in detail and how to incorporate the proposed fuzzy Pareto dominance relation into the designs of NSGA-II and SPEA2. Section IV details the experimental setting and findings for nine selected scalable benchmark problems. Finally, a conclusion is drawn in Section V along with pertinent observations.

II. Literature Review

This section reviews some fitness evaluation approaches in literature. The majority of existing multiobjective optimization algorithms exclusively use the concept of Pareto domination. These MOEAs compare two solutions on the basis of whether or not one dominates the other.

Pareto Optimality: An individual $\vec{x} \in \Omega$ is said to be Pareto optimal with respect to Ω if and only if there is no $\vec{x} \in \Omega$ for which $\vec{v} = f(\vec{x})$ dominates $\vec{u} = f(\vec{x})$. $f(\vec{x})$ is then called the Pareto optimal (objective) vector. *Remarks:* Any improvement in a Pareto optimal individual in one objective must lead to deterioration in at least one other objective.

Pareto Optimal Set: The set of all the Pareto optimal individuals in the decision space is called the Pareto optimal set (PS), $PS = \{\vec{x} \in \Omega \mid \exists \vec{y} \in \Omega,\ f(\vec{y}) < f(\vec{x})\}$.

Pareto Front: The image of the Pareto optimal set (PS) in the objective space through multiobjective vector function f is called the Pareto front (PF), $PF = \{f(\vec{x}) \mid \vec{x} \in PS\}$.

The definitions of Pareto dominance and Pareto optimality play an important role in the development of effective MOEAs. However, in MOP, Pareto domination does not define a complete ordering among all individuals in the objective space. In addition, it does not measure how much one solution is better than another one.

A. Existing Approaches to Fitness Evaluation

In literature, three main types of approaches are proposed as opposed to the Pareto dominance for fitness evaluation. The first class of the approaches for fitness evaluation uses scalar methods instead of Pareto dominance to assign each individual a fitness value and compare individuals based on these values. This class of designs can be further divided into four different categories. The first category uses predefined weighting coefficients such as weighted sum (WS) [10] and weighted min–max (Wmin–max) [11]. The second category focuses on the extreme values of individuals, for example, maximum ranking (MR) [10], global detriment (GD) [10], and profit (PF) [10]. The third directly compares individuals, including favor relation (FR) [10], k-dominance (KD) [12], and L-dominance (LD) [12]. The last category transforms objectives into constraints, for instance, constraint [11] and goal attainment (GAt) [11]. The second class of methods modifies the Pareto dominance concept to adapt it to higher dimensional problems, such as Pareto α-Dominance [13], Pareto ε-Dominance [13], and Pareto cone ε-Dominance [13]. Heuristically chosen parameters are incorporated into all of these methods. Each modified Pareto dominance design is a relaxing form of the Pareto dominance in that it makes one individual dominate another more easily in higher dimensional optimization problems. The third class is based on the idea of performance metrics. IBEA [14] is probably the most successful implementation of this class in that it has been shown to be more effective than other MOEAs for higher dimensional MOPs. There are other methods of this type: volume dominance (VD) [12] is based on the volume of dominated objective space by the individual; contraction/expansion of dominated area (CE) [12] adjusts the selection process by changing the size of individuals' dominance area and the distance to the best known solution. GB [10] ranks the population by a reference point that is composed by the best known objective value in each dimension. The fitness of a solution will be assigned based on its distance to the reference point. Given the above discussions, researchers mainly focus on two different aspects in the fitness evaluation. First, the Pareto dominance is replaced by another design, such as a scalar method or performance metrics. Both approaches assign each approximation front an exact score and use this score for comparison. However, both only consider one specific characteristic of the front and cannot quantify the approximation front comprehensively. Second, the Pareto dominance is modified by some predefined parameters. The choice of these parameters greatly affects the performance of the fitness assignment process. However, there exists no standard way or effective guideline to determine these parameters. In response to these challenges encountered by the existing approaches, a new fitness evaluation measure based on fuzzy logic is proposed in this paper.

B. Previous Works Related to Fuzzy Dominance

There exist three known works following a similar research line. Farina and Amato [15] designed the fuzzy-based definitions of optimality and domination for human decision making in many-objective optimization problems. The fuzzy definitions process preference information provided by the decision maker and generate a parameter whose value ranges from zero to one in order to compute different subsets of the Pareto optimal set. When the value of the above parameter is set to zero, the introduced definition is the same as the original Pareto optimality. When this parameter value is changed, a different subset of Pareto optimal solutions can be obtained corresponding to different degrees of optimality. Nasir *et al.* [16] suggested a fuzzy Pareto dominance concept to compare two solutions and used the scalar decomposition method of MOEA/D only when one of the solutions fails to dominate the other in terms of a fuzzy dominance level. Köppen *et al.* [17] studied the fuzzification of the Pareto dominance relation to yield a practically usable numerical representation of the dominance relation between two individuals. They used concepts from fuzzy fusion theory to combine the mutual degrees of dominance within a set of individuals and assigned a ranking value to each individual within this set. The ranking values reflect dominance degrees of individuals among themselves. Based on this, an extension of the standard genetic algorithm, fuzzy-dominance-driven GA (FDD-GA), was proposed.

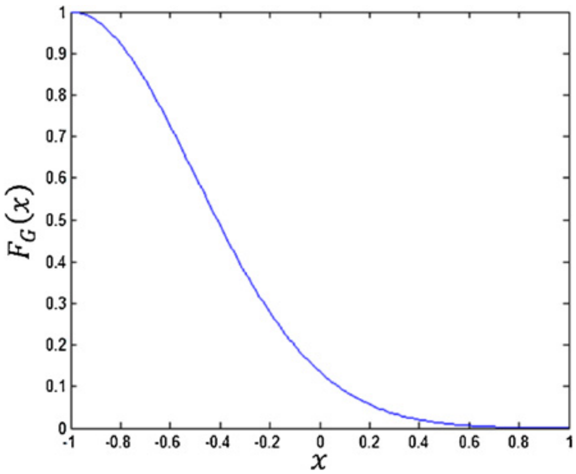

Fig. 2. Left Gaussian function.

III. Proposed Method

A. Background Knowledge of Fuzzy Set

According to the Pareto domination principle, when two individuals, $\vec{u}$ and $\vec{v}$, are compared, either $\vec{u}$ dominates $\vec{v}$, $\vec{u}$ is dominated by $\vec{v}$, or $\vec{u}$ and $\vec{v}$ are non-dominated with respect to each other. When the number of objectives exceeds five, the majority of the population becomes non-dominated with respect to each other and presents little selection pressure to facilitate evolution search in a many-objective optimization problem. A fuzzy set is applied here to quantify the degrees of domination, from dominate to being dominated and in between with various degree of domination in each objective. The set theoretic operator is then used to combine multiple fuzzy sets to allow the comparison of two individuals in many-objective problems. We define the fuzzy set based on the left Gaussian function (F_G) [18], as shown in Fig. 2, with $c = -1$ and $\sigma = 0.5$

$$F_G(x) = \begin{cases} 1 \text{ if } x \leq c \\ \exp\left(-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2\right) & \text{otherwise.} \end{cases} \quad (1)$$

The left Gaussian function is a monotonically decreasing function. The range is between zero and one, representing the domination degree that one individual is better than the other in one objective. The domain, on the other hand, is between minus one and plus one, corresponding to the normalized difference between two individuals in one objective. This difference is normalized by the absolute value of the maximum difference among all pairs of individuals in that objective. Because x is normalized, it does not depend on the range of each objective function. The mapping from the normalized difference between two individuals to the domination degree is defined by the chosen parameters (i.e., c and σ). In this paper, we choose c equal to the lower boundary of x (i.e., c = -1). As shown in Fig. 2, if the objective value of an individual in one dimension is much less than the other such that the normalized difference is closer to –1 (i.e., for a minimization problem), the degree of this individual being better than the other is close to one, which corresponds to the condition that this individual Pareto dominates the other in that dimension. On the other hand, if the objective value of one individual is much larger than the other such that the normalized difference is closer to 1, the degree of this individual being better than the other is closer to zero, which corresponds to the condition that this individual is Pareto dominated by the other in that dimension. Therefore, the left Gaussian function allows the proposed fuzzy Pareto dominance relation to be generalized beyond the original definition of Pareto dominance. σ defines the spread of the Gaussian function. Setting it too large will lead to the inability to discriminate $x \in [-1, 0]$. On the other hand, setting it too small will lead to the inability to discriminate $x \in [0, 1]$. Setting σ at 0.5 appears to be a good compromise to allow sufficient discrimination power for the complete domain of x. There exist several definitions in set theoretic operations that could be used to aggregate multiple fuzzy sets [18]. In this design, the individual's performance in each objective is considered as a fuzzy set and all these fuzzy sets from multiple objectives are combined through a product operator as the intersection of these fuzzy sets. For a two-objective MOP, assume fuzzy set A describes how individual a is better than individual b in the first objective, while fuzzy set B quantifies how individual a is better than individual b in the second objective. $A \cap B$ is the intersection of fuzzy sets A and B, implying how individual a is better than individual b in a two-objective MOP. The truth degree of $A \cap B$, $\mu_{A\cap B} = \mu_A \times \mu_B$, provides a measure of domination degree as how individual a is better than individual b in a two-objective MOP. This concept can be extended to M objectives in a straightforward manner.

B. Fuzzy Pareto Dominance Relation

Without loss of generality, the definition of the fuzzy Pareto dominance relation between a pair of individuals is given in the following.

