Seventh International Conference on Information Science and Technology
Da Nang, Vietnam; April 16-19, 2017

Solving Multimodal Optimization Problems through a Multiobjective Optimization Approach

Jing-Yu Ji*, Wei-Jie Yu (Corresponding Author)*, Wei-Neng Chen†, Zhi-Hui Zhan†, and Jun Zhang†
*Sun Yat-sen University, Guangzhou, China
Email: ywj21c@163.com
†South China University of Technology, Guangzhou, China

***Abstract*—This paper proposes a novel multiobjective optimization approach for solving multimodal optimization problems (MMOPs). An MMOP at hand is first transformed into a bi-objective optimization problem. The two objectives are constructed totally conflict by using the distance information and the objective function value. In this way, multiple optima of an MMOP are converted into the nondominated solutions of the transformed bi-objective optimization problem. Then, multiobjective optimization techniques based on differential evolution are applied to solve the bi-objective problem. In addition, a modified solution comparison criterion is proposed to improve the accuracy level of the final solutions. The performance of the proposed approach is evaluated on a suite of benchmark functions. Experimental results show that the proposed approach is very competitive compared with six state-of-the-art multimodal optimization algorithms on most of the benchmark functions.**

***Keywords*—multimodal optimization problems, multiobjective optimization, differential evolution**

I. Introduction

Multimodal optimization problem (MMOP) is a kind of global optimization problems with multimodal property. There are increasing real-world practical applications involved MMOPs which need to locate all the multiple optima simultaneously [1]–[3]. However, it is more difficult to solve an MMOP than a single-optimum optimization problem, even though they have the same mathematical formula.

In the field of evolutionary computation, evolutionary algorithm (EA) is a promising way to deal with MMOPs. However, most of classical EAs cannot seek all the multiple optima simultaneously for an MMOP, since they lack the ability of distinguishing solutions only from objective function value. When these algorithms solve MMOPs, their evolutionary operators lead the population converge to one of the multiple optima, and thus lose the others. Therefore, a lot of strategies based on niching method [4] have been proposed to enhance classical EAs for MMOPs. The niching method is a classical techniques to handle multimodal property. It mainly contains four different strategies, i.e., clearing [5], sharing [6], crowding [7] and restricted tournament selection [8]. In general, these strategies use distance information from decision space to maintain individuals with identical objective function values but locating at different peaks.

In recent years, some research work [9], [10] has been conducted to solve MMOPs based on multiobjective optimization techniques. These multiobjecitve approaches can locate multiple optima simultaneously by transforming an MMOP into a multiobjective optimization problem (MOP), which means all the optimal solutions of an MMOP need to be transformed into the nondominated solutions of a MOP. This paper proposes a bi-objective optimization algorithm for multimodal optimization, namely, BiMO. An MMOP is first transformed into a MOP with two strong conflicting objectives. The two objectives are constructed by using distance information and objective function value of the original MMOP. With the transformation, the multiple optima of the MMOP are converted into the nondominated solutions of the bi-objective optimization algorithm. Then, differential evolution (DE) [11]–[13] and multiobjective optimization techniques of nondominated sorting and truncation method [14] are utilized to optimize the bi-objective optimization algorithm. Specifically, DE as a search engine generates offspring population. Nondominated sorting and truncation method select the best solutions from the parent and offspring populations. As a result, BiMO can locate the multiple optima of an MMOP through finding the nondominated solutions of the transformed MOP. The proposed BiMO is tested on 20 benchmark multimodal functions from IEEE CEC 2013 [15]. The performance of BiMO compares favorably with six state-of-the-art multimodal optimization algorithms.

The remainder of this paper is organized as follows. Section II presents related work on MOP, and DE algorithm. Section III develops the proposed BiMO in detail. Experimental results are reported in Section IV. Finally, Section V draws the conclusion.

II. Related Work

A. Multiobjective Optimization Techniques for MMOPs

When applying multiobjective optimization techniques to solving an MMOP, the MMOP at hand is first transformed into a MOP which can be stated as follows:

$$\begin{array}{ll}\text{Minimize} & F(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_m(\mathbf{x})) \\ \text{subeject to} & \mathbf{x} = (x_1, \ldots, x_D) \in \Re\end{array} \quad (1)$$

This work was supported by the National Natural Science Foundation of China (Grant Nos. 61502544 and 61332002).

where $\mathbf{x}$ is a decision vector with D dimensions, m is the number of objectives and $\Re$ is the decision space.

