2016 International Conference on Computing, Networking and Communications, Mobile Computing and Vehicle Communications

# Sum of Arbitrarily Correlated Gamma Random Variables with Unequal Parameters and Its Application in Wireless Communications

Yizhi Feng[1,2], Miaowen Wen[1,2], Jun Zhang[1,2], Fei Ji[1,2], and Geng-xin Ning[1,2]
[1]School of Electronic and Information Engineering,
South China University of Technology, Guangzhou, 510640, China
[2]National Engineering Technology Research Center for Mobile Ultrasonic Detection,
South China University of Technology,Guangzhou, 510640, China
Email: yzfeng@scut.edu.cn, eemwwen@scut.edu.cn, eejzhang@scut.edu.cn,eefeiji@scut.edu.cn,ninggx@scut.edu.cn

***Abstract*—Characterizing the distribution for the sum of correlated Gamma random variables (RVs), especially for the sum of those with unequal fading and power parameters, is still an open issue. In this paper, based on the Cholesky factorization on the covariance matrix and moments matching method, we propose an approximate expression for the probability density function (PDF) of the sum of correlated Gamma RVs with unequal fading and power parameters and arbitrary correlation matrix. The proposed PDF expression is simple, accurate, and closed-form and thus can be conveniently used for general performance analysis in wireless communications. Simulation results are used to confirm the validity of the proposed PDF expression. The performance analysis of maximal-ratio combining (MRC) diversity system and cellular mobile radio system in wireless communications using the proposed PDF expression is also presented.**

***Index Terms*—Correlated Nakagami fading channels, sum of Gamma variables, unequal fading and power parameters, arbitrary covariance matrix, diversity system.**

## I. Introduction

Multipath fading is one of the main obstacles which seriously affect the performance of wireless communication system. To model the multipath fading statistics of the channel, Nakagami-$m$ distribution [1] is widely used and has attracted continued interest due to its wide versatility, experimental validity, and analytical tractability [2]-[5]. It has been a very popular fading model for performance analysis investigations in many important topics of wireless communications, such as diversity schemes, cochannel interference in cellular mobile radio systems, multihop relay networks, and so on [6]-[8].

As an efficient and powerful fading mitigation method, diversity technique has marked an important impact in the arena of wireless communication systems, in which the maximal-ratio combining (MRC) diversity scheme has been considered to be one of the optimal diversity combining schemes [8]. The performance analysis problem of MRC diversity combining system in a Nakagami-$m$ environment requires determination of the statistics (usually denoted by the probability density function, PDF) of the signal-to-noise ratio (SNR) at the combiner's output, which is the sum of the squared Nakagami-$m$ random variables (RVs) or equivalently the sum of the Gamma RVs since the square of a Nakagami-$m$ variable follows a Gamma distribution [1], [2], [6].

The PDF of the sum of the Gamma RVs has long been of interest in mathematics and wireless communications [9], and has been extensively studied over the past five decades. There are numerous papers on the PDF of the sum of the Gamma RVs, where most of the recent works focus on the correlated Gamma RVs [4], [8]-[11]. In [4], Alouini *et. al* derive an infinite-series representation for the PDF of the sum of arbitrarily correlated Gamma RVs with equal shape and unequal scale parameters. In [8], the PDF of the sum of independent but not necessarily identical distributed (i.n.i.d.) Gamma RVs is expressed in terms of Fox's $\overline{\mathrm{H}}$ function, which can be extended for the correlated case; however, the extending for the correlated case only applied to the Gamma RVs with equal shape parameter (also called fading parameter) $m$. In [9], the PDF of the sum of arbitrarily correlated and non-identically distributed Gamma RVs with non-identical and non-integer fading orders is derived but only the integer and half-integer values of fading parameter $m$ are considered and the PDF of the sum is also expressed as an infinite-series similar to [4]. The PDF of the sum of non-identical correlated Gamma RVs with integer fading parameters is derived in [10] whereas the result also involves a series of nested summations. In [11], the PDFs of the sum of correlated Gamma RVs with constant and exponential correlation are respectively given but only identical shape and scale parameters are considered. The characteristic function (CF) of the sum of correlated Gamma RVs is considered in [2], [3], [5], and [12], whereas no explicit form of the PDF is presented therein thus analyzing some performance measures may be complicated [9]. The multivariate Nakagami-$m$ PDF is presented using Green's matrix approximation in [13], whereas the identical shape and scale parameters are considered; moreover, it is very difficult, if not impossible, to obtain the PDF of the sum of the correlated Gamma RVs form the presented multivariate Nakagami-$m$ PDF therein.



