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Research article

A decision making algorithm for wind power plant based on q-rung orthopair hesitant fuzzy rough aggregation information and TOPSIS

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Abstract: Wind energy is one of the most significant renewable energy sources due to its widespread availability, low environmental impact, and great cost-effectiveness. The effective design of ideal wind energy extraction areas to generate electricity is one of the most critical issues in the exploitation of wind energy. The appropriate site selection for wind power plants is based on the concepts and criteria of sustainable environmental advancement, resulting in a low-cost and renewable energy source, as well as cost-effectiveness and job creation. The aim of this article is to introduce the idea of q-rung orthopair hesitant fuzzy rough set (q-ROHFRS) as a robust fusion of q-rung orthopair fuzzy set, hesitant fuzzy set, and rough set. A q-ROHFRS is a new approach towards modeling uncertainties in the multi-criteria decision making (MCDM). Various key properties of q-ROHFRS and some elementary operations on q-ROHFRSs are established. A list of novel q-rung orthopair hesitant fuzzy rough weighted geometric aggregation operators are developed on the basis of defined operational laws for q-ROHFRSs. Further, a decision making algorithm is developed to handle the uncertain and incomplete information in real word decision making problems. Then, a multi-attribute decision making method is established using q-rung orthopair hesitant fuzzy rough aggregation operators. Afterwards, a practical case study on evaluating the location of wind power plants is presented to validate the potential of the proposed technique. Further, comparative analysis based on the novel extended TOPSIS method is presented to demonstrate the capability of the proposed technique.

Keywords: q-rung orthopair hesitant fuzzy rough sets; aggregation operators; wind energy; decision making
Mathematics Subject Classification: 03B52, 03E72

1. Introduction

As the complexity of systems grows, it becomes increasingly difficult for the decision makers to choose the best option from a list of all feasible alternatives. It is difficult to summarize how difficult it is to achieve a single alternative, but it is not impossible. Many organizations have a tough time in establishing the motivation, aims, and opinions of their workforce. Consequently, no matter if the decision is made by a single person or a committee, it has the same effect on the organization's overall goals. This observation argues that in actual problems, criteria solved optionally prohibit each decision maker from achieving an optimal solution-optimum under each criterion. As a result, decision-makers are putting more effort into developing more practical and reliable ways for figuring out what the finest choice is. Classical or crisp strategies do not always work well when dealing with data that contains ambiguity and uncertainty in decision making situations. Consequently, how does one can handle a scenario like this? According to Zadeh [11], a fuzzy set was first proposed in 1965. Zadeh used fuzzy sets to assign membership grades in the unit interval [0,1] to elements of a set. It is noteworthy that many of the set theoretic features of crisp situations were described for fuzzy sets in Zadeh's work. Fuzzy set theory has piqued the interest of scientists and has found applications in decision sciences [12], artificial intelligence [13], medical diagnosis [14] and its enormous applications are discussed in [15]. In the universe, hesitancy is a natural phenomenon. Identifying one of the better alternatives with the same characteristics in real life is difficult. Due to the uncertainty and hesitancy of the results, experts are having difficulty in decisions making. To tackle hesitancy, Torra and Narukawa [16] developed the concept of hesitant fuzzy set (HFS), which allows an element to be a set of multiple possible values. The hesitant fuzzy set can be used to solve a variety of decision making problems. Many researcher used HFS to solve problems by aggregating operators in group decision-making [17–19]. Liao and Xu [21] identified generalized forms of hesitant fuzzy hybrid weighted averaging operator, hesitant fuzzy hybrid weighted geometric operator, generalized form of quasi-hesitant fuzzy hybrid weighted averaging and geometric operators.

Addressing numerous applications of fuzzy set and hesitant fuzzy set theories, Atanassov [1] identified flaws in these notions and introduced the concept of intuitionistic fuzzy set to broaden the aforementioned ideas. In intuitionistic fuzzy set each element is expressed by an ordered pair, and each pair is characterized by membership and non-membership grades with the condition that sum of their grades are less or equal to 1. In the last several decades, the intuitionistic fuzzy sets (IFSs) have been noteworthy and widely used by researchers to comprehend ambiguous and imprecise data. To cumulate all the executive of criteria for alternatives, aggregation operators, play vital character throughout the information merging procedure. Xu [19] presented weighted averaging operator while Xu and Yager [27] developed geometric aggregation operator for aggregating the different intuitionistic fuzzy numbers. Deschrijver [28] developed the IFS representation of t-norms and t-conorms. In some decision theories, the decision makers deal with the situation of particular attributes where values of their summation of membership and non-membership degrees exceeds 1. In such situation, IFSs have no ability to obtain any satisfactory result. To overcome this situation Yager [29] developed the idea of Pythagorean fuzzy set (PyFS) as a generalization of IFS, which satisfies that the value of summation of square of its membership and non membership degrees is less then or equals to 1. Clearly, PyFS is more flexible than IFS to deal with the imprecision and ambiguity in the practical multi-criteria decision making (MCDM) problems. Zhang and Xu [30] established an extension of TOPSIS to MADM with PyFS information. For MADM problems under Pythagorean fuzzy environment Yager and Abbasov [32] developed a series of aggregation operators. Peng and Yang [33] explained their relationship among these aggregation operators and established the superiority and inferiority ranking (SIR) for MAGDM method. Using Einstein operation Garg [34] generalized Pythagorean fuzzy information aggregation. Gou et al. [35] studied many Pythagorean fuzzy functions and investigated their fundamental properties. Zhang [36] put forward a ranked qualitative flexible (QUALIFLEX) multi-criteria approach with the closeness index-based ranking methods for multi-criteria Pythagorean fuzzy decision analysis. Zeng et al. [37] explored a hybrid method for Pythagorean fuzzy MCDM. Zeng [43], applied the Pythagorean fuzzy probabilistic OWA (PFPOWA) operator for MAGDM problem. The PyFS theory has been effectively implemented in a variety of fields, although there are some situations in real world problems that cannot be expressed utilizing this notion. Afterwards, Khan et al. [44] introduced the concept of Pythagorean hesitant fuzzy sets (PyHFSs). They established an evaluation method and identified operators for data aggregation. Xu and Zhou [45] identified a novel concept of probabilistic hesitant fuzzy sets (PHFSs). Inspired by the compatibility of PyHFSs, researchers extensively investigated the concept in multi-attribute decision-making problems (MADM) [38–42, 46, 48–50]. However, it is observed that there are still some limitations for PyHFSs, although they have proven to be efficient when dealing with complex fuzzy information in some practical applications. To overcome such limitations, Yager [51] established a new idea called q-rung orthopair fuzzy sets (q-ROFSs), in which the sum of the qth exponent of support for membership and the qth exponent of support non-membership is restricted to unity and demonstrated that the q-ROFS are more flexible than the previously discussed notions. The q-ROFSs provide a broader range of fuzzy information and are more versatile and appropriate approach to deal with unpredictable situations. Yager and Alajlan [52] explored the fundamental properties of these q-ROFSs and discussed that how they can be used in information representation. Subsequently, the authors in [53] put forward the notation of q-rung orthopair hesitant fuzzy set (q-ROHFS) and discussed the operational laws, which exist for any two q-ROHFSs. Wang et al. [54] investigated the Heronian mean operators in multiple attribute decision-making in a q-ROHFS framework. They also proposed the Hamacher norm-based aggregation operators (AOps) under dual hesitant q-ROFSs and discussed their usefulness in decision making problems. Hussain et al. [55] measured the entropy for hesitant fuzzy information using the Hausdorff metric and the structure of hesitant fuzzy TOPSIS approach. Zhan et al. [71] proposed a three-way multi-attribute decision-making (MADM) model based on an outranking relationship.

To overcome data uncertainty, several approaches have been developed and exist in the literature. Pawlak [9] is pioneer who studied the dominant concept of rough set theory. The classical set theory which deals with inconsistent and imprecise information is extended by rough set theory. In recent decades, research on the rough set has progressed significantly, both in terms of theoretical implementations and theory itself. In recent decades, researchers have demonstrated TOPSIS technique in a number of rough sets information. Chen et al. [56] studied rough set theory-based on fuzzy TOPSIS on serious game design assessment procedure. Khan et al. [57] implemented a rough set and the TOPSIS approach for the site selection of food distribution. Jiang et al. [68] developed the MADM technique to covering-based variable precision fuzzy rough sets in a medical diagnosis application. Zhan et al. [69] developed a unique technique for solving multi-attribute decision-making (MADM) problems by combining the covering-based variable precision fuzzy rough sets model with two conventional decision-making methods that are the PROMETHE method and the DEAS method. Lu et al. [58] investigated improved TOPSIS method based on rough set theory for selection of suppliers. Jiang et al. [70] established a novel methodology, the PROMETHEE-II method, based on variable precision fuzzy rough sets with fuzzy neighbours, and applied it to MADM in the context of medical diagnosis using the suggested rough approximation operators. Khoshaim et al. [47] investigated emergency decision-making utilizing q-rung orthopair fuzzy rough aggregation information.

The concept of rough sets has been extended by several researchers around the world in different directions. Using the fuzzy relation in place of the crisp binary relation, Dubois et al. [59] initiated the notion of fuzzy rough sets. The hybrid structure of IFSs and rough sets and intuitionistic fuzzy rough (IFR) sets are introduced by Cornelis et al. [60]. The IFRSs play a significant role like a bridge between these two theories. By utilizing IFR approximations, Zhou and Wu [61] established a novel decision making technique under IFR environment to address their constrictive and axiomatic analysis in detail. Zhan et al. [62] presented the decision making methodology under intuitionistic fuzzy rough environment and explored their applications in real word problems. Different extensions of IFRS are being investigated, see for example, [63, 64] to tackle the uncertainty in multi criteria group decision making problems. Chinram et al. [65] established the algebraic norm based AOPs based on EDAS technique under IFR information and discussed their applications in MAGDM. Jiang [72] suggested a decision-theoretic fuzzy rough set (DTFRS) model in hesitant fuzzy information systems and discussed its application in multi-attribute decision-making.

In some real-life circumstances, there exist numerous cases when decision-makers (DMs) have their strong points of view about ranking or rating of plans, projects, or political statements of a government. The q-rung orthopair hesitant fuzzy rough sets (q-ROHFRSs), a hybrid intelligent structure of rough sets and q-ROHFSs, is an improved classification approach that has captivated our interest in dealing with imprecise and ambiguous information. From the analysis, it is concluded that in decision-making, aggregation operators play a significant role in aggregating the collective data from different sources to a single value. In accordance with the best available knowledge to date, the development of AOps with the hybridization of q-ROHFS and a rough set is not observed in the q-ROF setting. As a result, the current q-ROHF rough structure is inspired, and we define a list of algebraic aggregation operators depending on rough data, such as q-rung orthopair hesitant fuzzy weighted averaging, order weighted averaging, hybrid weighted averaging, weighted geometric, ordered weighted geometric and hybrid weighted geometric aggregation operators, under the algebraic t-norm and t-conorm.

The main contributions of this article are as follows:

(1) To construct new notion of q-rung orthopair hesitant fuzzy rough sets and investigate their basic operational laws.

(2) To develop a list of aggregation operators based on algebraic t-norm and t-conorm and to discuss their related properties in detail.

(3) To develop a decision making methodology using proposed aggregation operators to aggregate the uncertain information in real word decision making problems.

(4) A numerical case study of a real-life DM problem concerning wind power project site selection evaluation is constructed based on the established operators.

(5) Finally, comparisons with the q-ROHFR-TOPSIS method are made to interpret the outcomes. The ranking of the obtained results are presented graphically.

The rest of this manuscript is organized as follows: Section 2 briefly retrospects some basic concepts of IFS, q-ROFSs, HFSs and rough set theory. A novel notion of q-rung orthopair hesitant fuzzy sets is presented and their basic interesting operational laws are defined in Section 3. Section 4 highlights a list of algebraic aggregation operators are established to aggregate the uncertain information. Section 5 is devoted to a decision making methodology based on the developed aggregation operators. Section 6 presents the numerical illustration concerning the agriculture farming. Further this section deals with the applicability of the developed methodology. Section 7 establishes the q-ROHFR-TOPSIS methodology to validate the proposed aggregation operators based multi attribute decision making methodology. Section 8 concludes this manuscript.

2. Preliminaries

This section describes the basic terminologies, that is intuitionistic fuzzy sets (IFS), q-rung orthopair fuzzy sets (q-ROFS), hesitant fuzzy sets (HFS), q-rung ortopair hesitant fuzzy sets (q-ROHFS), rough sets (RS) and q-rung orthopair fuzzy rough set (q-ROFRS).

Definition 1. *[1] For a universal set* $\aleph$, *an IFS* F *over* $\aleph$ *is defined as:*

$$F = \{\langle x, \wp_F(x), \Im_F(x)\rangle | x \in \aleph\},$$

for each $x \in F$ *the functions* $\wp_F : \aleph \to [0\,,1]$ *and* $\Im_F : \aleph \to [0, 1]$ *denote the degree of membership and non membership, respectively, which must satisfy the property that* $0 \leq \wp_F(x) + \Im_F(x) \leq 1$.

Definition 2. *[10] For a universal set* $\aleph$, *a q-ROFS* $\mathcal{F}$ *over* $\aleph$ *is defined as:*

$$\mathcal{F} = \{\langle x, \wp_{\mathcal{F}}(x), \Im_{\mathcal{F}}(x)\rangle | x \in \aleph\}$$

for each $x \in \aleph$ *the functions* $\wp_{\mathcal{F}} : \aleph \to [0, 1]$ *and* $\Im_{\mathcal{F}} : \aleph \to [0, 1]$ *denote the degrees of membership and non membership, respectively, which must satisfy* $(\Im_{\mathcal{F}}(x))^q + (\wp_{\mathcal{F}}(x))^q \leq 1, (q > 2 \in \mathbb{Z})$.