Given two vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u}$ Pareto dominates $\mathbf{v}$, if $f_i(u) \leq f_i(v)$ for all $i \in \{1, \ldots, m\}$ and $F(\mathbf{u}) \neq F(\mathbf{v})$. Moreover, vector $\mathbf{t}$ is said to be a nondominated solution, if there is no $\mathbf{x} \in \Re$ such that $\mathbf{x}$ Pareto dominates $\mathbf{t}$. In a MOP, one objective always conflicts with the others, and thus there is no $\mathbf{x} \in \Re$ which can optimize all the objectives at the same time. The set contains all the nondominated solutions of a MOP is called Pareto set, denoted by *PS*.

Multiobjective optimization techniques can make a tradeoff between population convergence and diversity, which is very attractive to solve MMOPs. With proper transformation, the multiple optima of an MMOP can be located simultaneously through multiobjective optimization techniques. In [16] and [9], the objective function of MMOP and the gradient information of an individual are utilized to transform an MMOP into a MOP. In [10], an MMOP is transformed into a bi-objective optimization problem. One objective is the original objective function of MMOP, and another is constructed by average distance information for population diversity. In [17], an MMOP is transformed into a set of bi-objective optimization problems based on the variables. The two objectives in each problem are totally conflicting.

B. Differential Evolution

Differential evolution (DE), first proposed by Storn and Price [11], is a simple and efficient evolutionary algorithm. It has been widely developed and applied to a wide range of optimization problems. A DE algorithm includes three evolutionary operators, namely, mutation, crossover and selection. The classical and most frequently used algorithm, i.e., DE/rand/1, is briefly introduced as follows.

1) Mutation: For each target vector $\mathbf{x}_i$, three different vectors are randomly selected in current population to generate a mutant vector $\mathbf{v}_i$, which can be expressed as follows:

$$\mathbf{v}_i = \mathbf{x}_{r_1} + F \cdot (\mathbf{x}_{r_2} + \mathbf{x}_{r_3}) \tag{2}$$

where i denotes the index number of the target vector, F is the scale factor and r_1, r_2, r_3 are distinct integers randomly selected from the indexes of vectors, and they are all different from the index i.

2) Crossover: Crossover operator is applied to produce a trial vector $\mathbf{u}_i$ which involves the mixed information of target vector $\mathbf{x}_i$ and mutant vector $\mathbf{v}_i$. The process can be expressed as follows:

$$\mathbf{u}_{i,j} = \begin{cases} \mathbf{v}_{i,j}, & \text{if } rand_j(0,1) \leq C_r, \text{ or } j == j_{rand} \\ \mathbf{x}_{i,j}, & \text{otherwise} \end{cases} \tag{3}$$

where $j = 1, \ldots, n$, C_r is the crossover rate and j_{rand} is an randomly integer generated from $[1, D]$. The condition $j = j_{rand}$ ensures that the trial vector has at least one component different from the target vector.

3) Selection: Selection operator decides whether the trial vector will replace the target vector and enter the next generation. It works as follows:

$$\mathbf{x}_i = \begin{cases} \mathbf{u}_i, & \text{if } f(\mathbf{u}_i) \leq f(\mathbf{x}_i) \\ \mathbf{x}_i, & \text{otherwise} \end{cases} \tag{4}$$

III. BiMO Algorithm

In this section, a novel bi-objective optimization algorithm named BiMO is proposed for MMOPs. BiMO first transforms an MMOP into a MOP with two objectives, and then solve the transformed bi-objective optimization problem through multiobjective optimization techniques.

A. A Bi-objective Opmization Model for MMOP

To solve an MMOP with multiobjective optimization techniques, an MMOP first needs to be transformed into a MOP. Inspired by the suggestion in [17], the transformed MOP should have strong conflicting objectives. We propose an optimization model with two totally conflicting objectives in this section. In this model, the objectives are composite of objective function value and distance information of individuals. The objective function value is derived from the original function value of MMOP. For each individual i, distance information is calculated from the rest of individuals in the current population, which can be formulated as follows:

$$\Omega_i = \sum_{j=1, j\neq i}^{N} \|\mathbf{x}_i - \mathbf{x}_j\| \tag{5}$$

where i denotes the index number of individual, and $\|\mathbf{x}_i$ - $\mathbf{x}_j\|$ is Euclidean distance between $\mathbf{x}_i$ and $\mathbf{x}_j$.

If Ω_i is considered as an independent objective, as done in [5], there is no conflict between the two objectives. Hence, it cannot ensure the multiple optima of an MMOP are totally converted into the nondominated solutions of the transformed MOP. Therefore, it needs extra techniques to archive some potential global optima in case the search leaves them out. To address this issue, BiMO constructs two totally conflicting objectives:

$$\begin{cases} f_1(\mathbf{x}) = \Omega(\mathbf{x})_{norm} + \xi * f(\mathbf{x})_{norm} \\ f_2(\mathbf{x}) = 1.0/\Omega(\mathbf{x})_{norm} + \xi * f(\mathbf{x})_{norm} \end{cases} \tag{6}$$

where ξ is a scaling factor, $\Omega(\mathbf{x})_{norm}$ and $f(\mathbf{x})_{norm}$ represent the normalized values of $\Omega(\mathbf{x})$ and $f(\mathbf{x})$ respectively, which are in the range $[0.0, 1.0]$. If the value of $\Omega(\mathbf{x})_{norm}$ is zero, it will be set to a very small value, e.g., 0.001.