978-1-4673-8579-4/16/$31.00 ©2016 IEEE

In this paper, we propose a simple, accurate, and closed-form approximation for the PDF expression of the sum of correlated Gamma RVs based on the moments matching method and the Cholesky factorization on the covariance matrix. Since unequal fading and power parameters and arbitrary correlation is considered for the Gamma RVs, the proposed PDF expression is very general and convenient for the performance evaluation in wireless communications.

The rest of this paper is organized as follows. The approximate PDF expression of the sum of correlated Gamma RVs based on the moments matching method and the Cholesky factorization on the covariance matrix is derived in section II. Section III gives numerical examples to illustrate the high accuracy and validity of the proposed PDF approximate expression. In section IV, the proposed PDF expression is applied to derive the average bit-error-rate (BER) for MRC diversity system with M-ary phase-shift (MPSK) and M-ary quadrature amplitude modulation (MQAM), as well as the outage probability for cellular mobile radio systems. The last section concludes this paper.

## II. PDF of the SUM of Correlated Gamma Variables with Arbitrary Parameters

Let $\gamma = [\gamma_1, \gamma_2, \ldots, \gamma_N]^T$ be an $N$ dimensional real column vector, where$[\cdot]^T$ means transposition, $\gamma_n \, (n = 1, 2, \ldots, N)$ are non-identically distributed and correlated Gamma RVs and the PDF of $\gamma_n$ is given by

$$f_{\gamma_n}(x) = \frac{1}{\Gamma(m_n)} \left(\frac{m_n}{\Omega_n}\right)^{m_n} x^{m_n - 1} \exp\left(-\frac{m_n}{\Omega_n} x\right) \quad (1)$$

where $\Gamma(\cdot)$ is the Gamma function [14], $\Omega_n$ is the mean fading power given by $\Omega_n = E[\gamma_n]$ with $E[\cdot]$ being the expectation operator, $m_n \geq 1/2$ is called as the fading parameter as well as the shape parameter and $\Omega_n / m_n$ the scale parameter. In what follows, we will use the notation $W \sim G(m, \Omega)$ to denote that $W$ follow Gamma distribution with fading parameter $m$ and mean fading power (also called as power parameter) $\Omega$. Let $\gamma$ be the sum of $\gamma_n, n = 1, 2, \ldots, N$, i.e.,

$$\gamma = \sum_{n=1}^{N} \gamma_n \quad (2)$$

We will show that $\gamma$ can be rewritten as the sum of a set of independent Gamma RVs. Let $\mathbf{R}_\gamma$ denote the covariance matrix of $\gamma$, then $\mathbf{R}_\gamma$ is given by

$$\mathbf{R}_\gamma = E\left[(\gamma - E[\gamma])(\gamma - E[\gamma])^T\right] \quad (3)$$

and the $(l, n)$th element of $\mathbf{R}_\gamma$ is given by

$$\mathbf{R}_\gamma(l, n) = \mathrm{cov}(\gamma_l, \gamma_n) = E[\gamma_l \gamma_n] - E[\gamma_l] E[\gamma_n] \quad (4)$$

