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Development and optimization of a resonance-based mechanical dynamic absorber structure for multiple frequencies

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Abstract

This paper presents a new dynamic absorber attenuating the vibrations at three resonance frequencies simultaneously. The dynamic vibration responses of mechanical systems with dynamic absorbers are mainly influenced by how close the eigenfrequencies of the installed dynamic absorbers are to the eigenfrequencies of the hosting structure. To suppress structural vibration at single target frequency, it is enough to install a single mass tuned dynamic absorber whose eigenfrequency is tuned to the excitation frequency. To suppress structural vibrations at multiple frequencies, multiple dynamic absorbers whose eigenfrequencies are tuned differently can be installed. Inevitably this approach increases the total weight of the dynamic absorbers and the manufacturing cost. To overcome the limitation and to attenuate vibrations over a wide range of frequencies, a new dynamic absorber without a damper tuned for multiple frequencies was presented in this research. The present dynamic absorber consists of a mass and two arm-beams connected to a hosting structure. By changing the geometric dimensions, it is possible to precisely tune the eigenfrequencies of the dynamic absorbers to those of hosting structure. The present study also adopts an optimization process to tune the geometric parameters based on the numerical optimization algorithm. With several numerical examples and an experiment, the validity of the present dynamic absorber is presented.

Keywords

Dynamic absorber, multiple frequencies, size optimization, tuned mass-spring damper

Introduction

Exploiting the availability of efficient dynamic absorber, finite element (FE) procedure, and structural optimization, this research develops a new dynamic absorber for multiple frequencies while reducing the mass of the dynamic absorber. Dynamic absorber with mass-spring (sometimes with damping) system illustrated in Figure 1 has been widely used to mitigate structural vibration in mechanical and civil structures for the effectiveness and the easy installation. The natural frequencies of dynamic vibration absorber are chosen to be equal or near to excitation frequencies and resonance frequencies of hosting structure. By tuning the frequencies of passive dynamic absorber and making the reaction force equal to the primary structure equal but opposite to the disturbing force, it is theoretically possible for the primary structure not to vibrate at all. With time-varying frequency of external force, however, the passive vibration absorber with fixed eigenfrequencies is not effective. To improve the effectiveness, it is common to add a damper to a passive vibration absorber. It is also an important subject in engineering to attenuate some vibration modes simultaneously. In order to achieve this purpose, it is possible to

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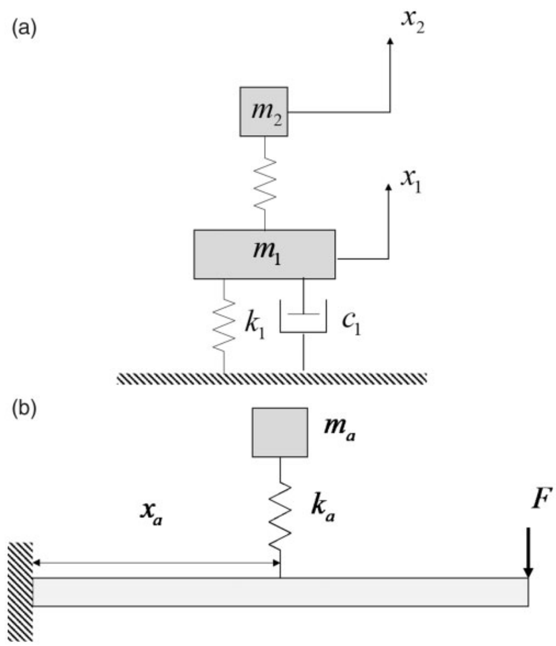

Figure 1. Passive dynamic absorber: (a) passive tuned mass damper and (b) dynamic absorbers attached to a beam.

add or install several different dynamic absorbers tuned for the vibration modes of a hosting structure. Inevitably high-priced equipment and a large space are required. To resolve this issue, this research aims to tune the eigenmodes of a dynamic absorber to the eigenmodes of a hosting structure and attenuates the vibration modes simultaneously only with the tuned dynamic absorber.

It has been an important subject to control the vibration of structure in both narrow and broad band frequencies using dynamic vibration absorber.[1–3] Besides the mass-spring dynamic absorber, a various type of dynamic absorbers have been proposed. A magnetorheological damper was proposed to suppress the vibration.[4] To control the vibration of a structure, an active shut technique using piezoelectric laminated structure was developed and validated experimentally.[5] The Belleville washer was implemented to form a nonlinear spring for a dynamic absorber to suppress vibrations of machine.[6] An experimental investigation of the dynamic absorber to an 11-story reinforced concrete shear-core office building was studied.[7,8] The nonlinear vibrating mechanical system described by the Duffing equation was presented.[9] Some active control methods to suppress vibrations also can be found.[10] The rotational absorber was also proposed.[3,11–13] To design a multidynamic vibration absorber and various dynamic absorbers, the optimization algorithm was applied.[14–19] The closed-form solution of the optimal design of dynamic vibration absorbers was proposed[20] and an optimization problem minimizing the standard deviation of difference among vibration amplitudes was presented. To suppress the vibration of beam-type structure, two inerter-based passive vibration configurations were also proposed.[21]

Several types of dynamic absorbers also have been proposed. When the frequencies of the excitation loads are known in prior, the tuned mass damper system is effective and robust and it shows various applications in mechanical and civil engineering applications.[22–24] However, it is also noticed that if the mass ratio between the masses of a hosting structure and a vibration damper is below 1%, the vibration reduction magnitude may be insufficient.[25–27] To overcome this limitation, the actively controlled tuned mass damper, the semi-actively controlled tuned mass damper, the actively controlled vibration absorbers and the semi-actively controlled vibration absorbers have been researched.[25–27] It was proposed that employing active tuned mass damper and active actuator, spring and damper are placed parallel together and the force exerted by the active actuator can be controlled to modify the working frequency and the damping effect. With the semi-actively controlled tuned mass damper, the damping force for each half-vibration cycle can be modulated with a passive mass-spring system and semi-active damper. Recently, the metamaterial-based resonators have been proposed.[28–30] Depending on the excitation load condition and the type of hosting structure, various dynamic absorbers have been proposed.

This research presents a new passive dynamic absorber suppressing vibrations at different frequencies simultaneously. One of the shortcomings of the conventional dynamic vibration absorber may be that it only absorbs the vibration at a certain frequency of a linear continuous structure. This characteristic of the dynamic absorber can be advantageous when the loading condition, i.e. force or moment, contains one single excitation frequency. But eventually, external loads in realistic vibrating mechanical systems contain several excitation frequencies and this becomes inefficient to use one kind of dynamic absorber. In this case, an active vibration absorber is recommended for variable frequency operation conditions. To overcome this shortcoming, this research presents a new type of passive dynamic absorber attenuating two bending modes and one torsional mode. This dynamic absorber consists of two beams connected a hosting structure. The size of the dynamic absorber studied here is about 10 cm but its sizes can be tuned and optimized to the dynamic absorber application of another size system.

To change the eigenfrequencies of the dynamic absorber, the relationship between the first three eigenfrequencies and the geometric parameters characterizing the present dynamic absorber is considered. Relying on the theories of solid mechanics and mechanical vibration, it may be possible to derive the approximate eigenfrequencies of the dynamic absorber for the passive vibration absorber. Recently, the developments of FE theory, computer hardware and software make it enable to compute the eigenfrequencies effectively and fast. For the application of the present dynamic absorber for mechanical vibration systems with different external vibrations, the parametric curves of the first three eigenfrequencies, i.e. two bending modes and one torsional mode, of the present dynamic absorber with respect to the geometric parameters are obtained. Although the relationship (curves) is complex, it is possible to change the geometric parameters to change the eigenfrequencies to desired frequencies. However, when very different frequencies should be considered or the different sizes of the dynamic absorber should be found, some limitations still exist with these curves. Furthermore, as the characteristic length of the dynamic absorber is about 10 cm, it may be difficult to find out very different size dynamic absorbers. To overcome this difficulty, this research suggests to apply the structural optimization scheme optimizing the geometric dimensions to minimize the proof mass subject to the frequency constraints. The objective function is chosen to minimize the proof mass to minimize the material cost. Using the sequential quadratic programming (SQP) method, it is possible to find out the geometric values of the dynamic absorber for different frequencies. Some numerical simulation and experiment results are provided in order to show the efficient performance of the present passive vibration absorber for multiple vibration modes.