Due to the first part in each equation of (6), if the first objective $f_1(\mathbf{x})$ increases, the second objective $f_2(\mathbf{x})$ must decrease, and vice versa. Thus, $f_1(\mathbf{x})$ always conflicts with $f_2(\mathbf{x})$. Based on the conflict, a set of nondominated solutions can be constructed. But only depends on the first part in each equation of (6), any solution will be a nondominated solution. In order to eliminate the solutions with bad performance of objective function values, the second part in each equation of (6) is introduced. As a result, if a multiobjective optimization algorithm can find the whole nondominated solutions of the transformed MOP, it can also locate the multiple optima of the original MMOP.

Algorithm 1: BiMO

Input:
- N: population size.
- $MaxGen$: maximum number of generations.

Output:
- PS: the nondominated solutions.

Initialization:
- Randomly generate an initial population of N individuals from decision space S.
- Compute the two objective values for each individual in initial population.

while $g < Max\_gen$ **do**
  Generate offspring population (set Q) from parent population (set P);
  $P = P \bigcup Q$;
  Compute the two objective values for each individual in P;
  Apply the nondominant sorting and pruning methods to truncate the population size of P from $2N$ to N.
  $g = g + 1$;
end
Find the nondominated solutions in P and output them.

B. Modified Solution Comparison Criterion

After an MMOP is transformed into a bi-objective optimization problem, DE/rand/1, nondominated sorting and truncation method are utilized to optimize the bi-objective optimization algorithm. In order to improve the accuracy level of solutions, we further design a modified solution comparison criterion for nondominated sorting. Given a niche radius σ, the proposed solution comparison criterion performs on two individuals $\mathbf{u}$ and $\mathbf{v}$ as follows:

- If $\mathbf{u}$ and $\mathbf{v}$ are in the different niches, the Pareto-dominance rule between $(f_1(\mathbf{u}), f_2(\mathbf{u}))$ and $(f_1(\mathbf{v}), f_2(\mathbf{v}))$ is used.
- If $\mathbf{u}$ and $\mathbf{v}$ are in the same niche, the one with smaller objective function value dominates the other one.

The niche radius σ is set to be very small value so that if $\mathbf{u}$ and $\mathbf{v}$ are in the same niche, they can be considered in the same peak. Hence, the one with worse performance will be eliminated directly. As a result, fitness evaluations would not waste on redundant individuals which are in the same peak.

C. Overall Process of BiMO

BiMO maintains the following information in each generation g:

1) A population P with size N, $P_g = \mathbf{x}_{1,g}, \ldots, \mathbf{x}_{N,g}$;
2) Two objective values of each individual: $f_1(\mathbf{x}_{i,g})$, $f_2(\mathbf{x}_{i,g})$, $i = 1, \ldots, N$;
3) The scaling factor ξ_g.

The pseudocode of BiMO is presented in Algorithm 1.

In the initialization phase, each dimension of an individual is randomly generated within its range. Then, DE/rand/1 is adopted as the search engine to generate offspring population Q. After that, nondominated sorting and truncation method are utilized to select N individuals from P and Q as a new population into the next generation. Multiobjective optimization techniques based on these two methods make a good balance between population diversity and convergence.

TABLE I
PARAMETER SETTING

Benchmark Function	Maximum Number of Fitness Evaluations	Population Size
F_1 - F_5	5.0E+04	80
F_6	2.0E+05	100
F_7	2.0E+05	300
F_8 - F_9	4.0E+05	300
F_{10}	2.0E+05	100
F_{11} - F_{13}	2.0E+05	200
F_{14} - F_{20}	4.0E+05	200

IV. Experimental Study

In this section, we compare BiMO with other six state-of-the-art multimodal optimization algorithms on 20 benchmark functions with different characteristics from CEC 2013 [15]. The first ten functions are classical, low-dimensional multimodal functions. The last ten ones are multimodal composition functions with scaling dimension. More details of these benchmark functions can be found in [15].

A. Parameter Settings

Each experiment is performed 50 times independently on each benchmark function. The recommended maximum number of fitness evaluations ($MaxFEs$) [15] and population size N for each test function are presented in Table I.