Definition 3. *[52] For a universal set* $\aleph$, *a q-rung orthopair hesitant fuzzy set (q-ROHFS)* $\mathcal{G}$*is defined as:*

$$\mathcal{G} = \{\left\langle x, \wp_{h_{\mathcal{G}}}(x), \Im_{h_{\mathcal{G}}}(x)\right\rangle | x \in \aleph\},$$

where $\wp_{h_{\mathcal{G}}}(x)$ *and* $\Im_{h_{\mathcal{G}}}(x)$ *are sets of some values in* $[0, 1]$ *denote the membership and non membership grades, respectively, which must satisfy the following properties:* $\forall x \in \aleph$, $\forall \mu_{\mathcal{G}}(x) \in \wp_{h_{\mathcal{G}}}(x)$, $\forall \nu_{\mathcal{G}}(x) \in \Im_{h_{\mathcal{G}}}(x)$ *with* $\left(\max\left(\wp_{h_{\mathcal{G}}}(x)\right)\right)^q + \left(\min\left(\Im_{h_{\mathcal{G}}}(x)\right)\right)^q \leq 1$ *and* $\left(\min\left(\wp_{h_{\mathcal{G}}}(x)\right)\right)^q + \left(\max\left(\Im_{h_{\mathcal{G}}}(x)\right)\right)^q \leq 1$. *For simplicity, we will use a pair* $\mathcal{G} = (\wp_{h_{\mathcal{G}}}, \Im_{h_{\mathcal{G}}})$ *to mean q-rung orthopair hesitant fuzzy number (q-ROHFN).*

Definition 4. *[52] Let* $\mathcal{G}_1 = (\wp_{h_{\mathcal{G}_1}}, \Im_{h_{\mathcal{G}_1}})$ *and* $\mathcal{G}_2 = (\wp_{h_{\mathcal{G}_2}}, \Im_{h_{\mathcal{G}_2}})$ *be two q-ROHFNs. Then their basic set theoretic operations are defined as follows:*

(1) $\mathcal{G}_1 \cup \mathcal{G}_2 = \left\{ \bigcup_{\substack{\mu_1 \in \wp_{h_{\mathcal{G}_1}} \\ \mu_2 \in \wp_{h_{\mathcal{G}_2}}}} \max(\mu_1, \mu_2), \bigcup_{\substack{\nu_1 \in \Im_{h_{\mathcal{G}_1}} \\ \nu_2 \in \Im_{h_{\mathcal{G}_2}}}} \min(\nu_1, \nu_2) \right\}$;

(2) $\mathcal{G}_1 \cap \mathcal{G}_2 = \left\{ \bigcup_{\substack{\mu_1 \in \wp_{h_{\mathcal{G}_1}} \\ \mu_2 \in \wp_{h_{\mathcal{G}_2}}}} \min(\mu_1, \mu_2), \bigcup_{\substack{\nu_1 \in \Im_{h_{\mathcal{G}_1}} \\ \nu_2 \in \Im_{h_{\mathcal{G}_2}}}} \max(\nu_1, \nu_2) \right\};$

(3) $\mathcal{G}_1^c = \left\{ \Im_{h_{\mathcal{G}_1}}, \wp_{h_{\mathcal{G}_1}} \right\}.$

Definition 5. *[52] Let $\mathcal{G}_1 = (\wp_{h_{\mathcal{G}_1}}, \Im_{h_{\mathcal{G}_1}})$ and $\mathcal{G}_2 = (\wp_{h_{\mathcal{G}_2}}, \Im_{h_{\mathcal{G}_2}})$ be two q-ROHFNs and $q > 2$ and $\gamma > 0$ be any real number. Then the operational laws are:*

(1) $\mathcal{G}_1 \oplus \mathcal{G}_2 = \left\{ \bigcup_{\substack{\mu_1 \in \wp_{h_{\mathcal{G}_1}} \\ \mu_2 \in \wp_{h_{\mathcal{G}_2}}}} \left\{ \sqrt[q]{\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q} \right\}, \bigcup_{\substack{\nu_1 \in \Im_{h_{\mathcal{G}_1}} \\ \nu_2 \in \Im_{h_{\mathcal{G}_2}}}} \{\nu_1 \cdot \nu_2\} \right\};$

(2) $\mathcal{G}_1 \otimes \mathcal{G}_2 = \left\{ \bigcup_{\substack{\mu_1 \in \wp_{h_{\mathcal{G}_1}} \\ \mu_2 \in \wp_{h_{\mathcal{G}_2}}}} \{\mu_1 \cdot \mu_2\}, \bigcup_{\substack{\nu_1 \in \Im_{h_{\mathcal{G}_1}} \\ \nu_2 \in \Im_{h_{\mathcal{G}_2}}}} \left\{ \sqrt[q]{\nu_1^q + \nu_2^q - \nu_1^q \nu_2^q} \right\} \right\};$

(3) $\gamma \mathcal{G}_1 = \left\{ \bigcup_{\mu_1 \in \wp_{h_{\mathcal{G}_1}}} \left\{ \sqrt[q]{1 - (1 - \mu_1^q)^\gamma} \right\}, \bigcup_{\nu_1 \in \Im_{h_{\mathcal{G}_1}}} \left\{ \nu_1^\gamma \right\} \right\};$

(4) $\mathcal{G}_1^\gamma = \left\{ \bigcup_{\mu_1 \in \wp_{h_{\mathcal{G}_1}}} \left\{ \mu_1^\gamma \right\}, \bigcup_{\nu_1 \in \Im_{h_{\mathcal{G}_1}}} \left\{ \sqrt[q]{1 - (1 - \nu_1^q)^\gamma} \right\} \right\}.$

Definition 6. *Let $\aleph$ be the universal set and $\mathcal{R} \subseteq \aleph \times \aleph$ be a (crisp) relation. Then:*
(1) $\mathcal{R}$ is reflexive if $(\jmath, \jmath) \in \mathcal{R}$, for each $\jmath \in \aleph$;
(3) $\mathcal{R}$ is symmetric if $\forall \jmath, a \in \aleph$, $(\jmath, a) \in \mathcal{R}$ then $(a, \jmath) \in \mathcal{R}$;
(4) $\mathcal{R}$ is transitive if $\forall\ \jmath, a, b \in \aleph$, $(\jmath, a) \in \aleph$ and $(a, b) \in \mathcal{R}$ implies $(\jmath, b) \in \mathcal{R}$.

Definition 7. *[9] Let $\aleph$ be a universal set and $\mathcal{R}$ be any relation on $\aleph$. Define a set valued mapping $\mathcal{R}^* : \aleph \rightarrow M(\aleph)$ by $\mathcal{R}^*(\jmath) = \{a \in \aleph | (\jmath, a) \in \mathcal{R}\}$, for all $\jmath \in \aleph$, where $\mathcal{R}^*(\jmath)$ is called a successor neighborhood of the element $\jmath$ with respect to relation $\mathcal{R}$.The pair $(\aleph, \mathcal{R})$ is called (crisp) approximation space. Now for any set $\mathcal{U} \subseteq \aleph$, the lower and upper approximation of $\mathcal{U}$ with respect to approximations space $(\aleph, \mathcal{R})$ is defined as:*

$$\begin{aligned} \underline{\mathcal{R}}(\mathcal{U}) &= \{\jmath \in \aleph | \mathcal{R}^*(\jmath) \subseteq \mathcal{U}\}; \\ \overline{\mathcal{R}}(\mathcal{U}) &= \{\jmath \in \aleph | \mathcal{R}^*(\jmath) \cap \mathcal{U} \neq \phi\}. \end{aligned}$$

The pair $\left(\underline{\mathcal{R}}(\mathcal{U}), \overline{\mathcal{R}}(\mathcal{U})\right)$ is called rough set and both $\underline{\mathcal{R}}(\mathcal{U}), \overline{\mathcal{R}}(\mathcal{U}) : M(\aleph) \rightarrow M(\aleph)$ are upper and lower approximation operators.

Definition 8. *[61] Let $\aleph$ be the universal set and $\mathcal{R} \in IFS\,(\aleph \times \aleph)$ be an IF relation. Then*

(1) *$\mathcal{R}$ is reflexive if $\mu_{\mathcal{R}}(\jmath, \jmath) = 1$ and $\nu_{\mathcal{R}}(\jmath, \jmath) = 0$, for all $\jmath \in \aleph$;*

(2) *$\mathcal{R}$ is symmetric if for all$(\jmath, a) \in \aleph \times \aleph$, $\mu_{\mathcal{R}}(\jmath, a) = \mu_{\mathcal{R}}(a, \jmath)$ and $\nu_{\mathcal{R}}(\jmath, a) = \nu_{\mathcal{R}}(a, \jmath)$;*

(3) *$\mathcal{R}$ is transitive if for all$(\jmath, b) \in \aleph \times \aleph$,*

$$\mu_{\mathcal{R}}(\jmath, b) \geq \bigvee_{a \in \aleph} \left[\mu_{\mathcal{R}}(\jmath, a) \wedge \mu_{\mathcal{R}}(a, b)\right];$$

and

$$\nu_{\mathcal{R}}(\jmath, b) = \bigwedge_{a \in \aleph} \left[\nu_{\mathcal{R}}(\jmath, a) \wedge \nu_{\mathcal{R}}(a, b)\right].$$

Definition 9. *Let $\aleph$ be the universal set. Then any $\mathcal{R} \in$ q-$RFS\,(\aleph \times \aleph)$ is called q-rung relation. The pair $(\aleph,\ \mathcal{R})$ is said to be q-rung approximation space. Now for any $\mathcal{U} \subseteq$ q-$RFS\,(\aleph)$, the upper and lower approximations of $\mathcal{U}$ with respect to q-RF approximation space $(\aleph, \mathcal{R})$ are two q-RFSs, which are denoted by $\overline{\mathcal{R}}(\mathcal{U})$ and $\underline{\mathcal{R}}(\mathcal{U})$, respectively and is defined as:*

$$\begin{aligned}
\overline{\mathcal{R}}(\mathcal{U}) &= \{\left\langle \jmath, \mu_{\overline{\mathcal{R}}(\mathcal{U})}(\jmath), \nu_{\overline{\mathcal{R}}(\mathcal{U})}(\jmath)\right\rangle | \jmath \in \aleph\}; \\
\underline{\mathcal{R}}(\mathcal{U}) &= \{\left\langle \jmath, \mu_{\underline{\mathcal{R}}(\mathcal{U})}(\jmath), \nu_{\underline{\mathcal{R}}(\mathcal{U})}(\jmath)\right\rangle | \jmath \in \aleph\};
\end{aligned}$$

where

$$\begin{aligned}
\mu_{\overline{\mathcal{R}}(\mathcal{U})}(\jmath) &= \bigvee_{g \in \aleph} [\mu_{\mathcal{R}}(\jmath, g) \bigvee \mu_{\mathcal{U}}(g)]; \\
\nu_{\overline{\mathcal{R}}(\mathcal{U})}(\jmath) &= \bigwedge_{g \in \aleph} [\nu_{\mathcal{R}}(\jmath, g) \bigwedge \nu_{\mathcal{U}}(g)]; \\
\mu_{\underline{\mathcal{R}}(\mathcal{U})}(\jmath) &= \bigwedge_{g \in \aleph} [\mu_{\mathcal{R}}(\jmath, g) \bigwedge \mu_{\mathcal{U}}(g)]; \\
\nu_{\underline{\mathcal{R}}(\mathcal{U})}(\jmath) &= \bigvee_{g \in \aleph} [\nu_{\mathcal{R}}(\jmath, g) \bigvee \nu_{\mathcal{U}}(g)];
\end{aligned}$$

such that $0 \leq (\mu_{\overline{\mathcal{R}}(\mathcal{U})}(\jmath))^q + (\nu_{\overline{\mathcal{R}}(\mathcal{U})}(\jmath))^q \leq 1$, and $0 \leq \left(\mu_{\underline{\mathcal{R}}(\mathcal{U})}(\jmath)\right)^q + \left(\nu_{\underline{\mathcal{R}}(\mathcal{U})}(\jmath)\right)^q \leq 1$. As $\left(\underline{\mathcal{R}}(\mathcal{U}), \overline{\mathcal{R}}(\mathcal{U})\right)$ are q-RFS s, so $\underline{\mathcal{R}}(\mathcal{U}), \overline{\mathcal{R}}(\mathcal{U})$: q-$RFS\,(\aleph) \to$ q-$RFS\,(\aleph)$ are upper and lower approximation operators. The pair $\mathcal{R}(\mathcal{U}) = (\underline{\mathcal{R}}(\mathcal{U}), \overline{\mathcal{R}}(\mathcal{U})) = \{\left\langle \jmath, (\mu_{\underline{\mathcal{R}}(\mathcal{U})}(\jmath), \nu_{\underline{\mathcal{R}}(\mathcal{U})}(\jmath), (\mu_{\overline{\mathcal{R}}(\mathcal{U})}(\jmath), \nu_{\overline{\mathcal{R}}(\mathcal{U})}(\jmath))\right\rangle | \jmath \in \mathcal{U}\}$ is known as q-rung orthopair rough set. For simplicity $\mathcal{R}(\mathcal{U}) = \{\left\langle \jmath, \mu_{\underline{\mathcal{R}}(\mathcal{U})}(\jmath), \nu_{\underline{\mathcal{R}}(\mathcal{U})}(\jmath), (\mu_{\overline{\mathcal{R}}(\mathcal{U})}(\jmath), \nu_{\overline{\mathcal{R}}(\mathcal{U})}(\jmath))\right\rangle | \jmath \in \aleph\}$ is represented as $\mathcal{R}(\mathcal{U}) = ((\underline{\mu}, \underline{\nu}), (\overline{\mu}, \overline{\nu}))$ and is known as q-RFRV.