By performing Cholesky factorization on the covariance matrix $\mathbf{R}_\gamma$, a lower triangular matrix $\mathbf{L}$ can be uniquely obtained such that [15]

$$\mathbf{R}_\gamma = \mathbf{L}\mathbf{L}^T \quad (5)$$

Let $\{w_n\}_{n=1}^{N}$ be a set of independent Gamma RVs where $w_n \sim G(m_{w,n}, \Omega_{w,n})$ and $\mathbf{w} = [w_1, w_2, \ldots, w_N]^T$ be an $N$ dimensional column vector with an $(N \times N)$ identity covariance matrix. Then $\gamma_n$ can be approximated by the sum of weighted independent Gamma RVs $w_k, k = 1, 2, \ldots, n$ [15]

$$\gamma_n = \sum_{k=1}^{n} l_{nk} w_k \quad (6)$$

where the fading parameter $m_{w,k}$ and power parameter $\Omega_{w,k}$ of $w_k$ can be obtained by matching the first and second moments of both sides of (6), $l_{nk}$ is the $(n, k)$th element of $\mathbf{L}$. It should be pointed out that in our method, the fading and power parameters of $w_k$ is not necessary for the derivation of the PDF of $\gamma = \gamma_1 + \gamma_2 + \cdots + \gamma_N$, which will be shown in the following part of this paper. From (6), $\gamma$ can be written as

$$\gamma = \sum_{n=1}^{N} \gamma_n = \sum_{n=1}^{N} \left(\sum_{k=1}^{n} l_{nk} w_k\right) \quad (7)$$

Combining the similar items, (7) can be rewritten as

$$\gamma = \sum_{n=1}^{N} \left(\sum_{k=n}^{N} l_{kn}\right) w_n \quad (8)$$

It's easy to show if $X \sim G(m, \Omega)$, then $aX \sim G(m, a\Omega)$, where $a$ is an constant number. Let

$$x_n = \left(\sum_{k=n}^{N} l_{kn}\right) w_n \quad (9)$$

Then $x_n \sim G\left(m_{w,n}, \left(\sum_{k=n}^{N} l_{kn}\right) \Omega_{w,n}\right)$ and we have

$$\gamma = \sum_{n=1}^{N} x_n \quad (10)$$

Since $\{w_n\}_{n=1}^{N}$ is a set of independent Gamma RVs, from (9), $\{x_n\}_{n=1}^{N}$ is also an independent Gamma RVs set. Since the sum of independent Gamma RVs can be approximated as a new Gamma distributed RV [1], it's easy to show that $\gamma \sim G(m_\gamma, \Omega_\gamma)$, namely, the PDF of $\gamma = \gamma_1 + \gamma_2 + \cdots + \gamma_N$ can be expressed as

$$f_\gamma(y) = \frac{1}{\Gamma(m_\gamma)} \left(\frac{m_\gamma}{\Omega_\gamma}\right)^{m_\gamma} y^{m_\gamma - 1} \exp\left(-\frac{m_\gamma}{\Omega_\gamma} y\right) \quad (11)$$

where the fading parameter $m_\gamma$ and mean fading power $\Omega_\gamma$ is given by [1. eq.(80)]

$$\begin{cases} m_\gamma = \dfrac{\left(\sum\limits_{n=1}^{N} \Omega_{x,n}\right)^2}{\sum\limits_{n=1}^{N} \frac{\Omega_{x,n}^2}{m_{x,n}}} \\ \Omega_\gamma = \sum\limits_{n=1}^{N} \Omega_{x,n} \end{cases} \quad (12)$$

where $m_{x,n}$ and $\Omega_{x,n}$ is the fading parameter and the mean fading power of $x_n$, respectively. Utilizing (6) and substituting

$m_{x,n} = m_{w,n}$ and $\Omega_{x,n} = \left(\sum_{k=n}^{N} l_{kn}\right)\Omega_{w,n}$ into (12), we obtain

$$\begin{cases} \Omega_\gamma = \sum_{n=1}^{N} \Omega_{x,n} = \sum_{n=1}^{N}\left(\sum_{k=n}^{N} l_{kn}\right)\Omega_{w,n} = \sum_{n=1}^{N}\Omega_n \\ m_\gamma = \dfrac{\left(\sum_{n=1}^{N}\Omega_{x,n}\right)^2}{\sum_{n=1}^{N}\frac{\Omega_{x,n}^2}{m_{x,n}}} = \dfrac{\left(\sum_{n=1}^{N}\Omega_n\right)^2}{\sum_{n=1}^{N}\frac{\left(\sum_{k=n}^{N} l_{kn}\right)^2\Omega_{w,n}^2}{m_{w,n}}} = \dfrac{\left(\sum_{n=1}^{N}\Omega_n\right)^2}{\sum_{n=1}^{N}\left(\sum_{k=n}^{N} l_{kn}\right)^2} \end{cases} \tag{13}$$