At each generation, N combinations of F and Cr are randomly generated from $[0.1, 0.2, 0.5, 1.0]$ and $[0.3, 0.5, 0.7]$ respectively for DE/rand/1 to produce N corresponding offspring. The niche radius σ in the modified solution comparison criterion is set to 0.01. The ξ_g in each generation is calculated by the following scheme:

$$\xi_g = 199 * (g/MaxGen)^2 \tag{7}$$

According to (6), the weight of $f(\mathbf{x})_{norm}$ in two objectives $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ increases smoothly during the evolution. At the early stage, $f(\mathbf{x})_{norm}$ has a low weight in $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$. Individuals with relatively bad performance of objective function values but good locations have chances to survive. At the later stage, $f(\mathbf{x})_{norm}$ has a high weight in $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ and thus solutions can be found at high accuracy level.

B. Performance Metrics

Two metrics, peak ratio (PR) and success rate (SR), and five different accuracy levels, i.e., ε = 1E-01, 1E-02, 1E-03, 1E-04, 1E-05, are utilized to measure the performance of an approach for MMOPs. The methods to compute the number of global optima, PR and SR can be found in [15].

TABLE II
SUCCESS RATES OBTAINED BY dADE, NVMO, MOMMOP, R2PSO, NCDE, NSDE AND BiMO FOR EACH PROBLEM

F	ε	dADE	NVMO	MOMMOP	R2PSO	NCDE	NSDE	BiMO	F	ε	dADE	NVMO	MOMMOP	R2PSO	NCDE	NSDE	BiMO
F_1	1E-01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	F_2	1E-01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	1E-02	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-02	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.96	**1.00**
	1E-03	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-03	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.80	**1.00**
	1E-04	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-04	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.67	**1.00**
	1E-05	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-05	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.63	**1.00**
F_3	1E-01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	F_4	1E-01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.98
	1E-02	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-02	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.98	0.98
	1E-03	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-03	**1.00**	**1.00**	0.86	**1.00**	**1.00**	0.02	0.98
	1E-04	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-04	**1.00**	**1.00**	0.78	**1.00**	**1.00**	0.00	0.98
	1E-05	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-05	**1.00**	**1.00**	0.62	**1.00**	**1.00**	0.00	0.96
F_5	1E-01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.98	**1.00**	F_6	1E-01	**1.00**	**1.00**	**1.00**	0.00	0.22	0.00	**1.00**
	1E-02	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.90	**1.00**		1E-02	**1.00**	**1.00**	**1.00**	0.00	0.00	0.00	**1.00**
	1E-03	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.70	**1.00**		1E-03	**1.00**	0.36	**1.00**	0.00	0.00	0.00	**1.00**
	1E-04	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.49	**1.00**		1E-04	0.78	0.00	**1.00**	0.00	0.00	0.00	**1.00**
	1E-05	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.24	**1.00**		1E-05	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**
F_7	1E-01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	F_8	1E-01	0.02	0.00	**1.00**	0.00	0.00	0.00	**1.00**
	1E-02	0.24	**1.00**	**1.00**	0.00	0.00	0.00	**1.00**		1E-02	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**
	1E-03	0.02	0.14	**1.00**	0.00	0.00	0.00	**1.00**		1E-03	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**
	1E-04	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**		1E-04	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**
	1E-05	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**		1E-05	0.00	0.00	**1.00**	0.00	0.00	0.00	**1.00**
F_9	1E-01	**1.00**	**1.00**	**1.00**	0.00	0.00	0.00	0.98	F_{10}	1E-01	0.50	**1.00**	**1.00**	0.82	**1.00**	0.00	**1.00**
	1E-02	**1.00**	0.00	**1.00**	0.00	0.00	0.00	0.98		1E-02	0.38	**1.00**	**1.00**	0.67	0.99	0.00	**1.00**
	1E-03	**1.00**	0.00	**1.00**	0.00	0.00	0.00	0.98		1E-03	0.28	**1.00**	**1.00**	0.55	0.90	0.00	**1.00**
	1E-04	**1.00**	0.00	0.94	0.00	0.00	0.00	0.98		1E-04	0.14	**1.00**	**1.00**	0.53	0.86	0.00	**1.00**
	1E-05	**1.00**	0.00	0.10	0.00	0.00	0.00	0.98		1E-05	0.02	0.66	**1.00**	0.12	0.80	0.00	**1.00**
F_{11}	1E-01	0.64	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.48	F_{12}	1E-01	**0.98**	0.22	0.96	0.00	0.78	0.00	0.36
	1E-02	0.00	0.00	**0.94**	0.00	0.22	0.00	0.48		1E-02	0.44	0.00	**0.88**	0.00	0.12	0.00	0.36
	1E-03	0.00	0.00	**0.64**	0.00	0.08	0.00	0.48		1E-03	0.00	0.00	**0.74**	0.00	0.00	0.00	0.36
	1E-04	0.00	0.00	0.02	0.00	0.06	0.00	**0.48**		1E-04	0.00	0.00	**0.70**	0.00	0.00	0.00	0.36
	1E-05	0.00	0.00	0.00	0.00	0.04	0.00	**0.48**		1E-05	0.00	0.00	0.12	0.00	0.00	0.00	**0.36**
F_{13}	1E-01	0.14	**1.00**	0.78	0.98	0.63	0.49	0.2	F_{14}	1E-01	0.70	**1.00**	0.00	**1.00**	**1.00**	0.98	0.00
	1E-02	0.00	0.00	**0.64**	0.00	0.00	0.00	0.16		1E-02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1E-03	0.00	0.00	0.00	0.00	0.00	0.00	**0.12**		1E-03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1E-04	0.00	0.00	0.00	0.00	0.00	0.00	**0.04**		1E-04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1E-05	0.00	0.00	0.00	0.00	0.00	0.00	**0.04**		1E-05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F_{15}	1E-01	**1.00**	**1.00**	0.00	0.69	**1.00**	**1.00**	0.00	F_{16}	1E-01	0.54	**1.00**	0.00	0.57	**1.00**	**1.00**	0.00
	1E-02	0.00	**0.02**	0.00	0.00	0.00	0.00	0.00		1E-02	0.00	**0.02**	0.00	0.00	0.00	0.00	0.00
	1E-03	0.00	0.00	0.00	0.00	0.00	0.00	0.00		1E-03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1E-04	0.00	0.00	0.00	0.00	0.00	0.00	0.00		1E-04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1E-05	0.00	0.00	0.00	0.00	0.00	0.00	0.00		1E-05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F_{17}	1E-01	0.76	**1.00**	0.00	0.04	**1.00**	0.86	0.00	F_{18}	1E-01	0.08	0.96	0.00	0.31	**1.00**	0.98	0.00
	1E-02	0.00	0.00	0.00	0.00	0.00	0.00	0.00		1E-02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1E-03	0.00	0.00	0.00	0.00	0.00	0.00	0.00		1E-03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1E-04	0.00	0.00	0.00	0.00	0.00	0.00	0.00		1E-04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1E-05	0.00	0.00	0.00	0.00	0.00	0.00	0.00		1E-05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F_{19}	1E-01	0.00	0.22	0.00	0.00	**0.67**	0.57	0.00	F_{20}	1E-01	0.00	0.00	0.00	0.00	**1.00**	**1.00**	0.00
	1E-02	0.00	0.00	0.00	0.00	0.00	0.00	0.00		1E-02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1E-03	0.00	0.00	0.00	0.00	0.00	0.00	0.00		1E-03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1E-04	0.00	0.00	0.00	0.00	0.00	0.00	0.00		1E-04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1E-05	0.00	0.00	0.00	0.00	0.00	0.00	0.00		1E-05	0.00	0.00	0.00	0.00	0.00	0.00	0.00