3. Construction of q-rung orthopair hesitant fuzzy rough sets

Herein, we propose the notion of q-rung orthopair hesitant fuzzy rough set (q-ROHFRS) which is the hybrid structure of rough set and q-ROFS. Also, we initiate the new score and accuracy functions to rank the q-ROHFRS and to put forward its basic operational laws.

Definition 10. *Let $\aleph$ be the universal set and for any subset $\mathcal{R} \in$ q-$ROHFS\,(\aleph \times \aleph)$ is said to be an q-rung hesitant fuzzy relation. The pair $(\aleph, \mathcal{R})$ is said to be q-ROHF approximation space. If $\mathcal{U} \subseteq$ q-$ROHFS\,(\aleph)$, then the upper and lower approximations of $\mathcal{U}$ with respect to q-ROHF approximation space $(\aleph, \mathcal{R})$ are two q-ROHFSs, which are denoted by $\overline{\mathcal{R}}(\mathcal{U})$ and $\underline{\mathcal{R}}(\mathcal{U})$, respectively and defined as:*

$$\overline{\mathcal{R}}(\mathcal{U}) = \left\{\left\langle \jmath, \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath), \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath)\right\rangle | \jmath \in \aleph\right\};$$

$$\underline{\mathcal{R}}(\mathcal{U}) = \left\{\left\langle \jmath, \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath), \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath)\right\rangle | \jmath \in \aleph\right\};$$

where

$$\begin{aligned}
\wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath) &= \bigvee_{k\in\aleph}\left[\wp_{h_{\mathcal{R}}}(\jmath,k)\bigvee\wp_{h_{\mathcal{U}}}(k)\right];\\
\Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath) &= \bigwedge_{k\in\aleph}\left[\Im_{h_{\mathcal{R}}}(\jmath,k)\bigwedge\Im_{h_{\mathcal{U}}}(k)\right];\\
\wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath) &= \bigwedge_{k\in\aleph}\left[\wp_{h_{\mathcal{R}}}(\jmath,k)\bigwedge\wp_{h_{\mathcal{U}}}(k)\right];\\
\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath) &= \bigvee_{k\in\aleph}\left[\Im_{h_{\mathcal{R}}}(\jmath,k)\bigvee\Im_{h_{\mathcal{U}}}(k)\right];
\end{aligned}$$

such that $0 \leq \left(\max(\wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath))\right)^q + \left(\min(\Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath))\right)^q \leq 1$ *and* $0 \leq \left(\min(\wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath))\right)^q + \left(\max(\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath))\right)^q \leq 1$. *As* $\left(\overline{\mathcal{R}}(\mathcal{U}), \underline{\mathcal{R}}(\mathcal{U})\right)$ *are q-ROHFS s, so* $\overline{\mathcal{R}}(\mathcal{U}), \underline{\mathcal{R}}(\mathcal{U})$: *q-ROHFS* $(\aleph) \to$ *q-RFS* $(\aleph)$ *are upper and lower approximation operators. The pair*

$$\mathcal{R}(\mathcal{U}) = \left(\underline{\mathcal{R}}(\mathcal{U}), \overline{\mathcal{R}}(\mathcal{U})\right) = \left\{\left\langle \jmath, \left(\wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath), \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath)\right), \left(\wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath), \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath)\right)\right\rangle | \jmath \in \mathcal{U}\right\}$$

will be called q-rung orthopair hesitant fuzzy rough set. For simplicity

$$\mathcal{R}(\mathcal{U}) = \left\{\left\langle \jmath, \left(\wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath), \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath)\right), \left(\wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath), \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath)\right)\right\rangle | \jmath \in \mathcal{U}\right\}$$

is represented as $\mathcal{R}(\mathcal{U}) = \left((\underline{\wp}, \underline{\Im}), (\overline{\wp}, \overline{\Im})\right)$ *and is known as q-ROHFR value.*

Definition 11. *Let* $\mathcal{R}(\mathcal{U}_1) = (\underline{\mathcal{R}}(\mathcal{U}_1), \overline{\mathcal{R}}(\mathcal{U}_1))$ *and* $\mathcal{R}(\mathcal{U}_2) = (\underline{\mathcal{R}}(\mathcal{U}_2), \overline{\mathcal{R}}(\mathcal{U}_2))$ *be two q-ROHFRSs. Then*

(1) $\mathcal{R}(\mathcal{U}_1) \cup \mathcal{R}(\mathcal{U}_2) = \{(\underline{\mathcal{R}}(\mathcal{U}_1) \cup \underline{\mathcal{R}}(\mathcal{U}_2)), (\overline{\mathcal{R}}(\mathcal{U}_1) \cup \overline{\mathcal{R}}(\mathcal{U}_2))\}$

(2) $\mathcal{R}(\mathcal{U}_1) \cap \mathcal{R}(\mathcal{U}_2) = \{(\underline{\mathcal{R}}(\mathcal{U}_1) \cap \underline{\mathcal{R}}(\mathcal{U}_2)), (\overline{\mathcal{R}}(\mathcal{U}_1) \cap \overline{\mathcal{R}}(\mathcal{U}_2))\}$.

Definition 12. *Let* $\mathcal{R}(\mathcal{U}_1) = (\underline{\mathcal{R}}(\mathcal{U}_1), \overline{\mathcal{R}}(\mathcal{U}_1))$ *and* $\mathcal{R}(\mathcal{U}_2) = (\underline{\mathcal{R}}(\mathcal{U}_2), \overline{\mathcal{R}}(\mathcal{U}_2))$ *be two q-ROHFRSs. Then*

(1) $\mathcal{R}(\mathcal{U}_1) \oplus \mathcal{R}(\mathcal{U}_2) = \{(\underline{\mathcal{R}}(\mathcal{U}_1) \oplus \underline{\mathcal{R}}(\mathcal{U}_2)), (\overline{\mathcal{R}}(\mathcal{U}_1) \oplus \overline{\mathcal{R}}(\mathcal{U}_2))\}$

(2) $\mathcal{R}(\mathcal{U}_1) \otimes \mathcal{R}(\mathcal{U}_2) = \{(\underline{\mathcal{R}}(\mathcal{U}_1) \otimes \underline{\mathcal{R}}(\mathcal{U}_2)), (\overline{\mathcal{R}}(\mathcal{U}_1) \otimes \overline{\mathcal{R}}(\mathcal{U}_2))\}$

(3) $\mathcal{R}(\mathcal{U}_1) \subseteq \mathcal{R}(\mathcal{U}_2) = \{(\underline{\mathcal{R}}(\mathcal{U}_1) \subseteq \underline{\mathcal{R}}(\mathcal{U}_2))$ *and* $(\overline{\mathcal{R}}(\mathcal{U}_1) \subseteq \overline{\mathcal{R}}(\mathcal{U}_2))\}$

(4) $\gamma\mathcal{R}(\mathcal{U}_1) = (\gamma\underline{\mathcal{R}}(\mathcal{U}_1), \gamma\overline{\mathcal{R}}(\mathcal{U}_1))$ *for* $\gamma \geq 1$

(5) $(\mathcal{R}(\mathcal{U}_1))^\gamma = ((\underline{\mathcal{R}}(\mathcal{U}_1))^\gamma, (\overline{\mathcal{R}}(\mathcal{U}_1))^\gamma)$ *for* $\gamma \geq 1$

(6) $\mathcal{R}(\mathcal{U}_1)^c = (\underline{\mathcal{R}}(\mathcal{U}_1)^c, \overline{\mathcal{R}}(\mathcal{U}_1)^c)$, *where* $\underline{\mathcal{R}}(\mathcal{U}_1)^c$ *and* $\overline{\mathcal{R}}(\mathcal{U}_1)^c$ *shows the complement of q-rung fuzzy rough approximation operators* $\underline{\mathcal{R}}(\mathcal{U}_1)$ *and* $\overline{\mathcal{R}}(\mathcal{U}_1)$,*that is* $\underline{\mathcal{R}}(\mathcal{U}_1)^c = \left(\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}, \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}\right)$.

(7) $\mathcal{R}(\mathcal{U}_1) = \mathcal{R}(\mathcal{U}_2)$ *iff* $\underline{\mathcal{R}}(\mathcal{U}_1) = \underline{\mathcal{R}}(\mathcal{U}_2)$ *and* $\overline{\mathcal{R}}(\mathcal{U}_1) = \overline{\mathcal{R}}(\mathcal{U}_2)$.

For the comarison/ranking of two or more q-ROHFR values, we utilize the score function. Superior the score value of q-ROHFR value greater that value is, and smaller the score value inferior that q-ROHFR value is. We will use the accuracy function when the score values are equal.

Definition 13. *The score function for q-ROHFRV $\mathcal{R}(\mathcal{U}) = (\underline{\mathcal{R}}(\mathcal{U}), \overline{\mathcal{R}}(\mathcal{U})) = ((\underline{\wp}, \underline{\Im}), (\overline{\wp}, \overline{\Im}))$ is given by*

$$\eth(\mathcal{R}(\mathcal{U})) = \frac{1}{4}\left(\begin{array}{c} 2 + \frac{1}{M_{\mathcal{G}}} \sum_{\underline{\mu_i} \in \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \left\{\underline{\mu_i}\right\} + \frac{1}{N_{\mathcal{G}}} \sum_{\overline{\mu_i} \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \{\overline{\mu_i}\} - \\ \frac{1}{M_{\mathcal{G}}} \sum_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} (\underline{\nu_i}) - \frac{1}{M_{\mathcal{G}}} \sum_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} (\overline{\nu_i}) \end{array} \right),$$

The accuracy function for q-ROHFR value $\mathcal{R}(\mathcal{U}) = (\underline{\mathcal{R}}(\mathcal{U}), \overline{\mathcal{R}}(\mathcal{U})) = ((\underline{\wp}, \underline{\Im}), (\overline{\wp}, \overline{\Im}))$ is given by

$$\mathbf{AC}\mathcal{R}(\mathcal{U}) = \frac{1}{4}\left(\begin{array}{c} \frac{1}{M_{\mathcal{G}}} \sum_{\mu_i \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} (\overline{\mu_i}) + \frac{1}{M_{\mathcal{G}}} \sum_{\mu_i \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} (\overline{\mu_i}) + \\ \frac{1}{M_{\mathcal{G}}} \sum_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} (\underline{\nu_i}) + \frac{1}{M_{\mathcal{G}}} \sum_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} (\overline{\nu_i}) \end{array} \right),$$

where $M_{\mathcal{G}}$ and $N_{\mathcal{G}}$ represent the number of elements in $\wp_{h_g}$ and $\Im_{h_g}$, respectively.

Definition 14. *Suppose $\mathcal{R}(\mathcal{U}_1) = (\underline{\mathcal{R}}(\mathcal{U}_1), \overline{\mathcal{R}}(\mathcal{U}_1))$ and $\mathcal{R}(\mathcal{U}_2) = (\underline{\mathcal{R}}(\mathcal{U}_2), \overline{\mathcal{R}}(\mathcal{U}_2))$ are two q-ROHFR values. Then:*

(1) *If $\eth(\mathcal{R}(\mathcal{U}_1)) > \eth(\mathcal{R}(\mathcal{U}_2))$, then $\mathcal{R}(\mathcal{U}_1) > \mathcal{R}(\mathcal{U}_2)$;*

(2) *If $\eth(\mathcal{R}(\mathcal{U}_1)) < \eth(\mathcal{R}(\mathcal{U}_2))$, then $\mathcal{R}(\mathcal{U}_1) < \mathcal{R}(\mathcal{U}_2)$;*

(3) *If $\eth(\mathcal{R}(\mathcal{U}_1)) = \eth(\mathcal{R}(\mathcal{U}_2))$, then*
(a) If $\mathbf{AC}\mathcal{R}(\mathcal{U}_1) > \mathbf{AC}\mathcal{R}(\mathcal{U}_2)$ then $\mathcal{R}(\mathcal{U}_1) > \mathcal{R}(\mathcal{U}_2)$;
(b) If $\mathbf{AC}\mathcal{R}(\mathcal{U}_1) < \mathbf{AC}\mathcal{R}(\mathcal{U}_2)$ then $\mathcal{R}(\mathcal{U}_1) < \mathcal{R}(\mathcal{U}_2)$;
(c) If $\mathbf{AC}\mathcal{R}(\mathcal{U}_1) = \mathbf{AC}\mathcal{R}(\mathcal{U}_2)$ then $\mathcal{R}(\mathcal{U}_1) = \mathcal{R}(\mathcal{U}_2)$.

4. q-Rung orthopair hesitant fuzzy rough geometric aggregation operators

Herein, we introduce the new idea of q-ROHF rough aggregation operators by embedding the notions of rough sets and q-ROHF aggregation operators to get aggregation concepts of q-rung orthopair hesitant fuzzy rough weighted geometric aggregation (q-ROHFRWGA) operator, q-rung orthopair hesitant fuzzy rough ordered weighted geometric aggregation (q-ROHFROWGA) operator and q-rung orthopair hesitant fuzzy rough hybrid weighted geometric aggregation (q-ROHFRHWGA) operator. Some fundamental properties of these notions are discussed.

4.1. q-Rung orthopair hesitant fuzzy rough weighted geometric aggregation operator

In this subsection, we discuss q-ROHFRWGA operator by employing the idea of rough sets into q-ROHF geometric operators. The detail of important characteristics of the developed operator are illustrated.