This paper is organized as follows: the upcoming section describes the basic equations of the dynamic absorber in connection to a beam. Next section presents a template of the dynamic absorber and an optimization process to determine the geometric parameters of the absorber for three frequencies. Subsequently, several numerical and experiment examples are presented. Finally, the conclusions and future research topics are provided.

Theory of resonance-based dynamic absorber

Before presenting the dynamic absorber for three frequencies, this section provides the review of the basic equations and the characteristics of the resonance-based dynamic absorber. To understand the characteristics of the dynamic absorber, sometimes called a tuned mass damper system, we can consider the classical mechanical system of Figure 2.[1] Without the loss of generality, the vibration behavior of the beam of length L often can be modeled with the Bernoulli–Euler beam theory and a dynamic vibration absorber with m mass and k stiffness is

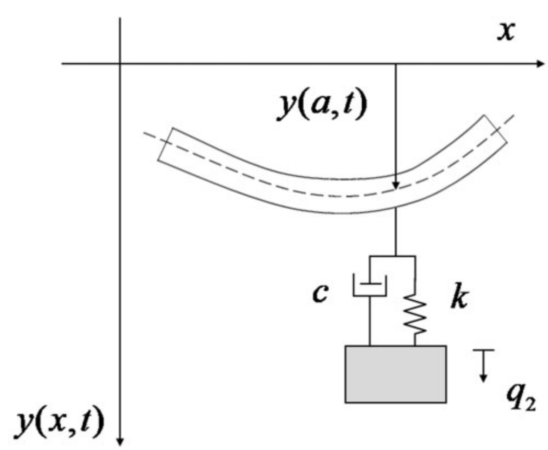

Figure 2. Vibration of a beam attached with a dynamic vibration absorber.[1]

assumed to be attached at a point $x = a$ on the beam. Assuming that the install of the dynamic absorber does not affect the free vibration mode of the hosting structure that is one of the main assumptions, the free vibration modes $\phi_i(x)$ can be obtained with the following conditions.

$$\frac{d^4\phi_i}{dx^4} = \beta_i\phi_i, \beta_i^4 = \frac{\omega_i^2 m_b}{EI} \tag{1}$$

$$\int_0^L \phi_i\phi_j \mathrm{d}x = \begin{cases} L\, i = j \\ 0\, i \neq j \end{cases}, \int_0^L \left(\frac{d^2\phi_i}{dx^2}\right)^2 \mathrm{d}x = \beta_i^4 L \tag{2}$$

where the mass, the Young's modulus, and the second moment of inertia are denoted by m_b, E, and I, respectively. The absolute coordinate of the absorber mass motion is denoted by q_2. The transverse vibration of the beam $y(x, t)$ with the ith eigenmode is formulated as follows

$$y(x, t) = q_1(t)\phi_i(x) \tag{3}$$

Neglecting the effect of the damping in Figure 2, the governing equation of the system can be formulated by the Lagrange approach as follows

$$\begin{aligned} KE &= \frac{1}{2}m\dot{q}_2^2 + \frac{1}{2}\rho A\int_0^L \dot{y}^2 \mathrm{d}x \\ PE &= \frac{1}{2}k\Big(y(a, t) - q_2\Big)^2 + \frac{1}{2}EI\int_0^L \left(\frac{d^2y}{dx^2}\right)^2 \mathrm{d}x \end{aligned} \tag{4}$$

Using the virtual work done by a harmonic externally applied force $f(x)e^{j\omega t}$, the virtual work is simplified as follows

$$\begin{aligned} \delta W &= \int_0^L f(x)e^{j\omega t}\delta y \mathrm{d}x \\ \delta W &= \delta q_1 \varepsilon e^{j\omega t} \end{aligned} \tag{5}$$

where ε is a force-dependent parameter to simply the integration.[1] Then the governing equation in the frequency domain can be formulated as follows

$$\begin{bmatrix} \rho AL & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} k\phi_i^2(a) + EI\beta_i{}^4 L & -k\phi_i(a) \\ -k\phi_i(a) & k \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} \varepsilon e^{i\omega t} \\ 0 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} k\phi_i^2(a) + EI\beta_i{}^4 L - \omega^2\rho AL & -k\phi_i(a) \\ -k\phi_i(a) & k - m\omega^2 \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} \varepsilon \\ 0 \end{bmatrix} \tag{7}$$

$$\begin{aligned} Q_1 &= \frac{(k - m\omega^2)\varepsilon}{\text{Determinant}}, \quad Q_2 = \frac{k\phi_i(a)\varepsilon}{\text{Determinant}} \\ \text{Determinant} &= EI\beta_i{}^4 Lk - \Big(A\rho Lk + mk\phi_i^2(a) + EI\beta_i{}^4 mL\Big)\omega^2 + Am\rho L\omega^4 \end{aligned} \tag{8}$$

where Q_1 and Q_2 are the solutions of q_1 and q_2 in the frequency domain. With the following nondimensional numbers, the governing equation can be further simplified as follows

$$\text{Dimensionless frequency}: \lambda = \frac{\omega}{\beta_i^2}\sqrt{\frac{\rho A}{EI}} \tag{9}$$

$$\text{Tuning ratio}: T^2 = \left(\frac{k}{m}\right)\left(\frac{\rho A}{EI\beta_i^4}\right) \tag{10}$$

$$\text{Mass ratio}: \mu = \left(\frac{m}{\rho AL}\right) \tag{11}$$

$$Q_1 = \frac{\varepsilon}{EI\beta_1^4 L}\left\{\frac{T^2-\lambda^2}{\lambda^4-\lambda^2\left(1+T^2\left(1+\mu\phi_i^2(a)\right)\right)+T^2}\right\} \tag{12}$$

$$y(x,\lambda) = \frac{\varepsilon\phi_i(x)}{EI\beta_1^4 L}\left\{\frac{T^2-\lambda^2}{\lambda^4-\lambda^2\left(1+T^2\left(1+\mu\phi_i^2(a)\right)\right)+T^2}\right\} \tag{13}$$

The above equations can be used as metrics for tuning the dynamic absorber. From the above classical dynamic absorber theory, we can observe the followings:

Note 1: With $T=\lambda$ or $\frac{k}{m}=\omega^2$ and nonzero for $\phi_i(a)$, the response y or Q_1 mathematically become zeros.

$$\begin{bmatrix} k\phi_i^2(a)+EI\beta_i{}^4L-\frac{k}{m}\rho AL & -k\phi_i(a) \\ -k\phi_i(a) & 0 \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} \varepsilon \\ 0 \end{bmatrix} \tag{14}$$

$$Q_1 = \frac{(k-m\omega^2)\varepsilon}{k^2\phi_i^2(a)}, \quad Q_2 = -\frac{\varepsilon}{k\phi_i(a)} \tag{15}$$

Note 2: With $\frac{k}{m}=\omega^2$ and nonzero $\phi_i(a)$, the response Q_1 is independent on the displacement at the dynamic absorber in theory. In other words, the displacement of the junction of the dynamic absorber, i.e. $\phi_i(a)$, does not affect the system with the assumption that the install of the dynamic absorber does not affect the free vibration mode of the hosting structure.

Note 3: Adversely, the note 2 may suggest that some optimal points exist for the location of the dynamic absorber; in terms of the definition of the optimization, the optimal values of $\phi_i(a)$ can be differentiated.

Note 4: When the dynamic absorber is ideally located at the node points ($\phi_i(a)=0$ or a point along a standing wave where the wave has minimum amplitude) of eigenmodes, the systems are decoupled and the displacement of the dynamic absorber mathematically does not affect to Q_1.

$$\begin{gathered}\begin{bmatrix} EI\beta_i{}^4L-\omega^2\rho AL & 0 \\ 0 & k-m\omega^2 \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} \varepsilon \\ 0 \end{bmatrix} \\ Q_1 = \frac{\varepsilon}{EI\beta_i{}^4L-\omega^2\rho AL}, \quad Q_2 = 0\end{gathered} \tag{16}$$

Note 5: With complex three-dimensional structures, it is essentially required not to place the dynamic absorber to the nodal points ($\phi_i(a)=0$) of hosting structure.

Note 6: The above equations assume that the attachment of the dynamic absorber does not affect the responses of the hosting structure but there are some situations when the mass of the dynamic absorber is heavier enough to affect the response of the hosting structure.

