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NUMERICAL STUDY ON RUN-UP HEIGHTS OF SOLITARY WAVE WITH HYDRODYNAMIC PRESSURE MODEL

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ABSTRACT: For many shallow water flows, it is sufficient to consider the depth-averaged equations, referred as the shallow water equations, which are two-dimensional in the horizontal plane, since the length scale of the vertical direction is much smaller than that of the horizontal directions. Assuming that the pressure distribution is hydrostatic, the mathematical formulation and its numerical implementation are considerably simplified. In this study, a numerical model is newly developed to investigate various free surface fl ow problems. The governing equations are the Navier–Stokes equations with the pressure decomposed into the sum of a hydrostatic and a hydrodynamic components. The equation for the free surface movement is a depth–averaged continuity equation which is a free surface equation. These governing equations are simultaneously solved by using a finite difference method with a semi–implicit method and fractional step method. At the first step, the vertical momentum equations are discretized by using an implicit method over the vertical direction. In the second step, the discrete horizontal momentum equations are projected on to the free surface equation. Finally, the hydrodynamic pressure and final velocity field are calculated. To verify the accuracy and stability, the present numerical model is applied to move practical problems such as the run–up process of solitary waves attacking a circular island. The numerically obtained maximum run–up heights around a circular island are compared with available laboratory measurements. A very reasonable agreement is observed.

Keywords: Hazard map, run-up, hydrodynamic pressure, semi-implicit method, fractional step method

INTRODUCTION

Numerical simulation of shallow water flows based on incompressible Navier–Stokes equations is extensively applied in hydraulic and coastal studies such as waves interacting with structures or over varying topographies. Due to an increase in computer capacities in recent years, three–dimensional numerical models are developed extensively and applied for many hydraulic and coastal problems. In many numerical shallow water models, it is simulated assuming a hydrostatic pressure distribution in depth. In most cases, where the vertical acceleration component is small, sufficient accuracy for simulating most free surface flows is obtained by using numerical models, which assume the hydrostatic pressure approximation. It simplifies the coupled three–dimensional set of equations to be solved. However, hydrostatic pressure approximation is only applicable for simulating flows where the horizontal scale of motion is much larger than its vertical scale. Hydrostatic pressure assumption is no longer valid for flows over bed topographies of abrupt variations, flows with sharp density gradients and short wave motions, where the ratio of the vertical to horizontal scales of motion is not sufficiently small and for above such cases, a hydrodynamic pressure distribution should be considered (Koçyigit et al., 2002).

Hydrostatic pressure models based on the Navier–Stokes equations are widely applied too many hydraulic and coastal problems. If a hydrostatic approximation is assumed, the vertical momentum equation is omitted and thus, the vertical velocity is calculated from the continuity equation. However, it is well known that the hydrostatic pressure assumption is no longer valid in many physical problems such as flows over rapidly varying topographies and short surface waves where the ratio of the vertical to horizontal scales of motion is not very small, the effect of vertical acceleration become important. Therefore, a hydrodynamic pressure becomes significant and critical for modeling free surface flows when the vertical motion of flows is no longer negligible.

As a numerical application of wetting and drying algorithm, the present model applied to simulate solitary wave run–up on a circular island. This test has been widely used to verify wetting and drying algorithm for wave run–up heights. Numerical results of the present model obtained for maximum wave run–up heights around a circular island by incident solitary waves and compared with both laboratory measurements and

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existing numerical results. Laboratory experiments were performed at the Coastal Engineering Research Center of the US Army corps of Engineers.

GOVERNNING EQUATIONS

For incompressible flows, the governing equations describing the free surface flows are the three–dimensional Navier–Stokes equations (NSE), which are the conservation of mass and momentum. The governing equations can be written in primitive variables in a Cartesian coordinates as the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\nu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left(\nu\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\nu\frac{\partial v}{\partial z}\right) \tag{3}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}\left(\nu\frac{\partial w}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu\frac{\partial w}{\partial y}\right)+\frac{\partial}{\partial z}\left(\nu\frac{\partial w}{\partial z}\right)-g \tag{4}$$

To modeling of the free surface flow, integrating the continuity equation over the water depth, leading to

$$\frac{\partial\eta}{\partial t}+\frac{\partial}{\partial x}\left[\int_{-h}^{\eta}u\,dz\right]+\frac{\partial}{\partial y}\left[\int_{-h}^{\eta}v\,dz\right]=0 \tag{5}$$

With the boundary conditions for the horizontal velocities at the bottom and the surface, wind stresses can be incorporated as

$$\left[\nu\frac{\partial u}{\partial z}\right]_{i+1/2,j,k}=\tau_{sx} \tag{6}$$

$$\left[\nu\frac{\partial v}{\partial z}\right]_{i,j+1/2,k}=\tau_{sy} \tag{7}$$

where, $\tau_{sx}=\rho_a c_f u_w\sqrt{u_w^2+v_w^2}$, $\tau_{sy}=\rho_a c_f v_w\sqrt{u_w^2+v_w^2}$, ρ_a is the air density, c_f is the drag coefficient, and u_w, v_w are wind velocities in the $x-$ and $y-$ axis directions.