Definition 15. *Let $\mathcal{R}(\mathcal{U}_i) = (\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i))$ $(i = 1, 2, 3, ..., n)$ be the collection of q-ROHFR values with weight vector $w = (w_1, w_2, ..., w_n)^T$ such that $\sum_{i=1}^{n} w_i = 1$ and $0 \leq w_i \leq 1$. Then q-ROHFRWGA operator is determined as:*

$$q - ROHFRWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) = \left(\sum_{i=1}^{n} \left(\underline{\mathcal{R}}(\mathcal{U}_i)\right)^{w_i}, \sum_{i=1}^{n} \left(\overline{\mathcal{R}}(\mathcal{U}_i)\right)^{w_i}\right).$$

Based on the aforementioned definition, the aggregated result for q-ROHFRWGA operator is presented in the following theorem.

Theorem 1. *Let $\mathcal{R}(\mathcal{U}_i) = (\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i))$ $(i = 1, 2, 3, ..., n)$ be the collection of q-ROHFR values with weight vectors $w = (w_1, w_2, ..., w_n)^T$ such that $\sum_{i=1}^{n} w_i = 1$ and $0 \leq w_i \leq 1$. Then the q-ROHFRWG operator is described as:*

$$
\begin{aligned}
& q - ROHFRWG\,(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)) \\
= & \left(\sum_{i=1}^{n} \left(\underline{\mathcal{R}}(\mathcal{U}_i)\right)^{w_i}, \sum_{i=1}^{n} \left(\overline{\mathcal{R}}(\mathcal{U}_i)\right)^{w_i} \right) \\
= & \left(\begin{array}{c} \left\{ \bigcup_{\underline{\mu_i} \in \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{n} \left(\underline{\mu_i}\right)^{w_i}, \bigcup_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{1 - \prod_{i=1}^{n} \left(1 - \left(\underline{\nu_i}\right)^q\right)^{w_i}} \right\} \\ \left\{ \bigcup_{\overline{\mu_i} \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{n} (\overline{\mu_i})^{w_i}, \bigcup_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{1 - \prod_{i=1}^{n} (1 - (\overline{\nu_i})^q)^{w_i}} \right\} \end{array} \right).
\end{aligned}
$$

Proof. Utilizing mathematical induction to find the the desired proof. Using the operational law, it follows that

$$
\mathcal{R}(\mathcal{U}_1) \oplus \mathcal{R}(\mathcal{U}_2) = \left[\underline{\mathcal{R}}(\mathcal{U}_1) \oplus \underline{\mathcal{R}}(\mathcal{U}_2), \overline{\mathcal{R}}(\mathcal{U}_1) \oplus \overline{\mathcal{R}}(\mathcal{U}_2)\right]
$$

and

$$
\gamma\mathcal{R}(\mathcal{U}_1) = \left(\gamma\underline{\mathcal{R}}(\mathcal{U}_1), \gamma\overline{\mathcal{R}}(\mathcal{U}_1)\right)
$$

If $n = 2$, then

$$
\begin{aligned}
& q - ROHFRWA\,(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2)) \\
= & \left(\sum_{i=1}^{2} w_i \underline{\mathcal{R}}(\mathcal{U}_i), \sum_{i=1}^{2} w_i \overline{\mathcal{R}}(\mathcal{U}_i) \right) \\
= & \left(\begin{array}{c} \left(\bigcup_{\underline{\mu_i} \in \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{2} \left(\underline{\mu_i}\right)^{w_i}, \bigcup_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{2} \left(1 - \left(\underline{\nu_i}\right)^q\right)^{w_i}\right)} \right) \\ \left(\bigcup_{\overline{\mu_i} \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{2} (\overline{\mu_i})^{w_i}, \bigcup_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{2} (1 - (\overline{\nu_i})^q)^{w_i}\right)} \right) \end{array} \right).
\end{aligned}
$$

The result is true for $n = 2$. Let it be true for $n = k$, that is

$$
\begin{aligned}
& q - ROHFRWA\,(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_k)) \\
= & \left(\sum_{i=1}^{k} w_i \underline{\mathcal{R}}(\mathcal{U}_i), \sum_{i=1}^{k} w_i \overline{\mathcal{R}}(\mathcal{U}_i) \right) \\
= & \left(\begin{array}{c} \left(\bigcup_{\underline{\mu_i} \in \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{k} \left(\underline{\mu_i}\right)^{w_i}, \bigcup_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{k} \left(1 - \left(\underline{\nu_i}\right)^q\right)^{w_i}\right)} \right), \\ \left(\bigcup_{\overline{\mu_i} \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{k} (\overline{\mu_i})^{w_i}, \bigcup_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{k} (1 - (\overline{\nu_i})^q)^{w_i}\right)} \right) \end{array} \right).
\end{aligned}
$$

Now, we have to show that the result is true for $n = k + 1$, we have

$$q - ROHFRWA\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_{k+1})\right)$$

$$= \begin{pmatrix} \left(\sum_{i=1}^{k} w_i \underline{\mathcal{R}}(\mathcal{U}_i) \oplus w_{k+1} \underline{\mathcal{R}}(\mathcal{U}_{k+1})\right), \\ \left(\sum_{i=1}^{k} w_i \overline{\mathcal{R}}(\mathcal{U}_i) \oplus w_{k+1} \overline{\mathcal{R}}(\mathcal{U}_{k+1})\right) \end{pmatrix}$$

$$= \begin{pmatrix} \left(\bigcup_{\underline{\mu_i} \in \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{K+1} \left(\underline{\mu_i}\right)^{w_i}, \bigcup_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{K+1} \left(1 - \left(\underline{\nu_i}\right)^q\right)^{w_i}\right)} \right) \\ \left(\bigcup_{\overline{\mu_i} \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{K+1} \left(\overline{\mu_i}\right)^{w_i}, \bigcup_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{K+1} \left(1 - \left(\overline{\nu_i}\right)^q\right)^{w_i}\right)} \right) \end{pmatrix}.$$

Thus the result is true for $n = k + 1$. Hence, the result is true for all $n \geq 1$. From the above analysis $\underline{\mathcal{R}}(\mathcal{U})$ and $\overline{\mathcal{R}}(\mathcal{U})$ are q-ROHFR values. So, $\sum_{i=1}^{k} w_i \underline{\mathcal{R}}(\mathcal{U}_i)$ and $\sum_{i=1}^{k} w_i \overline{\mathcal{R}}(\mathcal{U}_i)$ are also q-ROHFR values. Therefore, q-ROHFRWA $(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n))$ is a q-ROHFR value under q-ROHF approximation space $(\aleph, \mathcal{R})$. □

Theorem 2. *Let $\mathcal{R}(\mathcal{U}_i) = (\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i))$ $(i = 1, 2, 3, ..., n)$ be the collection of q-ROHFR values with weight vectors $w = (w_1, w_2, ..., w_n)^T$ such that $\sum_{i=1}^{n} w_i = 1$ and $0 \leq w_i \leq 1$. Then q-ROHFRWG operator satisfy the following properties:*

(1) ***Idempotency:*** *If $\mathcal{R}(\mathcal{U}_i) = \mathcal{H}(\mathcal{U})$ for $i = 1, 2, 3, ..., n$ where $\mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}), \overline{\mathcal{H}}(\mathcal{U})\right) = \left((\underline{b}, \underline{d}), (\overline{b}, \overline{d})\right)$. Then*

$$q - ROHFRWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ...\mathcal{R}(\mathcal{U}_n)\right) = \mathcal{H}(\mathcal{U}).$$

(2) ***Boundedness:*** *Let $(\mathcal{R}(\mathcal{U}))^- = \left(\min_i \underline{\mathcal{R}}(\mathcal{U}_i), \max_i \overline{\mathcal{R}}(\mathcal{U}_i)\right)$ and $(\mathcal{R}(\mathcal{U}))^+ = \left(\max_i \underline{\mathcal{R}}(\mathcal{U}_i), \min_i \overline{\mathcal{R}}(\mathcal{U}_i)\right)$. Then*

$$(\mathcal{R}(\mathcal{U}))^- \leq q - ROHFRWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) \leq (\mathcal{R}(\mathcal{U}))^+.$$

(3) ***Monotonicity:*** *Suppose $\mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}_i), \overline{\mathcal{H}}(\mathcal{U}_i)\right)$ $(i = i, 2, ..., n)$ is another collection of q-ROHFRVs such that $\underline{\mathcal{H}}(\mathcal{U}_i) \leq \underline{\mathcal{R}}(\mathcal{U}_i)$ and $\overline{\mathcal{H}}(\mathcal{U}_i) \leq \overline{\mathcal{R}}(\mathcal{U}_i)$. Then*

$$q - ROHFRWG\left(\mathcal{H}(\mathcal{U}_1), \mathcal{H}(\mathcal{U}_2), ..., \mathcal{H}(\mathcal{U}_n)\right) \leq q - ROHFRWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right).$$

(4) ***Shiftinvariance:*** *Let q-ROHFRV $\mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}), \overline{\mathcal{H}}(\mathcal{U})\right) = \left((\underline{b}, \underline{d}), (\overline{b}, \overline{d})\right)$. Then*

$$q - ROHFRWG\left(\mathcal{R}(\mathcal{U}_1) \oplus \mathcal{H}(\mathcal{U}), \mathcal{R}(\mathcal{U}_2) \oplus \mathcal{H}(\mathcal{U}), ..., \mathcal{R}(\mathcal{U}_n) \oplus \mathcal{H}(\mathcal{U})\right) = \\ q - ROHFRWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) \oplus \mathcal{H}(\mathcal{U}).$$

(5) ***Homogeneity:*** *For any real number $\gamma > 0$;*

$$q - ROHFRWG\left(\gamma\mathcal{R}(\mathcal{U}_1), \gamma\mathcal{R}(\mathcal{U}_2), ..., \gamma\mathcal{R}(\mathcal{U}_n)\right) = \gamma \cdot q - ROHFRWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right).$$

(6) ***Commutativity:*** *Suppose $\mathcal{R}'(\mathcal{U}_i) = \left(\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}'}(\mathcal{U}_i)\right)$ and $\mathcal{R}(\mathcal{U}_i) = (\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i))$, $(i = 1, 2, 3, ..., n)$ is a collection of q-ROHFR values. Then*

$$q - ROHFRWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) = q - ROHFRWG\left(\mathcal{R}'(\mathcal{U}_1), \mathcal{R}'(\mathcal{U}_2), ..., \mathcal{R}'(\mathcal{U}_n)\right).$$

Proof. **(1) Idempotency:** As $\mathcal{R}(\mathcal{U}_i) = \mathcal{H}(\mathcal{U})$ (for all $i = 1,2,3,...,n$) where $\mathcal{H}(\mathcal{U}_i) = \left(\underline{\mathcal{H}}(\mathcal{U}), \overline{\mathcal{H}}(\mathcal{U})\right) = \left((\underline{b_i}, \underline{d_i}), (\overline{b_i}, \overline{d_i})\right)$

$$
\begin{aligned}
& q-ROHFRWA\,(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)) \\
= & \left(\sum\nolimits_{i=1}^{n} w_i \underline{\mathcal{R}}\,(\mathcal{U}_i)\,, \sum\nolimits_{i=1}^{n} w_i \overline{\mathcal{R}}(\mathcal{U}_i)\right) \\
= & \left[\begin{array}{c}
\left\{\bigcup\limits_{\underline{\mu_i}\in\wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \prod\limits_{i=1}^{n}\left(\underline{\mu_i}\right)^{w_i}, \bigcup\limits_{\underline{\nu_i}\in\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{1-\prod\limits_{i=1}^{n}\left(1-\left(\underline{\nu_i}\right)^{q}\right)^{w_i}}\right\} \\
\left\{\bigcup\limits_{\overline{\mu_i}\in\wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \prod\limits_{i=1}^{n}\left(\overline{\mu_i}\right)^{w_i}, \bigcup\limits_{\overline{\nu_i}\in\Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{1-\prod\limits_{i=1}^{n}\left(1-\left(\overline{\nu_i}\right)^{q}\right)^{w_i}}\right\}
\end{array}\right]
\end{aligned}
$$

for all i, $\mathcal{R}(\mathcal{U}_i) = \mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}), \overline{\mathcal{H}}(\mathcal{U})\right) = \left((\underline{b_i}, \underline{d_i}), (\overline{d_i}, \overline{d_i})\right)$. Therefore,

$$
\begin{aligned}
= & \left[\begin{array}{c}
\bigcup\limits_{\underline{b_i}\in\wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \prod\limits_{i=1}^{n}\left(\underline{b_i}\right)^{w_i}, \bigcup\limits_{\underline{d_i}\in\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1-\prod\limits_{i=1}^{n}\left(1-\left(\underline{d_i}\right)^{q}\right)^{w_i}\right)} \\
\bigcup\limits_{\overline{b_i}\in\wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \prod\limits_{i=1}^{n}\left(\overline{b_i}\right)^{w_i}, \bigcup\limits_{\overline{d_i}\in\Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1-\prod\limits_{i=1}^{n}\left(1-\left(\overline{d_i}\right)^{q}\right)^{w_i}\right)}
\end{array}\right] \\
= & \left[\left(\underline{b_i}, \left(1-\left(1-\underline{d_i}\right)\right)\right), \left(\overline{b_i}, \left(1-\left(1-\overline{d_i}\right)\right)\right)\right] = \left(\underline{\mathcal{H}}(\mathcal{U}), \overline{\mathcal{H}}(\mathcal{U})\right) = \mathcal{H}(\mathcal{U}).
\end{aligned}
$$

Hence q-ROHFRWA$(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)) = \mathcal{H}(\mathcal{U})$.

