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Bridge Connecting Multiobjetive and Multimodal: A New Approach for Multiobjetive Optimization via Multimodal Optimization

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***Abstract*—Multimodal optimization problem (MMOP) and multiobjective optimization problem (MOP) are two kinds of widely-studied problems in the optimization and evolutionary computation (EC) community. Although the MMOP and the MOP share a common characteristic that they both require the EC algorithms to obtain a set of solutions, this interesting relationship has not arisen sufficient attentions in the EC research community. The two branches of MMOP and MOP almost develop independently in the EC community. In this paper, we make the first attempt to fill the gap by building a bridge to connect the MOP to the MMOP, with the following contributions. Firstly, a novel and innovative idea is proposed to solve MOP by connecting the MOP to the MMOP. Secondly, an example of transformation method is illustrated to show the feasibility of the bridge connecting the MOP to the MMOP. Thirdly, experiments are conducted and the results show the effectiveness of using MMOP algorithms to obtain solutions that can be well mapped back to reflect the Pareto front of the MOP. Last but not least, this new perspective on connecting MOP to MMOP will inspire more diversity and more efficient future works on the topic of deep researches into both MMOP and MOP.**

Keywords—Multiobjective optimization problem (MOP), multimodal optimization problem (MMOP), evolutionary computation (EC), bridge, transformation

I. Introduction

Multimodal optimization and multiobjective optimization are two fast developing branches in evolutionary computation (EC) community. The multimodal optimization problem (MMOP) has several optima with the same fitness value so that EC algorithms are required to find the global optimal solutions as many as possible [1]. The multiobjective optimization problem (MOP) has several objective functions so that generally there does not exist a single solution that is better than all the other solutions on all the objectives [2]. Therefore, the MOP requires the EC algorithms to find a set of non-dominated solutions along the Pareto front (PF). Note that the MOP with more than 3 objectives is also called many-objective optimization problem (MaOP) [3].

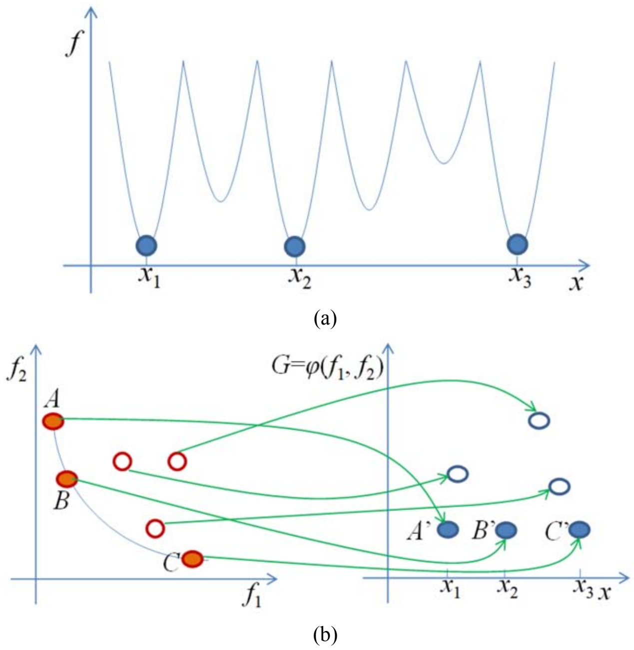

Fig. 1. Illustration of multiple global optimal solutions distribution on MMOP and MOP. (a) Multiple global optimal solutions of MMOP. (b) Mapping of multiple Pareto optimal solutions in objective space and decision space.

The researches on MMOP and MOP are relative independent during a long time. Generally speaking, the researches on MMOP mainly focus on how to divide the population into different niches so that each niche can locate a promising region to find an optimal solution [4]–[6]. While for the MOP, many researches focus on using Pareto-based method [7], decomposition-based method [8], indicator-based method [9], and/or multi-population-based method [2] to generate, select, and push the solutions to approach the PF.