Multi-degree-of-freedom dynamic absorber

For the sake of mitigating structural vibrations at three frequencies, this section proposes a new dynamic absorber consisting of one mass and two side arms whose first three eigenmodes are two bending modes and one twisting mode. It aims to attenuate the dynamic vibrations of three modes simultaneously. In order to observe the dynamic characteristics of this absorber, the geometric parameters are varied and the eigenfrequencies are plotted versus the varied variables. This parametric study can be employed as a heuristic algorithm tuning the eigenfrequencies with small frequency differences among the eigenfrequencies of a hosting structure and the dynamic absorber. In addition, to cope with large differences in eigenfrequencies not covered by this heuristic tuning algorithm, this section also proposes a size optimization framework minimizing the tip mass subject to the errors between the eigenfrequencies of the absorber and a hosting structure.

A template of a dynamic absorber for three modes

Conventional dynamic absorber in mechanical and civil engineering aims to attenuate vibration at a single resonance. It is efficient and widely used for the mitigation of structural vibrations in the field of mechanical and civil engineering. When the suppression of several vibration modes is required, multiple dynamic absorbers with different eigenmodes can be installed. However, the increased total weight of several dynamic absorbers inevitably affects the vibration modes of the hosting structure and some side effects such as the cost increase, the complex installation or the space confinement can also occur. Therefore, it inevitabely can deteriorate the performance of the dynamic absorber. To resolve this side effect, it is possible to develop a single dynamic absorber for multiple vibration modes of hosting structure. To achieve this purpose, this research proposes to use the dynamic absorber in Figure 3 with one mass and two side arms whose first frequencies with a given geometry are about 4.5221 Hz, 25.4584 Hz, and 74.2961 Hz, respectively. The ends of the two arms of this dynamic absorber can be attached to the surfaces of hosting structure. As shown in Figure 3, the first and third modes are the bending modes and the second mode is the twisting/torsional mode. Not to mention, the eigenfrequencies and the mode shapes are determined by the geometric dimensions as illustrated in Figure 4.

The geometric parameters of the dynamic absorber affect the eigenfrequencies. The first and third eigenfrequencies are decreased with respect to the third power of the length of the arms l_s as the bending rigidity is decreased with respect to the three power of the arm length. The second twisting eigenfrequency is decreased with respect to the length as the twisting rigidity is proportionally decreased with respect to the length. By increasing the width b_s, the eigenfrequencies are increased however due to the mass effect of the arms, some nonlinear behaviors are observed as shown in Figure 4(c). By increasing the thickness, t_s, the eigenfrequencies are also increased (Figure 4(d)). By increasing the dimensions of the mass, i.e. l_m, b_m, and t_m, the eigenfrequencies are decreasing. Compared with the other design variables, the design variable, d_s, allows us to tune the eigenfrequencies in detail. Note that these curves can make it possible to tune this dynamic absorber by an engineer.

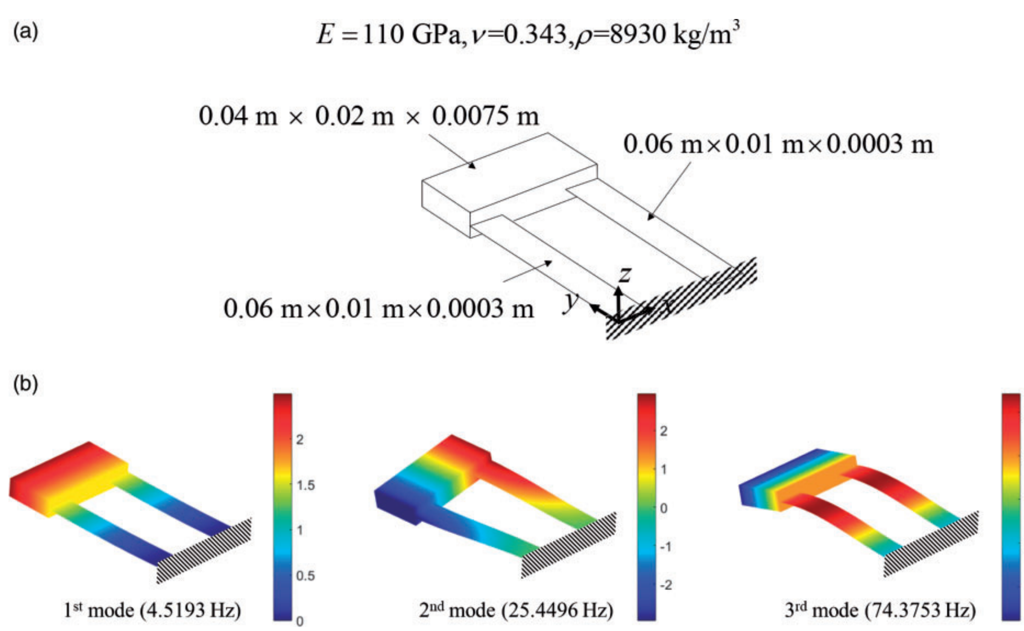

Figure 3. A design of multifrequency vibration absorber and the mode shapes: (a) a design of multifrequency vibration absorber and (b) the mode shapes.

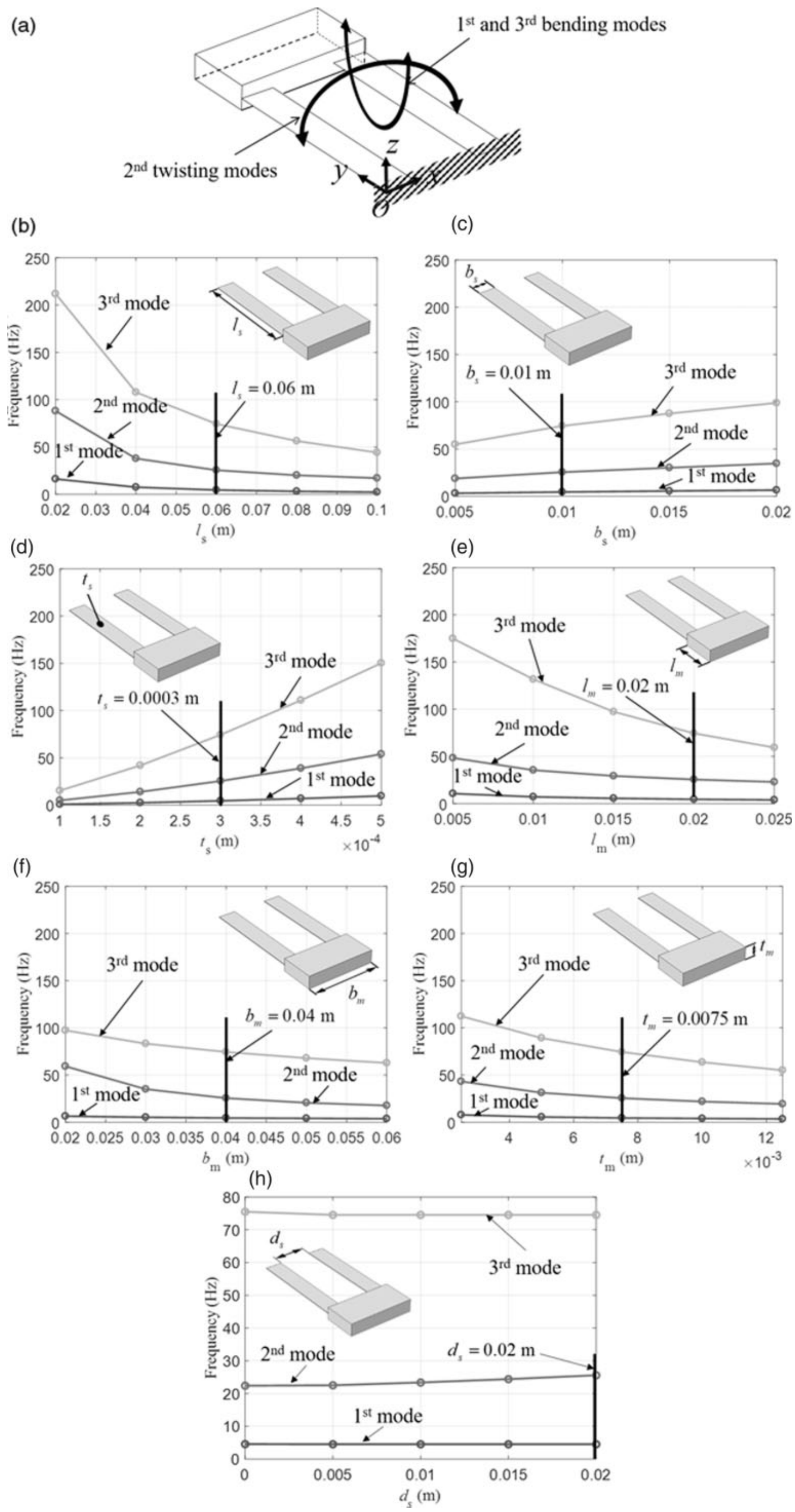

Figure 4. Analysis of the effect of each parameter on the first, the second, and the third eigenfrequencies: (a) mode shapes of the dynamic absorber and (b)–(f) eigenfrequencies according to seven geometric parameters.