C. Comparison With Six State-of-the-Art Approaches

The performance of BiMO is compared with six state-of-the-art multimodal optimization algorithms, i.e., dADE [18], NVMO [19], MOMMOP [17], R2PSO [20], NCDE [21] and NSDE [21]. Among these algorithms, dADE, NCDE and NSDE, are based on different strategies of DE. R2PSO, NVNO and MOMOP are PSO-based, mesh population-based and multiobjective optimization based multimodal optimization algorithms, respectively. All the six algorithms are set with the same population size and MaxFEs as presented in Table I. The rest of parameters in these algorithms are set as recommended in their corresponding articles.

Table II and Table III report the experiment results of SR and PR at all five accuracy levels for each benchmark function. The best results obtained by the seven algorithms are highlighted in bold. From Table II and Table III, it can be seen that the proposed BiMO obtains 100% SRs and 100% PRs for F_1-F_3, F_5-F_9 and F_{10} at all five accuracy levels. For F_4 and F_9, BiMO also achieves relatively high SRs and PRs at five accuracy levels. However, for F_6-F_9, dADE, NVMO, R2PSO, DCDE and NSDE cannot obtain good SRs and PRs at the last two accuracy levels. It is worth noting that locating all the global optima for F_6-F_9 with limited computational resources is usually very difficult for an EA, since these four functions contain 18, 36, 81 and 216 optimal solutions, respectively. For the last ten complex functions, all of the seven EAs cannot consistently find all the optimal optima in all runs. Still, BiMO provides competitive performance with the other six algorithms on most of these functions.

Overall, the proposed BiMO can locate the multiple optima of MMOPs efficiently by taking the advantages of multiobjective optimization.