where we use the fact that $\mathrm{var}[w_n] = \Omega_{w,n}^2/m_{w,n} = 1$( $\mathbf{w} = [w_1, w_2, \ldots, w_N]^T$ have an identity covariance matrix). From (5), it can be easily obtained that

$$\sum_{n=1}^{N}\left(\sum_{k=n}^{N} l_{kn}\right)^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}\mathbf{R}_\gamma(i,j) \tag{14}$$

Therefore, we have

$$\gamma \sim G\left(\left(\sum_{n=1}^{N}\Omega_n\right)^2 \Big/ \sum_{i=1}^{N}\sum_{j=1}^{N}\mathbf{R}_\gamma(i,j), \sum_{n=1}^{N}\Omega_n\right) \tag{15}$$

On the other hand, it's easy to show that the expectation and variance of $\gamma = \gamma_1 + \gamma_2 + \cdots + \gamma_N$ is given by

$$\begin{cases} E[\gamma] = E\left[\sum_{n=1}^{N}\gamma_n\right] = \sum_{n=1}^{N}\Omega_n \\ var[\gamma] = var\left[\sum_{n=1}^{N}\gamma_n\right] = \sum_{i=1}^{N}\sum_{j=1}^{N}\mathbf{R}_\gamma(i,j) \end{cases} \tag{16}$$

From (15) and (16), we can draw an interesting and useful conclusion that the sum of arbitrary correlated Gamma RVs with unequal fading and power parameters can be approximated as a new Gamma RV whose fading parameter and mean fading power can be also directly and simply obtained by matching their first and second moments, which is the same result as that can be obtained for the sum of independent Gamma RVs.

## III. Numerical Examples

In this section, sample numerical examples are given to illustrate the high accuracy of the proposed PDF approximate expression by comparing the proposed PDF expression with the exact expression offered in [4] and the approximate expression proposed in [16] for constant and circular correlation model with equal shape and scale parameters, as well as with Monte Carlo simulations for arbitrary correlation model with arbitrarily unequal shape and scale parameters. The correlated Gamma RV set $\{\gamma_n\}$ for Monte Carlo simulation are generated 100000 times by using the algorithm in [15].

Fig. 1 shows a comparison of the PDF obtained from the proposed expression (11), the exact expression [4, eq. (5)], and the approximate expression in [16] for five correlated Gamma RVs with equal fading and power parameters and constant correlation. The parameters and correlation are the same as those in [4], i.e.,$N = 5$, $m = 2.5$, $\Omega = 1$, $\rho = \rho_{ij} = 0.64, i \neq j$, where

$$\rho_{ij} = \frac{\mathbf{R}_\gamma(i,j)}{\sqrt{\mathrm{var}(\gamma_i)\,\mathrm{var}(\gamma_j)}} \tag{17}$$

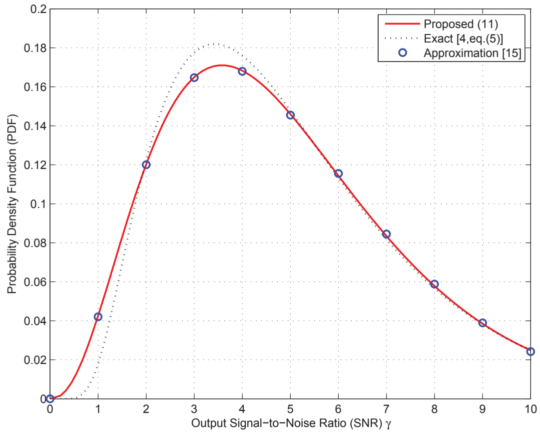


Fig. 1. Comparison between proposed, exact, and approximate PDFs for constant correlation with $N = 5$, $\Omega = 1$, $m = 2.5$, and correlation coefficients $\rho = \rho_{ij} = 0.64, i \neq j$.

denotes the correlation coefficient between $\gamma_i$ and $\gamma_j$. It can be observed perfect match between the proposed approximation (11) and the approximate solution in [16] for the PDF of the sum of correlated Gamma RVs with constant correlation model. It can be also observed that the PDF in Fig. 1 given by the proposed method match very well with the exact solution offered in [4], especially for high SNR.