Parametric optimization of the dynamic absorber

The dynamic absorber in Figure 3 can attenuate the three vibration modes about 4 Hz, 25 Hz, and 74 Hz, simultaneously. By changing the dimensions of this dynamic absorber, it can be tuned for another hosting structure with different resonance frequencies. Figure 4(b) to (h) illustrate the variations of the eigenfrequencies with respect to the geometric dimensions and as discussed engineers can rely on them to change the geometric parameters for different frequencies. This approach can be effective in case of small differences among the eigenfrequencies. On the other hand, it is possible to use the size optimization process to determine the geometric dimensions of the dynamic absorber by solving the following optimization problem in the equation (17) minimizing the mass of the dynamic absorber subject to the constraints for the eigenfrequencies. This optimization problem can be efficiently

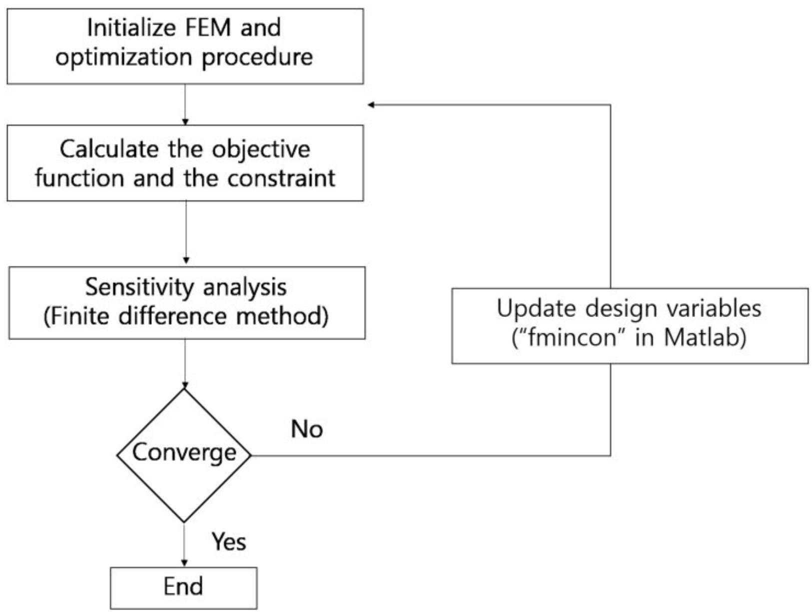

Figure 5. Optimization procedure.

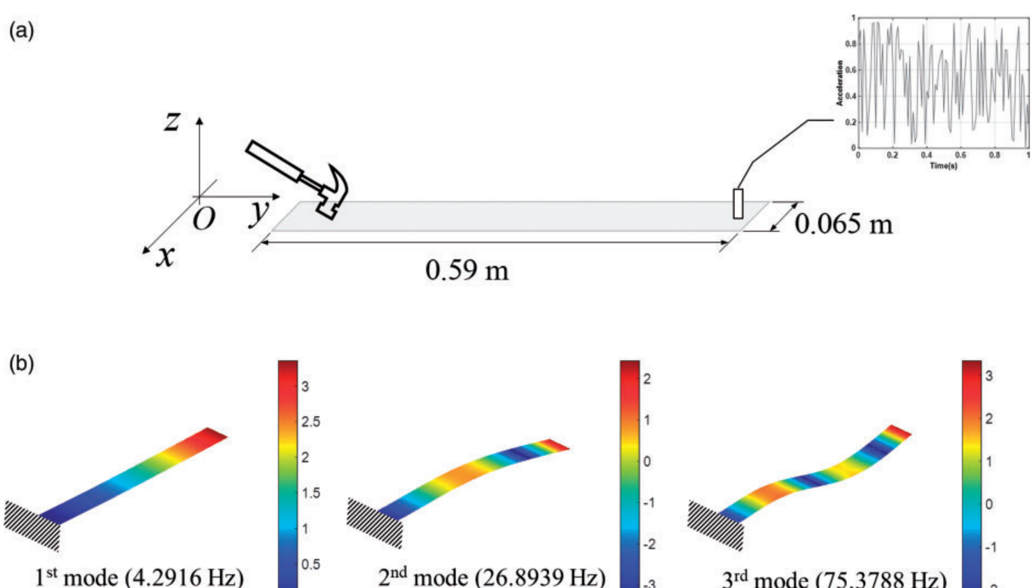

Figure 6. (a) The FE model of fix-free beam and (b) the simulated three frequency modes (material property: 183 GPa, Poisson's ratio: 0.3, Density: 7840 kg/m^3, 0.59 m $\times$ 0.065 m $\times$ 0.0019 m).

solved by the gradient optimizer and this research employs the SQP approach, i.e., *fmincon* in Matlab, for an optimizer. For the sensitivity analysis, the finite difference method is employed. Figure 5 presents the overall optimization procedure. With this size optimization framework, it is possible to effectively determine the geometric parameters of the dynamic absorber for different eigenfrequencies. The existence of an optimal solution of the size optimization problem in the equation (17) should be considered from a mathematical point of view. In case of the nonexistence of an optimal solution with the present dynamic absorber with one mass and two arms,

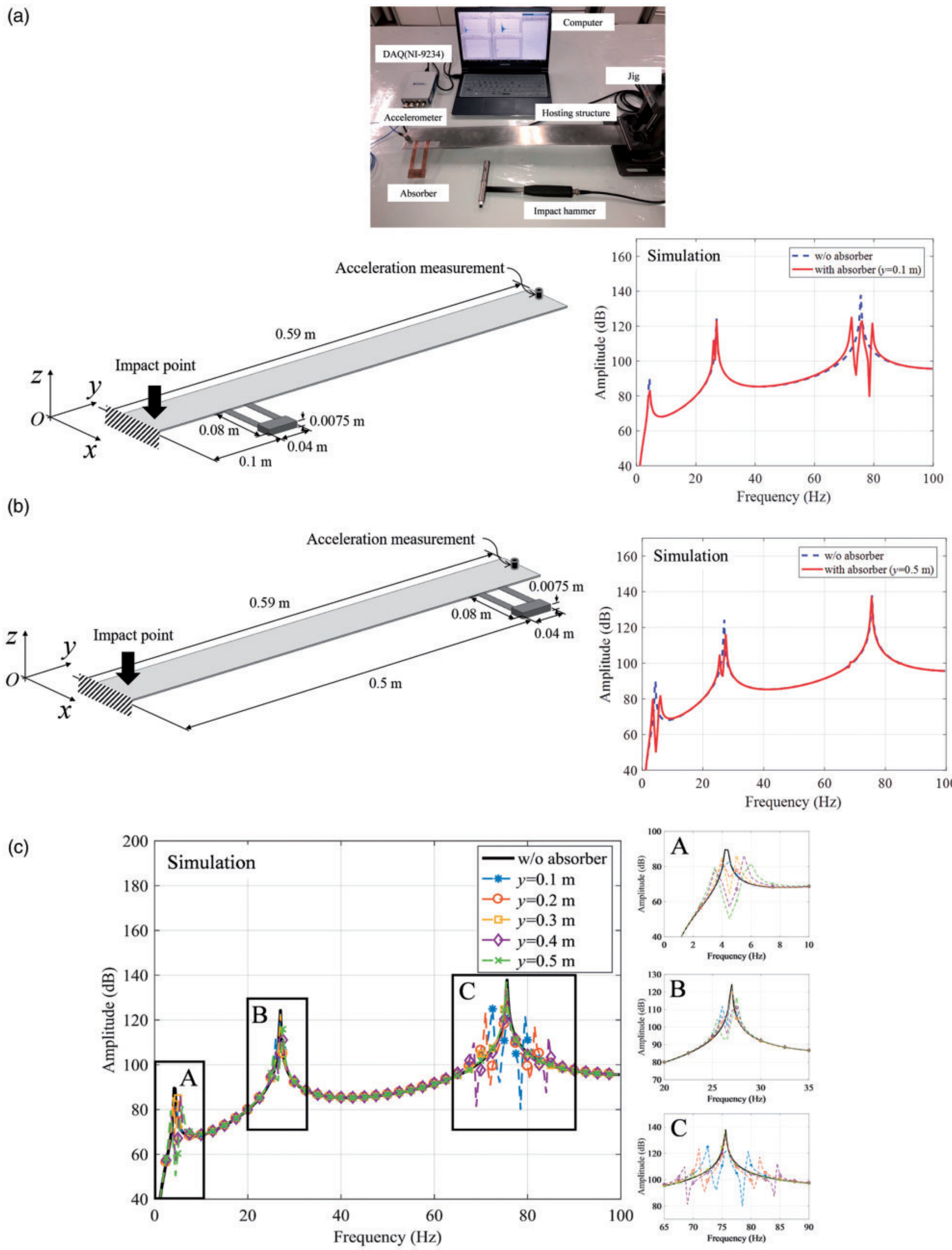

Figure 7. The effect of the location of the dynamic absorber: (a) the FE simulated frequency response functions at $y = 0.1$ m, (b) the FE simulated frequency response functions at $y = 0.5$ m, and (c) the response curves with several locations.