V. CONCLUSION

In this paper, a novel approach to transform an MMOP into a bi-objective optimization problem has been proposed. Based on this approach, we have developed a bi-objective optimization algorithm called BiMO for multimodal optimization. As analyzed in section III-A, when all the nondominated solutions of the transformed MOP are founded by BiMO, it is equivalent to find all the optimal solutions of the MMOP. Moreover, a modified solution comparison criterion has also been proposed

TABLE III
PEAK RITOS OBTAINED BY DADE, NVMO, MOMMOP, R2PSO, NCDE, NSDE AND BIMO FOR EACH PROBLEM

F	ε	dADE	NVMO	MOMMOP	R2PSO	NCDE	NSDE	BiMO	F	ε	dADE	NVMO	MOMMOP	R2PSO	NCDE	NSDE	BiMO
F_1	1E-01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	F_2	1E-01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	1E-02	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-02	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.98	**1.00**
	1E-03	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-03	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.87	**1.00**
	1E-04	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-04	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.78	**1.00**
	1E-05	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-05	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.75	**1.00**
F_3	1E-01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	F_4	1E-01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.99
	1E-02	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-02	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.99	0.99
	1E-03	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-03	**1.00**	**1.00**	**1.00**	0.97	**1.00**	0.27	0.99
	1E-04	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-04	**1.00**	**1.00**	**1.00**	0.95	**1.00**	0.24	0.99
	1E-05	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**		1E-05	**1.00**	**1.00**	**1.00**	0.91	**1.00**	0.24	0.97
F_5	1E-01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.99	**1.00**	F_6	1E-01	**1.00**	**1.00**	**1.00**	0.79	0.91	0.06	**1.00**
	1E-02	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.95	**1.00**		1E-02	**1.00**	**1.00**	**1.00**	0.71	0.73	0.06	**1.00**
	1E-03	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.85	**1.00**		1E-03	**1.00**	0.94	**1.00**	0.62	0.51	0.06	**1.00**
	1E-04	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.75	**1.00**		1E-04	0.98	0.67	**1.00**	0.54	0.31	0.06	**1.00**
	1E-05	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.61	**1.00**		1E-05	0.00	0.00	**1.00**	0.46	0.16	0.05	**1.00**
F_7	1E-01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	F_8	1E-01	0.84	0.41	**1.00**	0.18	0.60	0.01	**1.00**
	1E-02	0.96	**1.00**	**1.00**	0.58	0.88	0.05	**1.00**		1E-02	0.60	0.29	**1.00**	0.10	0.11	0.01	**1.00**
	1E-03	0.89	0.945	**1.00**	0.55	0.87	0.05	**1.00**		1E-03	0.55	0.27	**1.00**	0.05	0.01	0.01	**1.00**
	1E-04	0.82	0.90	**1.00**	0.48	0.87	0.05	**1.00**		1E-04	0.43	0.20	**1.00**	0.02	0.00	0.01	**1.00**
	1E-05	0.73	0.81	**1.00**	0.43	0.87	0.05	**1.00**		1E-05	0.36	0.03	**1.00**	0.01	0.00	0.01	**1.00**