The boundary conditions at the bottom are given by expressing the bottom stress in terms of the velocity components taken from values of the layer adjacent to the sediment–water interface. The bottom stress can be related to the turbulent law of the wall, a drag coefficient associated with quadratic velocity, or using the Manning or Chezy formula such as

$$\left[\nu\frac{\partial u}{\partial z}\right]_{i+1/2,j,1}=\gamma_b u \tag{8}$$

$$\left[\nu\frac{\partial v}{\partial z}\right]_{i,j+1/2,1}=\gamma_b v \tag{9}$$

where, $\gamma_b=g\sqrt{u^2+v^2}/Cz^2$ and Cz is the Chezy friction coefficient.

NUMERICAL SIMULATION

For a certain class of problems where the governing equations can be solved sequentially, it is better to use a staggered grid system in finite difference formulations. In Cartesian coordinate system, a staggered arrangement, suggested by Harlow and Welch (1965), offers several benefits over a collocated grid. The grid system is shown in Fig. 1.

Hydrostatic Pressure Step

The first step of calculations is performed by neglecting the implicit contribution of the hydrodynamic pressure. The resulting velocity field and water surface elevation at a new time level are not yet finalized and denoted by $\tilde{u}$, $\tilde{v}$, $\tilde{w}$, and $\tilde{\eta}$. A general semi–implicit discretization of the momentum equations (10 to 12) takes the following form

$$\frac{\tilde{u}^{n+1}_{i+1/2,j,k}-u^{n}_{i+1/2,j,k}}{\Delta t}+F=-\theta g\frac{\left(\tilde{\eta}^{n+1}_{i+1,j}-\tilde{\eta}^{n+1}_{i,j}\right)}{\Delta x}-(1-\theta)\left[\frac{g\left(\eta^{n}_{i+1,j}-\eta^{n}_{i,j}\right)}{\Delta x}+\frac{q^{n}_{i+1,j,k}-q^{n}_{i,j,k}}{\Delta x}\right]+\nu\frac{\dfrac{\tilde{u}^{n+1}_{i+1/2,j,k+1}-\tilde{u}^{n+1}_{i+1/2,j,k}}{\Delta z^{n}_{i+1/2,j,k+1/2}}-\dfrac{\tilde{u}^{n+1}_{i+1/2,j,k}-\tilde{u}^{n+1}_{i+1/2,j,k-1}}{\Delta z^{n}_{i+1/2,j,k-1/2}}}{\Delta z^{n}_{i+1/2,j,k}} \tag{10}$$

$$\frac{\tilde{v}^{n+1}_{i,j+1/2,k}-v^{n}_{i,j+1/2,k}}{\Delta t}+F=-\theta g\frac{\left(\tilde{\eta}^{n+1}_{i,j+1}-\tilde{\eta}^{n+1}_{i,j}\right)}{\Delta y}-\left(1-\theta\right)\left[\frac{g\left(\eta^{n}_{i,j+1}-\eta^{n}_{i,j}\right)}{\Delta y}+\frac{q^{n}_{i,j+1,k}-q^{n}_{i,j,k}}{\Delta y}\right]+\nu\frac{\dfrac{\tilde{v}^{n+1}_{i,j+1/2,k+1}-\tilde{v}^{n+1}_{i,j+1/2,k}}{\Delta z^{n}_{i,j+1/2,k+1/2}}-\dfrac{\tilde{v}^{n+1}_{i,j+1/2,k}-\tilde{v}^{n+1}_{i,j+1/2,k-1}}{\Delta z^{n}_{i,j+1/2,k-1/2}}}{\Delta z^{n}_{i,j+1/2,k}} \tag{11}$$

$$\frac{\tilde{w}^{n+1}_{i,j,k+1/2}-w^{n}_{i,j,k+1/2}}{\Delta t}+F=-\left(1-\theta\right)\frac{q^{n}_{i,j,k+1}-q^{n}_{i,j,k}}{\Delta z^{n}_{i,j,k+1/2}}+\nu\frac{\dfrac{\tilde{w}^{n+1}_{i,j,k+3/2}-\tilde{w}^{n+1}_{i,j,k+1/2}}{\Delta z^{n}_{i,j,k+1}}-\dfrac{\tilde{w}^{n+1}_{i,j,k+1/2}-\tilde{w}^{n+1}_{i,j,k-1/2}}{\Delta z^{n}_{i,j,k}}}{\Delta z^{n}_{i,j,k+1/2}} \tag{12}$$