Fuzzy Pareto Dominance Relation: For an M-objective minimization problem, given two individuals $\vec{u} = f(\vec{x_u}) = (u_1, \ldots, u_M)$ and $\vec{v} = f(\vec{x_v}) = (v_1, \ldots, v_M)$, define $\vec{p}(\vec{u}) = \vec{u} - \vec{v}$ as the performance of $\vec{u}$ with respect to $\vec{v}$ and $\vec{p}(\vec{v}) = \vec{v} - \vec{u}$ as the performance of $\vec{v}$ compared with $\vec{u}$, respectively. The left Gaussian function transforms each dimension of $\vec{p}(\vec{u})$ and $\vec{p}(\vec{v})$ into a measure in [0, 1]. That is $\vec{\varphi}(\vec{u}) = F_G(\vec{p}(\vec{u})) = (\varphi_1^u, \ldots, \varphi_M^u)$ and $\vec{\varphi}(\vec{v}) = F_G(\vec{p}(\vec{v})) = (\varphi_1^v, \ldots, \varphi_M^v)$. Here, in $\vec{\varphi}(\vec{u})$, each $\varphi_i^u, i = 1, \ldots, M$, is considered a fuzzy set and set theoretic operator is applied to the product of all these fuzzy sets. Then we obtain a fuzzy product value of $\vec{u}$, which is $\varphi_{\text{PRODUCT}}^u = \varphi_1^u \times \varphi_2^u \times \cdots\ldots \times \varphi_M^u$. Similarly, a fuzzy product value of $\vec{v}$ is defined accordingly as $\varphi_{\text{PRODUCT}}^v = \varphi_1^v \times \varphi_2^v \times \cdots\ldots \times \varphi_M^v$. Finally, from the comparison of $\varphi_{\text{PRODUCT}}^u$ and $\varphi_{\text{PRODUCT}}^v$, a vector $\vec{u}$ is said to fuzzy-dominate $\vec{v}$ (denoted by $\vec{u} <_F \vec{v}$) if $\varphi_{\text{PRODUCT}}^u > \varphi_{\text{PRODUCT}}^v$. From the definition of φ_{PRODUCT}, two individuals are unlikely to have the same fuzzy product value for a many-objective optimization problem. Therefore, the proposed fuzzy Pareto dominance relation can differentiate a pair of individuals in most cases to maintain the selection pressure. This characteristic makes the fuzzy Pareto dominance relation more effective than the original Pareto dominance relation in many-objective optimization problems in which the Pareto dominance principle cannot preserve the needed selection pressure throughout the evolutionary process. Moreover, the proposed fuzzy Pareto dominance relation can be formulated to continue classifying high-dimensional individuals into different degrees of optimality when incorporating the domination degree. The domination degree between individuals $\vec{u}$ and $\vec{v}$ can be derived from $\varphi_{\text{PRODUCT}}^u$ and $\varphi_{\text{PRODUCT}}^v$. When $\varphi_{\text{PRODUCT}}^u$ is much larger than $\varphi_{\text{PRODUCT}}^v$, $\varphi_{\text{PRODUCT}}^u/(\varphi_{\text{PRODUCT}}^u + \varphi_{\text{PRODUCT}}^v) \approx 1$. This implies that the domination degree is closer to 1 in which $\vec{u}$ fuzzy Pareto dominates $\vec{v}$ infers $\vec{u}$ Pareto dominates $\vec{v}$. When $\varphi_{\text{PRODUCT}}^u$ is much smaller than $\varphi_{\text{PRODUCT}}^v$, $\varphi_{\text{PRODUCT}}^u/(\varphi_{\text{PRODUCT}}^u + \varphi_{\text{PRODUCT}}^v) \approx 0$. This implies that the domination degree is closer to 0 in which $\vec{u}$ fuzzy Pareto dominates $\vec{v}$ (or $\vec{u}$ is fuzzy Pareto dominated by $\vec{v}$ with the domination degree of 1), which suggests $\vec{u}$ is Pareto dominated by $\vec{v}$. Under an extreme condition in a two-objective optimization problem, if $\vec{u}$ is much better than $\vec{v}$ in one objective, but much worse in the other objective, $\varphi_{\text{PRODUCT}}^u \approx \varphi_{\text{PRODUCT}}^v \approx 0$, but $\varphi_{\text{PRODUCT}}^u/(\varphi_{\text{PRODUCT}}^u + \varphi_{\text{PRODUCT}}^v) \approx 0.5$. This implies that the domination degree equals 0.5 in which $\vec{u}$ fuzzy Pareto dominates $\vec{v}$ means $\vec{u}$ is non-dominated with respect to $\vec{v}$. Therefore, from the above discussions, the Pareto dominance can be regarded as a special case of the proposed fuzzy Pareto dominance relation.

C. Fitness Assignment Based on Fuzzy Pareto Dominance

Assuming there are n individuals in the competition pool, every individual is paired with every other n–1 individual respectively, forming n–1 pairs of competitions. For individual $\vec{u}$, in each of n–1 pairs, it is compared with the other individual $\vec{v}$ using the fuzzy Pareto dominance relation and generates both fuzzy product values of $\varphi_{\text{PRODUCT}}^u$ and $\varphi_{\text{PRODUCT}}^v$. The fuzzy product value of $\vec{u}$ is then normalized as $\varphi_{\text{PRODUCT}}^u/(\varphi_{\text{PRODUCT}}^u + \varphi_{\text{PRODUCT}}^v)$. This new normalized value is considered the performance of $\vec{u}$ compared with $\vec{v}$. After calculating each pair, add all n–1 performance values of $\vec{u}$ together to obtain a sum value. This sum value is then divided by (n–1) and is regarded as the fuzzy fitness measure (FFM) of individual $\vec{u}$. Fig. 3 shows this fuzzy fitness assignment process among a pool of individuals. The definition of Pareto dominance only considered relations between two individuals. Here, by incorporating the fuzzy fitness assignment process, we extend this definition to all individuals in a competition pool at the same time. The new definitions of fuzzy Pareto optimality, fuzzy Pareto optimal set, and fuzzy Pareto front are given below.

Fuzzy Pareto Optimality: An individual $\vec{x} \in \Omega$ is said to be fuzzy Pareto optimal with respect to Ω if its fuzzy fitness measure, $FEM(\vec{x})$, is greater than a predefined threshold, α. $f(\vec{x})$ is called the fuzzy Pareto optimal (objective) vector.

Fuzzy Pareto Optimal Set: The set of all the fuzzy Pareto optimal individuals in the decision space is called the fuzzy Pareto optimal set (FPS),

```
Input:  a competition pool containing n individuals
FOR i = 1:n
   % calculate FFM of individual i
   FFM(i) = 0;     % initialize FFM of individual i
   FOR j = 1:n (j ≠ i )
      % calculate performance value of i w.r.t. j
      p = φ^i_PRODUCT/(φ^i_PRODUCT + φ^j_PRODUCT)
      FFM(i) = FFM(i) + p;
   END
   FFM(i) = FFM(i)/(n − 1)
END
Output: a n-dimension vector FFM with each dimension
        representing the FFM of an individual
```

Fig. 3. Fuzzy fitness assignment process.

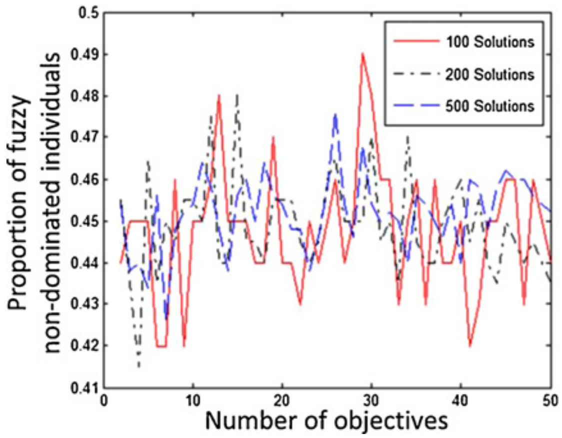

Fig. 4. Proportion of fuzzy Pareto optimal set in the population.

$FPS = \{\vec{x} \in \Omega | FEM(\vec{x}) > \alpha\}$.

Fuzzy Pareto Front: The image of the fuzzy Pareto optimal set (FPS) in the objective space is called the fuzzy Pareto front (FPF), $FPS = \{f(\vec{x}) | \vec{x} \in FPS\}$.

When the same experiment is rerun, as in Fig. 1 (i.e., initial populations of 100, 200, and 500 for the scalable benchmark function DTLZ2), Fig. 4 shows the proportion of fuzzy non-dominated individuals in the population with respect to the varying numbers of objectives from two to 50 with a step size of one. Every data point in Fig. 4 was averaged over 50 independent runs of the same initialization. In this case, the proposed fuzzy Pareto dominance relation with a heuristically chosen predefined threshold (i.e., α) of 0.5 is applied as opposed to the original Pareto dominance relation used to generate Fig. 1. In Fig. 1, the proportion of non-dominated individuals based on the original Pareto domination definition grows at a fast rate as the number of objectives becomes larger. On the other hand, the proportion of non-dominated individuals based on the fuzzy Pareto domination definition proposed in this paper remains at the same level and is independent of the number of objectives. Therefore, the proposed fuzzy Pareto dominance relation effectively differentiates individuals into different degrees of optimality beyond the classification of the original Pareto dominance. Furthermore, the fuzzy Pareto dominance relation can be made adaptable by adjusting the threshold value α to control the size of the fuzzy Pareto optimal set throughout the evolution process. This potential will be exploited in our future research.

```
Input:  a competition pool containing n individuals
r=1;   % set initial rank value
% set the initial number of population that has been
   assigned rank value
c = 0;  % set the initial number of population that has
        been assigned rank value
α = 0.5;  % threshold value is heuristically chosen
While  c < n
  FFM=Fuzzy fitness assignment process(P);
  FOR i = 1:(n − c)
       % if fuzzy fitness measure of i is larger than a
         predefined threshold, it will be added to front r
       IF FFM(i) > α
          r_value(i) = r;     % assign rank to i
          c = c + 1;
          P = P − {i};
       END
    END
   r=r+1;
END
Output: rank values of n individuals
```

Fig. 5. Rank value assignment process.

D. Fuzzy Dominance NSGA-II

In the original NSGA-II [8], two individuals are compared based on their rank values and crowding-distances. The Pareto dominance relation determines the rank value of each individual. In the proposed improved design, we will use the fuzzy fitness assignment method instead, which is based on the fuzzy Pareto dominance relation to assign each individual a rank value. After the rank value is determined, the same crowding-distance is used as the original design of NSGA-II. Specifically, the rank value assignment process based on the fuzzy dominance works as follows. For every individual, if its FFM is greater than a predefined threshold, α, then place it in the first non-dominated rank. After assigning rank one, remove all individuals in the first rank and consider the remaining individuals. Do a recalculation of the fuzzy fitness measure for each of the remaining individuals. If the recalculated FFM of an individual is greater than the same predefined threshold, α, then place it in the second rank. Continue with the above process until all individuals have rank values assigned. In this process, all FFMs are generated by the fuzzy fitness assignment process outlined in Fig. 3 and the predefined threshold is not changed throughout the complete rank value assignment process. This process ensures one individual is fuzzy non-dominated with respect to others in the same rank. After all individuals have rank values assigned, calculate the crowding-distance for each individual. Fig. 5 explains how to assign the rank value to each individual and perform the fuzzy non-dominated sorting. The parameter, α, directly affects the rank value assignment process in the proposed design. The variation of α determines the number of ranks resulted and the proportion of non-dominated individuals in the population. The smaller the value of α, the more individuals will be placed in the first non-dominated front, resulting in fewer ranks. The larger the value of α, the more individuals will be classified into the last front, which makes them non-discriminable. Take the 5-D DTLZ2 as an example. Given an initial population of 200, the proposed fuzzy fitness assignment process will place

TABLE I
RANK VALUE ASSIGNMENT RESULTS IN DTLZ2

α	Number of Ranks			Proportion of Fuzzy Non-dominated Individuals (%)		
	$M = 5$	$M = 10$	$M = 20$	$M = 5$	$M = 10$	$M = 20$
0.1	2	2	2	98	97	96
0.2	3	3	3	86	84	82
0.3	4	4	4	72	71	68
0.4	5	5	5	59	56	57
0.5	8	8	8	48	47	47
0.6	9	9	9	35	39	38
0.7	10	10	10	27	27	29
0.8	10	10	10	16	20	19
0.9	10	10	10	9	9	10

98% of the population into the first non-dominated front, if α is chosen to be 0.1. This is clearly undesirable as these non-dominated individuals will present little selection pressure during the evolutionary process. On the other hand, if α is chosen to be large, say 0.9, this will result in ten ranks and the majority of the population, nearly 70% of the population in this case, will be assigned into the last front (i.e., the 10th front). This is clearly undesirable as they cannot be differentiated by the proposed fuzzy Pareto domination principle. In Table I, where all the results are averaged over 30 independent runs and rounded to whole numbers, the resulting number of ranks and the proportion of individuals placed in the non-dominated front versus the choice of parameter α between 0 and 1 are shown, given the population size of 200. M is referred to as the number of objectives chosen in the scalable benchmark function DTLZ2. As can be observed, the resulting number of ranks and the proportion of individuals placed in the non-dominated front are independent of the number of objectives, M. Similar experiments made on other benchmark functions (i.e., DTLZ1, DTLZ3, DTLZ6, DTLZ7, WFG1, and WFG2) show consistent findings.