It is interesting that both MMOP and MOP are actually multiple solutions problems. That is, both of them have multiple solutions that can be regarded as the same important, i.e., all can be regarded as global optimal solution. Fig. 1 gives the illustration. Fig. 1(a) is very easy to understand that for a minimization MMOP, it has three global optima x_1, x_2,

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and x_3. The left side of Fig. 1(b) shows the PF of an MOP. Noted that the Pareto optimal solutions A, B, C, and many others along the PF are non-dominated with each other and therefore they can be regarded as having the same fitness value with respect to all objectives. The two objective functions of the MOP are denoted as f_1 and f_2, respectively. We can define a new function G to combine f_1 and f_2 as

$$\min G = \varphi(f_1, f_2) \tag{1}$$

so that the G values of the solutions A', B', and C' in the decision space are the same and are all the global minima. Herein the solutions A', B', and C' in the right side of Fig. 1(b) are corresponding to the Pareto optimal solutions A, B, and C in the left side of Fig. 1(b). If we only consider the Pareto optimal solutions A, B, and C in MOP, we can say that this MOP also has three global optima x_1, x_2, and x_3.

In this sense, we find a very interesting phenomenon that the MMOP and MOP have some things in common and they can be connected by a bridge. That is, as both the MMOP and MOP aim to find multiple optimal solutions, the MMOP algorithms may be useful for finding the Pareto optimal solutions in MOP and the MOP algorithms may be promising to help locate different global optima in MMOP. In the literatures, we can only find some researches that proposed to transform the MMOP into MOP and used MOP algorithms to help solve the problem [10]–[15]. In the transformation, a two-objectives MOP is commonly formulated, where one objective is the original objective of the MMOP and the other objective is usually defined as a diversity indicator that helps the algorithm to spread the population to search for more global optima [10]–[14]. However, these MOPs may not be the standard MOPs because their objectives are not conflicting with each other. Therefore, the research in [15] designed two conflicting objectives in each dimension to form a set of MOPs. Our previous work [16] further defined two conflicting objectives for the MMOP to transform it to a standard MOP.

Although some works mentioned above have made bridge somehow to connect the MMOP to the MOP, however, as far as we know, there is still no work on the research topic to make bridge to connect the MOP to the MMOP. Therefore, this paper makes the first attempt to fill this gap. That is, we give an example to transform an MOP to an MMOP and propose to use EC-based MMOP algorithms to solve the formed MMOP to obtain a set of global optimal solutions. Then these global optimal solutions to the MMOP are actually also the Pareto optimal solutions to the MOP. This way, the original MOP is solved.

The contributions of our work include:

(1) As far as we know, our work is the first time to try to make a bridge that connects the MOP to the MMOP. This is a novel and innovative way to solve MOP.

(2) An example of transformation method is illustrated to show the feasibility of the bridge connecting the MOP to the MMOP.

(3) The transformed MMOP is well solved and the obtained solutions are actually the corresponding Pareto optimal solutions to the original MOP. Significantly, these solutions are promising for the original MOP when compared with the state-of-the-art MOP algorithms.

(4) This new perspective on connecting MOP to MMOP has shown the effectiveness and will inspire more diversity and more efficient future research works on this topic.

The rest of the paper is organized as follows. Some related works on both the MMOP and the MOP are reviewed in Section II. Section III describes the transformation method to make bridge connecting the MOP to the MMOP. Section IV presents the experimental studies. In Section V, conclusions of this paper and future work are given.

II. Related Works on MMOP and MOP

A. Related Works on MMOP

As there are a number of global optima in the MMOP, it requires the EC algorithms, e.g., the genetic algorithm (GA) [17], particle swarm optimization (PSO) [18][19], estimation of distribution algorithm [20], and/or differential evolution (DE) [21][22], to obtain the global optima as many as possible. For example, the species-based DE [5] and the crowding-based DE [6] are the two classic niching-based EC algorithms. However, the challenging issue of the speciation and crowding niching methods is the high sensitivity of the niche parameters such as the species radius and the crowding size, which are very difficult to be configured. Therefore, Li [23] proposed a parameter-free ring topology niching method to design the R2PSO algorithm.