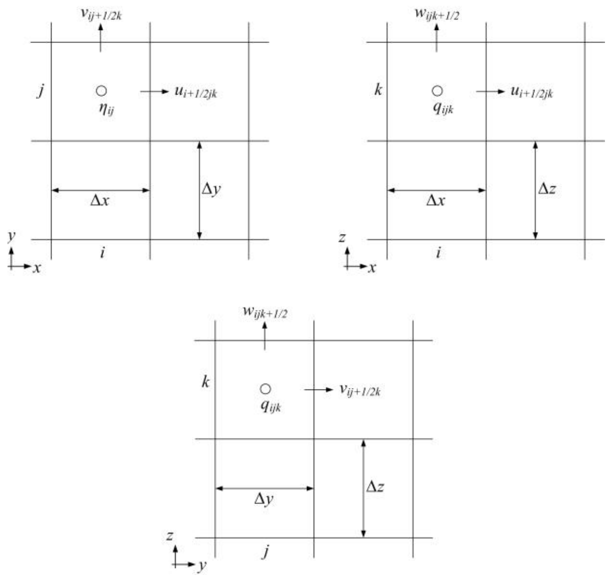

Fig. 1 A definitive sketch of a staggered grid system

where, the vertical space increment Δz is usually defined as the distance between two consecutive level surfaces except near the bottom and near the free surface where, Δz is the distance between a level surface and bottom or free surface, respectively. F is a finite difference operator that includes the explicit discretization of the convective and horizontal viscosity terms.

Hydrodynamic Pressure Step

In the hydrodynamic pressure correction step of calculations, the new velocity fields and the new water surface elevation are computed by correcting the provisional values, after including the provisional hydrodynamic pressure terms. Specifically, the discrete momentum equations are taken to be

$$u^{n+1}_{i+1/2,j,k}=\tilde{u}^{n+1}_{i+1/2,j,k}-\theta\frac{\Delta t}{\Delta x}\left(\tilde{q}^{n+1}_{i+1,j,k}-\tilde{q}^{n+1}_{i,j,k}\right) \tag{13}$$

$$v^{n+1}_{i,j+1/2,k}=\tilde{v}^{n+1}_{i,j+1/2,k}-\theta\frac{\Delta t}{\Delta y}\left(\tilde{q}^{n+1}_{i,j+1,k}-\tilde{q}^{n+1}_{i,j,k}\right) \tag{14}$$

$$w^{n+1}_{i,j,k+1/2}=\tilde{w}^{n+1}_{i,j,k+1/2}-\theta\frac{\Delta t}{\Delta z^{n}_{i,j,k+1/2}}\left(\tilde{q}^{n+1}_{i,j,k+1}-\tilde{q}^{n+1}_{i,j,k}\right) \tag{15}$$

where, $\tilde{q}$ denotes the hydrodynamic pressure correction which combines the provisional free surface elevation.

The new free surface elevation is obtained by hydrostatic relation as follows:

$$\eta^{n+1}_{i,j}=\tilde{\eta}^{n+1}_{i,j}+\frac{\tilde{q}^{n+1}_{i,j,M}}{g} \tag{16}$$

and thus, the final hydrodynamic pressure component $q^{n+1}_{i,j,k}$ is obtained by the following equation:

$$\begin{aligned} q^{n+1}_{i,j,k}&=g\left(\tilde{\eta}^{n+1}_{i,j}-\eta^{n+1}_{i,j}\right)+\tilde{q}^{n+1}_{i,j,k}\\ &=\tilde{q}^{n+1}_{i,j,k}-\tilde{q}^{n+1}_{i,j,M}\end{aligned} \tag{17}$$

APPLICATION OF NUMERICAL MODEL

During the last decades, huge tsunami run–up heights were occurred at many countries and islands. Therefore, losses of human beings and property damage were occurred. Several studies of maximum run–up heights have been performed (Liu et al., 1991; Liu et al., 1995; Cho, 1995).