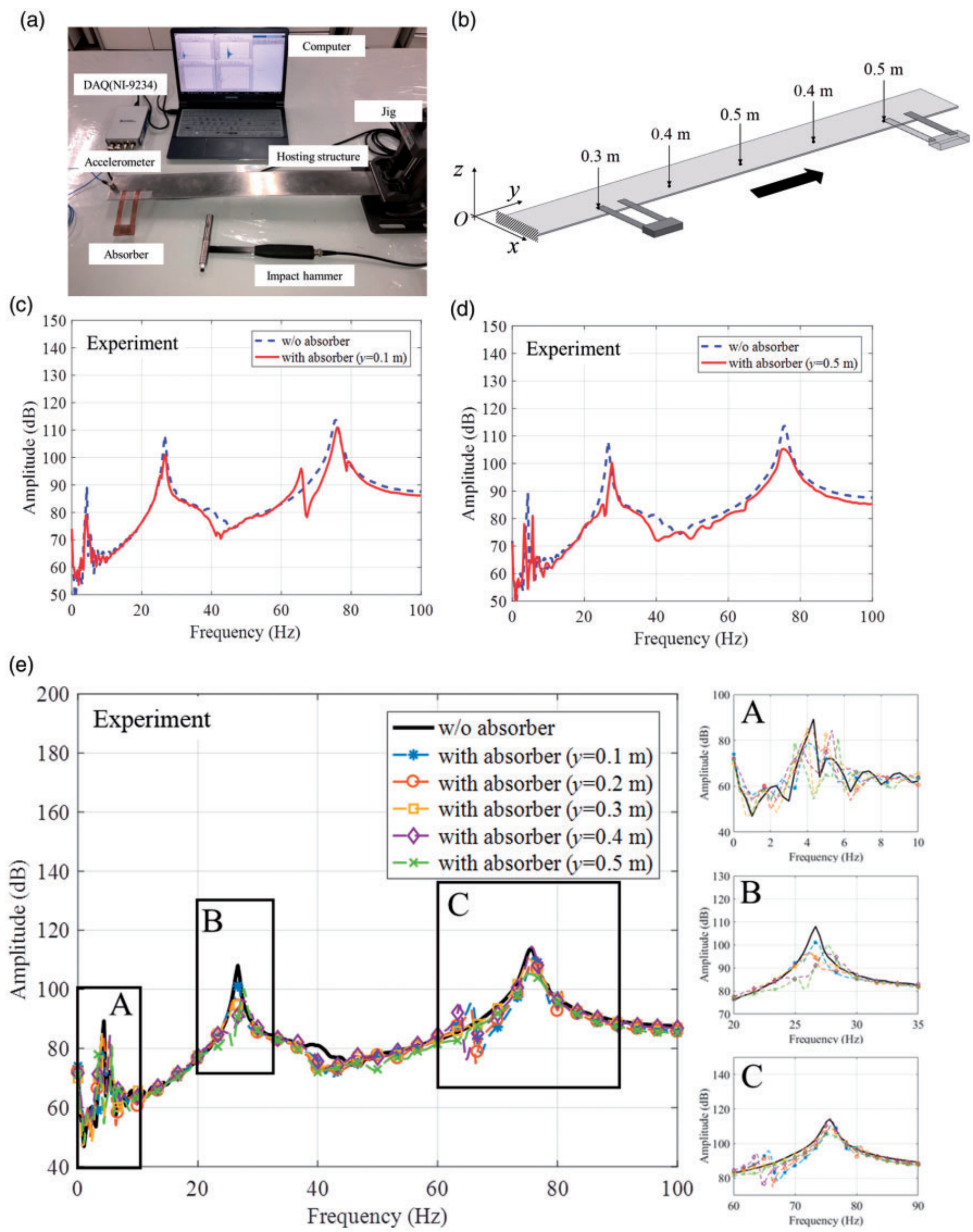

Figure 8. Vibration reduction effect by the multifrequency vibration absorber at the different positions: (a) experiment setup, (b) the position of the dynamic absorber, (c) an experimental frequency response function result at 0.1 m, (d) an experimental frequency response function result at 0.5 m and (e) vibration experiment results at different positions.

it is possible to increase the degrees of freedom in geometry to the absorber by adding arms or changing the optimization parameters.

$$\begin{aligned} &\underset{\mathbf{X}}{\text{Minimize}}\ l_m \times b_m \times t_m \\ &\text{Subject to}\ \left(f_1 - f_1^t\right)^2 \leqslant \varepsilon \\ &\qquad\qquad \left(f_2 - f_2^t\right)^2 \leqslant \varepsilon \\ &\qquad\qquad \left(f_3 - f_3^t\right)^2 \leqslant \varepsilon \\ &\mathbf{X} = [l_s, b_s, t_s, l_m, b_m, t_m, d_s] \end{aligned} \tag{17}$$

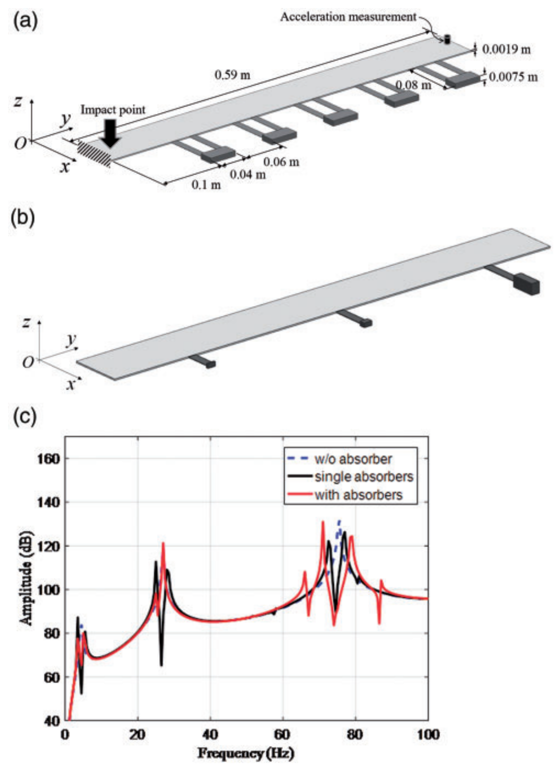

Figure 9. The equally distributed multiple dynamic vibration absorbers: (a) an FE model with five dynamic absorbers, (b) an FE model with single absorbers, and (c) the FE simulated frequency response functions.

where the objective function is the total mass $l_m \times b_m \times t_m$ and the constraints for the differences of the eigenfrequencies (f_i, $i = 1, 2, 3$) and the target frequencies ($f_i^t, i = 1, 2, 3$) are constrained in the optimization formulation (17). As the enforcing of the equal constraints exactly is impossible, an allowable error bound is denoted by ε.

Simulation and experiment verifications

To show the effectiveness and application of the present dynamic absorber, this section conducts FE simulations and an experiment to bring the resonant amplitude down by attaching the dynamic absorber to hosting structure.