1) The smaller the value of α, the more individuals will be assigned in the first non-dominated front, resulting in fewer ranks. The larger the value of α, the more individuals will be grouped into the last front.
2) The resulting number of ranks and the proportion of individuals placed in the non-dominated front are independent of the number of objectives.

As is commonly known, the original Pareto dominance criterion is effective when the number of objectives is less than three. Fig. 1 indicates that the proportion of non-dominated individuals in the initial population is between 40% and 50% when the number of objectives is two. This proportion (40–50%) heuristically chosen is believed to provide sufficient selection pressure during the early stage of the evolutionary process and corresponds to α at 0.5 (for Table I). When a different problem is dealt with, a similar experiment on varying α (from 0.1 to 0.9 in the increment of 0.1) will be made for the given number of objective functions, M. A table similar to Table I will be produced. The α, which corresponds to 40–50% of fuzzy non-dominated individuals in the initial population, will be chosen *ad hoc* to provide the best performances for a given problem at hand. The NSGA-II is modified by adopting

Input: population size; number of generations; fitness function; Gaussian function;
Step1: Initialize population
A random parent population P_0 is created. The population is sorted based on the fuzzy Pareto dominance. Each solution is assigned a rank equal to its fuzzy-domination level where 1 is the best level. Binary tournament selection, SBX recombination, and polynomial mutation operators are used to create a child population Q_0 of the same size N.Set $t = 0$
Step2: $R_t = P_t \cup Q_t$
Combine parent and children population.
The population R_t will have size $2N$.
Step3: F=fuzzy-nondominated-sort (R_t) until $|P_{t+1}| < N$. Population R_t is sorted based on the non-domination sorting. The new parent population P_{t+1} is formed by adding solutions from the first front to the next best front before the size exceeds N.
Step4: Calculate crowding distance-assignment (F_i) and include k-th non-dominated front in the parent population $P_{t+1} = P_{t+1} \cup F_i$
Step5: Sort in descending order (P_{t+1}, $\geq_n$) and choose the first N elements of P_{t+1}:
$P_{t+1} = P_{t+1}[0\colon N]$
Step6: Q_{t+1}=make-new-pop (P_{t+1})
This population of size N is now used for selection, crossover, and mutation to create a new population.

Fig. 6. Fuzzy dominance FD-NSGA-II.

the fuzzy Pareto dominance relations and the corresponding fuzzy fitness assignment process. In order to fairly compare the performance by using the proposed fuzzy Pareto dominance relation and the original Pareto dominance relation, we use the same structure as the original NSGA-II. The only difference is that in the original NSGA-II, the fitness assignment is completed by the Pareto dominance, while in the modified NSGA-II (named FD-NSGA-II), the fuzzy Pareto dominance principle is applied. Fig. 6 explains each step of the modified FD-NSGA-II in detail.

E. Fuzzy Dominance SPEA2

In the proposed design, the fuzzy Pareto dominance relation is incorporated into the original SPEA2 [9]. Two main steps of the original SPEA2, fitness assignment and environment selection, are modified according to the fuzzy Pareto dominance. After the fitness value is assigned and environment selection is completed, the same mating selection and variation operators are applied as the original design of SPEA2. In the original SPEA2, the fitness value of individual $\vec{x}$ is assigned as

$$f(\vec{x}) = R(\vec{x}) + D(\vec{x}) \tag{2}$$

where $R(\vec{x})$ is the sum of strength values of individuals that dominates$\vec{x}$ and $D(\vec{x})$ is the density value of $\vec{x}$. The strength value of one individual represents the number of individuals it dominates and the density value is generated by an adaptation of the kth nearest neighbor method, where the density at any point is a decreasing function of the distance to the kth nearest data point. If $F(\vec{x}) < 1$, $\vec{x}$ is treated as non-dominated individuals and is assigned rank one. In SPEA2 based on the fuzzy Pareto dominance, the fitness value of individual $\vec{x}$ is

Input: population size; archive size $\overline{N}$;
number of generations T;
Step1: Initialize population
Generate an initial population P_0 and create the empty archive (external set) $\overline{P_0} = \emptyset$. Set = 0 .
Step2: Fuzzy fitness assignment:
Calculate the fuzzy fitness measure of each individual $\vec{x}$ in P_t and $\overline{P_t}$: $F(\vec{x}) = s(\vec{x}) - D(\vec{x})$
Step 3: Environmental selection:
Sort individuals in P_t and $\overline{P_t}$ based on their fuzzy fitness values $F(x)$. First $\overline{N}$ individuals in the sorted population are treated as non-dominated individuals. Then, copy all non-dominated individuals to $\overline{P_{t+1}}$.
Step 4: Termination:
If $t > T$ or another stopping criterion is satisfied, then stop.
Step 5: Mating selection:
Perform binary tournament selection with replacement on $\overline{P_{t+1}}$ in order to fill the mating pool.
Step 6: Variation:
Apply recombination and mutation operators to the mating pool and set P_{t+1} to the resulting population. Increment generation counter $t = t + 1$ and go to fuzzy fitness assignment step again.

Fig. 7. Fuzzy dominance FD-SPEA2.

assigned as:

$$F(\vec{x}) = s(\vec{x}) - D(\vec{x}) \quad (3)$$

where $s(\vec{x})$ is generated from the fuzzy fitness assignment process detailed in Fig. 3 and $D(\vec{x})$ is the same as the original SPEA2. We have known that the larger the value of $s(\vec{x})$, the better the convergence of $\vec{x}$. Meanwhile, the smaller the value of $D(x)$, the smaller the density of $\vec{x}$. Therefore, the larger the value of $F(\vec{x})$, the better the performance of $\vec{x}$. As far as the environment selection is concerned, in the original approach there are three different selection ways; each corresponds to one of three situations: the number of non-dominated individuals generated from the fitness assignment process is smaller, equal, or larger than the archive size. However, in many-objective optimization problems, this selection process incurs a huge computational cost. In the proposed design, individuals are directly sorted based on their fuzzy fitness measures and the number of selected individuals is the same as the archive size. This process has a lower computational complexity than the original design. Fig. 7 explains each step of the modified FD-SPEA2 in detail.

IV. Experimental Results

A. Selected MOEAs for Comparison

In the experiment, seven state-of-the-art MOEAs are chosen for comparison. They are the original NSGA-II [8], NSGA-II based on fuzzy Pareto dominance relation (FD-NSGA-II), the original SPEA2 [9], SPEA2 based on fuzzy Pareto dominance relation (FD-SPEA2), MOEA/D [19], NSGA-II with fuzzy-optimality (NSGA-II-FO) [15], and fuzzy-dominance-driven GA (FDD-GA) [17]. MOEA/D is chosen due to its superior performance in many-objective optimization problems [20].

TABLE II
Problem Characteristics of the DTLZ Test Suite

MOP	Multimodal	Bias	Disconnect	Many-to-One Mapping
DTLZ1	Yes	No	No	Yes
DTLZ2	No	No	No	Yes
DTLZ3	Yes	No	No	Yes
DTLZ4	No	Yes	No	Yes
DTLZ5	No	No	Unknown	Yes
DTLZ6	No	Yes	Unknown	Yes
DTLZ7	Yes	No	Yes	No

B. Selected Benchmark Functions

Nine widely used scalable many-objective benchmark problems, DTLZ1, DTLZ2, DTLZ3, DTLZ4, DTLZ5, DTLZ6, and DTLZ7 [2], WFG1, and WFG2 [21], are chosen to evaluate the performance of the MOEAs considered. In this experiment, chosen MOEAs are tested in 5-, 10-, and 20-D objective space of these benchmark problems. The DTLZ test suite contains a variety of problem characteristics that present various degrees of complications for the underlying MOEAs. These problem characteristics are summarized in Table II [21]. Specifically, "multimodal" implies there are many local fronts in the search space; "bias" means the Pareto optimal solutions are non-uniformly distributed along the global Pareto front; "disconnect" refers to the disconnected global Pareto front; and "many-to-one mapping" shows that several different variables in decision space are mapped to the same solutions in objective space. According to [22], DTLZ1, DTLZ3, and DTLZ6 introduce a large number of local Pareto fronts and DTLZ2 presents a spherical Pareto front so as to test MOEA's ability to converge to the global Pareto front. DTLZ4 generates a nonuniform distribution of points along the Pareto front, and therefore it challenges MOEA's ability to maintain a good distribution of solutions. The Pareto front of DTLZ5 is a degenerated hypersurface. DTLZ6 has disconnected Pareto-optimal regions and it will test an algorithm's ability to maintain subpopulations in disconnected portions of the objective space. DTLZ7 is constructed with the constraint surface approach. The Pareto front of DTLZ7 is the intersection of a straight line and a hyperplane. The tested MOEA may find it difficult to converge to the Pareto front and to maintain a good distribution of solutions along it. The WFG test suite [21] shares similar problem characteristics as the DTLZ test suite [2], but allows control via a series of composable transformations. We consider two of the more popular test functions (i.e., WFG1 and WFG2) often seen in literature. From [21], the problem characteristics of these two test functions, WFG1 and WFG2, are with high-dimensional objective space and multiple global optima.

C. Selected Performance Metrics

In this experiment, two performance metrics are chosen to quantify the performance. Inverted Generational Distance (IGD) [23] considers both convergence and diversity at the same time. Spacing [24] measures the distribution of individuals in the Pareto front. The fewer the IGD and Spacing values, the better is the algorithm's performance. For the IGD metric, we randomly generate a large number of reference points, which is 100 000 samples in all benchmark problems, as suggested in [25] and [26] to maintain a fair comparison throughout the simulation study. In order to provide statistical quantifications on both performance metrics (i.e., IGD and Spacing), we apply the Mann–Whitney–Wilcoxon rank-sum test [27] to quantify whether one of two approximation fronts by independent observations tends to have better performance than the other in a statistical meaningful sense, especially when the performance metric values of two approximation fronts are very close to each other or even in differentiable.

D. Parameter Setting in Experiment

The population size in all seven MOEAs is set to be 200 for all test instances. The stopping criterion is set at 200 generations. Initial populations are generated by uniform random sampling from the search space in all MOEAs considered. The simulated binary crossover (SBX) and polynomial mutation are used. The crossover operator generates one offspring, which is then modified by the mutation operator. Following the practice in [8], the distribution indexes in SBX and the polynomial mutation are set to be 20. The crossover rate is 1.00 (to be consistent with what were used in NSGA-II [8] and MOEA/D [19]), while the mutation rate is $1/m$ and m is the number of decision variables. As suggested in [2], for DTLZ1, m is chosen to be 9 and 14 for 5- and 10-D problems, respectively, while for DTLZ2-7, m is chosen to be 14 and 19 for 5- and 10-D problems, respectively. From [21], m is set to be 24 for all WFG test functions in both 5- and 10-D problems.

E. Experimental Results

Detailed comparison results are presented in the following. Here, let X1, X2, X3, X4, X5, X6, and X7 denote FD-NSGA-II, FD-SPEA2, MOEA/D, NSGA-II, SPEA2, NSGA-II-FO, and FDD-GA, respectively. Tables III–XI present the performance results on metrics IGD and Spacing, given mean values and standard deviations (in brackets) for comparing the seven chosen MOEAs in all nine benchmark problems with 5-, 10-, and 20-D objectives. These results (mean and standard deviation) are averaged over 30 independent trials with the same experimental setup.