A series of laboratory experiments were performed in a large–scale basin at the Coastal Engineering Research Center (CERC) of the US Army Corps of Engineers (Briggs et al., 1994). The center of a circular island was located at $x = 15$ m and $y = 13$ m. The surface of the island and the floor of the basin were smoothly finished with concrete mortar. A directional spectral wave generator was installed along the $x-$ axis direction and used to generate incident solitary waves. The total length of the wave generator is 27.432 m and it consists of 60 individual paddles moving parallel to the water surface; each of them can be driven independently and electronically. A schematic sketch of the island geometry is shown in Fig. 2, and the location of the gage points of wave run–up heights is shown in Fig. 3.

In general, the maximum run–up height decreases as the gage number increases, except in the lee side (gage of solitary waves 8) of the island. Such results are due to the trapped solitary waves approaching from the opposite directions, which collide and generate a higher run–up heights in the lee side of the island (Cho, 1995; Liu et al., 1995).

In laboratory experiments and numerical simulations, three different initial solitary wave heights are used with $h=0.016$ m, 0.032 m, and 0.064 m. The nonlinearity of the incident solitary wave is defined as $\varepsilon=h/H_0$ with H_0 being a still water depth. The wave run–up heights, R are normalized by incident wave heights.

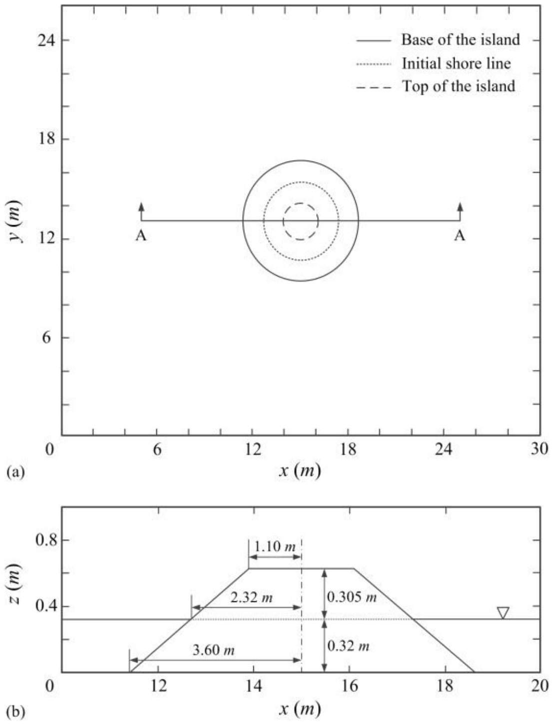

Fig. 2. (a) Top view of the wave basin with the island; (b) the vertical view of the circular island on the cross–section A–A (Cho, 1995)

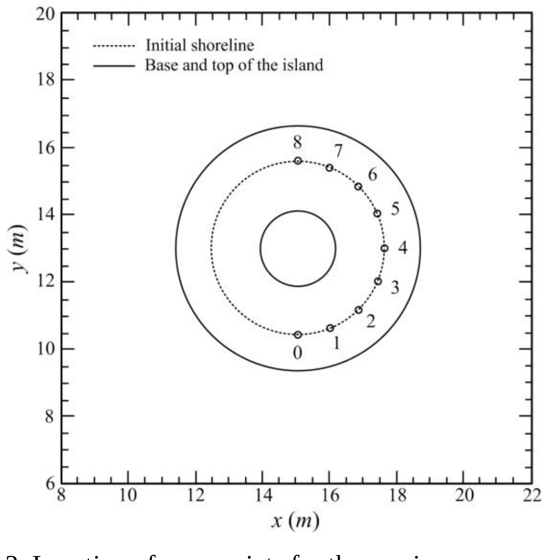

Fig. 3. Location of gage points for the maximum run–up heights measurements

The wave run–up heights are measured around the island from the incident wave direction with an interval $\pi/8$ (Fig. 3).

In laboratory experiments and numerical simulations, the constant water depth was fixed at $H_0=0.32$ m. In the present study, the numerical simulations are focused on the cases of $\varepsilon=0.05$($h=0.016$ m) and $\varepsilon=0.1$ ($h=0.032$ m). The computational domain is consisted by using a horizontal grid spacing of $\Delta x=\Delta y=0.1$ m and a vertical grid spacing of $\Delta z_k=0.0032$ m.

Fig. 4 and Fig. 5 present a comparison of the maximum run–up heights between the present model and laboratory measurements. When $\varepsilon=0.05$, the present numerical model and the two–dimensional numerical model based on the shallow water theory (Cho, 1995; Liu et al., 1995) well represent laboratory measurements. However, when the incident solitary wave height increases, two–dimensional numerical model shows a large discrepancy while the present numerical model provides reasonable results with the laboratory measurements. The present model slightly underestimates the laboratory measurements.