Beam vibration example

The first example verifies the performance of the dynamic absorber through the FE simulation and an experiment with the fixed-free beam in Figure 6 whose first eigenfrequencies are 4.2916 Hz, 26.8939 Hz, and 75.3788 Hz. At one end, a unit force is applied and at the other end, the vertical acceleration is computed using the FE model. The frequency response functions with and without the vibration absorber are computed and compared in Figure 7(a: $y = 0.1$ m), (b: $y = 0.5$ m), and (c). By attaching the dynamic absorber, the reductions of the three resonance amplitudes are observed in Figure 7. The amplitude of the third resonance is little affected as the location and position are not optimized. The theory of the classical dynamic absorber does not consider the mass of the dynamic absorber as noted in an earlier section. To investigate the effect of the location of the dynamic absorbers, Figure 7(c) simulates the frequency response functions with the different locations and it shows that the third resonance can be effectively attenuated by changing the location; this is due to the change of the effective mass for the third eigenmodes. By investigating the modes of the two structures, it can be observed that when the mode shape of the hosting structure is matched to the relative mode shape of the dynamic absorber, the huge attenuation can be achievable.

To validate the performance of the dynamic absorber, we conduct an experiment in Figure 8. Compared with the FE simulation in Figure 7, almost the same responses can be obtained with some discrepancies due to the

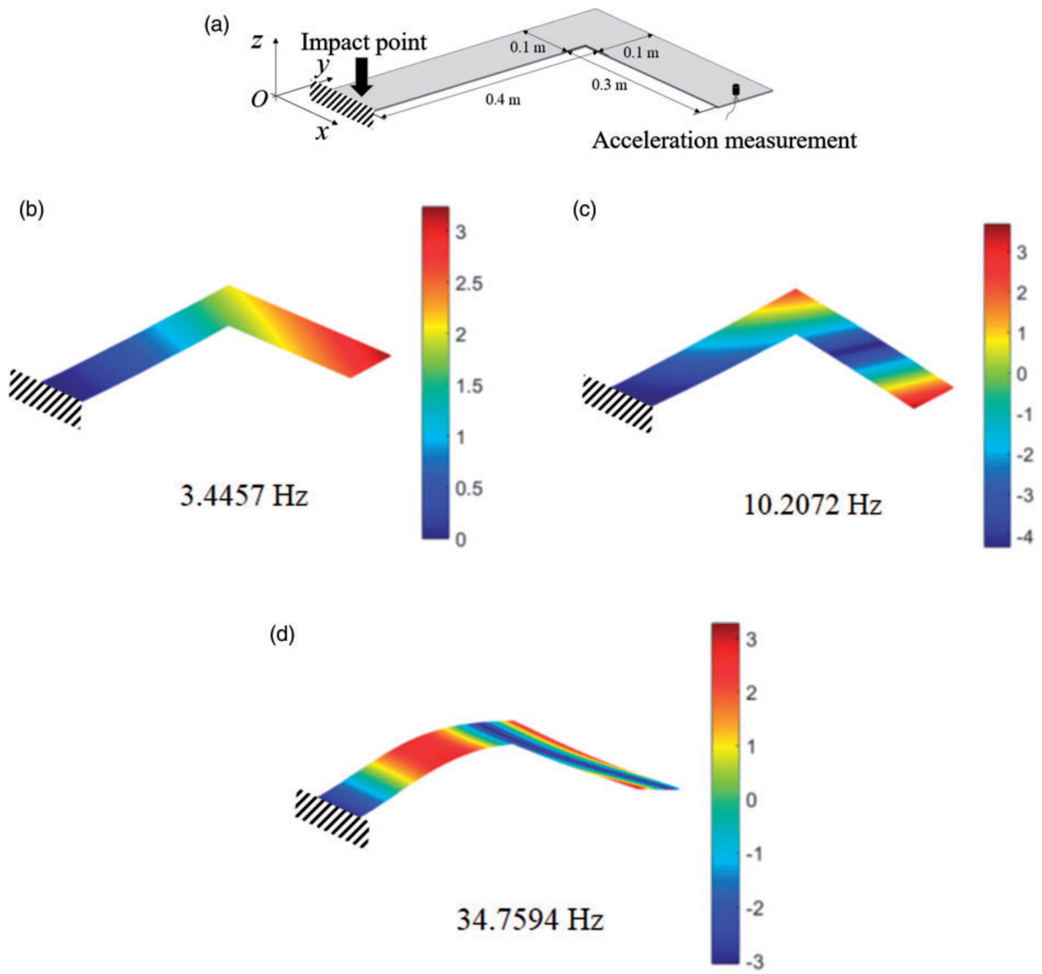

Figure 10. The first three vibration modes of a *L*-beam: (a) a geometry, (b) the first mode (3.4457 Hz), (c) the second mode (10.2072 Hz), and (d) the third mode (34.7549 Hz).

small differences in the material properties and the boundary conditions. It was an issue on how to efficiently install the dynamic absorber, i.e. location and angle configuration. The theory of the tuned mass damper states that the damper should be placed in parallel to the vibration of hosting structure. Therefore, it is not clear how to in install these dampers aiming to attenuate the vibrations of multiple frequencies. Indeed, to find out the optimal location of the dynamic absorber, the location of the dynamic absorber can be varied in Figure 8(c) and (e). As illustrated, the position of the dynamic absorber affects the performance of the dynamic absorber. This implies that an engineer should determine a location effectively attenuating the three frequencies considering its application area and its effective masses at each eigenfrequencies.

To overlay the vibration attenuation effect with the multifrequency vibration absorber at each location shown in Figure 7, the five multifrequency vibration absorbers are simultaneously arranged at intervals of 0.1 m as in Figure 9(a). The significant response attenuation can be achieved in Figure 9. The results in Figure 9(a) and (c) confirm that each dynamic absorber attached to each location has a different vibration damping effect and also shows that the multiple dynamic absorbers become effective in reducing the vibration.

Furthermore this example also shows the importance of the locations of the vibration absorbers to maximize the attenuation. The classical theory of absorber is based on a system with one degree of freedom for the optimization of mass/spring constants and control parameters of dynamic absorber. In order to compare the performance with a dynamic absorber for single frequency absorbers, the model with three absorbers in Figure 9(b) and its performance are compared. As shown, the present dynamic absorber can attenuate the resonances simultaneously.