A non-parametric statistical hypothesis test, the Mann–Whitney–Wilcoxon rank-sum test, is applied on all benchmark functions with 5-, 10-, and 20-objectives, comparing FD-NSGA-II (X1) and FD-SPEA2 (X2) with respect to each other chosen MOEAs (i.e., X3 for MOEA/D, X4 for NSGA-II, X5 for SPEA2, X6 for NSGA-II-FO, and X7 for FDD-GA) over 30 independent runs. The results are shown in Tables XII–XX. From the performance metrics results by all chosen MOEAs, we test the hypothesis that there is no significant difference between IGD and Spacing values from X1 to X3–X7 and from X2 to X3–X7. In each of these tables, H_0 represents the null hypothesis. The p-value is the probability that the null hypothesis is true, which is computed using an inferential statistical testing. If a p-value is lower than the chosen significance level, it implies that the null hypothesis can be rejected in favor of an alternative test H_1 [28]. This significance level is user-defined and we choose it at 5% in this study. For DTLZ1, which possesses a linear Pareto front and a large number of local Pareto fronts, both FD-NSGA-II and FD-SPEA2 obtain better IGD scores than the other five competing MOEAs according to Table III. Although the Spacing score of MOEA/D is slightly better than those of FD-NSGA-II and FD-SPEA2 for the 5- and 10-D problems, the Mann–Whitney–Wilcoxon rank-sum test results given in

TABLE III
PERFORMANCE ON IGD AND SPACING METRICS FOR DTLZ1

MOP	MOEA	IGD	Spacing
DTLZ1 (5-D)	X1	3.3281(0.5153)	4.2104(1.3291)
	X2	3.4285(0.4408)	3.9958(1.2560)
	X3	3.6313(0.4017)	3.8880(1.7292)
	X4	3.7522(1.0616)	5.3872(0.4314)
	X5	3.7293(1.2132)	5.2032(0.2263)
	X6	3.5596(0.4209)	5.4901(0.2195)
	X7	3.6310(0.5982)	5.3991(1.5267)
DTLZ1 (10-D)	X1	3.3307(0.3998)	14.4632(2.8026)
	X2	3.2182(0.3017)	15.1500 (3.5709)
	X3	3.5263(0.6461)	11.1856(10.1242)
	X4	3.9157(0.9372)	16.3901(1.4234)
	X5	3.8201(0.7451)	17.4592 (2.1953)
	X6	3.3726(0.4902)	19.0912(6.4821)
	X7	3.6506(0.7930)	18.7800(6.5913)
DTLZ1 (20-D)	X1	3.3619(0.4690)	26.9580(4.9202)
	X2	3.5183(0.5892)	27.1127(3.7805)
	X3	3.5322(0.3044)	27.5756(4.4583)
	X4	3.9200(1.1302)	28.2073(3.3891)
	X5	3.8921(1.0920)	27.1173(2.7391)
	X6	3.8724(0.9342)	29.1023(7.1026)
	X7	4.1547(0.6129)	28.1903(8.4045)

TABLE IV
PERFORMANCE ON IGD AND SPACING METRICS FOR DTLZ2

MOP	MOEA	IGD	Spacing
DTLZ2 (5-D)	X1	0.0041(0.0019)	0.2148(0.0198)
	X2	0.0066(0.0003)	0.2283(0.0412)
	X3	0.0088(0.0012)	0.2164(0.1040)
	X4	0.0072(0.0033)	0.4642(0.0453)
	X5	0.0097(0.0073)	0.3129(0.0003)
	X6	0.0150(0.0081)	0.2388(0.0763)
	X7	0.0124(0.0033)	0.3269(0.1301)
DTLZ2 (10-D)	X1	0.0068(0.0017)	0.2295(0.0727)
	X2	0.0074(0.0025)	0.2371(0.0507)
	X3	0.0091(0.0056)	0.2399(0.0863)
	X4	0.0109(0.0029)	0.4274(0.1892)
	X5	0.0128(0.0016)	0.3321(0.0027)
	X6	0.0141(0.0053)	0.3991(0.0792)
	X7	0.0140(0.0019)	0.5294(0.1095)
DTLZ2 (20-D)	X1	0.0066(0.0016)	0.1552(0.0214)
	X2	0.0098(0.0068)	0.1719(0.0098)
	X3	0.0104(0.0027)	0.1713(0.0504)
	X4	0.0142(0.0038)	0.4235(0.2001)
	X5	0.0129(0.0040)	0.3015(0.0009)
	X6	0.0119(0.0029)	0.3201(0.0438)
	X7	0.0157(0.0066)	0.2294(0.1102)

TABLE V
PERFORMANCE ON IGD AND SPACING METRICS FOR DTLZ3

MOP	MOEA	IGD	Spacing
DTLZ3 (5-D)	X1	0.7233(0.0512)	26.9898(5.2886)
	X2	0.7513(0.1521)	21.5531(4.3912)
	X3	0.7730(0.1973)	23.9980(9.4332)
	X4	0.7594(0.1290)	29.5120(5.0396)
	X5	0.7948(0.0002)	31.1043(1.1342)
	X6	0.9619(0.3907)	32.0944(6.7126)
	X7	0.7962(0.0018)	29.0435(5.1042)
DTLZ3 (10-D)	X1	0.7346(0.0547)	27.1094(3.1735)
	X2	0.7728(0.1944)	25.0926(4.1569)
	X3	0.7990(0.1496)	24.0926(7.2763)
	X4	0.9180(0.1764)	28.7201(3.2093)
	X5	0.7951(0.0003)	30.0832(1.0932)
	X6	1.0141(0.4780)	27.1450(5.9823)
	X7	0.7987(0.0009)	28.0139(4.1703)
DTLZ3 (20-D)	X1	0.6834(0.0571)	15.2105(3.4821)
	X2	0.7235(0.1425)	15.1472(2.4052)
	X3	0.7262(0.0449)	14.3092(5.1035)
	X4	0.9257(0.1536)	16.4092(3.0213)
	X5	0.7953(0.0000)	18.1094(2.0003)
	X6	0.7957(0.0007)	16.8617(3.0256)
	X7	0.7969(0.0021)	18.9802(3.0129)

TABLE VI
PERFORMANCE ON IGD AND SPACING METRICS FOR DTLZ4

MOP	MOEA	IGD	Spacing
DTLZ4 (5-D)	X1	0.0073(0.0018)	0.0463(0.0032)
	X2	0.0068(0.0020)	0.0377(0.0194)
	X3	0.0105(0.0019)	0.0484(0.0089)
	X4	0.0124(0.0000)	0.0764(0.0129)
	X5	0.0123(0.0004)	0.1092(0.0023)
	X6	0.0129(0.0015)	0.1618(0.1011)
	X7	0.0153(0.0039)	0.0414(0.0193)
DTLZ4 (10-D)	X1	0.0037(0.0004)	0.0226(0.0014)
	X2	0.0046(0.0086)	0.0164(0.0101)
	X3	0.0050(0.0004)	0.0196(0.0117)
	X4	0.0054(0.0005)	0.0429(0.0079)
	X5	0.0056(0.0006)	0.0228(0.0037)
	X6	0.0083(0.0054)	0.0616(0.0191)
	X7	0.0067(0.0022)	0.0162(0.0177)
DTLZ4 (20-D)	X1	0.0377(0.0109)	0.3901(0.1027)
	X2	0.0377(0.0093)	0.4070(0.1022)
	X3	0.0478(0.0029)	0.5064(0.1429)
	X4	0.6767(1.3038)	0.5654(0.2011)
	X5	0.0583(0.0032)	0.4174(0.1217)
	X6	0.0599(0.0050)	0.4144(0.0935)
	X7	0.0682(0.0247)	0.3860(0.1937)

TABLE VII
PERFORMANCE ON IGD AND SPACING METRICS FOR DTLZ5

MOP	MOEA	IGD	Spacing
DTLZ5 (5-D)	X1	0.0018(0.0009)	0.0875(0.0102)
	X2	0.0063(0.0011)	0.0964(0.0310)
	X3	0.0099(0.0011)	0.0912(0.0412)
	X4	0.0072(0.0019)	0.1098(0.0219)
	X5	0.0168(0.0004)	0.1429(0.0125)
	X6	0.0148(0.0033)	0.1172(0.0304)
	X7	0.0146(0.0035)	0.1134(0.0125)
DTLZ5 (10-D)	X1	0.0035(0.0025)	0.0822(0.0055)
	X2	0.0077(0.0010)	0.0725(0.0103)
	X3	0.0109(0.0021)	0.0859(0.0205)
	X4	0.0107(0.0024)	0.1299(0.0174)
	X5	0.0183(0.0000)	0.1584(0.0183)
	X6	0.0175(0.0015)	0.0963(0.0391)
	X7	0.0162(0.0036)	0.1703(0.0298)
DTLZ5 (20-D)	X1	0.0058(0.0020)	0.1029(0.0381)
	X2	0.0087(0.0013)	0.1124(0.0127)
	X3	0.0109(0.0025)	0.1227(0.0424)
	X4	0.0124(0.0009)	0.1278(0.0482)
	X5	0.0186(0.0000)	0.1279(0.0373)
	X6	0.0095(0.0021)	0.1662(0.0501)
	X7	0.0121(0.0061)	0.1384(0.0624)

TABLE VIII
PERFORMANCE ON IGD AND SPACING METRICS FOR DTLZ6

MOP	MOEA	IGD	Spacing
DTLZ6 (5-D)	X1	0.0164(0.0052)	2.0684(0.2443)
	X2	0.0100(0.0027)	2.1610(0.3662)
	X3	0.0188(0.0136)	2.1603(0.9433)
	X4	0.0228(0.0051)	2.6906(0.6276)
	X5	0.0377(0.0031)	3.0923(0.9871)
	X6	0.0375(0.0029)	2.2404(2.9707)
	X7	0.0290(0.0140)	2.1241(0.8573)
DTLZ6 (10-D)	X1	0.0132(0.0046)	1.9552(0.5742)
	X2	0.0096(0.0028)	2.2845(0.6278)
	X3	0.0379(0.0013)	2.4365(0.0664)
	X4	0.0353(0.0044)	3.1348(0.9717)
	X5	0.0421(0.0000)	3.2300(0.0000)
	X6	0.0382(0.0087)	2.5782(2.2036)
	X7	0.0337(0.0145)	2.7950(1.3051)
DTLZ6 (20-D)	X1	0.0085(0.0024)	2.2857(0.4717)
	X2	0.0071(0.0014)	2.3579(0.3730)
	X3	0.0369(0.0007)	2.3920(0.0304)
	X4	0.0367(0.0004)	3.2753(0.5845)
	X5	0.0420(0.0010)	3.4359(0.0213)
	X6	0.0398(0.0035)	3.5489(2.5810)
	X7	0.0402(0.0042)	2.4510(0.6652)

Table XII show that there is no appreciable difference among the Spacing values of these three MOEAs for the 5- and 20-D DTLZ1, while both FD-NSGA-II and FD-SPEA2 are better than MOEA/D in the 10-D DTLZ1. This implies that MOEA/D cannot be guaranteed to generate a better distributed approximated front than FD-NSGA-II and FD-SPEA2 in the statistical sense according to the quality measures used. Also, results in Table XII confirm the findings in Table III that FD-NSGA-II and FD-SPEA2 attain better performance on IGD scores than the other five algorithms except for the 20-D DTLZ1, in which the performances of FD-SPEA2 and MOEA/D are indifferent.