The comparison between the proposed PDF obtained from (11), the exact PDF obtained from [4, eq. (5)], and the approximate PDF in [16] for circular correlation model with equal fading and power parameters is shown in Fig. 2, where $N = 5$, $m = 2.7$, $\Omega = 1$, and the correlation coefficient is given by

$$\rho_\gamma = \begin{pmatrix} 1 & 0.64 & 0.36 & 0.36 & 0.64 \\ 0.64 & 1 & 0.64 & 0.36 & 0.36 \\ 0.36 & 0.64 & 1 & 0.64 & 0.36 \\ 0.36 & 0.36 & 0.64 & 1 & 0.64 \\ 0.64 & 0.36 & 0.36 & 0.64 & 1 \end{pmatrix} \tag{18}$$

where the $(i,j)$th element of $\rho_\gamma$ is $\rho_{ij}$. Similar to the results for the PDFs with constant correlation model shown in Fig.1, it is shown that for the PDF of the sum of correlated Gamma RVs with circular correlation model, the proposed approximation (11) matches perfectly with the approximate solution in [16], and matches quite well with the exact solution offered in [4] in particular for high SNR.

Fig. 3 compares the PDFs obtained from the proposed expression (11) with the one obtained via Monte Carlo simulation for the sum of arbitrarily correlated Gamma RVs with unequal parameters. In order to check the accuracy of the proposed PDF approximation with more comprehensive insight, the different number of correlated Gamma RVs, say, $N = 3$, 4, and 5 is considered. For $N = 5$, we use the linearly arbitrary correlation model whose correlation matrix was given

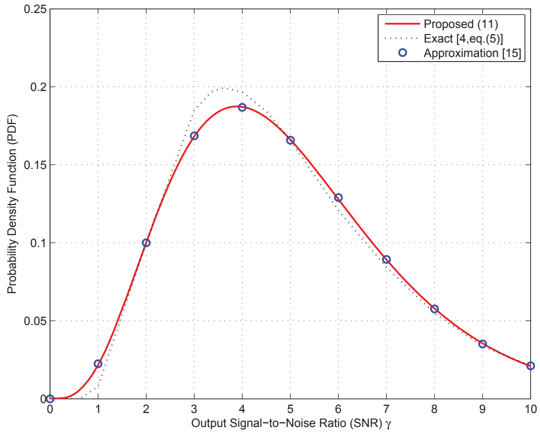


Fig. 2. Comparison between proposed, exact, and approximate PDFs for circular correlation with $N = 5$, $\Omega = 1$, $m = 2.7$, and correlation coefficients matrix (18).

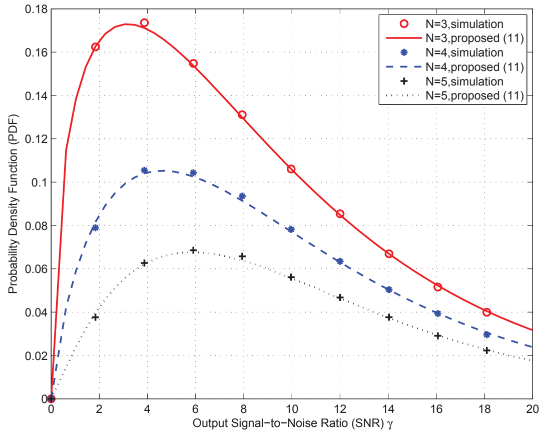


Fig. 3. Comparison between proposed and simulation PDFs for arbitrary correlation and unequal parameters with different number of Gamma variables $N$.