(2) Boundedness: As

$$
\begin{aligned}
\left(\underline{\mathcal{R}}\,(\mathcal{U})\right)^{-} &= \left[\left(\min_i\{\underline{\mu_i}\}, \max_i\left\{\underline{\nu_i}\right\}\right), \left(\min_i\{\overline{\mu_i}\}, \max_i\left\{\overline{\nu}_i\right\}\right)\right] \\
\left(\underline{\mathcal{R}}\,(\mathcal{U})\right)^{+} &= \left[\left(\max_i\{\underline{\mu_i}\}, \min_i\left\{\underline{\nu_i}\right\}\right), \left(\max_i\{\overline{\mu_i}\}, \min_i\left\{\overline{\nu}_i\right\}\right)\right]
\end{aligned}
$$

and $\mathcal{R}(\mathcal{U}_i) = \left[\left(\underline{\wp_i}, \underline{\Im_i}\right), \left(\overline{\wp_i}, \overline{\Im}_i\right)\right]$. To prove that

$$
(\mathcal{R}(\mathcal{U}))^{-} \leq q-ROHFRWA\,(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)) \leq (\mathcal{R}(\mathcal{U}))^{+}.
$$

Since for each $i = 1,2,3,...,n$, it follows that

$$
\begin{aligned}
\min_i\left\{\underline{\mu_i}\right\} &\leq \left\{\underline{\mu_i}\right\} \leq \max_i\left\{\underline{\mu_i}\right\} \Longleftrightarrow \prod_{i=1}^{n}\left(\min_i\left\{\underline{\mu_i}\right\}\right)^{w_i} \\
&\leq \prod_{i=1}^{n}\left(\underline{\mu_i}\right)^{w_i} \leq \prod_{i=1}^{n}\left(\max_i\left\{\underline{\mu_i}\right\}\right)^{w_i}.
\end{aligned}
$$

This implies that

$$
\min_i\left\{\underline{\mu_i}\right\} \leq \prod_{i=1}^{n}\left\{\underline{\mu_i}\right\}^{w_i} \leq \max_i\left\{\underline{\mu_i}\right\}. \tag{4.1}
$$

Likewise, we can show that

$$\min_i \{\overline{\mu_i}\} \leq \prod_{i=1}^{n} \{\overline{\mu_i}\}^{w_i} \leq \max_i \{\overline{\mu_i}\} \tag{4.2}$$

Next for each $i = 1, 2, 3, ..., n$, we have

$$\begin{aligned}
\min_i\{\underline{\nu_i}\} &\leq \{\underline{\nu_i}\} \leq \max_i\{\underline{\nu_i}\} \Longleftrightarrow 1 - \max_i\{\underline{\nu_i}\} \leq 1 - \{\underline{\nu_i}\} \leq 1 - \{\underline{\nu_i}\} \\
&\Longleftrightarrow \prod_{i=1}^{n} \left(1 - \max_i\{\underline{\nu_i}\}\right)^{w_i} \leq \prod_{i=1}^{n} \left(1 - \{\underline{\nu_i}\}\right)^{w_i} \leq \prod_{i=1}^{n} \left(1 - \min_i\{\underline{\nu_i}\}\right)^{w_i} \\
&\Longleftrightarrow \left(1 - \max_i\{\underline{\nu_i}\}\right) \leq \prod_{i=1}^{n} \left(1 - \{\underline{\nu_i}\}\right)^{w_i} \leq \left(1 - \min_i\{\underline{\nu_i}\}\right) \\
&\Longleftrightarrow 1 - \left(1 - \min_i\{\underline{\nu_i}\}\right) \leq 1 - \prod_{i=1}^{n} \left(1 - \{\underline{\nu_i}\}\right)^{w_i} \leq 1 - \left(1 - \max_i\{\underline{\nu_i}\}\right).
\end{aligned}$$

Hence,

$$\min_i\{\underline{\nu_i}\} \leq 1 - \prod_{i=1}^{n} \left(1 - \{\underline{\nu_i}\}\right)^{w_i} \leq \max_i\{\underline{\nu_i}\}. \tag{4.3}$$

and

$$\min_i\{\overline{\nu_i}\} \leq 1 - \prod_{i=1}^{n} (1 - \{\overline{\nu_i}\})^{w_i} \leq \max_i\{\overline{\nu_i}\}. \tag{4.4}$$

So from Eqs (4.1)–(4.4) we have

$$\left(\underline{\mathcal{R}}(\mathcal{U})\right)^{-} = \left[\left(\min_i\{\underline{\mu_i}\}, \max_i \left\{\underline{\nu_i}\right\}\right), \left(\min_i\{\overline{\mu_i}\}, \max_i \{\overline{\nu}_i\}\right)\right].$$

(3) Monotonicity: Since $\mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}_i), \overline{\mathcal{H}}(\mathcal{U}_i)\right) = \left(\left(\underline{b}, \underline{d}\right), \left(\overline{b}, \overline{d}\right)\right)$ and $\mathcal{R}(\mathcal{U}_i) = \left(\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i)\right)$ to show that $\underline{\mathcal{H}}(\mathcal{U}_i) \leq \underline{\mathcal{R}}(\mathcal{U}_i)$ and $\overline{\mathcal{H}}(\mathcal{U}_i) \leq \overline{\mathcal{R}}(\mathcal{U}_i)$ (for $i = 1, 2, 3, ..., n$), so

$$\begin{aligned}
\underline{d_i} &\leq \underline{\nu_i} \Rightarrow 1 - \underline{d_i} \leq 1 - \underline{\nu_i} \Rightarrow \prod_{i=1}^{n} \left(1 - \underline{\nu_i}\right)^{w_i} \leq \prod_{i=1}^{n} \left(1 - \underline{d_i}\right)^{w_i} \\
&\Rightarrow 1 - \prod_{i=1}^{n} \left(1 - \underline{d_i}\right)^{w_i} \leq 1 - \prod_{i=1}^{n} \left(1 - \underline{\nu_i}\right)^{w_i}.
\end{aligned} \tag{4.5}$$

next

$$\underline{b}_i \geq \underline{\mu_i} \Rightarrow \prod_{i=1}^{n} \underline{b}_i^{w_i} \geq \prod_{i=1}^{n} \underline{\mu_i}^{w_i}. \tag{4.6}$$

Likewise, we can show that

$$1 - \prod_{i=1}^{n} \left(1 - \overline{d}_i\right)^{w_i} \leq 1 - \prod_{i=1}^{n} (1 - \overline{\nu_i})^{w_i}. \tag{4.7}$$

and
$$\prod_{i=1}^{n}\left(\overline{b}_i\right)^{w_i} \geq \prod_{i=1}^{n}\left(\overline{\mu}_i\right)^{w_i}. \tag{4.8}$$

Hence, from Eqs (4.5)–(4.8), we get $\underline{\mathcal{H}}(\mathcal{U}_i) \leq \underline{\mathcal{R}}(\mathcal{U}_i)$ and $\overline{\mathcal{H}}(\mathcal{U}_i) \leq \overline{\mathcal{R}}(\mathcal{U}_i)$. Therefore,

$$q-ROHFRWA\,(\mathcal{H}(\mathcal{U}_1), \mathcal{H}(\mathcal{U}_2), ..., \mathcal{H}(\mathcal{U}_n)) \leq q-ROHFRWA\,(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)).$$

(4) Shiftinvariance: As $\mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}), \overline{\mathcal{H}}(\mathcal{U})\right) = \left((\underline{b_i}, \underline{d_i}), (\overline{b}_i, \overline{d}_i)\right)$ is a q-ROHFR value and $\mathcal{R}(\mathcal{U}_i) = \left(\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i)\right) = \left[\left(\underline{\wp_i}, \underline{\Im_i}\right), \left(\overline{\wp_i}, \overline{\Im}_i\right)\right]$ is the collection of q-ROHFR values, so

$$\mathcal{R}(\mathcal{U}_1) \oplus \mathcal{H}(\mathcal{U}) = \left[\underline{\mathcal{R}}(\mathcal{U}_1) \oplus \underline{\mathcal{H}}(\mathcal{U}), \overline{\mathcal{R}}(\mathcal{U}_i) \oplus \overline{\mathcal{H}}(\mathcal{U})\right].$$

As
$$\left(\underline{\mu_i}\underline{b_i}, \left(1-\left(1-\underline{\nu_i}\right)\left(1-\underline{d_i}\right)\right), \left(\overline{\mu}_i\overline{b_i}, \left(1-(1-\overline{\nu_i})\left(1-\overline{d_i}\right)\right)\right)\right).$$

Thus, q-ROHFR value $\mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}), \overline{\mathcal{H}}(\mathcal{U})\right) = \left((\underline{b_i}, \underline{d_i}), (\overline{b_i}, \overline{d}_i)\right)$. It follows that

$$\begin{aligned}
& q-ROHFRWA\,(\mathcal{R}(\mathcal{U}_1) \oplus \mathcal{H}(\mathcal{U}), \mathcal{R}(\mathcal{U}_2) \oplus \mathcal{H}(\mathcal{U}), ..., \mathcal{R}(\mathcal{U}_n) \oplus \mathcal{H}(\mathcal{U})) \\
= & \left[\sum_{i=1}^{n} w_i \underline{\mathcal{R}}(\mathcal{U}_i) \oplus \mathcal{H}(\mathcal{U}), \sum_{i=1}^{n} w_i \left(\overline{\mathcal{R}}(\mathcal{U}_i) \oplus \mathcal{H}(\mathcal{U})\right)\right] \\
= & \left[\begin{array}{l}
\left\{\bigcup_{\underline{\mu_i} \in \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{n}\left(\underline{\mu_i}\right)^{w_i} \underline{b}_i^{w_i}, \bigcup_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\underline{\nu_i}\right)^q\right)^{w_i}\left(1-\underline{d_i}\right)^{w_i}\right)}\right\}, \\
\left\{\bigcup_{\overline{\mu}_i \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \underline{b}_i \prod_{i=1}^{n}\left(\overline{\mu}_i\right)^{w_i}, \bigcup_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\overline{\nu_i}\right)^q\right)^{w_i}\right)\left(1-\overline{d_i}\right)^{w_i}}\right\}
\end{array}\right] \\
= & \left[\begin{array}{l}
\left\{\bigcup_{\underline{\mu_i} \in \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \underline{b}\prod_{i=1}^{n}\left(\underline{\mu_i}\right)^{w_i}, \bigcup_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1-\left(1-\underline{d}\right)\prod_{i=1}^{n}\left(1-\left(\underline{\nu_i}\right)^q\right)^{w_i}\right)}\right\}, \\
\left\{\bigcup_{\overline{\mu}_i \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \overline{b}\prod_{i=1}^{n}\left(\overline{\mu}_i\right)^{w_i}, \bigcup_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1-\overline{d}\right)\left(1-\prod_{i=1}^{n}\left(1-\left(\overline{\nu_i}\right)^q\right)^{w_i}\right)}\right\}
\end{array}\right] \\
= & \left[\begin{array}{l}
\left\{\left(\bigcup_{\underline{\mu_i} \in \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{n}\left(\underline{\mu_i}\right)^{w_i}, \bigcup_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\underline{\nu_i}\right)^q\right)^{w_i}\right)}\right) \oplus \left(\underline{b_i}, \underline{d_i}\right)\right\}, \\
\left\{\left(\bigcup_{\overline{\mu}_i \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{n}\left(\overline{\mu}_i\right)^{w_i}, \bigcup_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\overline{\nu_i}\right)^q\right)^{w_i}\right)}\right) \oplus \left(\overline{b_i}, \overline{d_i}\right)\right\}
\end{array}\right] \\
= & \left[\begin{array}{l}
\left(\bigcup_{\underline{\mu_i} \in \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{n}\left(\underline{\mu_i}\right)^{w_i}, \bigcup_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\underline{\nu_i}\right)^q\right)^{w_i}\right)}\right), \\
\left(\bigcup_{\overline{\mu}_i \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{n}\left(\overline{\mu}_i\right)^{w_i}, \bigcup_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\overline{\nu_i}\right)^q\right)^{w_i}\right)}\right)
\end{array}\right] \oplus \left[\left(\underline{b_i}, \underline{d_i}\right), \left(\overline{b_i}, \overline{d_i}\right)\right]
\end{aligned}$$

$$= q-ROHFRWA\left(\mathcal{R}(\mathcal{U}_1),\mathcal{R}(\mathcal{U}_2),...,\mathcal{R}(\mathcal{U}_n)\right)\oplus\mathcal{H}(\mathcal{U}).$$

(5) Homogeneity: For a real number $\gamma > 0$ and $\mathcal{R}(\mathcal{U}_i) = \left(\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i)\right)$ be a q-ROHFRVs. Consider

$$\begin{aligned}\gamma\mathcal{R}(\mathcal{U}_i) &= \left(\gamma\underline{\mathcal{R}}(\mathcal{U}_i), \gamma\overline{\mathcal{R}}(\mathcal{U}_i)\right)\\ &= \left[\begin{array}{c}\left\{\bigcup_{\underline{\mu}_i\in\wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}}\left(\underline{\mu}_i^{\gamma}\right), \bigcup_{\underline{\nu}_i\in\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}}\left(\sqrt[q]{\left(1-\left(1-\underline{\nu}_i^{q}\right)^{\gamma}\right)}\right)\right\},\\ \left\{\bigcup_{\overline{\mu}_i\in\wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}}\left(\overline{\mu}_i^{\gamma}\right), \bigcup_{\overline{\nu}_i\in\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}}\left(\sqrt[q]{\left(1-(1-\overline{\nu}_i^{q})^{\gamma}\right)}\right)\right\}\end{array}\right]\end{aligned}$$