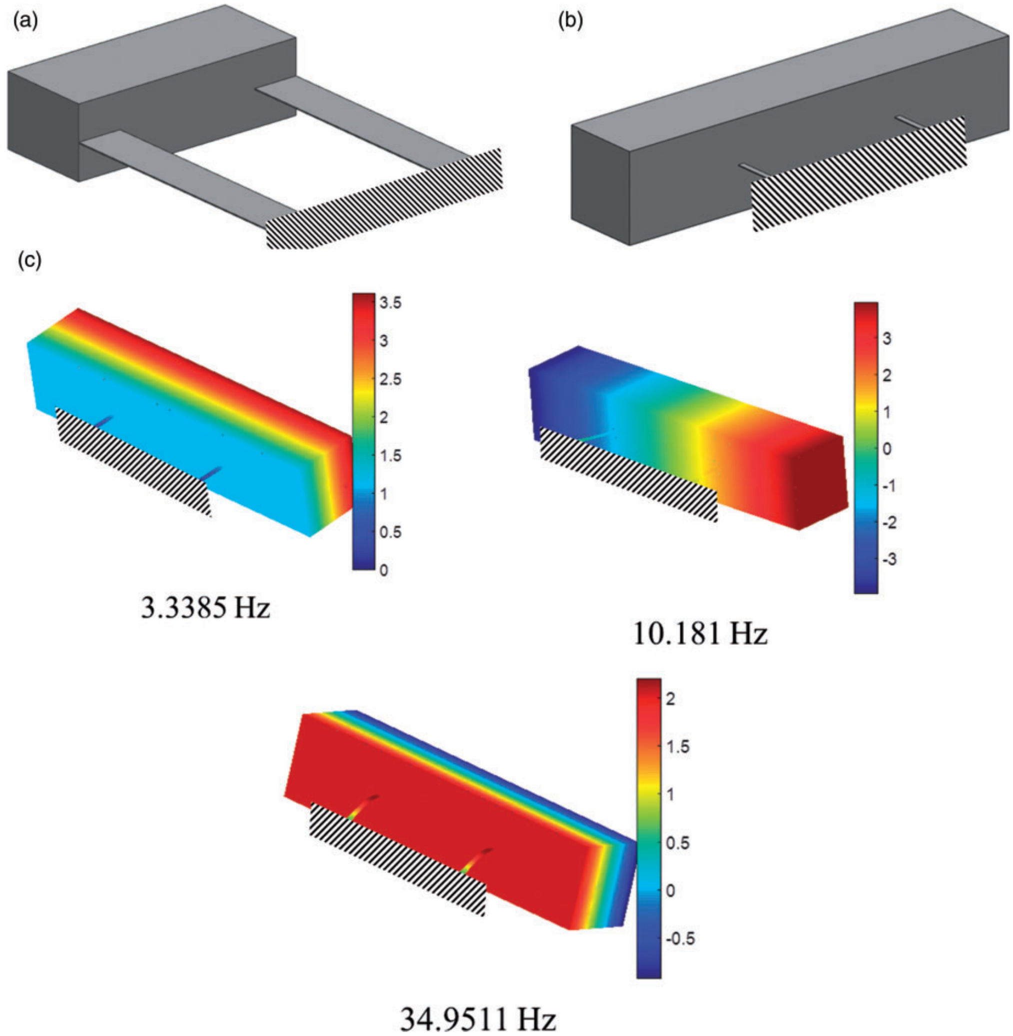

Figure 11. An optimized multifrequency vibration absorber for the *L*-beam in Figure 10: (a) an initial design and an optimized design ($l_s = 31.8440$ mm, $b_s = 10.4789$ mm, $t_s = 1.5000$ mm, $l_m = 5.0569$ mm, $b_m = 24.7000$ mm, $t_m = 0.5266$ mm, $d_s = 21.6039$ mm) and (b) the first three modes of the optimized design (3.3385 Hz, 10.1810 Hz, and 34.94 Hz).

L-*beam vibration example*

To further study the vibration attenuation effect of the multifrequency vibration absorber and to demonstrate how to adjust the geometry of the dynamic absorber, the vibrations of the *L*-beam in Figure 10(a) are considered. The eigenfrequencies and their associated eigenmodes of the *L*-beam are computed in Figure 10; the first three eigenfrequencies are 3.4457 Hz, 10.2072 Hz, and 34.7549 Hz, respectively. In case of the application of the dynamic absorber for a single frequency, it is easy to modify the mass or the spring constant to precisely match the vibration modes of the hosting structure with those of the dynamic absorber. When more than two eigenfrequencies are of interest, however, it is not straightforward. In other words, it often becomes cumbersome to match the eigenfrequencies of the dynamic absorber and the hosting structure as a change of one geometric parameter can make one eigenfrequency close to the target eigenfrequency but an adverse effect to another target eigenfrequency. It can be a serious issue with the three eigenfrequencies.

As the eigenfrequencies of the dynamic absorber in the previous example are different to the eigenfrequencies of the *L*-beam, the direct application of the previous dynamic absorber is not possible and some modifications of the geometry are essential in order to adjust the eigenfrequencies of the dynamic absorber to those of the *L*-beam. After some trials and errors with the heuristic tuning algorithm using the curves of Figure 4, we can come up with the new design (right of Figure 11(a)) whose eigenmodes and eigenfrequencies in Figure 11(b) are within 10%

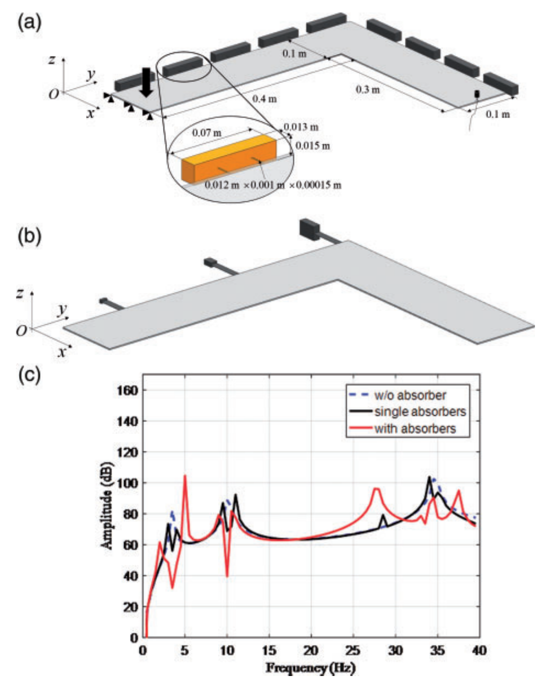

Figure 12. The FE simulation with the multiple dynamic absorber: (a) the FE simulation model with the present absorber, (b) the FE simulation model with the absorbers tuned to each frequency (single absorbers), and (c) the FE simulated frequency response functions.

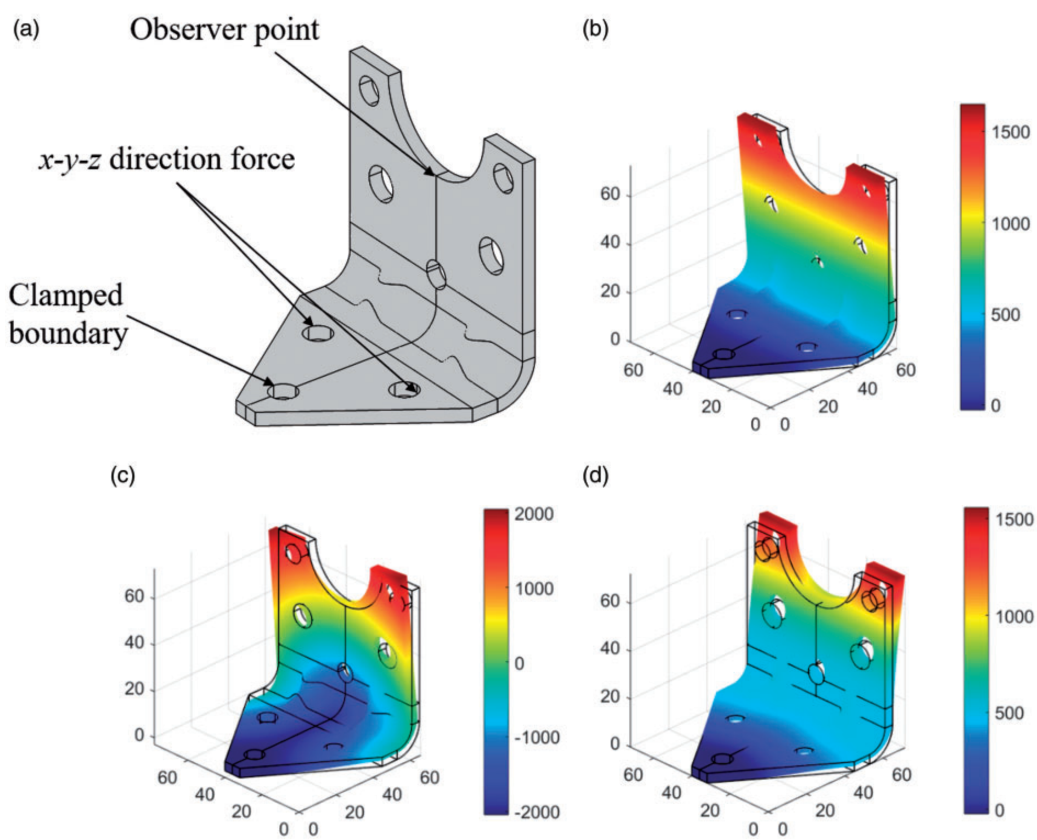

Figure 13. The three fundamental vibration modes of a bracket: (a) a geometry, (b) the first mode (30.9958 Hz), (c) the second mode (44.0977 Hz), and (d) the third mode (109.0059 Hz).

errors with the eigenfrequencies of the L-beam. It is the best design with the heuristic design method and the design with 10% error still can dramatically reduce the performance of the dynamic absorber. To compensate this error, Figure 12(a) shows the attachment of the nine dynamic absorbers; the dynamic absorbers are equally distributed at the boundaries of the beam. Figure 12(b) shows the beam with the absorbers tuned for the each frequency (the single absorbers). Figure 12(c) shows the comparison between the frequency response functions with/without the absorber and with the single absorbers; the impact force is applied near to the clamp boundary condition and the acceleration at the end of the beam is measured. It shows that the peaks at the resonances are significantly reduced. In order to compare the performance with a dynamic absorber for single-frequency absorbers, the model in Figure 12(b) with single absorbers is compared with the present absorber of Figure 12(a).