in [2] as

$$\rho_\gamma = \begin{pmatrix} 1 & 0.795 & 0.605 & 0.375 & 0.283 \\ 0.795 & 1 & 0.795 & 0.605 & 0.375 \\ 0.605 & 0.795 & 1 & 0.795 & 0.605 \\ 0.375 & 0.605 & 0.795 & 1 & 0.795 \\ 0.283 & 0.375 & 0.605 & 0.795 & 1 \end{pmatrix} \quad (19)$$

The fading parameters and the mean fading powers of the five Gamma RVs are $m = [1.0\ 1.2\ 1.4\ 1.6\ 1.8]$, $\Omega = [1.0\ 1.3\ 1.9\ 2.7\ 4.1]$, respectively. The correlation matrices for $N = 3$ and $N = 4$ can both be obtained from the above matrix in (19), which are respectively the 3rd and 4th order leading principle submatrices of the matrix in (19). Also, the fading and power parameters for $N = 3$ and $N = 4$ are respectively the 3rd and 4th order leading principle subvectors and can be obtained from those for $N = 5$. Again perfect match can be observed between the proposed method and Monte Carlo simulation, which demonstrates the high accuracy of the proposed approximation for the PDF of the sum of Gamma RVs.

## IV. Applications to the Performance of Wireless Communication Systems

The derived approximate PDF for the sum of correlated Gamma RVs in (11) can be conveniently used and give tractable results for the general performance investigation in many topics of wireless communications, such as the analysis of average BER for MRC diversity systems and outage probability for cellular mobile radio systems.

### A. Average BER for MRC Diversity Systems

Consider an MRC diversity receiver with $N$ diversity branches go through correlated Nakagami-$m$ fading channels. The branch SNRs are denoted by $\gamma_1, \gamma_2, \ldots, \gamma_N$ , respectively. The instantaneous SNR of the $l$th branch is $\gamma_l \sim G(m_l, \Omega_l)$, where $\Omega_l$ is the average SNR of the $l$th branch and $m_l$ is the fading parameter of the $l$th branch. The SNR at the MRC output is then given by $\gamma = \gamma_1 + \gamma_2 + \cdots + \gamma_N$. Since the branches are correlated, traditionally, the evaluation of the average BER requires the knowledge of the joint PDF of $\{\gamma_n\}_{n=1}^{N}$ and an $N$-fold integration [19]. Here we present a very simple method for the evaluation of the average BER. According to section II, $\gamma = \gamma_1 + \gamma_2 + \cdots + \gamma_N$ can be approximated as a new Gamma distributed RV such that $\gamma \sim G(m_\gamma, \Omega_\gamma)$, where $m_\gamma$ and $\Omega_\gamma$ can be obtained from (13) or (15). With the help of [17, eq. (17)] and the CF expression of Gamma variable, the average BER for the MRC receiver with coherent detection of MPSK signals can be expressed as

$$\begin{aligned} P_e &= \int_0^\infty \left[\frac{1}{\pi}\int_0^{\pi-\pi/M} \exp\left(\frac{-\gamma\sin^2(\pi/M)}{\sin^2\theta}\right)d\theta\right] f(\gamma)\,d\gamma \\ &= \frac{1}{\pi}\int_0^{\pi-\pi/M} \varphi_\gamma\left(\frac{-\sin^2(\pi/M)}{\sin^2\theta}\right)d\theta \\ &= \frac{1}{\pi}\int_0^{\pi-\pi/M}\left(1+\frac{\Omega_\gamma}{m_\gamma}\frac{\sin^2(\pi/M)}{\sin^2\theta}\right)^{-m_\gamma}d\theta \end{aligned} \quad (20)$$

where $\varphi_\gamma(s)$ is the CF of $\gamma$. Similarly, with the help of [17, eq. (25)], the average BER for the MRC receiver with coherent detection of MQAM can be expressed as