Recently, several powerful niching methods are proposed, e.g., the affinity propagation clustering-based automatic niching [24], the local binary operator-based niching [25], and the distributed individuals for multiple peaks (DIMP)-based niching [26].

B. Related Works on MOP

For the MOP, many researches focus on using the Pareto-based method [7], decomposition-based method [8], indicator-based method [9], and/or multi-population-based method [2] to generate, select, and push the solutions to approach the PF.

Firstly, as any two solutions to the MOP have different values on different objectives, it is difficult to determine which solution is better to survive into the next generation. This brings a serious problem to EC algorithms because the algorithms are driven by the fitness value according to the "survive of fitness" natural selection principle. Therefore, the Pareto-based method uses Pareto domination relationship to determine the rank of solutions and to select those with better ranks into the next generation to drive the evolution [7].

Secondly, as considering the multiple objectives is difficult, the decomposition-based method dose not deal with all the objectives simultaneously, but uses weights to decompose the MOP into a set of single-objective optimization subproblems and solves these subproblems to obtain the Pareto optimal solutions [8].

Moreover, as the purpose of MOP is to find a set of non-dominated solutions along the PF, a promising method is to use some indicators that can measure how the obtained solutions approximate the PF. Therefore, the third popular MOP method is the indicator-based method where the diversity indicator and convergence indicator are always used to help generate, select, and drive the solutions [9].

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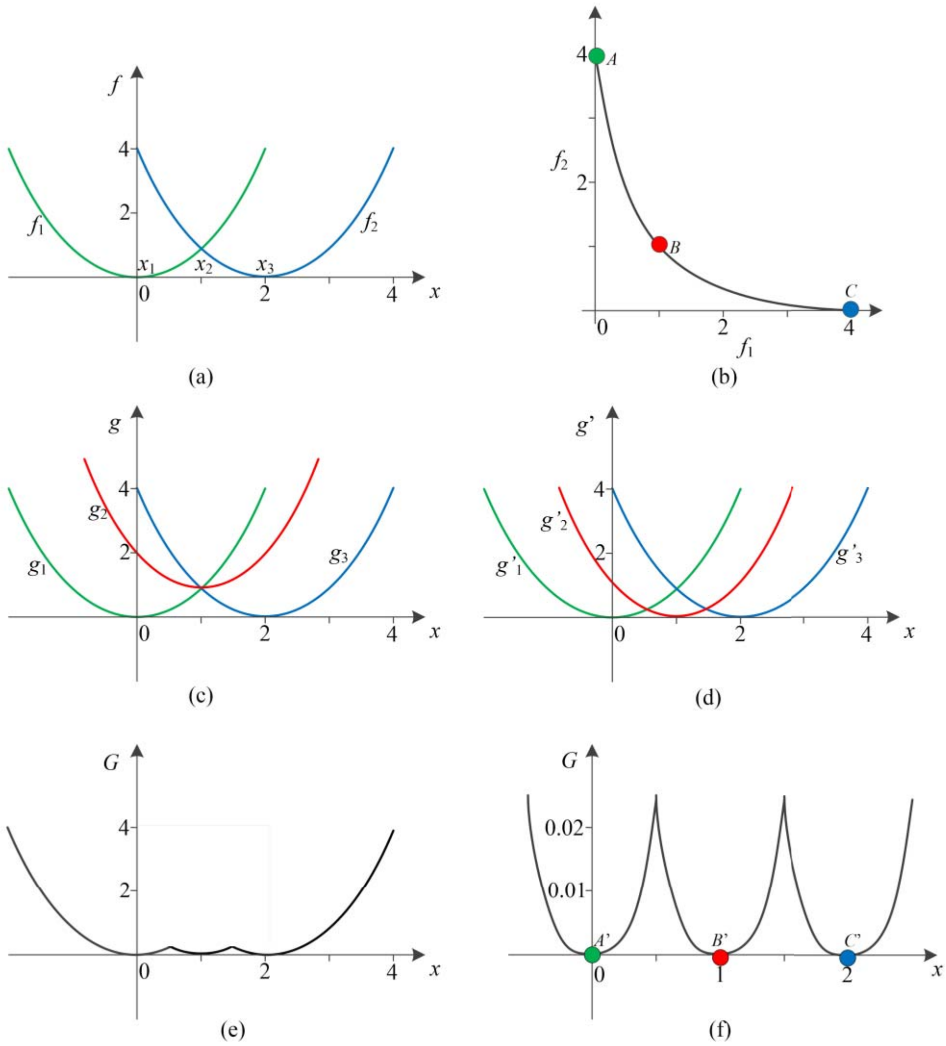