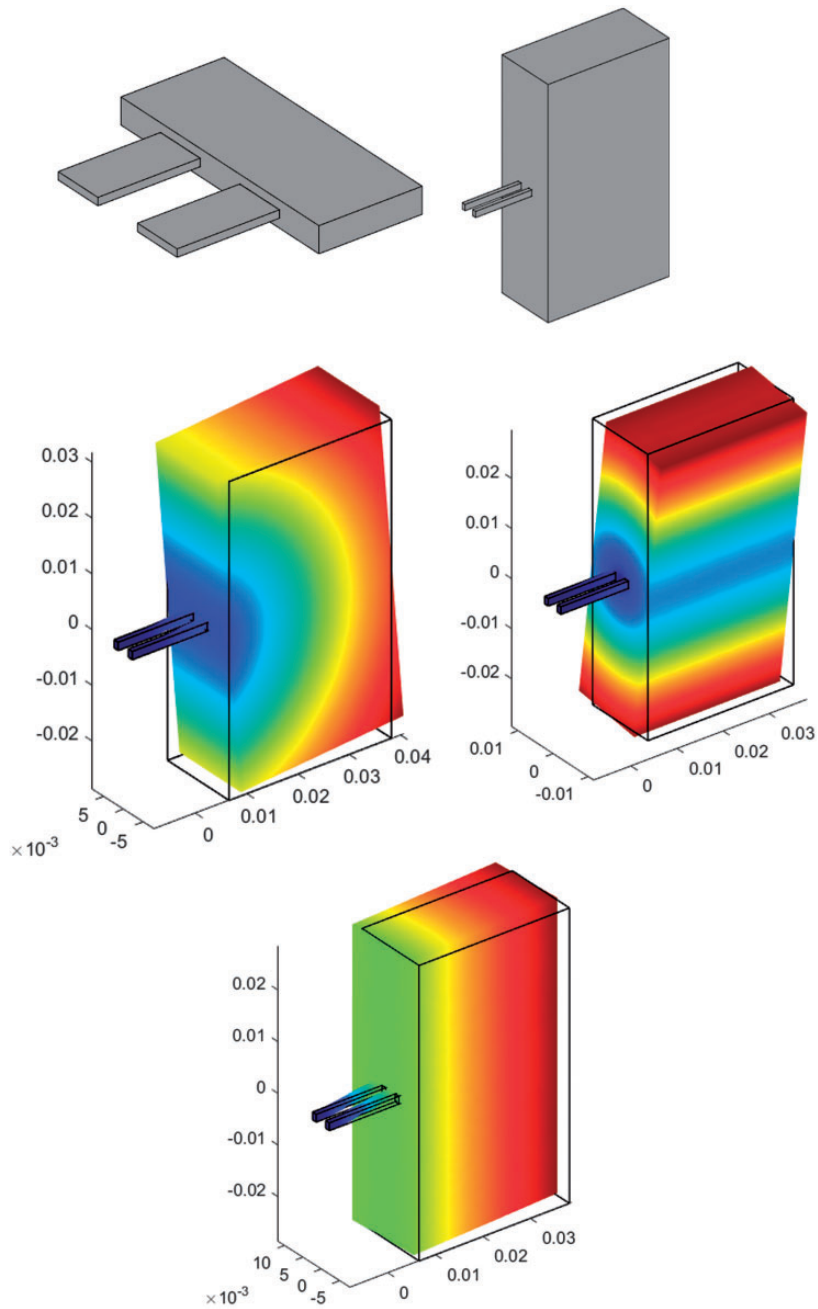

Figure 14. An optimized multifrequency vibration absorber for the bracket in Figure 13: (a) an initial design and an optimized design ($l_s = 31.84400$ mm, $b_s = 10.4789$ mm, $t_s = 1.5000$ mm, $l_m = 5.0569$ mm, $b_m = 24.7000$ mm, $t_m = 0.5266$ mm, $d_s = 21.6039$ mm) and (b) the first three modes of the optimized design (31.1779 Hz, 43.8442 Hz, 109.0759 Hz).

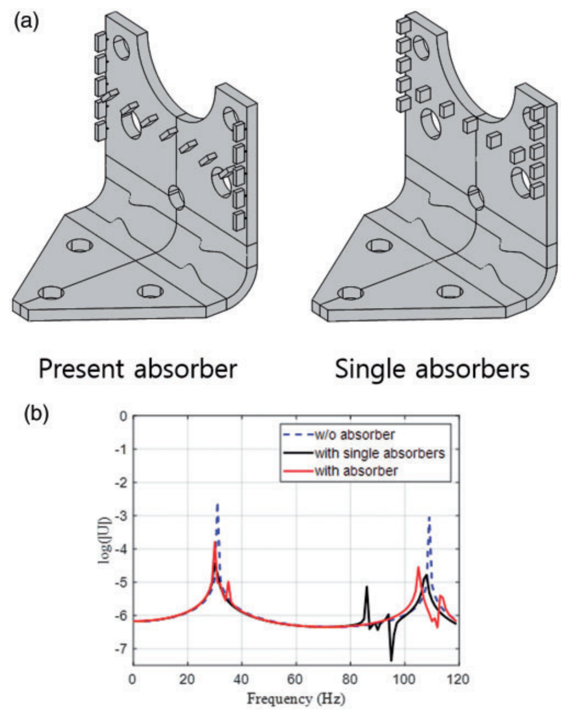

Figure 15. A bracket with the optimized dynamic absorber: (a) the bracket with the optimized dynamic absorbers and the conventional single absorbers and (b) the frequency response functions of the observer point.

The comparison is shown in Figure 12(c). As illustrated, the present dynamic absorber can attenuate the resonances simultaneously. It is also possible to use the three different dynamic absorbers. However, it can cause some difficulties in finding optimal layout and can increase the cost.

3D Bracket vibration example

The dynamic absorber in Figure 3 aims to attenuate the three modes about 4 Hz, 25 Hz, and 74 Hz, simultaneously. By changing the dimensions of this dynamic absorber, it can be applied to other structures with different resonance frequencies. To show this opportunity, the application of the dynamic absorber for the bracket with the associated boundary conditions in Figure 13(a) is considered. This bracket shows the first three eigenfrequencies of 30.9958 Hz, 44.0977 Hz, and 109.0059 Hz in Figure 13(b), (c), and (d), respectively. Compared with the eigenfrequencies of Figure 3, they are significantly different and it is hard to manually and heuristically change the geometry parameters of Figure 3 to attenuate the vibrations. To resolve this difficulty systematically, the optimization process in equation (17) can be applied and the optimal design in Figure 14 with 31.1779 Hz, 43.8442 Hz, 109.0759 Hz can be obtained. Then, to attenuate the vibrations at the observer point, several vibration absorbers are mounted to the bracket in Figure 15(a) (left). Figure 15(a) (right) shows the bracket with the single absorbers tuned for each frequencies. The frequency response functions are shown in Figure 15(b) illustrating that the optimized dynamic damper is effective in reducing the vibrations at the observer point. This example shows that the present dynamic absorber and the optimization algorithm can be useful in reducing the vibration.

Conclusions

This research studies a new type of dynamic absorber for three frequencies. The dynamic absorber theories are based on the eigenfrequencies and eigenmodes of hosting structure and mainly the vibration reduction of one resonance frequency has been researched. From an engineering point of view, it is one of the important and complex subjects to consider multiple frequency vibrations only with a passive tuned mass damper for complex-shaped hosting structure. To contribute this research field, this study presents a new dynamic absorber attenuating two bending modes and one twisting mode simultaneously. With the present dynamic absorber, the vibration reduction for three frequencies of hosting structure can be possible. It was noticed that the manual adjustments, i.e. trial and error, of the geometric parameters are cumbersome because the modification of one geometric parameter for a particular frequency causes the changes of the other frequencies of the vibration damper of interest. To effectively optimize the geometric parameters of the dynamic absorber, the approach based on the frequency response curves and the size optimization method were presented. To show the validity of the present dynamic absorber, the suppression of the resonance oscillations of the three structures is considered, i.e. two beam problems and a bracket problem using FE simulation and experiment. The present dynamic absorber is effective in attenuating the vibrations at three frequencies simultaneously. Compared with the conventional dynamic absorbers tuned for single frequencies, it has some advantages in compactness. For a specific frequency range, the reductions of the vibrations by the present absorber can be smaller than that of the conventional dynamic absorbers. For future research topic, it is possible to combine the developed dynamic absorber and the optimization schemes (a heuristic and a size optimization scheme) for the application of mechanical metamaterial. Furthermore, a research to find out the optimal location of dynamic absorbers maximizing the effective mass should be researched.

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