For DTLZ2, which has a spherical Pareto-optimal front, FD-NSGA-II and FD-SPEA2 show better performance than the other five algorithms on both IGD and Spacing metrics in most conditions as shown in Table IV, except FD-SPEA2 performs slightly worse than MOEA/D on Spacing in the 5- and 20-D problems. However, the Mann–Whitney–Wilcoxon rank-sum

TABLE IX
PERFORMANCE ON IGD AND SPACING METRICS FOR DTLZ7

MOP	MOEA	IGD	Spacing
DTLZ7 (5-D)	X1	0.0887(0.0063)	0.0188(0.0080)
	X2	0.1035(0.0209)	0.0751(0.0306)
	X3	0.1044(0.0172)	1.9310(0.0271)
	X4	0.1061(0.0318)	0.3779(0.0327)
	X5	0.1208(0.0148)	0.2021(0.0123)
	X6	0.1234(0.0531)	1.9142(1.7280)
	X7	0.1254(0.0531)	1.2918(1.4145)
DTLZ7 (10-D)	X1	0.2161(0.0175)	0.0470(0.0277)
	X2	0.2178(0.0233)	0.2680(0.0850)
	X3	0.2236(0.1025)	2.2467(2.1536)
	X4	0.2882(0.0187)	0.9042(0.0821)
	X5	0.2615(0.0203)	1.0014(0.0035)
	X6	0.2852(0.1069)	2.8118(2.8106)
	X7	0.2817(0.1124)	2.9495(1.0605)
DTLZ7 (20-D)	X1	0.1079(0.0011)	0.0320(0.0167)
	X2	0.1064(0.0012)	0.2322(0.0413)
	X3	0.1101(0.0051)	0.7805(0.6089)
	X4	0.1143(0.0004)	0.7791(0.1168)
	X5	0.1160(0.0005)	0.4025(0.0195)
	X6	0.1136(0.0039)	0.8441(0.4113)
	X7	0.1162(0.0005)	0.5001(0.1596)

TABLE X
PERFORMANCE ON IGD AND SPACING METRICS FOR WFG1

MOP	MOEA	IGD	Spacing
WFG1 (5-D)	X1	0.0033(0.0001)	0.0186(0.0067)
	X2	0.0064(0.0128)	0.1109(0.1721)
	X3	0.0102(0.0086)	0.1182(0.1679)
	X4	0.0074(0.0001)	0.3423(0.2546)
	X5	0.0081(0.0001)	0.2901(0.0010)
	X6	0.0079(0.0003)	0.1459(0.0751)
	X7	0.0083(0.0005)	0.1205(0.1789)
WFG1 (10-D)	X1	0.0154(0.0021)	0.1434(0.0076)
	X2	0.0153(0.0084)	0.2911(0.0789)
	X3	0.0173(0.0002)	0.0514(0.0179)
	X4	0.0155(0.0023)	0.4849(0.0852)
	X5	0.0166(0.0005)	0.2021(0.0130)
	X6	0.0162(0.0007)	0.3765(0.3465)
	X7	0.0170(0.0004)	1.2852(1.7167)
WFG1 (20-D)	X1	0.0076(0.0005)	1.1567(1.4540)
	X2	0.0304(0.0003)	1.0686(1.4233)
	X3	0.0301(0.0006)	1.2166(0.1917)
	X4	0.0331(0.0011)	2.2047(0.2174)
	X5	0.0360(0.0006)	1.3509(0.0021)
	X6	0.0342(0.0045)	1.2029(0.4092)
	X7	0.0477(0.0022)	1.7845(0.3024)

TABLE XI
PERFORMANCE ON IGD AND SPACING METRICS FOR WFG2

MOP	MOEA	IGD	Spacing
WFG2 (5-D)	X1	0.0014(0.0001)	0.4368(0.0569)
	X2	0.0126(0.0096)	0.1661(0.1962)
	X3	0.0030(0.0007)	0.4370(0.0711)
	X4	0.0031(0.0007)	0.4430(0.0664)
	X5	0.0030(0.0006)	0.4846(0.0700)
	X6	0.0035(0.0010)	0.5346(0.0893)
	X7	0.0062(0.0053)	0.8673(0.6500)
WFG2 (10-D)	X1	0.0033(0.0012)	1.2007(0.1450)
	X2	0.0080(0.0016)	0.9931(0.1798)
	X3	0.0086(0.0024)	1.5202(0.1098)
	X4	0.0099(0.0024)	1.0346(0.0947)
	X5	0.0095(0.0027)	1.3074(0.1408)
	X6	0.0078(0.0055)	1.1975(0.1674)
	X7	0.0162(0.0143)	1.4234(0.9366)
WFG2 (20-D)	X1	0.0052(0.0035)	1.7351(0.3006)
	X2	0.0125(0.0054)	1.2833(0.2092)
	X3	0.0133(0.0011)	1.6847(0.5317)
	X4	0.0189(0.0049)	2.3179(0.6803)
	X5	0.0170(0.0087)	1.8932(0.4769)
	X6	0.0192(0.0069)	1.9258(0.7025)
	X7	0.0201(0.0072)	2.0015(0.4501)

test results in Table XIII show that both have no noticeable difference on Spacing scores for the 20-D DTLZ2 and FD-SPEA2 is better than MOEA/D in the 5-D DTLZ2.

Meanwhile, Table XIII shows that there is no significant difference in IGD value between FD-SPEA2 and MOEA/D in the statistical sense. Also, for IGD, FD-SPEA2 performs equally well as NSGA-II and SPEA2 for the 5-D DTLZ2, while FD-NSGA-II performs equally well in IGD as MOEA/D for the 10-D DTLZ2. Therefore, combining findings in Tables IV and XIII, we can draw the conclusion that both FD-NSGA-II and FD-SPEA2 perform competitively with respect to MOEA/D for the 10- and 20-D DTLZ2, while they perform better than others (i.e., NSGA-II, SPEA2, NSGA-II-FO, and FDD-GA) in nearly all dimensions. For DTLZ3, which introduces many local Pareto-optimal fronts, FD-NSGA-II and FD-SPEA2 have better IGD scores in Table V but hypothesis test results in Table XIV indicate that there is no significant difference between FD-NSGA-II and MOEA/D for the 5- and 20-D problems. On the other hand, although FD-NSGA-II and FD-SPEA2 achieve worse Spacing scores than MOEA/D from Table V, hypothesis test results in Table XIV assure they perform equally well. Therefore, we can say that FD-SPEA2 performs no worse than the five chosen state-of-the-art MOEAs and FD-NSGA-II performs as well as MOEA/D and a little bit better than the remaining four chosen MOEAs with respect to both performance metrics.

For DTLZ4, which generates a nonuniform distribution of individuals along the Pareto front, both FD-NSGA-II and FD-SPEA2 according to Table VI offer a better IGD and Spacing scores in most cases except for the 10-D problem, in which FD-NSGA-II's Spacing value is slightly worse than MOEA/D. However, statistical results in Table XV show there is no considerable difference among FD-NSGA-II, FD-SPEA2, MOEA/D, SPEA2, and FDD-GA on the Spacing metric. Also, for the 10-D DTLZ4, FD-SPEA2 has the same performance as others on IGD. Therefore, for DTLZ4, both FD-NSGA-II and FD-SPEA2 present significantly better IGD scores than others, while they are no worse than others for the Spacing metric. For DTLZ5, whose Pareto front is a degenerated hypersurface, both FD-NSGA-II and FD-SPEA2 perform much better than all the other chosen MOEAs in the

TABLE XII

RESULTS OF MANN–WHITNEY–WILCOXON RANK-SUM TEST IN DTLZ1

		$5-\mathrm{D}:\mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi} \quad VSH_1: \mu_{X1}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
MOEA	IGD			Spacing		
	H	*p-Value*	Winner	H	*p-Value*	Winner
		$5-\mathrm{D}:\mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi} \quad VSH_1: \mu_{X1}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
X3	1	0.0003	X1	0	0.9012	None
X4	1	3.0129E–04	X1	1	2.4721E–04	X1
X5	1	3.0091E–04	X1	1	1.9012E–04	X1
X6	1	6.0712E–04	X1	1	1.3279E–04	X1
X7	1	0.0011	X1	1	5.0925E–04	X1
		$5-\mathrm{D}:\mathrm{X2}: H_0: \mu_{X2}=\mu_{Xi} \quad VSH_1: \mu_{X2}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
X3	1	7.0129E–04	X2	0	0.3981	None
X4	1	0.0039	X2	1	1.8267E–04	X2
X5	1	4.7098E–04	X2	1	2.3671E–04	X2
X6	1	2.3569E–04	X2	1	1.0092E–04	X2
X7	1	0.0001	X2	1	0.5708	X2
		$10-\mathrm{D}:\mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi} \quad VSH_1: \mu_{X1}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
X3	1	7.2901E–04	X1	1	0.0015	X1
X4	1	8.0134E–04	X1	1	1.5982E–04	X1
X5	1	3.1584E–04	X1	1	2.0186E–04	X1
X6	1	0.0299	X1	1	0.0233	X1
X7	1	1.9901E–04	X1	1	1.0923E–04	X1
		$10-\mathrm{D}:\mathrm{X2}: H_0: \mu_{X2}=\mu_{Xi} \quad VSH_1: \mu_{X2}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
X3	1	9.2091E–04	X2	1	3.1078E–04	X2
X4	1	5.6231E–04	X2	1	0.0155	X2
X5	1	4.8513E–04	X2	1	2.1091E–04	X2
X6	1	1.4309E–04	X2	1	1.0012E–04	X2
X7	1	8.3800E–04	X2	1	5.9807E–04	X2
		$20-\mathrm{D}:\mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi} \quad VSH_1: \mu_{X1}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
X3	1	1.0012E–04	X1	0	0.7825	None
X4	1	1.9904E–04	X1	1	2.6712E–04	X1
X5	1	3.6701E–04	X1	0	0.4041	None
X6	1	9.0127E–04	X1	1	3.0921E–04	X1
X7	1	0.0004	X1	1	4.1267E–04	X1
		$20-\mathrm{D}:\mathrm{X2}: H_0: \mu_{X2}=\mu_{Xi} \quad VSH_1: \mu_{X2}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
X3	0	0.3566	None	0	0.9001	None
X4	1	0.0096	X2	1	1.8267E–04	X2
X5	1	0.0211	X2	0	0.4595	None
X6	1	3.6709E–04	X2	1	4.3902E–04	X2
X7	1	0.0102	X2	1	5.3087E–04	X2