$$\begin{aligned} P_e &= \frac{4q}{\pi}\int_0^{\pi/2}\left(1+\frac{\Omega_\gamma}{m_\gamma}\frac{p}{\cos^2\theta}\right)^{-m_\gamma}d\theta \\ &- \frac{4q^2}{\pi}\int_{\pi/4}^{\pi/2}\left(1+\frac{\Omega_\gamma}{m_\gamma}\frac{p}{\cos^2\theta}\right)^{-m_\gamma}d\theta \end{aligned} \quad (21)$$

where $p = 1.5/(M-1)$ and $q = 1 - 1\big/\sqrt{M}$ [17]. Obviously, by using the proposed PDF expression for the sum of correlated Gamma RVs, the derivation of the average BER using $N$-branch MRC in correlated Nakagami fading reduces to a single integral, which becomes a very simple and closed-form expression.

### B. Outage Probability for Cellular Mobile Radio Systems

Consider a cellular mobile radio system where $N$ simultaneously active mobiles communicate with a single base station and all users are assumed to go through Nakagami-$m$ fading with their instantaneous signal power $s_n \sim G(m_{s,n}, \Omega_{s,n}), n = 0, 1, \ldots, N-1$. The $N-1$ interfering signals $\{s_n\}_{n=1}^{N-1}$ are assumed to be correlated each other whereas independent from the desired signal $s_0$ and the correlation coefficient between $s_i$ and $s_j$ is $\rho_{ij} = \rho_{ij}(i, j = 1, \ldots, N-1)$[4]. Let $s_I = \sum_{n=1}^{N-1} s_n$. Then $s_I \sim G(m_I, \Omega_I)$ and its PDF and fading parameters can be obtained from (11)-(14). Let $\lambda$ be the carrier-to-interference ratio (CIR), i.e., $\lambda = s_0/s_I$. The PDF of $\lambda$ can be easily obtained with the help of [18, eq.(6-60)] and [14, eq. (3.381.4)]. The outage probability is the probability that the CIR falls below a predetermined threshold $\lambda_{th}$[4], which can be derived with the help of [14, eq. (3. 194.1)] and given by

$$\begin{aligned} P_{out} = & \frac{\Gamma(m_{s,0}+m_I)}{\Gamma(m_{s,0})\Gamma(m_I)} \left(\frac{m_{s,0}\Omega_I}{m_I\Omega_{s,0}}\right)^{m_{s,0}} \frac{\lambda_{th}^{m_{s,0}}}{m_{s,0}} \\ & \cdot {}_2F_1\left(m_{s,0}+m_I, m_{s,0}; m_{s,0}+1; -\frac{m_{s,0}\Omega_I}{m_I\Omega_{s,0}}\lambda_{th}\right) \end{aligned} \tag{22}$$

where ${}_2F_1(a, b; c; z)$ is the Gauss hyper-geometric function [14, eq. (9.14.2)] . It's evident that the outage probability can be easily evaluated using (22).

## V. Conclusion

In this paper, we proposed a simple, accurate, and closed-form expression for the PDF of the sum of arbitrarily correlated Gamma RVs with unequal fading and power parameters. It's found that the sum of correlated Gamma RVs with unequal fading and power parameters and arbitrary correlation matrix can be approximated as a new Gamma RV whose fading parameter and mean fading power can be directly and simply obtained by matching their first and second moments, which is the same result as the sum of independent Gamma RVs. The proposed PDF expression for the sum of arbitrarily correlated Gamma RVs with unequal parameters can be conveniently used to investigate the general performance in wireless communications over arbitrarily correlated Nakagami-*m* fading channels, since it not only greatly simplifies the analysis and can give tractable and closed-form expressions of a number of performance measures, such as the average BER for MRC diversity systems and the outage probability of the cellular mobile radio systems, but also provides a PDF-based approach for the derivation of some performance measures which are harder analyze via moment generating function (MGF) or CF-based approaches.


## Acknowledgement

The authors would like to thank the editor and the anonymous reviewers for their constructive and helpful comments and suggestions in improving the manuscript. This work was supported in part by the National Natural Science Foundation of People's Republic of China (61372080, 61372083, 61431005), the Guangdong Natural Science Foundation (S2013010013376), and the Guangzhou Science and Technology Project (No.2012J2200005).


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