Fig. 2. Illustration of bridge that connects MOP to MMOP. (a) The objective functions curves of the SCH MOP in the decision space. (b) Three Pareto optimal solutions on the PF in the objective space. (c) Three aggregative functions whose global optima are corresponding to the three Pareto solutions in (b). (d) The modifications of aggregative functions from (c). (e) Combine the modified aggregative functions in (d) to obtain the MMOP. (f) The detailed landscape of the MMOP.

Recently, the fourth MOP method based on multiple populations for multiple objectives (MPMO) has gained great successes and attentions in the EC community [2]. Under the MPMO framework, each objective of the MOP is tackled with by a population, so that the selection difficulty can be avoided because the solution fitness values of this population are determined by its corresponding objective. This not only makes the EC algorithm in each population very easy to perform its evolutionary operators, but also is beneficial to sufficiently search all the objectives space of the whole MOP. Moreover, the MPMO framework also contains an information share mechanism to share the information among all the populations so as to approach the whole PF. Due to the simple and easy implementation for each EC algorithm in each population and the promising performance in solving MOP, the MPMO framework has been widely followed by researchers in EC community [27] and has been extended to MaOP [3] and real-world MOP applications [28].

III. BRIDGE CONNECTING MOP TO MMOP

A. Relation Analysis between MOP and MMOP

To illustrate and analyze the relationship between MOP and MMOP, a simple yet representative MOP named SCH problem [7] is adopted, with the formulation as

$$F(x)=\begin{cases}\min f_1(x)=x^2\\ \min f_2(x)=(x-2)^2\end{cases} \tag{2}$$

where x belongs to [–1000, 1000].

As shown in Fig. 2(a), the figures of f_1 and f_2 are two parabolas. Moreover, the interval of [0, 2] is the Pareto optimal solutions interval of this MOP. That is, any solution in this interval is corresponding to a Pareto optimal solution on the PF in the objective space. Fig. 2(b) shows the PF of the SCH MOP and three Pareto optimal solutions.

Therefore, in this sense, all the Pareto optimal solutions on the PF of Fig. 2(b) have their corresponding decision variable x whose value is in the interval [0, 2] in Fig. 2(a). If we can find out these x values in the decision space, the Pareto optimal solutions in the objective space can thus be found.

The Fig. 2(a) and (b) show three Pareto optimal solutions x_1=0, x_2=1, and x_3=2, which are corresponding to the Pareto optimal solutions A, B, and C on the PF. The objective values (f_1, f_2) of A, B, and C are (4, 0), (1, 1), and (0, 4), respectively. In fact, according to the decomposition method [8], a Pareto optimal solution on the PF is corresponding to the global optimum of a decomposed single-objective problem which is the aggregation of the multiple objectives via a special weight. Herein, similar to Eq. (1), we define three aggregative functions as

$$g_1(x) = 1\times f_1(x) + 0\times f_2(x) = f_1(x) = x^2 \quad (3)$$

$$g_2(x) = 0.5\times f_1(x) + 0.5\times f_2(x) = (x-1)^2 + 1 \quad (4)$$

$$g_3(x) = 0\times f_1(x) + 1\times f_2(x) = f_2(x) = (x-2)^2 \quad (5)$$

The curves of these three functions are plotted in Fig. 2(c), clearly showing that the global optimal solutions of them are x_1=0, x_2=1, and x_3=2, respectively.