TABLE XIII

RESULTS OF MANN–WHITNEY–WILCOXON RANK-SUM TEST IN DTLZ2

		$5-\mathrm{D}:\mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi} \quad VSH_1: \mu_{X1}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
MOEA	IGD			Spacing		
	H	*p-Value*	Winner	H	*p-Value*	Winner
		$5-\mathrm{D}:\mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi} \quad VSH_1: \mu_{X1}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
X3	1	2.6501E–04	X1	1	0.0033	X1
X4	1	7.3098E–04	X1	1	3.9417E–04	X1
X5	1	5.4781E–04	X1	1	1.1309E–04	X1
X6	1	3.4201E–04	X1	1	3.0912E–04	X1
X7	1	1.8267E–04	X1	1	2.0913E–04	X1
		$5-\mathrm{D}:\mathrm{X2}: H_0: \mu_{X2}=\mu_{Xi} \quad VSH_1: \mu_{X2}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
X3	0	0.5024	None	1	4.3277E–04	X2
X4	0	0.8102	None	1	1.8267E–04	X2
X5	0	0.4098	None	1	1.8267E–04	X2
X6	1	0.0012	X2	1	1.4501E–04	X2
X7	1	0.0257	X2	1	0.0091	X2
		$10-\mathrm{D}:\mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi} \quad VSH_1: \mu_{X1}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
X3	0	0.5817	None	0	0.2902	None
X4	1	5.0982E–04	X1	1	4.3091E–04	X1
X5	1	7.8493E–04	X1	1	1.7399E–04	X1
X6	1	4.0912E–04	X1	1	1.0017E–04	X1
X7	1	0.0016	X1	1	0.0199	X1
		$10-\mathrm{D}:\mathrm{X2}: H_0: \mu_{X2}=\mu_{Xi} \quad VSH_1: \mu_{X2}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
X3	0	0.4552	None	0	0.3092	None
X4	1	0.0031	X2	1	1.8267E–04	X2
X5	1	0.0036	X2	1	2.1237E–04	X2
X6	1	4.1435E–04	X2	1	3.0911E–04	X2
X7	1	2.3571E–04	X2	1	0.0019	X2
		$20-\mathrm{D}:\mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi} \quad VSH_1: \mu_{X1}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
X3	1	0.0026	X1	1	0.0391	X1
X4	1	4.1604E–04	X1	1	2.9260E–04	X1
X5	1	3.6891E–04	X1	1	0.0014	X1
X6	1	5.0124E–04	X1	1	3.0785E–04	X1
X7	1	1.6211E–04	X1	1	7.2019E–04	X1
		$20-\mathrm{D}:\mathrm{X2}: H_0: \mu_{X2}=\mu_{Xi} \quad VSH_1: \mu_{X2}\neq\mu_{Xi} \quad Xi\in\{X3, X4, X5, X6, X7\}$				
X3	0	0.4317	None	0	0.9012	None
X4	1	9.5601E–04	X2	1	3.2907E–04	X2
X5	1	0.0079	X2	1	4.3091E–04	X2
X6	1	8.1367E–04	X2	1	0.0004	X2
X7	1	2.8182E–04	X2	1	1.9101E–04	X2

TABLE XIV

RESULTS OF MANN−WHITNEY−WILCOXON RANK-SUM TEST IN DTLZ3

	$5-\mathrm{D}: \mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi}$	$VSH_1: \mu_{X1}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
MOEA	IGD			Spacing		
	H	*p-Value*	Winner	H	*p-Value*	Winner
	$5-\mathrm{D}: \mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi}$	$VSH_1: \mu_{X1}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
X3	0	0.4026	None	0	0.9698	None
X4	1	6.1431E–04	X1	1	1.8267E–04	X1
X5	1	5.4391E–04	X1	1	1.4973E–04	X1
X6	1	4.3722E–04	X1	1	5.2091E–04	X1
X7	1	1.6211E–04	X1	1	3.7649E–04	X1
	$5-\mathrm{D}: \mathrm{X2}: H_0: \mu_{X2}=\mu_{Xi}$	$VSH_1: \mu_{X2}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
X3	0	0.6702	None	1	6.3092E–04	X2
X4	0	0.5054	None	1	3.2871E–04	X2
X5	0	0.4902	None	1	4.3091E–04	X2
X6	0	0.9802	None	1	0.0093	X2
X7	0	0.7935	None	1	0.0332	X2
	$10-\mathrm{D}: \mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi}$	$VSH_1: \mu_{X1}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
X3	1	0.0443	X1	0	0.4727	None
X4	1	8.3800E–04	X1	1	1.8267E–04	X1
X5	1	5.1456E–04	X1	1	1.9012E–04	X1
X6	1	3.1584E–04	X1	0	1	None
X7	1	1.8267E–04	X1	1	4.3964E–04	X1
	$10-\mathrm{D}: \mathrm{X2}: H_0: \mu_{X2}=\mu_{Xi}$	$VSH_1: \mu_{X2}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
X3	0	0.3012	None	0	0.4727	None
X4	0	0.7906	None	1	1.8267E–04	X2
X5	1	0.0036	X2	1	1.8267E–04	X2
X6	0	0.4435	None	0	1	None
X7	1	0.0257	X2	0	0.5708	None
	$20-\mathrm{D}: \mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi}$	$VSH_1: \mu_{X1}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
X3	0	0.4026	None	0	0.9698	None
X4	1	4.1604E–04	X1	1	1.8267E–04	X1
X5	1	6.3403E–04	X1	1	1.8267E–04	X1
X6	1	1.2777E–04	X1	0	0.1405	None
X7	1	1.6211E–04	X1	1	3.2984E–04	X1
	$20-\mathrm{D}: \mathrm{X2}: H_0: \mu_{X2}=\mu_{Xi}$	$VSH_1: \mu_{X2}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
X3	0	0.1634	None	0	0.5205	None
X4	0	0.2395	None	1	1.8267E–04	X2
X5	0	0.7096	None	1	1.8267E–04	X2
X6	0	0.8145	None	0	0.1405	None
X7	0	0.8182	None	1	0.0091	X2

TABLE XV

RESULTS OF MANN−WHITNEY−WILCOXON RANK-SUM TEST IN DTLZ4

	$5-\mathrm{D}: \mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi}$	$VSH_1: \mu_{X1}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
MOEA	IGD			Spacing		
	H	*p-Value*	Winner	H	*p-Value*	Winner
	$5-\mathrm{D}: \mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi}$	$VSH_1: \mu_{X1}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
X3	1	0.0017	X1	0	0.3298	None
X4	1	3.0127E–04	X1	1	0.9801E–04	X1
X5	1	1.9802E–04	X1	1	1.9012E–04	X1
X6	1	2.6704E–04	X1	1	3.4309E–04	X1
X7	1	1.8205E–04	X1	0	0.4209	None
	$5-\mathrm{D}: \mathrm{X2}: H_0: \mu_{X2}=\mu_{Xi}$	$VSH_1: \mu_{X2}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
X3	1	0.0022	X2	1	1.8267E–04	X2
X4	1	1.8267E–04	X2	1	3.2984E–04	X2
X5	1	1.8267E–04	X2	1	2.4613E–04	X2
X6	1	1.7265E–04	X2	1	2.4613E–04	X2
X7	1	1.8165E–04	X2	0	0.1620	None
	$10-\mathrm{D}: \mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi}$	$VSH_1: \mu_{X1}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
X3	1	0.0173	X1	0	0.8267	None
X4	1	1.7265E–04	X1	1	1.8267E–04	X1
X5	1	1.8165E–04	X1	0	0.0539	None
X6	1	1.7265E–04	X1	1	0.0211	X1
X7	1	1.7861E–04	X1	0	0.1620	None
	$10-\mathrm{D}: \mathrm{X2}: H_0: \mu_{X2}=\mu_{Xi}$	$VSH_1: \mu_{X2}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
X3	0	0.1829	None	1	0.0034	X2
X4	0	0.5044	None	1	2.4789E–04	X2
X5	0	0.3010	None	0	0.7913	None
X6	0	0.4710	None	1	0.0257	X2
X7	0	0.5702	None	0	0.1405	None
	$20-\mathrm{D}: \mathrm{X1}: H_0: \mu_{X1}=\mu_{Xi}$	$VSH_1: \mu_{X1}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
X3	1	0.0039	X1	1	1.8267E–04	X1
X4	1	1.6305E–04	X1	1	1.8267E–04	X1
X5	1	1.8267E–04	X1	0	0.1859	None
X6	1	1.8063E–04	X1	1	0.0376	X1
X7	1	3.2984E–04	X1	0	0.9097	None
	$20-\mathrm{D}: \mathrm{X2}: H_0: \mu_{X2}=\mu_{Xi}$	$VSH_1: \mu_{X2}\neq\mu_{Xi}$	$Xi\in\{X3, X4, X5, X6, X7\}$			
X3	1	0.0036	X2	1	1.8267E–04	X2
X4	1	1.6305E–04	X2	1	1.8267E–04	X2
X5	1	1.8267E–04	X2	0	0.7913	None
X6	1	1.8063E–04	X2	0	0.2413	None
X7	1	2.4613E–04	X2	0	0.3075	None

TABLE XVI

RESULTS OF MANN–WHITNEY–WILCOXON RANK-SUM TEST IN DTLZ5

MOEA	IGD			Spacing		
	H	*p-Value*	Winner	H	*p-Value*	Winner
$5-\text{D}:\text{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	0.0021	X1	1	3.2901E–04	X1
X4	1	7.8012E–04	X1	1	2.4041E–04	X1
X5	1	4.3091E–04	X1	1	1.9012E–04	X1
X6	1	2.6704E–04	X1	1	5.8723E–04	X1
X7	1	1.3901E–04	X1	1	0.0028	X1
$5-\text{D}:\text{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	0.0013	X2	0	0.4791	None
X4	0	0.7891	None	0	0.1859	None
X5	1	1.3173E–04	X2	1	1.8267E–04	X2
X6	1	2.2064E–04	X2	0	0.1405	None
X7	1	2.9692E–04	X2	0	0.8791	None
$10-\text{D}:\text{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	3.2984E–04	X1	0	0.5205	None
X4	1	4.3964E–04	X1	1	1.3276E–04	X1
X5	1	6.3864E–05	X1	1	1.2093E–04	X1
X6	1	1.3173E–04	X1	0	0.1405	None
X7	1	1.3173E–04	X1	1	1.4102E–04	X1
$10-\text{D}:\text{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	0.0036	X2	0	0.2903	None
X4	1	0.0058	X2	1	1.4309E–04	X2
X5	1	6.3864E–05	X2	1	0.0013	X2
X6	1	1.7970E–04	X2	0	0.4257	None
X7	1	1.3173E–04	X2	1	2.0912E–04	X2
$20-\text{D}:\text{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	0.0017	X1	1	7.5713E–04	X1
X4	1	2.4613E–04	X1	1	1.8267E–04	X1
X5	1	6.3864E–04	X1	1	1.8267E–04	X1
X6	1	0.0017	X1	1	2.4613E–04	X1
X7	1	0.0013	X1	1	2.1620E–04	X1
$20-\text{D}:\text{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	0.0075	X2	0	0.2413	None
X4	1	3.2984E–04	X2	1	1.8267E–04	X2
X5	1	6.3864E–04	X2	1	1.8267E–04	X2
X6	1	4.0442E–04	X2	1	1.9762E–04	X2
X7	1	2.1976E–04	X2	1	3.6319E–04	X2