Now

$$\begin{aligned}& q-ROHFRWA\left(\gamma\mathcal{R}(\mathcal{U}_1),\gamma\mathcal{R}(\mathcal{U}_2),...,\gamma\mathcal{R}(\mathcal{U}_n)\right)\\ &= \left[\begin{array}{c}\left(\bigcup_{\underline{\mu}_i\in\wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}}\prod_{i=1}^{n}\left(\underline{\mu}_i\right)^{\gamma}, \bigcup_{\underline{\nu}_i\in\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}}\sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\underline{\nu}_i\right)^{q}\right)^{\gamma}\right)}\right),\\ \left(\bigcup_{\overline{\mu}_i\in\wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}}\prod_{i=1}^{n}(\overline{\mu}_i)^{\gamma}, \bigcup_{\overline{\nu}_i\in\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}}\sqrt[q]{\left(1-\prod_{i=1}^{n}(1-(\overline{\nu}_i)^{q})^{\gamma}\right)}\right)\end{array}\right]\\ &= \gamma q-ROHFRWA\left(\mathcal{R}(\mathcal{U}_1),\mathcal{R}(\mathcal{U}_2),...,\mathcal{R}(\mathcal{U}_n)\right).\end{aligned}$$

(6) Commutativity: Suppose

$$\begin{aligned}& q-ROHFRWA\left(\mathcal{R}(\mathcal{U}_1),\mathcal{R}(\mathcal{U}_2),...,\mathcal{R}(\mathcal{U}_n)\right)\\ &= \left[\sum_{i=1}^{n}\gamma_i\underline{\mathcal{R}}(\mathcal{U}_i), \sum_{i=1}^{n}\gamma_i\overline{\mathcal{R}}(\mathcal{U}_i)\right]\\ &= \left[\begin{array}{c}\left(\bigcup_{\underline{\mu}_i\in\wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}}\prod_{i=1}^{n}\left(\underline{\mu}_i\right)^{\gamma_i}, \bigcup_{\underline{\nu}_i\in\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}}\sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\underline{\nu}_i\right)^{q}\right)^{\gamma_i}\right)}\right),\\ \left(\bigcup_{\overline{\mu}_i\in\wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}}\prod_{i=1}^{n}(\overline{\mu}_i)^{\gamma_i}, \bigcup_{\overline{\nu}_i\in\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}}\sqrt[q]{\left(1-\prod_{i=1}^{n}(1-(\overline{\nu}_i)^{q})^{\gamma_i}\right)}\right)\end{array}\right].\end{aligned}$$

Let $\left(\mathcal{R}'(\mathcal{U}_1),\mathcal{R}'(\mathcal{U}_2),...,\mathcal{R}'(\mathcal{U}_n)\right)$ be a permutation of $(\mathcal{R}(\mathcal{U}_1),\mathcal{R}(\mathcal{U}_2),...,\mathcal{R}(\mathcal{U}_n))$. Then we have $\mathcal{R}(\mathcal{U}_i) = \mathcal{R}'(\mathcal{U}_i)(i = 1,2,3,...,n)$

$$\begin{aligned}&= \left[\begin{array}{c}\left(\bigcup_{\underline{\mu}_i\in\wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}}\prod_{i=1}^{n}\left(\underline{\mu}'_i\right)^{\gamma_i}, \bigcup_{\underline{\nu}_i\in\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}}\sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\underline{\nu}'_i\right)^{q}\right)^{\gamma_i}\right)}\right),\\ \left(\bigcup_{\overline{\mu}_i\in\wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}}\prod_{i=1}^{n}\left(\overline{\mu}'_i\right)^{\gamma_i}, \bigcup_{\overline{\nu}_i\in\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}}\sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\overline{\nu}'_i\right)^{q}\right)^{\gamma_i}\right)}\right)\end{array}\right],\\ &= \left[\sum_{i=1}^{n}\gamma_i\underline{\mathcal{R}}'(\mathcal{U}_i), \sum_{i=1}^{n}\gamma_i\overline{\mathcal{R}}'(\mathcal{U}_i)\right]\\ &= q-ROHFRWA\left(\mathcal{R}'(\mathcal{U}_1),\mathcal{R}'(\mathcal{U}_2),...,\mathcal{R}'(\mathcal{U}_n)\right).\end{aligned}$$

This completes the proof. □

4.2. q-Rung orthopair hesitant fuzzy rough ordered weighted geometric aggregation operator

Definition 16. *Let $\mathcal{R}(\mathcal{U}_i) = (\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i))$ $(i = 1, 2, 3, ..., n)$ be the collection of q-ROHFRVs with weight vector $w = (w_1, w_2, ..., w_n)^T$ such that $\sum_{i=1}^{n} w_i = 1$ and $0 \leq w_i \leq 1$. Then q-ROHFROWG operator is described as:*

$$q - ROHFROWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) = \left(\sum_{i=1}^{n} \left(\underline{\mathcal{R}_{\Omega}}(\mathcal{U}_i)\right)^{w_i}, \sum_{i=1}^{n} \left(\overline{\mathcal{R}_{\Omega}}(\mathcal{U}_i)\right)^{w_i}\right).$$

Theorem 3. *Let $\mathcal{R}(\mathcal{U}_i) = (\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i))$ $(i = 1, 2, 3, ..., n)$ be the collection of q-ROHFR values with weight vectors $w = (w_1, w_2, ..., w_n)^T$ such that $\sum_{i=1}^{n} w_i = 1$ and $0 \leq w_i \leq 1$. Then q-ROHFROWG operator is a mapping defined as*

$$\begin{aligned}
& q - ROHFROWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) \\
= & \left(\sum_{i=1}^{n} \left(\underline{\mathcal{R}_{\Omega}}(\mathcal{U}_i)\right)^{w_i}, \sum_{i=1}^{n} \left(\overline{\mathcal{R}_{\Omega}}(\mathcal{U}_i)\right)^{w_i}\right) \\
= & \left(\begin{array}{c}
\left\{\bigcup_{\underline{\mu_i} \in \wp h_{\underline{\mathcal{R}}(\mathcal{U})}} \prod_{i=1}^{n} \left(\underline{\mu_{\Omega_i}}\right)^{w_i}, \bigcup_{\underline{\nu_i} \in \Im h_{\underline{\mathcal{R}}(\mathcal{U})}} \sqrt[q]{1 - \prod_{i=1}^{n} \left(1 - \left(\underline{\nu_{\Omega_i}}\right)^{q}\right)^{w_i}}\right\} \\
\left\{\bigcup_{\overline{\mu_i} \in \wp h_{\overline{\mathcal{R}}(\mathcal{U})}} \prod_{i=1}^{n} \left(\overline{\mu_{\Omega_i}}\right)^{w_i}, \bigcup_{\overline{\nu_i} \in \Im h_{\overline{\mathcal{R}}(\mathcal{U})}} \sqrt[q]{1 - \prod_{i=1}^{n} \left(1 - \left(\overline{\nu}_{\Omega_i}\right)^{q}\right)^{w_i}}\right\}
\end{array}\right),
\end{aligned}$$

where $\mathcal{R}_{\Omega}(\mathcal{U}_i) = (\underline{\mathcal{R}_{\Omega_i}}(\mathcal{U}_i), \overline{\mathcal{R}_{\Omega_i}}(\mathcal{U}_i))$ depicts the superior value of permutation from the collection of q-ROHFR values.

Proof. The proof is similar to the proof of Theorem 1. □

Theorem 4. *Let $\mathcal{R}(\mathcal{U}_i) = (\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i))$ $(i = 1, 2, 3, ..., n)$ be the collection of q-ROHFR values with weight vectors $w = (w_1, w_2, ..., w_n)^T$ such that $\sum_{i=1}^{n} w_i = 1$ and $0 \leq w_i \leq 1$. Then q-ROHFROWG operator is satisfy the following properties:*

(1) ***Idempotency:*** *If $\mathcal{R}(\mathcal{U}_i) = \mathcal{H}(\mathcal{U})$ for $i = 1, 2, 3, ..., n$ where $\mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}), \overline{\mathcal{H}}(\mathcal{U})\right) = \left((\underline{b}, \underline{d}), (\overline{b}, \overline{d})\right)$. Then*

$$q - ROHFROWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) = \mathcal{H}(\mathcal{U}).$$

(2) ***Boundedness:*** *Let $(\mathcal{R}(\mathcal{U}))^{-} = \left(\min_i \underline{\mathcal{R}}(\mathcal{U}_i), \max_i \overline{\mathcal{R}}(\mathcal{U}_i)\right)$ and $(\mathcal{R}(\mathcal{U}))^{+} = \left(\max_i \underline{\mathcal{R}}(\mathcal{U}_i), \min_i \overline{\mathcal{R}}(\mathcal{U}_i)\right)$. Then*

$$(\mathcal{R}(\mathcal{U}))^{-} \leq q - ROHFROWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) \leq (\mathcal{R}(\mathcal{U}))^{+}.$$

(3) ***Monotonicity:*** *Let $\mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}_i), \overline{\mathcal{H}}(\mathcal{U}_i)\right)(i = 1, 2, 3, ..., n)$ be another collection of q-ROHFRVs such that $\underline{\mathcal{H}}(\mathcal{U}_i) \leq \underline{\mathcal{R}}(\mathcal{U}_i)$ and $\overline{\mathcal{H}}(\mathcal{U}_i) \leq \overline{\mathcal{R}}(\mathcal{U}_i)$. Then*

$$q - ROHFROWG\left(\mathcal{H}(\mathcal{U}_1), \mathcal{H}(\mathcal{U}_2), ..., \mathcal{H}(\mathcal{U}_n)\right) \leq q - ROHFROWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right).$$

(4) ***Shiftinvariance:*** *Let q-ROHFRV* $\mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}), \overline{\mathcal{H}}(\mathcal{U})\right) = \left((\underline{b}, \underline{d}), (\overline{b}, \overline{d})\right)$. *Then*

$$q - ROHFROWG\left(\mathcal{R}(\mathcal{U}_1) \oplus \mathcal{H}(\mathcal{U}), \mathcal{R}(\mathcal{U}_2) \oplus \mathcal{H}(\mathcal{U}), ..., \mathcal{R}(\mathcal{U}_n) \oplus \mathcal{H}(\mathcal{U})\right) = q - ROHFROWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) \oplus \mathcal{H}(\mathcal{U}).$$

(5) ***Homogeneity:*** *For any real number* $\gamma > 0$;

$$q{-}ROHFROWG\left(\gamma\mathcal{R}(\mathcal{U}_1), \gamma\mathcal{R}(\mathcal{U}_2), ..., \gamma\mathcal{R}(\mathcal{U}_n)\right) = \gamma{\cdot}q{-}ROHFROWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right).$$

(6) ***Commutativity:*** *Let* $\mathcal{R}'(\mathcal{U}_i) = \left(\underline{\mathcal{R}'}(\mathcal{U}_i), \overline{\mathcal{R}'}(\mathcal{U}_i)\right)$ *and* $\mathcal{R}(\mathcal{U}_i) = (\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i))$, $(i = 1, 2, 3, ..., n)$ *is a collection of q-ROHFR values. Then*

$$q - ROHFROWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) = q - ROHFROWG\left(\mathcal{R}'(\mathcal{U}_1), \mathcal{R}'(\mathcal{U}_2), ..., \mathcal{R}'(\mathcal{U}_n)\right).$$

Proof. The proof is similar to the proof of Theorem 2. □

4.3. q-Rung orthopair hesitant fuzzy rough hybrid weighted geometric aggregation operator

Definition 17. *Let* $\mathcal{R}(\mathcal{U}_i) = (\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i))$ $(i = 1, 2, 3, ..., n)$ *be the collection of q-ROHFR values along weight vector* $w = (w_1, w_2, ..., w_n)^T$ *such that* $\sum_{i=1}^{n} w_i = 1$ *and* $0 \le w_i \le 1$. *Let* $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)^T$ *such that* $\sum_{i=1}^{n} \varrho_i = 1$ *and* $0 \le \varrho_i \le 1$ *be the given associated weight vector. Then q-ROHFRHWG operator is a mapping defined as:*

$$q - ROHFRHWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) = \sum_{i=1}^{n} \varrho_i \underline{\widetilde{\mathcal{R}}_{\Omega}}(\mathcal{U}_i) = \left(\sum_{i=1}^{n} \varrho_i \underline{\widetilde{\mathcal{R}}_{\Omega}}(\mathcal{U}_i), \sum_{i=1}^{n} \varrho_i \overline{\widetilde{\mathcal{R}}_{\Omega}}(\mathcal{U}_i)\right).$$

Theorem 5. *Let* $\mathcal{R}(\mathcal{U}_i) = (\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i))$ $(i = 1, 2, 3, ..., n)$ *be the collection of q-ROHFR values with weight vectors* $w = (w_1, w_2, ..., w_n)^T$ *such that* $\sum_{i=1}^{n} w_i = 1$ *and* $0 \le w_i \le 1$ *and* $\varrho = (\varrho_1, \varrho_2, ..., \varrho_n)^T$, $\sum_{i=1}^{n} \varrho_i = 1$ *and* $0 \le \varrho_i \le 1$ *be the associated weight vector. Then q-ROHFRHWGA operator is a mapping defined as*

$$\begin{aligned}
& q - ROHFRHWG\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) \\
= & \left(\sum_{i=1}^{n} \varrho_i \underline{\widetilde{\mathcal{R}}_{\Omega_i}}(\mathcal{U}_i), \sum_{i=1}^{n} \varrho_i \overline{\widetilde{\mathcal{R}}_{\Omega_i}}(\mathcal{U}_i)\right) \\
= & \left(\begin{array}{c}
\left\{\bigcup_{\underline{\mu_i} \in \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{n} \left(\underline{\widetilde{\mu}_{\Omega_i}}\right)^{\varrho_i}, \bigcup_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{1 - \prod_{i=1}^{n}\left(1 - \left(\underline{\widetilde{\nu}_{\Omega_i}}\right)^{q}\right)^{\varrho_i}}\right\}, \\
\left\{\bigcup_{\overline{\mu_i} \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{n} \left(\overline{\widetilde{\mu}_{\Omega_i}}\right)^{\varrho_i}, \bigcup_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{1 - \prod_{i=1}^{n}\left(1 - \left(\overline{\widetilde{\nu}_{\Omega_i}}\right)^{q}\right)^{\varrho_i}}\right\}
\end{array}\right),
\end{aligned}$$

where $\widetilde{\mathcal{R}_{\Omega}}(\mathcal{U}_i) = (\mathcal{R}(\mathcal{U}_i))^{n\varrho_i} = (\left(\underline{\mathcal{R}}(\mathcal{U}_i)\right)^{n\varrho_i}, \left(\overline{\mathcal{R}}(\mathcal{U}_i)\right)^{n\varrho_i})$ *depicts the superior value of permutation from the collection of q-ROHFR values and n represents the balancing coefficient.*