In order to construct the MMOP, we remove the constant after completing the square of the function g, so as to obtain a function g'. Note that the global optimal solutions of the two functions g and g' are the same. In this example, only the g_2 in Eq. (4) needs to remove the constant 1 to become $g'_2(x)=(x-1)^2$, while g_1 and g_3 are the same as g'_1 and g'_3, respectively. This way, we can obtain the Fig. 2(d) with three functions g'_1, g'_2, and g'_3.

At last, we can define an MMOP as

$$G(x) = \min\left\{g'_1(x), g'_2(x), g'_3(x)\right\} \quad (6)$$

whose curve is shown in Fig. 2(e) with three global optimal solution x_1=0, x_2=1, and x_3=2.

Moreover, in order to show the curve of the MMOP more clearly, the figure is magnified to Fig. 2(f).

Therefore, the relation between the MOP $F(x)$ and the MMOP $G(x)$ is illustrated in Fig. 2. The Fig.2 gives an example that makes the bridge to connect the MOP in Eq. (2) to the MMOP in Eq. (6).

B. Steps of Transforming MOP to MMOP

A more general transformation method for the MOP in Eq. (2) to an MMOP is described in this subsection as the following four steps.

Step 1: Determine a set of weights $W=\{w_1, w_2, \ldots w_K\}$.

Step 2: Using the set of weights to obtain a set of single-objective functions as

$$g_i(x) = w_i \times f_1(x) + (1-w_i)\times f_2(x) \quad (7)$$

where $1\le i\le K$.

Step 3: Complete the square of g_i and remove the constant to obtain g'_i.

Step 4: Define the MMOP $G(x)$ as

$$G(x) = \min\left\{g'_i(x)\right\}, \forall i, 1\le i\le K \quad (8)$$

After the transformation, we can use any MMOP algorithm to solve the transformed MMOP to obtain a set of global optima. Herein, the number of global optima is the same as K. Actually, these K global optimal solutions are all the Pareto optimal solutions of the original MOP. In this way, the original MOP is solved.

IV. Experimental Studies

A. Experimental Settings

The SCH problem is adopted as a case study in this paper. The problem definition is shown in Eq. (2) and the PF is shown in Fig. 2(b). To evaluate the algorithm, the inverted generational distance (IGD) metric is adopted to measure the convergence and diversity of the solutions obtained by different algorithms. The IGD value is calculated as

$$IGD(A,P) = \frac{\sum_{i=1}^{|P|} d(P_i, A)}{|P|} \quad (9)$$

where A is the solutions set obtained by the algorithm and P is the solutions set that uniformly sampled along the PF with size $|P|$. The $d(P_i, A)$ is the Euclidean distance between the solution P_i and the solution in A that is nearest to P_i. A smaller IGD value indicates that the obtained solutions in A are more closed to the PF and are better distributed along the PF. Especially, if every P_i is covered by A, the IGD will have the minimal value 0. Herein, as the Pareto optimal solution interval of the SCH MOP is [0, 2], we sample 101 uniformly distributed points along the PF to form the set P. That is, $x_i=(i-1)/50$, $1\le i\le 101$, to form 101 solutions in the set P.

In order to evaluate the effectiveness of using MMOP algorithm to solve the MMOP which is transformed from the SCH MOP, a simple MMOP algorithm named R2PSO [23] is adopted. The MMOP is formed by the transformation method shown in Section III. Note that different K values will result in different MMOPs with different number of global optima. In this experiment, we test 4 types of K values, including a small value K=11, two medium values K=101 and K=201, and a large value K=501. When K=11, 101, 201, and 501, the weights are set as W={0, 0.1, 0.2, …0.9, 1}, W={0, 0.01, 0.02, …0.99, 1}, W={0, 0.005, 0.01, …0.995, 1}, and W={0, 0.002, 0.004, …0.998, 1}, respectively. The parameters of R2PSO are set the same as the proposals in its original paper [23], with the population size 500.

The MMOP algorithm R2PSO is compared with the state-of-the-art MOP algorithm NSGA-II [7]. The parameters of NSGA-II are also set the same as the proposals in its original paper [7], with the population size 100. However, as the R2PSO uses population size 500, the NSGA-II is also tested with population size 500.