TABLE XVII

RESULTS OF MANN–WHITNEY–WILCOXON RANK-SUM TEST IN DTLZ6

MOEA	IGD			Spacing		
	H	*p-Value*	Winner	H	*p-Value*	Winner
$5-\text{D}:\text{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	0	0.4274	None	0	0.1212	None
X4	1	0.0211	X1	1	0.0452	X1
X5	1	1.8267E–04	X1	1	2.0912E–04	X1
X6	1	2.8123E–04	X1	1	1.4305E–04	X1
X7	1	0.0041	X1	1	1.9023E–04	X1
$5-\text{D}:\text{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	0	0.1705	None	0	0.4044	None
X4	1	2.4613E–04	X2	1	0.0313	X2
X5	1	1.8267E–04	X2	1	1.8267E–04	X2
X6	1	2.4981E–04	X2	1	2.3901E–04	X2
X7	1	0.0113	X2	1	1.3092E–04	X2
$10-\text{D}:\text{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	1.2783E–04	X1	0	0.5205	None
X4	1	1.1067E–04	X1	1	1.7462E–04	X1
X5	1	2.3901E–04	X1	1	1.9801E–04	X1
X6	1	2.5703E–04	X1	1	0.0092	X1
X7	1	0.0034	X1	1	2.3091E–04	X1
$10-\text{D}:\text{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	3.6123E–04	X2	1	0.0122	X2
X4	1	4.0921E–04	X2	1	1.5019E–04	X2
X5	1	1.6723E–05	X2	1	1.3298E–04	X2
X6	1	2.0915E–04	X2	1	4.2098E–04	X2
X7	1	3.0027E–04	X2	1	1.1101E–04	X2
$20-\text{D}:\text{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	1.8267E–04	X1	0	0.6381	None
X4	1	2.7014E–04	X1	1	0.0122	X1
X5	1	8.7450E–04	X1	1	4.3723E–04	X1
X6	1	1.4939E–04	X1	1	0.0211	X1
X7	1	1.3173E–04	X1	0	0.1405	None
$20-\text{D}:\text{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	3.1902E–04	X2	0	0.6019	None
X4	1	4.0126E–04	X2	1	1.0921E–04	X2
X5	1	1.8267E–04	X2	1	1.4703E–04	X2
X6	1	1.0932E–04	X2	1	1.9052E–04	X2
X7	1	2.1620E–04	X2	0	0.1709	None

TABLE XVIII

RESULTS OF MANN−WHITNEY−WILCOXON RANK-SUM TEST IN DTLZ7

		$5-\mathrm{D}:\mathrm{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
MOEA	IGD			Spacing		
	H	*p-Value*	Winner	H	*p-Value*	Winner
		$5-\mathrm{D}:\mathrm{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
X3	1	2.4724E–04	X1	1	1.9230E–04	X1
X4	1	0.0033	X1	1	2.4530E–04	X1
X5	1	0.0067	X1	1	1.3872E–04	X1
X6	1	1.2783E–04	X1	1	3.5289E–04	X1
X7	1	4.1093E–04	X1	1	0.0024	X1
		$5-\mathrm{D}:\mathrm{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
X3	0	0.4032	None	1	0.0023	X2
X4	0	0.5614	None	1	4.3092E–04	X2
X5	1	2.3094E–04	X2	1	1.8267E–04	X2
X6	1	1.3683E–04	X2	1	1.7982E–04	X2
X7	1	1.3498e–04	X2	1	4.9826E–04	X2
		$10-\mathrm{D}:\mathrm{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
X3	0	0.3884	None	1	4.5302E–04	X1
X4	1	0.0035	X1	1	2.1876E–04	X1
X5	1	1.2985E–04	X1	1	3.2085E–04	X1
X6	1	4.3026E–04	X1	1	2.9302E–04	X1
X7	1	1.0354E–04	X1	1	1.0034E–04	X1
		$10-\mathrm{D}:\mathrm{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
X3	0	0.6590	None	1	1.2235E–04	X2
X4	1	2.3704E–04	X2	1	0.0119	X2
X5	1	3.6209E–05	X2	1	2.0984E–04	X2
X6	1	1.0413E–04	X2	1	0.0093	X2
X7	1	3.8099E–04	X2	1	5.2904E–04	X2
		$20-\mathrm{D}:\mathrm{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
X3	1	2.4367E–04	X1	1	3.6894E–04	X1
X4	1	1.0937E–04	X1	1	7.0921E–04	X1
X5	1	0.0056	X1	1	2.3904E–04	X1
X6	1	3.5713E–04	X1	1	2.1136E–04	X1
X7	1	4.9083E–04	X1	1	4.3091E–04	X1
		$20-\mathrm{D}:\mathrm{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
X3	1	2.7094E–04	X2	1	4.6071E–04	X2
X4	1	0.0104	X2	1	1.7324E–04	X2
X5	1	2.3509E–04	X2	1	2.9063E–04	X2
X6	1	1.4327E–04	X2	1	0.0045	X2
X7	1	2.5603E–04	X2	1	2.3079E–04	X2

TABLE XIX

RESULTS OF MANN−WHITNEY−WILCOXON RANK-SUM TEST IN WFG1

		$5-\mathrm{D}:\mathrm{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
MOEA	IGD			Spacing		
	H	*p-Value*	Winner	H	*p-Value*	Winner
		$5-\mathrm{D}:\mathrm{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
X3	1	0.0015	X1	1	1.2109E–04	X1
X4	1	2.1896E–04	X1	1	4.5204E–04	X1
X5	1	1.7861E–04	X1	1	6.0791E–04	X1
X6	1	2.4095E–04	X1	1	2.6093E–04	X1
X7	1	0.0093	X1	1	0.0023	X1
		$5-\mathrm{D}:\mathrm{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
X3	0	0.4952	None	0	0.5957	None
X4	0	0.3075	None	1	0.0073	X2
X5	0	0.1399	None	1	1.3173E–04	X2
X6	0	0.4274	None	0	0.0620	None
X7	0	0.1405	None	0	0.0866	None
		$10-\mathrm{D}:\mathrm{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
X3	1	0.0023	X1	1	1.0092E–04	X3
X4	1	3.2901E–04	X1	1	2.6903E–04	X1
X5	1	4.0925E–05	X1	1	1.1723E–04	X1
X6	1	2.9018E–04	X1	1	4.1072E–04	X1
X7	1	3.6905E–04	X1	1	2.0981E–04	X1
		$10-\mathrm{D}:\mathrm{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
X3	0	0.4037	None	1	2.3691E–04	X3
X4	0	0.2119	None	1	0.0035	X2
X5	0	0.8120	None	1	3.1376E–04	X2
X6	0	0.3902	None	1	4.1092E–04	X2
X7	1	1.0021E–04	X2	1	5.0824E–04	X2
		$20-\mathrm{D}:\mathrm{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
X3	1	1.7733E–04	X1	1	7.5713E–04	X1
X4	1	4.0032E–04	X1	1	5.0147E–04	X1
X5	1	0.0064	X1	1	1.8267E–04	X1
X6	1	2.5031E–04	X1	1	6.0012E–04	X1
X7	1	1.3099E–04	X1	1	0.0026	X1
		$20-\mathrm{D}:\mathrm{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$				
X3	0	0.3075	None	1	2.4313E–04	X2
X4	1	1.5270E–04	X2	1	3.2975E–04	X2
X5	1	2.1091E–04	X2	1	3.0125E–04	X2
X6	1	3.2209E–04	X2	1	1.8572E–04	X2
X7	1	2.4525E–04	X2	1	3.2515E–04	X2

TABLE XX

RESULTS OF MANN–WHITNEY–WILCOXON RANK-SUM TEST IN WFG2

$5-\mathrm{D}:\mathrm{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
MOEA	IGD			Spacing		
	H	*p-Value*	Winner	H	*p-Value*	Winner
$5-\mathrm{D}:\mathrm{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	1.8267E–04	X1	1	0.0113	X1
X4	1	2.4613E–04	X1	0	0.7913	None
X5	1	3.2984E–04	X1	0	0.1620	None
X6	1	4.1762E–04	X1	1	0.0173	X1
X7	1	1.0419E–04	X1	1	1.2023E–04	X1
$5-\mathrm{D}:\mathrm{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	0	0.5823	None	1	0.0140	X2
X4	0	0.6776	None	0	1	None
X5	0	0.8501	None	1	1.6212E–04	X2
X6	0	0.4274	None	1	0.0139	X2
X7	0	0.2005	None	1	0.0192	X2
$10-\mathrm{D}:\mathrm{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	0.0122	X1	1	5.0321E–04	X1
X4	1	1.0092E–04	X1	0	0.4064	None
X5	1	0.0065	X1	1	2.3094E–04	X1
X6	1	0.0134	X1	0	0.5209	None
X7	1	5.2091E–04	X1	1	1.2098E–04	X1
$10-\mathrm{D}:\mathrm{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	3.6702E–04	X2	1	0.0137	X2
X4	1	2.5809E–04	X2	1	3.2091E–04	X2
X5	1	0.0029	X2	1	1.3223E–04	X2
X6	1	4.3598E–04	X2	1	0.0189	X2
X7	1	1.4307E–04	X2	1	4.3982E–04	X2
$20-\mathrm{D}:\mathrm{X1}:H_0:\mu_{X1}=\mu_{Xi}\quad VSH_1:\mu_{X1}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	1	1.3793E–04	X1	1	4.1092E–04	X1
X4	1	2.5099E–04	X1	1	2.1374E–04	X1
X5	1	2.0933E–04	X1	1	1.0095E–04	X1
X6	1	0.0045	X1	0	0.5012	None
X7	1	4.1309E–04	X1	0	0.4822	None
$20-\mathrm{D}:\mathrm{X2}:H_0:\mu_{X2}=\mu_{Xi}\quad VSH_1:\mu_{X2}\neq\mu_{Xi}\quad Xi\in\{X3,X4,X5,X6,X7\}$						
X3	0	0.6351	None	1	1.4592E–04	X2
X4	1	4.2981E–04	X2	1	2.0398E–04	X2
X5	1	5.8705E–04	X2	1	1.8902E–04	X2
X6	1	1.9840E–04	X2	1	1.4703E–04	X2
X7	1	3.7703E–04	X2	1	2.1099E–04	X2

IGD and Spacing metrics as shown in Table VII, except MOEA/D performs slightly better than FD-SPEA2 for the 5-D problem. Hypothesis test results in Table XVI show MOEA/D and FD-SPEA2 have no appreciable difference and FD-SPEA2 performs equally well with respect to nearly all others with respect to Spacing metric.

For DTLZ6, which involves disconnected Pareto-optimal regions and has a large number of local Pareto fronts, both FD-NSGA-II and FD-SPEA2 have a better performance than all other MOEAs in IGD and Spacing as given in Table VIII, except MOEA/D performs a little bit better than FD-SPEA2 for the 5-D problem. The FDD-GA was the second best for the Spacing measure. However, statistical results in Table XVII indicate that both FD-NSGA-II and FD-SPEA2 perform equally well as MOEA/D on Spacing in all dimensions and on IGD for the 5-D DTLZ6 problem.

For DTLZ7, where the Pareto front is the intersection of a straight line and a hyperplane, both FD-NSGA-II and FD-SPEA2 show better performance than all chosen MOEAs on IGD and Spacing metrics, as displayed in Table IX. In Table XVIII, although the IGD score of MOEA/D is as good as that of FD-SPEA2 for both 5- and 10-D problems, its Spacing score is much worse. This implies that MOEA/D cannot guarantee to generate a well distributed approximation front although this front may converge well to the true Pareto front. The hypothesis test results clearly validate the conclusion drawn in this regard.