Proof. The proof is similar to the proof of Theorem 1. □

Theorem 6. *Let $\mathcal{R}(\mathcal{U}_i) = (\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i))$ $(i = 1, 2, 3, ..., n)$ be the collection of q-ROHFR values with weight vectors $w = (w_1, w_2, ..., w_n)^T$ such that $\sum_{i=1}^{n} w_i = 1$ and $0 \leq w_i \leq 1$. Then q-ROHFRHWGA operator is satisfy the following properties:*

(1) ***Idempotency:*** *If $\mathcal{R}(\mathcal{U}_i) = \mathcal{H}(\mathcal{U})$ for $i = 1, 2, 3, ..., n$ where $\mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}), \overline{\mathcal{H}}(\mathcal{U})\right) = \left((\underline{b}, \underline{d}), (\overline{b}, \overline{d})\right)$. Then*

$$q - ROHFRHWG\,(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)) = \mathcal{H}(\mathcal{U}).$$

(2) ***Boundedness:*** *Let $(\mathcal{R}(\mathcal{U}))^- = \left(\min_i \underline{\mathcal{R}}\,(\mathcal{U}_i)\,, \max_i \overline{\mathcal{R}}(\mathcal{U}_i)\right)$ and $(\mathcal{R}(\mathcal{U}))^+ = \left(\max_i \underline{\mathcal{R}}\,(\mathcal{U}_i)\,, \min_i \overline{\mathcal{R}}(\mathcal{U}_i)\right)$. Then*

$$(\mathcal{R}(\mathcal{U}))^- \leq q - ROHFRHWG\,(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)) \leq (\mathcal{R}(\mathcal{U}))^+\,.$$

(3) ***Monotonicity:*** *Let $\mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}_i), \overline{\mathcal{H}}(\mathcal{U}_i)\right)(i = 1, 2, 3, ..., n)$ is another collection of q-ROHFR values such that $\underline{\mathcal{H}}(\mathcal{U}_i) \leq \underline{\mathcal{R}}\,(\mathcal{U}_i)$ and $\overline{\mathcal{H}}(\mathcal{U}_i) \leq \overline{\mathcal{R}}(\mathcal{U}_i)$. Then*

$$q - ROHFRHWG\,(\mathcal{H}(\mathcal{U}_1), \mathcal{H}(\mathcal{U}_2), ..., \mathcal{H}(\mathcal{U}_n)) \leq q - ROHFRHWG\,(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n))\,.$$

(4) ***Shiftinvariance:*** *Let q-ROHFR value $\mathcal{H}(\mathcal{U}) = \left(\underline{\mathcal{H}}(\mathcal{U}), \overline{\mathcal{H}}(\mathcal{U})\right) = \left((\underline{b}, \underline{d}), (\overline{b}, \overline{d})\right)$, then*

$$\begin{aligned} & q - ROHFRHWG\,(\mathcal{R}(\mathcal{U}_1) \oplus \mathcal{H}(\mathcal{U}), \mathcal{R}(\mathcal{U}_2) \oplus \mathcal{H}(\mathcal{U}), ..., \mathcal{R}(\mathcal{U}_n) \oplus \mathcal{H}(\mathcal{U})) = \\ & q - ROHFRHWG\,(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)) \oplus \mathcal{H}(\mathcal{U}). \end{aligned}$$

(5) ***Homogeneity:*** *For any real number $\gamma > 0$;*

$$q{-}ROHFRHWG\,(\gamma\mathcal{R}(\mathcal{U}_1), \gamma\mathcal{R}(\mathcal{U}_2), ..., \gamma\mathcal{R}(\mathcal{U}_n)) = \gamma{\cdot}q{-}ROHFRHWG\,(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n))\,.$$

(6) ***Commutativity:*** *Suppose $\mathcal{R}'(\mathcal{U}_i) = \left(\underline{\mathcal{R}}\,(\mathcal{U}_i)\,, \overline{\mathcal{R}}'(\mathcal{U}_i)\right)$ and $\mathcal{R}(\mathcal{U}_i) = (\underline{\mathcal{R}}(\mathcal{U}_i), \overline{\mathcal{R}}(\mathcal{U}_i))$, $(i = 1, 2, 3, ..., n)$ is a collection of q-ROHFR values. Then*

$$q - ROHFRHWG\,(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)) = q - ROHFRHWG\left(\mathcal{R}'(\mathcal{U}_1), \mathcal{R}'(\mathcal{U}_2), ..., \mathcal{R}'(\mathcal{U}_n)\right).$$

Proof. The proof is similar to the proof of Theorem 2. □

5. Multi-attribute decision making methodology

Here, we developed an algorithm for addressing uncertainty in multi-attribute group decision making (MAGDM) under q-rung orthopair hesitant fuzzy rough information. Consider a DM problem with $\{\jmath_1, \jmath_2, ..., \jmath_n\}$ a set of n alternatives and a set of attributes $\{c_1, c_2, ..., c_n\}$ with $(w_1, w_2, ..., w_n)^T$ the weights, that is, $w_i \in [0, 1]$, $\sum_{i=1}^{n} w_i = 1$. To test the reliability of kth alternative $\jmath_i$ under the the attribute c_i, let be a set of decision makers (DMs) $\left\{\mathring{D}_1, \mathring{D}_2, ..., \mathring{D}_{\hat{\jmath}}\right\}$ and $(\varrho_1, \varrho_2, ..., \varrho_i)^T$ be DMs weights such that $\varrho_i \in [0, 1]$, $\sum_{i=1}^{n} \varrho_i = 1$. The expert evaluation matrix is described as:

$$M = \left[\overline{\mathcal{R}}(\mathcal{U}_{ij}^{\hat{\jmath}})\right]_{m \times n}$$

$$= \begin{bmatrix} \left(\overline{\mathcal{R}}(\mathcal{U}_{11}), \underline{\mathcal{R}}(\mathcal{U}_{11})\right) & \left(\overline{\mathcal{R}}(\mathcal{U}_{12}), \underline{\mathcal{R}}(\mathcal{U}_{12})\right) & \cdots & \left(\overline{\mathcal{R}}(\mathcal{U}_{1j}), \underline{\mathcal{R}}(\mathcal{U}_{1j})\right) \\ \left(\overline{\mathcal{R}}(\mathcal{U}_{21}), \underline{\mathcal{R}}(\mathcal{U}_{21})\right) & \left(\overline{\mathcal{R}}(\mathcal{U}_{22}), \underline{\mathcal{R}}(\mathcal{U}_{22})\right) & \cdots & \left(\overline{\mathcal{R}}(\mathcal{U}_{2j}), \underline{\mathcal{R}}(\mathcal{U}_{2j})\right) \\ \left(\overline{\mathcal{R}}(\mathcal{U}_{31}), \underline{\mathcal{R}}(\mathcal{U}_{31})\right) & \left(\overline{\mathcal{R}}(\mathcal{U}_{32}), \underline{\mathcal{R}}(\mathcal{U}_{32})\right) & \cdots & \left(\overline{\mathcal{R}}(\mathcal{U}_{3j}), \underline{\mathcal{R}}(\mathcal{U}_{3j})\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\overline{\mathcal{R}}(\mathcal{U}_{i1}), \underline{\mathcal{R}}(\mathcal{U}_{i1})\right) & \left(\overline{\mathcal{R}}(\mathcal{U}_{i2}), \underline{\mathcal{R}}(\mathcal{U}_{i2})\right) & \cdots & \left(\overline{\mathcal{R}}(\mathcal{U}_{ij}), \underline{\mathcal{R}}(\mathcal{U}_{ij})\right) \end{bmatrix},$$

where $\overline{\mathcal{R}}(\mathcal{U}_{ij}) = \left\{\left\langle \jmath, \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath), \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath)\right\rangle | \jmath \in \aleph\right\}$ and $\underline{\mathcal{R}}(\mathcal{U}) = \left\{\left\langle \jmath, \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath), \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath)\right\rangle | \jmath \in \aleph\right\}$ such that $0 \le \left(\max(\wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath))\right)^q + \left(\min(\Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath))\right)^q \le 1$ and $0 \le \left(\min(\wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath)\right)^q + \left(\max(\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath))\right)^q \le 1$ are the q-ROHF rough values. The main steps for MAGDM are as follows:

Step 1. Construct the experts evaluation matrices as:

$$(E)^{\jmath} = \begin{bmatrix} \left(\overline{\mathcal{R}}(\mathcal{U}^{\jmath}_{11}), \underline{\mathcal{R}}(\mathcal{U}^{\jmath}_{11})\right) & \left(\overline{\mathcal{R}}(\mathcal{U}^{\jmath}_{12}), \underline{\mathcal{R}}(\mathcal{U}^{\jmath}_{12})\right) & \cdots & \left(\overline{\mathcal{R}}(\mathcal{U}^{\jmath}_{1j}), \underline{\mathcal{R}}(\mathcal{U}^{\jmath}_{1j})\right) \\ \left(\overline{\mathcal{R}}(\mathcal{U}^{\jmath}_{21}), \underline{\mathcal{R}}(\mathcal{U}^{\jmath}_{21})\right) & \left(\overline{\mathcal{R}}(\mathcal{U}^{\jmath}_{22}), \underline{\mathcal{R}}(\mathcal{U}^{\jmath}_{22})\right) & \cdots & \left(\overline{\mathcal{R}}(\mathcal{U}^{\jmath}_{2j}), \underline{\mathcal{R}}(\mathcal{U}^{\jmath}_{2j})\right) \\ \left(\overline{\mathcal{R}}(\mathcal{U}^{\jmath}_{31}), \underline{\mathcal{R}}(\mathcal{U}^{\jmath}_{31})\right) & \left(\overline{\mathcal{R}}(\mathcal{U}^{\jmath}_{32}), \underline{\mathcal{R}}(\mathcal{U}^{\jmath}_{32})\right) & \cdots & \left(\overline{\mathcal{R}}(\mathcal{U}^{\jmath}_{3j}), \underline{\mathcal{R}}(\mathcal{U}^{\jmath}_{3j})\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\overline{\mathcal{R}}(\mathcal{U}^{\jmath}_{i1}), \underline{\mathcal{R}}(\mathcal{U}^{\jmath}_{i1})\right) & \left(\overline{\mathcal{R}}(\mathcal{U}^{\jmath}_{i2}), \underline{\mathcal{R}}(\mathcal{U}^{\jmath}_{i2})\right) & \cdots & \left(\overline{\mathcal{R}}(\mathcal{U}^{\jmath}_{ij}), \underline{\mathcal{R}}(\mathcal{U}^{\jmath}_{ij})\right) \end{bmatrix}$$

where $\hat{\jmath}$ represents the number of experts.

Step 2. Evaluate the normalized experts matrices $(N)^{\hat{\jmath}}$, as

$$(N)^{\hat{\jmath}} = \begin{cases} \mathcal{R}(\mathcal{U}_{ij}) = \left(\underline{\mathcal{R}}\left(\mathcal{U}_{ij}\right), \overline{\mathcal{R}}\left(\mathcal{U}_{ij}\right)\right) = \left(\left(\underline{\mu_{ij}}, \underline{\nu_{ij}}\right), \left(\overline{\mu_{ij}}, \overline{\nu_{ij}}\right)\right) & \text{if} \quad \text{for benefit} \\ \left(\mathcal{R}(\mathcal{U}_{ij})\right)^c = \left(\left(\underline{\mathcal{R}}\left(\mathcal{U}_{ij}\right)\right)^c, \left(\overline{\mathcal{R}}\left(\mathcal{U}_{ij}\right)\right)^c\right) = \left(\left(\underline{\nu_{ij}}, \underline{\mu_{ij}}\right), \left(\overline{\nu_{ij}}, \overline{\mu_{ij}}\right)\right) & \text{if} \quad \text{for cost} \end{cases}$$

Step 3. Compute the collected q-rung orthopair hesitant fuzzy rough information of decision makers using the q-ROHFRWA aggregation operators.

$$\begin{aligned} & q-ROHFRWA\left(\mathcal{R}(\mathcal{U}_1), \mathcal{R}(\mathcal{U}_2), ..., \mathcal{R}(\mathcal{U}_n)\right) \\ = & \left(\sum_{i=1}^{n} w_i \underline{\mathcal{R}}(\mathcal{U}_i), \sum_{i=1}^{n} w_i \overline{\mathcal{R}}(\mathcal{U}_i)\right) \\ = & \begin{bmatrix} \bigcup_{\underline{\mu_i} \in \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{n} \left(\underline{\mu_i}\right)^{w_i}, \bigcup_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n} \left(1 - \left(\underline{\nu_i}\right)^q\right)^{w_i}\right)}, \\ \bigcup_{\overline{\mu_i} \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \prod_{i=1}^{n} \left(\overline{\mu_i}\right)^{w_i}, \bigcup_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n} \left(1 - (\overline{\nu_i})^q\right)^{w_i}\right)} \end{bmatrix}. \end{aligned}$$

Step 4. Evaluate the aggregated q-ROHFR values for each considered alternative with respect to the given list of criteria/attributes by utilizing the proposed aggregation information.