F_9	1E-01	**1.00**	**1.00**	**1.00**	0.23	0.47	0.01	0.95	F_{10}	1E-01	0.99	**1.00**	**1.00**	0.98	**1.00**	0.10	**1.00**
	1E-02	**1.00**	0.68	**1.00**	0.20	0.46	0.01	0.95		1E-02	0.98	**1.00**	**1.00**	0.95	0.99	0.10	**1.00**
	1E-03	**1.00**	0.40	**1.00**	0.16	0.46	0.01	0.95		1E-03	0.98	**1.00**	**1.00**	0.94	0.99	0.10	**1.00**
	1E-04	**1.00**	0.28	**1.00**	0.12	0.46	0.01	0.95		1E-04	0.97	**1.00**	**1.00**	0.91	0.99	0.10	**1.00**
	1E-05	**1.00**	0.19	0.98	0.09	0.46	0.01	0.95		1E-05	0.95	0.97	**1.00**	0.84	0.98	0.10	**1.00**
F_{11}	1E-01	0.893	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.69	F_{12}	1E-01	**1.00**	0.85	**1.00**	0.58	0.97	0.14	0.78
	1E-02	0.67	0.67	**0.99**	0.65	0.84	0.25	0.69		1E-02	0.89	0.75	**0.99**	0.49	0.72	0.14	0.78
	1E-03	0.67	0.67	**0.94**	0.64	0.75	0.25	0.69		1E-03	0.75	0.73	**0.97**	0.43	0.41	0.14	0.78
	1E-04	0.67	0.67	0.72	0.64	**0.73**	0.25	0.69		1E-04	0.74	0.71	**0.96**	0.40	0.25	0.14	0.77
	1E-05	0.67	0.67	0.67	0.63	**0.70**	0.25	0.69		1E-05	0.73	0.57	**0.84**	0.35	0.18	0.14	0.77
F_{13}	1E-01	0.74	**1.00**	0.96	**1.00**	0.89	0.61	0.65	F_{14}	1E-01	0.92	**1.00**	0.78	**1.00**	**1.00**	0.98	0.63
	1E-02	0.67	0.67	**0.93**	0.66	0.67	0.23	0.65		1E-02	**0.67**	**0.67**	0.63	0.54	**0.67**	0.19	**0.67**
	1E-03	**0.67**	**0.67**	**0.67**	0.63	**0.67**	0.23	0.65		1E-03	0.66	**0.67**	**0.67**	0.48	**0.67**	0.19	0.63
	1E-04	**0.67**	**0.67**	**0.67**	0.63	**0.67**	0.23	0.65		1E-04	0.66	**0.67**	**0.67**	0.41	**0.67**	0.19	0.63
	1E-05	**0.67**	0.66	**0.67**	0.61	0.66	0.23	0.65		1E-05	0.66	0.64	**0.67**	0.37	**0.67**	0.19	0.63
F_{15}	1E-01	**1.00**	**1.00**	0.68	0.81	**1.00**	**1.00**	0.28	F_{16}	1E-01	0.87	**1.00**	0.67	0.66	**1.00**	**1.00**	0.48
	1E-02	0.63	**0.71**	0.65	0.19	0.47	0.13	0.28		1E-02	0.67	**0.70**	0.67	0.18	0.67	0.17	0.48
	1E-03	0.62	**0.67**	0.62	0.19	0.35	0.13	0.28		1E-03	**0.67**	0.65	**0.67**	0.14	**0.67**	0.17	0.48
	1E-04	**0.62**	**0.62**	0.61	0.17	0.32	0.13	0.28		1E-04	**0.67**	0.65	**0.67**	0.10	**0.67**	0.17	0.48
	1E-05	**0.62**	0.39	0.59	0.15	0.31	0.13	0.28		1E-05	**0.67**	0.63	**0.67**	0.08	**0.67**	0.17	0.48
F_{17}	1E-01	0.94	**1.00**	0.53	0.11	**1.00**	0.86	0.13	F_{18}	1E-01	0.68	0.99	0.50	0.40	**1.00**	0.98	0.34
	1E-02	0.47	0.48	**0.53**	0.03	0.25	0.11	0.13		1E-02	**0.66**	0.48	0.50	0.05	0.51	0.16	0.34
	1E-03	0.42	0.44	**0.53**	0.02	0.25	0.11	0.13		1E-03	**0.63**	0.47	0.50	0.04	0.50	0.16	0.34
	1E-04	0.41	0.41	**0.52**	0.02	0.25	0.11	0.13		1E-04	**0.63**	0.47	0.50	0.04	0.50	0.16	0.34
	1E-05	0.40	0.32	**0.49**	0.01	0.25	0.11	0.13		1E-05	**0.63**	0.36	0.50	0.03	0.49	0.16	0.34
F_{19}	1E-01	0.42	0.34	0.25	0.02	**0.81**	0.10	0.13	F_{20}	1E-01	0.03	0.00	0.13	0.03	**1.00**	0.98	0.13
	1E-02	0.14	0.13	0.25	0.01	**0.37**	0.10	0.13		1E-02	0.00	0.00	0.13	0.01	**0.25**	0.12	0.13
	1E-03	0.06	0.13	0.25	0.01	**0.36**	0.10	0.13		1E-03	0.00	0.00	0.13	0.00	**0.25**	0.12	0.13
	1E-04	0.02	0.13	0.25	0.00	**0.35**	0.10	0.13		1E-04	0.00	0.00	0.13	0.00	**0.25**	0.12	0.13
	1E-05	0.00	0.10	**0.25**	0.00	0.03	0.10	0.13		1E-05	0.00	0.00	0.13	0.00	**0.25**	0.12	0.13