For WFG1, the global Pareto front is biased and the shape of the global Pareto front is mixed with concave and convex regions. MOEA/D performs better than FD-NSGA-II and FD-SPEA2 on the Spacing metric for the 10-D problem and a little bit better than FD-SPEA2 on the IGD metric for the 20-D problem. The Mann–Whitney–Wilcoxon rank-sum test results given in Table XIX indicate that FD-SPEA2 shares the same performance as others on IGD for the 5- and 10-D problems but is better for the 20-D problem. FD-NSGA-II performs better than others, except that it performs as well as MOEA/D for the 20-D problem. For the Spacing metric, FD-NSGA-II and FD-SPEA2 perform worse than MOEA/D for the 10-D problem. This is by far the only poor performance of the proposed designs (FD-NSGA-II and FD-SPEA2) when compared to the well-regarded MOEA/D. It is worthy of further investigation. For WFG2, the Pareto optimal front consists of several non-contiguous convex parts. Table XI shows that FD-SPEA2 performs worse than all others on IGD for the 5-D problem, but is much better than other MOEAs on Spacing in the same dimension. However, statistical results displayed in Table XX indicate that there is no appreciable difference between FD-SPEA2 and others on IGD for the 5-D problem. From Table XI, FD-NSGA-II is worse than MOEA/D on Spacing for the 20-D problem. However, hypothesis test results in Table XX cannot support such a distinction. From the above analysis, FD-NSGA-II and FD-SPEA2 ensure a better or at least competitive performance compared to five state-of-the-art MOEAs in all nine benchmark problems with respect to both performance metrics from a statistical perspective.

V. Conclusion

Evolutionary algorithms perform poorly to solve MOPs with many objectives, due to loss of selection pressure in the fitness evaluation solely based upon the original Pareto optimality principle. In this paper, we introduced a new fitness evaluation mechanism to continuously differentiate individuals into different degrees of optimality beyond the classification of the original Pareto dominance. The concept of fuzzy logic was adopted to define a fuzzy Pareto domination relation. A fuzzy set based on the left Gaussian function was applied here to quantify the degrees of domination, from dominate to being dominated and in between with various degree of domination in each objective. The set theoretic operator was then used to combine multiple fuzzy sets to allow the comparison of two individuals in many-objective optimization problems. As a case study, the fuzzy concept was incorporated into the designs of NSGA-II and SPEA2. From the experimental results, given a large number of benchmark problems with various problem characteristics and the performance metrics IGD and Spacing, both NSGA-II and SPEA2 based on the fuzzy Pareto dominance relation ensured better performance in both convergence and diversity. The performance improvement gained from adopting the fuzzy Pareto dominance relation is clearly appreciable. Continuing research will be extended to other state-of-the-art MOEAs and to MOEAs designed specifically to handle constrained MaOPs [29], [30] and dynamic MaOPs [31], [32].

References

[1] H. Ishibuchi, N. Tsukamoto, and Y. Nojima, "Evolutionary many-objective optimization: A short review," in *Proc. IEEE Congr. Evol. Comput.*, Hong Kong, China, 2008, pp. 2424–2431.

[2] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scalable multi objective optimization test problems," in *Proc. IEEE Congr. Evol. Comput.*, Honolulu, HI, USA, 2002, pp. 825–830.

[3] D. Brockhoff and E. Zitzler, "Objective reduction in evolutionary multiobjective optimization: Theory and applications," *Evol. Comput.*, vol. 17, no. 2, pp. 135–166, 2009.

[4] K. Deb and D. Saxena, "On finding Pareto-optimal solutions through dimensionality reduction for certain large-dimensional multiobjective optimization problems," Tech. Rep., KanGAL No. 2005011, 2005.

[5] H. Singh, A. Isaacs, and T. Ray, "A Pareto corner search evolutionary algorithm and dimensionality reduction in many-objective optimization problems," *IEEE Trans. Evol. Comput.*, vol. 15, no. 4, pp. 539–556, Aug. 2011.

[6] L. B. Said, S. Bechikh, and K. Ghedira, "The r-dominance: A new dominance relation for interactive evolutionary multicriteria decision making," *IEEE Trans. Evol. Comput.*, vol. 14, no. 5, pp. 801–818, Oct. 2010.

[7] L. A. Zadeh, "Fuzzy sets," *Inform. Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965.

[8] K. Deb, A. Pratab, S. Agrawal, and T. Meyarivan, "A fast and elitist mult-iobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, Apr. 2002.

[9] E. Zitzler, M. Laumanns, and L. Thiele, "SPEA2: Improving the strength Pareto evolutionary algorithm," Swiss Federal Inst. Technol., Zurich, Switzerland, Tech. Rep., TIK-Rep. 103, 2001.

[10] M. Garza-Fabre, G. Toscano Pulido, and C. Coello Coello, "Ranking methods for many-objective optimization," in *Proc. 8th Mexican Int. Conf. Artif. Intell.*, Guanajuato, Mexico, 2009, pp. 633–645.

[11] M. Garza-Fabre, G.T. Pulido, and C. A. Coello Coello, "Alternative fitness assignment methods for many-objective optimization problems," in *Proc. Artif. Evol. Conf.*, Strasbourg, France, 2009, pp. 146–157.

[12] E. Hughes, "Fitness assignment methods for many-objective problems," in *Multi-Objective Problem Solving from Nature: From Concepts to Applications*, J. Knowles, D. Corne, and K. Deb, Eds. Berlin, Germany: Springer, 2008, pp. 307–329.

[13] L. Batista, F. Campelo, F. Guimaraes, and J. Ramirez, "A comparison of dominance criteria in many-objective optimization problems," in *Proc. IEEE Congr. Evol. Comput.*, Barcelona, Spain, 2010, pp. 2359–2366.

[14] E. Zitzler and S. Kunzli, "Indicator-based selection in multiobjective search," in *Proc. Int. Conf. Parallel Prob. Solving Nat.*, Birmingham, U.K., 2004, pp. 832–842.

[15] M. Farina and P. Amato, "A fuzzy definition of 'optimality' for many-criteria optimization problems," *IEEE Trans. Syst., Man, Cybern., A Syst. Hum.*, vol. 34, no. 3, pp. 315–326, May 2004.

[16] M. Nasir, A. K. Mondal, S. Sengupta, S. Das, and A. Abraham, "An improved multiobjective evolutionary algorithm based on decomposition with fuzzy dominance," in *Proc. IEEE Congr. Evol. Comput.*, New Orleans, LA, USA, 2011, pp. 765–772.

[17] M. Köppen, R. V. Garcia, and B. Nickolay, "Fuzzy-Pareto-dominance and its application in evolutionary multiobjective optimization," in *Proc. Int. Conf. Evol. Multi-Criterion Optimization*, Guanajuato, Mexico, 2005, pp. 399–412.

[18] J. Mendel, "Fuzzy logic systems for engineering: A tutorial," *Proc. IEEE*, vol. 83, pp. 345–377, 1995.

[19] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *IEEE Trans. Evol. Comput.*, vol. 11, no. 6, pp. 712–731, Dec. 2007.

[20] Z. He and G. G. Yen, "Performance metrics ensemble on multiobjective optimization algorithms," *IEEE Trans. Evol. Comput.*, to be published.

[21] S. Huband, P. Hingston, L. Barone, and L. While, "A review of multiobjective test problems and a scalable test problem toolkit," *IEEE Trans. Evol. Comput.*, vol. 10, no. 5, pp. 477–506, Oct. 2006.

[22] F. Pierro, S. Khu, and D. Savic, "An investigation on preference order ranking scheme for multiobjective evolutionary optimization," *IEEE Trans. Evol. Comput.*, vol. 11, no. 1, pp. 17–45, Feb. 2007.

[23] Q. Zhang, A. Zhou, and Y. Jin, "RM-MEDA: A regularity model based multiobjective estimation of distribution algorithm," *IEEE Trans. Evol. Comput.*, vol. 12, no. 1, pp. 41–63, Feb. 2008.

[24] J. Schott, "Fault tolerant design using single and multicriteria genetic algorithm optimization," M.S. thesis, Massachusetts Inst. Technol., Cambridge, MA, USA, Apr. 1995.

[25] I. Karahan, and M. Koksalan, "A territory defining multiobjective evolutionary algorithms and preference in corporation," *IEEE Trans. Evol. Comput.*, vol. 14, no. 4, pp. 636–664, Aug. 2010.

[26] J. Bader and E. Zitzler, "HypE: An algorithm for fast hypervolume-based many-objective optimization," Swiss Federal Inst. Technol., Zurich, Switzerland, Tech. Rep., TIK-Rep. 286, 2008.

[27] S. Robert, J. Torrie, and D. Dickey, *Principles and Procedures of Statistics: A Biometrical Approach*. New York, NY, USA: McGraw-Hill, 1997.

[28] J. Knowles, L. Thiele, and E. Zitzler, "A tutorial on the performance assessment of stochastic multiobjective optimizers," Swiss Federal Inst. Technol., Zurich, Switzerland, Tech. Rep., TIK-Rep. 214, 2005.

[29] S. Venkatraman and G. G. Yen, "A generic framework for constrained optimization using genetic algorithms," *IEEE Trans. Evol. Comput.*, vol. 9, no. 4, pp. 424–435, Aug. 2005.

[30] Y. G. Woldesenbet, G. G. Yen, and B. G. Tessema, "Constraint handling in multiobjective evolutionary optimization," *IEEE Trans. Evol. Comput.*, vol. 13, no. 3, pp. 514–525, Jun. 2009.

[31] M. Daneshyari and G. G. Yen, "Cultural-based multiobjective particle swarm optimization," *IEEE Trans. Syst., Man, Cybern. B, Cyberne.*, vol. 41, no. 2, pp. 553–567, Apr. 2011.

[32] Y. G. Woldesenbet and G. G. Yen, "Dynamic evolutionary algorithm with variable relocation," *IEEE Trans. Evol. Comput.*, vol. 13, no. 3, pp. 500–513, Jun. 2009.

Zhenan He received the B.E. degree in automation from the University of Science and Technology, Beijing, China, in 2008, and the M.S. degree in electrical and computer engineering from Oklahoma State University, Stillwater, OK, USA, in 2011.

He is with the School of Electrical and Computer Engineering, Oklahoma State University. His current research interests include multiobjective optimization using evolutionary algorithms, neural networks, and machine learning.

Gary G. Yen (S'87–M'88–SM'97–F'09) received the Ph.D. degree in electrical and computer engineering from the University of Notre Dame, Belmont, CA, USA, in 1992.

He is a Professor at the School of Electrical and Computer Engineering, Oklahoma State University, Stillwater, OK, USA. His current research interests include intelligent control, computational intelligence, conditional health monitoring, signal processing, and their industrial/defense applications.

Dr. Yen is currently serving as an Associate Editor for the IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION and the *International Journal of Swarm Intelligence Research*. He served as the Vice President for Technical Activities from 2005 to 2006 and the President in 2010–2011 of the IEEE Computational Intelligence Society and is the Founding Editor-in-Chief of the IEEE COMPUTATIONAL INTELLIGENCE MAGAZINE from 2006 to 2009. In 2011, he received the Andrew P. Sage Best Transactions Paper Award from the IEEE Systems, Man and Cybernetics Society. In 2013, he was a recipient of the Meritorious Service Award from the IEEE Computational Intelligence Society. He is a fellow of IET.

Jun Zhang (M'02–SM'08) received the Ph.D. degree in electrical engineering from the City University of Hong Kong, Hong Kong, in 2002.

Since 2004, he has been with Sun Yat-Sen University, Guangzhou, China, where he is currently a Professor and the Ph.D. Supervisor with the Department of Computer Science. He has authored seven research books and book chapters, and over 100 technical papers in his research areas. His current research interests include computational intelligence, cloud computing, data mining, wireless sensor networks, operations research, and power electronic circuits.