To make the comparisons fair, all the algorithms use the same maximum fitness evaluations (MaxFEs) budget as 200000. All the algorithms run 10 times independently.

B. Experimental Results

The experimental results are compared in Table I. The detailed IGD results of all the 10 runs together with their best, worst, mean (average), and median values are presented and compared. The results show that the MMOP algorithm R2PSO has promising performance to obtain solutions that can map back to the original MOP with very good IGD value. Moreover, the results obtained by R2PSO on the MMOP with K=101 have the best mean and median IGD values. Particularly, the best IGD value among its 10 runs reaches the E–4 level while all the IGD values of all the algorithms in all the runs are on E–3 or E–2 levels. The reason may be that a larger K value forms an MMOP with more global optima, which has two kinds of influences. On the one hand, the MMOP is with more global optima so that the obtained solutions can approximate the true PF with more diversity and more convergence. For example, the R2PSO solves the MMOP with K=101 global optimal can results in better IGD value than that it solves the MMOP with only K=11 global optimal. However, this will happen only if the MMOP algorithm is powerful enough to obtain the more global

TABLE I
THE IGD RESULTS OF MOP ALGORITHM AND MMOP ALGORITHM

Run	NSGA-II		R2PSO			
	Pop Size = 100	*Pop Size* = 500	*K*=11	*K*=101	*K*=201	*K*=501
1	1.766E–02	3.651E–03	1.940E–02	1.721E–03	4.796E–03	7.540E–03
2	1.649E–02	3.520E–03	1.887E–02	1.889E–03	5.044E–03	7.236E–03
3	1.613E–02	3.522E–03	1.970E–02	7.183E–04	4.395E–03	8.016E–03
4	1.689E–02	3.482E–03	1.744E–02	1.153E–03	3.898E–03	7.703E–03
5	1.570E–02	3.337E–03	1.631E–02	1.679E–03	6.084E–03	7.976E–03
6	1.962E–02	3.649E–03	2.342E–02	1.759E–03	5.751E–03	8.031E–03
7	1.744E–02	3.837E–03	1.866E–02	7.765E–04	5.337E–03	1.144E–02
8	1.608E–02	3.536E–03	2.061E–02	3.535E–03	6.384E–03	8.626E–03
9	1.704E–02	3.443E–03	2.334E–02	3.810E–03	4.311E–03	8.437E–03
10	1.836E–02	3.322E–03	2.412E–02	4.466E–03	6.177E–03	7.172E–03
Best	1.570E–02	3.322E–03	1.631E–02	**7.183E–04**	3.898E–03	7.172E–03
Worst	1.962E–02	**3.837E–03**	2.412E–02	4.466E–03	6.384E–03	1.144E–02
Mean	1.714E–02	3.530E–03	2.019E–02	**2.151E–03**	5.218E–03	8.218E–03
Median	1.697E–02	3.521E–03	1.955E–02	**1.740E–03**	5.191E–03	7.996E–03

optima. Otherwise, on the other hand, the large K value will cause that the transformed MMOP is too complex and too difficult for solving. In this situation, the MMOP algorithm performs poorly to obtain enough global optimal solutions of the complex MMOP, and at last makes the results mapping back to the MOP with poor IGD value. Therefore, a very large K value, e.g., K=201 and K=501 will make R2PSO result in poor IGD value than that of a medium value like K=101.

Nevertheless, the R2PSO algorithm generally outperforms the original NSGA-II algorithms with population size set to 100. Only when solving the MMOP with K=11 global optima, R2PSO obtains slightly worse IGD value than that of NSGA-II. To show a fairer comparison, the results obtained by NSGA-II upgraded with population size of 500 are also presented. The results show that R2PSO solving the MMOP with K=101 global optima still obtains better results than that of NSGA-II. The R2PSO performs better than NSGA-II in terms of best IGD value, mean IGD value, and median IGD value.

Therefore, the transformation of MOP to MMOP and then using MMOP algorithm to solve the problem is a feasible and effective new approach to solve the MOP.