to eliminate redundant individuals for a specific peak. As a result, the proposed BiMO can save many fitness evaluations and thus achieve the multiple solutions with a high level of accuracy. We have tested BiMO on 20 benchmark multimodal functions from CEC 2013. The performance of BiMO is compared with six state-of-the-art multimodal optimization algorithms. It can be concluded that BiMO performs better than, or at least comparably to, the other six algorithms in terms of peak ratio and success rate. Future work will focus on high-dimension multimodal optimization as well as the application of BiMO to multimodal engineering design problems.

REFERENCES

[1] K. Wong, K. Leung, and M. H. Wong, "Protein structure prediction on a lattice model via multimodal optimization techniques," in *Genetic and Evolutionary Computation Conference, GECCO 2010, Proceedings, Portland, Oregon, USA, July 7-11, 2010*, 2010, pp. 155–162.

[2] Q. Ling, G. Wu, and Q. Wang, "Restricted evolution based multimodal function optimization in holographic grating design," in *Proceedings of the IEEE Congress on Evolutionary Computation, CEC 2005, 2-4 September 2005, Edinburgh, UK*, 2005, pp. 789–794.

[3] E. Dilettoso and N. Salerno, "A self-adaptive niching genetic algorithm for multimodal optimization of electromagnetic devices," *IEEE Transactions on Magnetics*, vol. 42, no. 4, pp. 1203–1206, April 2006.

[4] D. E. Goldberg and J. Richardson, "Genetic algorithms with sharing for multimodal function optimization," in *International Conference on Genetic Algorithms on Genetic Algorithms and Their Application*, 1987, pp. 41–49.

[5] A. Pétrowski, "A clearing procedure as a niching method for genetic algorithms," in *International Conference on Evolutionary Computation*, 1996, pp. 798–803.

[6] B. L. Miller and M. J. Shaw, "Genetic algorithms with dynamic niche sharing for multimodal function optimization," in *International Conference on Evolutionary Computation*, 1996, pp. 786–791.

[7] R. Thomsen, "Multimodal optimization using crowding-based differential evolution," in *Proceedings of the IEEE Congress on Evolutionary Computation, CEC 2004, 19-23 June 2004, Portland, OR, USA*, 2004, pp. 1382–1389.

[8] G. R. Harik, "Finding multimodal solutions using restricted tournament selection," in *Proceedings of the 6th International Conference on Genetic Algorithms, Pittsburgh, PA, USA, July 15-19, 1995*, 1995, pp. 24–31.

[9] K. Deb and A. Saha, "Multimodal optimization using a bi-objective evolutionary algorithm," *Evolutionary Computation*, vol. 20, no. 1, pp. 27–62, 2012.

[10] A. Basak, S. Das, and K. C. Tan, "Multimodal optimization using a biobjective differential evolution algorithm enhanced with mean distance-based selection," *IEEE Transactions on Evolutionary Computation*, vol. 17, no. 5, pp. 666–685, Oct 2013.

[11] R. Storn and K. Price, *Differential evolution-a simple and efficient*

adaptive scheme for global optimization over continuous spaces. ICSI Berkeley, 1995, vol. 3.

[12] S. Das and P. N. Suganthan, "Differential evolution: a survey of the state-of-the-art," *IEEE Transactions on Evolutionary Computation*, vol. 15, no. 1, pp. 4–31, 2011.

[13] W.-J. Yu, M. Shen, W.-N. Chen, Z.-H. Zhan, Y.-J. Gong, Y. Lin, O. Liu, and J. Zhang, "Differential evolution with two-level parameter adaptation." *IEEE Transactions on Cybernetics*, vol. 44, no. 7, pp. 1080–1099, 2014.

[14] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, Apr 2002.

[15] X. Li, A. Engelbrecht, and M. G. Epitropakis, "Benchmark functions for cec2013 special session and competition on niching methods for multimodal function optimization," *RMIT University, Evolutionary Computation and Machine Learning Group, Australia, Tech. Rep*, 2013.

[16] J. Yao, N. N. Kharma, and P. Grogono, "Bi-objective multipopulation genetic algorithm for multimodal function optimization," *IEEE Transactions on Evolutionary Computation*, vol. 14, no. 1, pp. 80–102, 2010.

[17] Y. Wang, H. Li, G. G. Yen, and W. Song, "MOMMOP: multiobjective optimization for locating multiple optimal solutions of multimodal optimization problems," *IEEE Transactions on Cybernetics*, vol. 45, no. 4, pp. 830–843, 2015.

[18] M. G. Epitropakis, X. Li, and E. K. Burke, "A dynamic archive niching differential evolution algorithm for multimodal optimization," in *2013 IEEE Congress on Evolutionary Computation*. IEEE, 2013, pp. 79–86.

[19] D. Molina, A. Puris, R. Bello, and F. Herrera, "Variable mesh optimization for the 2013 cec special session niching methods for multimodal optimization," in *2013 IEEE Congress on Evolutionary Computation*. IEEE, 2013, pp. 87–94.

[20] X. Li, "Niching without niching parameters: particle swarm optimization using a ring topology," *IEEE Transactions on Evolutionary Computation*, vol. 14, no. 1, pp. 150–169, 2010.

[21] B.-Y. Qu, J. J. Liang, and P. N. Suganthan, "Niching particle swarm optimization with local search for multi-modal optimization," *Information Sciences*, vol. 197, pp. 131–143, 2012.