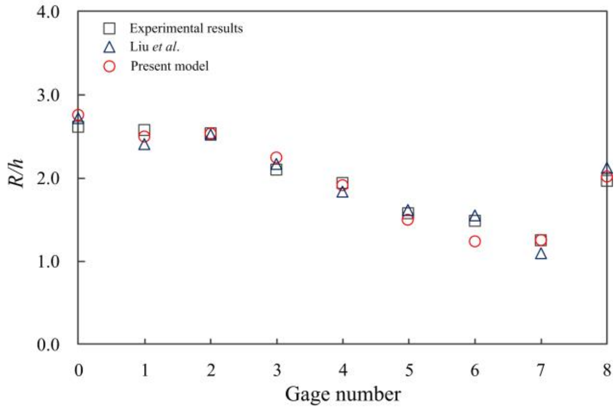

Fig. 4. Comparison of the maximum run–up heights with $\varepsilon=0.05$

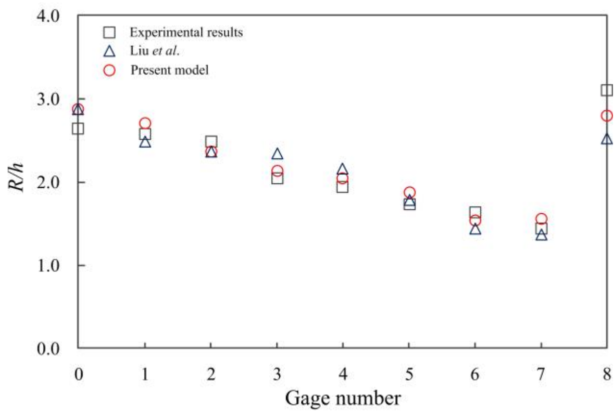

Fig. 5. Comparison of the maximum run–up heights with $\varepsilon=0.1$

CONCLUSIONS

Through the present study, a numerical model was developed, which can be used to examine the three–dimensional free surface flows with considering a hydrodynamic pressure. The numerical model solves the Navier–Stokes equations for three velocities and pressure, as well as the depth–averaged continuity equation for the free surface movement. The governing equations have been integrated by using a semi–implicit, fractional step method, where the hydrostatic pressure component is determined first and the hydrodynamic component of the pressure is then computed in a subsequent step. This method is relatively simple and numerically stable even at large Courant numbers; it is suitable for simulations of complex three–dimensional flows where a small deviation from a hydrostatic pressure is allowed. In the present study, the above scheme is further improved by including a correction for the free surface in such a fashion that a new free surface elevation is implicitly coupled with the hydrodynamic pressure along with the new velocity field. During the last decades, huge tsunami run–up heights were occurred at many countries and islands. Therefore, losses of human beings and property damage were occurred. Several studies of maximum run–up heights have been performed (Liu et al., 1991; Liu et al., 1995; Cho, 1995).

The present model is applied to simulate the run–up heights of solitary waves on a circular island. This test has been widely used to verify wetting and drying algorithm for wave run–up heights. Numerical results of the present model are obtained for the maximum wave run–up heights of incident solitary waves around a circular island and compared with both laboratory measurements and existing numerical results obtained from the shallow water equations model. The present numerical model shows reasonable results with the laboratory measurements.

REFERENCES

Briggs, M.J., Synolakis, C.E. and Harkins, G.S. (1994). Tsunami runup on a conical island. Proceedings of Waves-Physical and Numerical Modeling. University of British Columbia, Vancouver, Canada, 446-455.

Cho, Y.–S. (1995). Numerical simulations of tsunami propagation and run–up. Ph.D. Thesis, Cornell University, New York, USA.

Harlow, F. H. and Welch, J. E. (1965). Numerical calculation of time–dependent viscous incompressible flow. Physics of Fluids, 8, 2,182–2,189.

Koçyigit, M. B., Falconer, R. A. and Lin, B. (2002). Three–dimensional numerical modelling of free surface flows with non–hydrostatic pressure. International Journal for Numerical Methods in Fluids, 40(9), 1,145–1,162.

Liu, P.L.–F, Cho, Y.–S, Briggs, M. J., Kanoglu, U. and Synolakis, C. E. (1995). Run–up of solitary waves on a circular island. Journal of Fluid Mechanics, 302, 259–285.

Liu, P. L.–F., Cho, Y.–S., Yoon, S. B. and Seo, S. N. (1994). Numerical simulations of the 1960 Chilean tsunami propagation and inundation at Hilo, Hawaii. Tsunami: Progress in Prediction, Disaster Prevention and Warning, Recent Development in Tsunami Research, Edited by M.I. El–Sabh, Kluwer Academic Publishers.