C. Results Visulization

In order to show the final results obtained by the R2PSO algorithm on the MMOP landscape and the MOP objective space, the results are visualized in this subsection. The results of the best run obtained by R2PSO when solving the MMOP with K=101 global optimal are adopted.

On the one hand, the final population of R2PSO on the MMOP landscape is shown in Fig. 3. The figure shows that the final population and the found solutions have already located all the optimal regions. On the other hand, the results mapped back to the objective space of the MOP are plotted in Fig. 4. The figure shows that the obtained solutions also cover the PF of the original MOP. From these two figures, we can see that the solutions obtained by MMOP algorithms can well approach the PF of the MOP, which validates the feasibility of the bridge connecting MOP to MMOP.

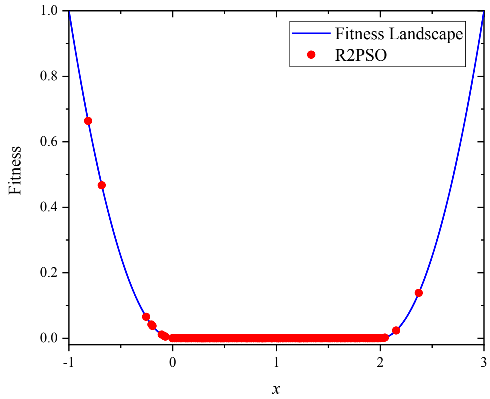

Fig. 3. Final results of R2PSO on MMOP landscape.

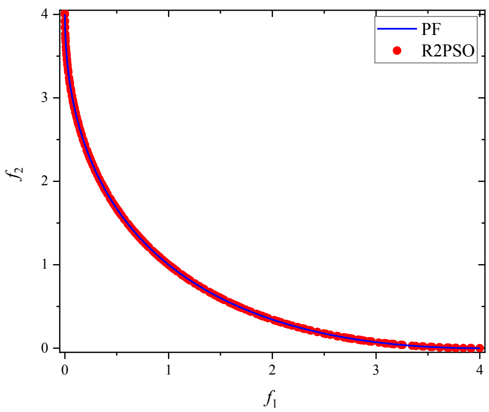

Fig. 4. Final results of R2PSO on MOP objective space.

V. CONCLUSION

This paper proposes a transformation method to make a bridge to connect the MOP to the MMOP, which presents a new idea and perspective to the researches into MOP. Although we have made relation analyses and discussions on the MOP and the MMOP in this paper and also given an example to transform the SCH MOP to an MMOP, there are still many open problems that can be considered in the future researches.

Firstly, although the common characteristic on "multiple global optima" of MMOP and MOP is intuitive, are there any theories that can guarantee their relation? This is a fundamental and interesting research direction.

Secondly, although this paper gives a case study on the SCH MOP, is the transformation method on this MOP also suitable for other MOPs? Or how to design a general transformation method that can be used on various kinds of MOPs is still a challenging research topic.

Thirdly, it is also remarkable to note that the number of global optima in MMOP is always countable, i.e., discrete. However, the actual PF of MOP is always continuous and therefore the number of Pareto optimal solutions is uncountable. So how this difference can be considered during the transformation? For example, the value of K in our work to indicate the number of weights (i.e., aggregative functions) indeed influences the complexity of the transformed MMOP and the performance of the MMOP algorithm. Therefore, this is also a meaningful research direction.

Last but not least, more MOPs can be tested to further evaluate the feasibility and effectiveness of the idea of connecting MOP to MMOP. Moreover, more well-performing MMOP algorithms like the dual-strategy DE (DSDE) [1], automatic niching DE (ANDE) [24], local binary operator-based adaptive DE (LBPADE) [25], and distributed individuals-based DE (DIDE) [26] can be adopted to obtain more promising solutions to the transformed MMOP, so that we can further compare them with the other well-performing MOP algorithms to further evaluate the efficiency and advantages of the idea of connecting MOP to MMOP.

In summary, this paper opens a new research idea of connecting the multiobjective optimization to the multimodal optimization, and has gained some promising results on a case study. In the future work, topics include the theoretical analyses, general transformation method, complexity control, and further performance promotion and evaluation are worthy of wide and deep research.

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