Step 5. Find the ranking of alternatives based on score function as:

$$\eth(\mathcal{R}(\mathcal{U})) = \frac{1}{4}\begin{pmatrix} 2 + \frac{1}{M_{\mathcal{G}}} \sum_{\mu_i \in \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} (\underline{\mu_i}) + \frac{1}{N_{\mathcal{G}}} \sum_{\mu_i \in \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} (\overline{\mu_i}) \\ \frac{1}{M_{\mathcal{G}}} \sum_{\underline{\nu_i} \in \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}} (\underline{\nu_i}) - \frac{1}{M_{\mathcal{G}}} \sum_{\overline{\nu_i} \in \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}} (\overline{\nu_i}) \end{pmatrix}.$$

Step 6. Rank all the alternative scores in descending order. The alternative having larger value will be superior/best.

6. Numerical application

In this section, we propose the numerical application related to choose the ideal location for wind power station to validate the established operators, we describe a numerical MCGDM example using the suggested aggregations technique with the q-ROHFR information.

6.1. Practical case study (the evaluation of site selection for wind power station)

Providing human populations with sustainable and widely available sources of energy has become one of the most difficult issues in recent decades. Global energy demand is predicted to grow by an average of 8% per year between 2000 AD and 2030 AD [3]. Fossil fuels provide a significant amount of the required energy and have the greatest impact. To minimize their dependency on fossil fuels, some industrialized countries have implemented laws that promote the use of renewable energy sources such as wind and solar power. Wind power is one of the most consistent and sustainable renewable sources of energy. Wind is now a significant, ecologically beneficial, and financially viable resource. Because of advancements and reductions in the cost of energy conversion technology, it has become even more attractive as a source of sustainable renewable energy. Wind energy has grown in popularity as a result of states implementing a variety of effective policies to promote the installation of wind power plants. The cost of producing power from wind energy is now competitive with that of generating energy from fossil fuels. Therefore, wind power is a well-accepted and risk-free type of renewable energy that may be economically feasible, environmentally safe, and contribute significantly to the reduction of hazardous gases. The objective of this research is to evaluate the requirement for new energy resources as a result of population growth. Selecting the most appropriate location to build the wind power plant is a crucial step before establishing the technical project. The improvement of facility location provides knowledge of analyzing challenges connected to installations in certain locations based on specified criteria [4, 5]. Wind power plants are among the most efficient and environmentally friendly energy sources, contributing significantly to the current energy supply. The site selection for wind farms is complicated, and many factors take into consideration such as the economy, slope of topography, transportation, environmental geological conditions, hydrogeological engineering conditions, surface hydrological conditions, distance from substations, and feasibility of wind power [6, 7].

(c_1) The distance from substations: Substations are essential for connecting a wind farm to the power system. Interconnection evaluations for a wind power plant might be expensive, based on the extent of the plan and the site suggested for interconnection. Therefore, it is more cost effective and technically feasible to put the wind energy station near distribution substations.

(c_2) The slope of the geography: The slope of a location is an important factor for economics and transportation. Wind turbines should preferably be installed on entirely horizontal terrains. However, if this is not available, this slope must be produced, which will take effort and time, rising installation expenditures.

(c_3) The distance from the power grid: It is intrinsically linked to the transmission of energy to power stations or transformers, because the closer the power grids, the shorter the distance and the less energy will be consumed. Moreover, reduced distances result in lower connection expenses.

(c_4) The distance to highways and roads: The transporting huge turbines may be complicated and costly therefore, accessibility to main roads are essential. To reduce expenses, the distance between a feasible wind power plant and major highways or railways should be minimized. Almost all analyses indicate that sites closer to roadways are more essential for developing a wind farm. Wind power plant near to highways and roads provide financial benefits.

Suppose an organization wants to assessment the procedure of choosing a site for wind power plant. They will invite a panel of experts to analyze an appropriate location for site selection for wind power station. The considered attribute weight vector is $w = (0.18, 0.25, 0.31, 0.26)^T$ and DM experts weight vector is $w = (0.23, 0.38, 0.39)^T$.

Step 1. The information of three professional experts are evaluated in Table 1 to Table 6 using q-ROHFR values.

Table 1. Expert 1 information.

	c_1	c_2
A_1	$\begin{pmatrix} \{(0.10, 0.20, 0.50), (0.30, 0.40)\}, \\ \{(0.30, 0.80, 0.90), (0.40, 0.60)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.5, 0.7, 0.9), (0.5, 0.6, 0.8)\}, \\ \{(0.3, 0.5, 0.6), (0.7, 0.9)\} \end{pmatrix}$
A_2	$\begin{pmatrix} \{(0.50, 0.60, 0.70), (0.70, 0.90)\}, \\ \{(0.30, 0.50, 0.70), (0.60, 0.70)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.2, 0.4, 0.5), (0.5)\}, \\ \{(0.6, 0.7), (0.3, 0.5, 0.9)\} \end{pmatrix}$
A_3	$\begin{pmatrix} \{(0.40, 0.50, 0.60), (0.60, 0.70, 0.80)\}, \\ \{(0.70, 0.80), (0.10, 0.40, 0.70)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.1), (0.5, 0.6)\}, \\ \{(0.4, 0.6, 0.7), (0.5, 0.7)\} \end{pmatrix}$
A_4	$\begin{pmatrix} \{(0.6, 0.7, 0.9), (0.3, 0.4, 0.6)\}, \\ \{(0.2, 0.7), (0.7, 0.8, 0.9)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.3, 0.4, 0.5), (0.4, 0.7, 0.9)\}, \\ \{(0.1, 0.2), (0.2, 0.3)\} \end{pmatrix}$

Table 2. Expert 1 information.

	c_3	c_4
A_1	$\begin{pmatrix} \{(0.2, 0.3, 0.4), (0.3, 0.4, 0.7)\}, \\ \{(0.1, 0.5), (0.3, 0.5)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.5, 0.6), (0.4, 0.5, 0.7)\}, \\ \{(0.6, 0.8, 0.9), (0.6, 0.7, 0.9)\} \end{pmatrix}$
A_2	$\begin{pmatrix} \{(0.4, 0.5, 0.8), (0.4, 0.5, 0.7)\}, \\ \{(0.2, 0.5), (0.4, 0.5)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.4, 0.6, 0.8), (0.3, 0.5)\}, \\ \{(0.7), (0.1, 0.3, 0.4)\} \end{pmatrix}$
A_3	$\begin{pmatrix} \{(0.3, 0.6, 0.7), (0.5, 0.7, 0.8)\}, \\ \{(0.5, 0.9), (0.5, 0.8)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.3, 0.6), (0.5, 0.6, 0.8)\}, \\ \{(0.1, 0.3, 0.7), (0.3, 0.4)\} \end{pmatrix}$
A_4	$\begin{pmatrix} \{(0.3, 0.4, 0.5), (0.7, 0.8, 0.9)\}, \\ \{(0.6, 0.7), (0.4, 0.7)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.2, 0.3, 0.4), (0.5, 0.6, 0.9)\}, \\ \{(0.3, 0.4), (0.7, 0.8)\} \end{pmatrix}$

Table 3. Expert 2 information.

	c_1	c_2
A_1	$\begin{pmatrix} (\{0.2, 0.3, 0.4\}, \{0.2, 0.5\}), \\ (\{0.4, 0.6\}, \{0.2, 0.5\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.4, 0.5, 0.6\}, \{0.3, 0.7, 0.8\}), \\ (\{0.2, 0.7, 0.8\}, \{0.2, 0.8, 0.9\}) \end{pmatrix}$
A_2	$\begin{pmatrix} (\{0.1, 0.3, 0.4\}, \{0.5, 0.8\}), \\ (\{0.5, 0.6\}, \{0.8, 0.9\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.3, 0.4, 0.6\}, \{0.7, 0.8\}), \\ (\{0.1, 0.5\}, \{0.3, 0.7, 0.8\}) \end{pmatrix}$
A_3	$\begin{pmatrix} (\{0.6, 0.7, 0.8\}, \{0.3, 0.4\}), \\ (\{0.3, 0.8, 0.9\}, \{0.2, 0.5, 0.7\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.3, 0.4, 0.5\}, \{0.4, 0.7, 0.9\}), \\ (\{0.4, 0.6, 0.7\}, \{0.7, 0.8, 0.9\}) \end{pmatrix}$
A_4	$\begin{pmatrix} (\{0.1, 0.2, 0.3\}, \{0.5, 0.7\}), \\ (\{0.2, 0.4\}, \{0.7, 0.8\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.5, 0.7, 0.8\}, \{0.3, 0.5, 0.7\}), \\ (\{0.3, 0.4, 0.6\}, \{0.4, 0.5, 0.7\}) \end{pmatrix}$

Table 4. Expert 2 information.

	c_3	c_4
A_1	$\begin{pmatrix} (\{0.2, 0.4\}, \{0.3, 0.5\}), \\ (\{0.4, 0.7, 0.8\}, \{0.2, 0.6\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.1, 0.2\}, \{0.4, 0.6\}), \\ (\{0.2, 0.5\}, \{0.7, 0.9\}) \end{pmatrix}$
A_2	$\begin{pmatrix} (\{0.3, 0.5, 0.7\}, \{0.2, 0.6\}), \\ (\{0.6, 0.7, 0.8\}, \{0.2, 0.8\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2, 0.3\}, \{0.4, 0.6, 0.7\}), \\ (\{0.1, 0.3, 0.5\}, \{0.2, 0.3, 0.5\}) \end{pmatrix}$
A_3	$\begin{pmatrix} (\{0.5, 0.6, 0.7\}, \{0.3, 0.5\}), \\ (\{0.7, 0.8, 0.9\}, \{0.2, 0.3, 0.5\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2, 0.7, 0.8\}, \{0.2, 0.7\}), \\ (\{0.1, 0.2\}, \{0.5, 0.6, 0.7\}) \end{pmatrix}$
A_4	$\begin{pmatrix} (\{0.6, 0.7, 0.9\}, \{0.2, 0.5\}), \\ (\{0.6, 0.9\}, \{0.2, 0.5\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.3, 0.5\}, \{0.4, 0.6, 0.7\}), \\ (\{0.2, 0.3, 0.6\}, \{0.4, 0.5, 0.7\}) \end{pmatrix}$

Table 5. Expert 3 information.

	c_1	c_2
A_1	$\begin{pmatrix} (\{0.4, 0.7, 0.9\}, \{0.3, 0.6, 0.8\}), \\ (\{0.2, 0.3, 0.8\}, \{0.7, 0.8, 0.9\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.4, 0.7, 0.8\}, \{0.7, 0.8\}), \\ (\{0.3, 0.5, 0.6\}, \{0.7, 0.8\}) \end{pmatrix}$
A_2	$\begin{pmatrix} (\{0.1, 0.3, 0.4\}, \{0.5, 0.6, 0.9\}), \\ (\{0.2, 0.3, 0.7\}, \{0.2, 0.6, 0.8\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2, 0.3, 0.7\}, \{0.3, 0.8, 0.9\}), \\ (\{0.1, 0.5, 0.8\}, \{0.2, 0.7, 0.8\}) \end{pmatrix}$
A_3	$\begin{pmatrix} (\{0.2, 0.3, 0.5\}, \{0.4, 0.8, 0.9\}), \\ (\{0.1, 0.8, 0.9\}, \{0.4, 0.7\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2, 0.3, 0.8\}, \{0.2, 0.8\}), \\ (\{0.5, 0.8, 0.9\}, \{0.2, 0.9\}) \end{pmatrix}$
A_4	$\begin{pmatrix} (\{0.1, 0.5, 0.7\}, \{0.5, 0.8\}), \\ (\{0.3, 0.5, 0.7\}, \{0.4, 0.9\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2, 0.3\}, \{0.5, 0.6\}), \\ (\{0.3, 0.8, 0.9\}, \{0.7, 0.8, 0.9\}) \end{pmatrix}$

Table 6. Expert 3 information.

	c_3	c_4
A_1	$\begin{pmatrix} (\{0.2, 0.3, 0.8\}, \{0.5, 0.6, 0.7\}), \\ (\{0.3, 0.5, 0.6\}, \{0.2, 0.8, 0.9\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2, 0.3, 0.7\}, \{0.2, 0.3, 0.7\}), \\ (\{0.2, 0.3, 0.8\}, \{0.5, 0.7\}) \end{pmatrix}$
A_2	$\begin{pmatrix} (\{0.1, 0.3, 0.6\}, \{0.4, 0.6, 0.8\}), \\ (\{0.6, 0.7, 0.9\}, \{0.3, 0.8, 0.9\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.1, 0.2, 0.3\}, \{0.2, 0.5\}), \\ (\{0.3, 0.4, 0.6\}, \{0.1, 0.2\}) \end{pmatrix}$
A_3	$\begin{pmatrix} (\{0.1, 0.2, 0.3\}, \{0.3, 0.5, 0.9\}), \\ (\{0.2, 0.3, 0.4\}, \{0.2, 0.4, 0.6\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2, 0.8, 0.9\}, \{0.1, 0.2\}), \\ (\{0.2, 0.4, 0.5\}, \{0.7, 0.8\}) \end{pmatrix}$
A_4	$\begin{pmatrix} (\{0.2, 0.3, 0.7\}, \{0.8, 0.9\}), \\ (\{0.2, 0.3, 0.8\}, \{0.1, 0.2, 0.3\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2, 0.3, 0.8\}, \{0.2, 0.3\}), \\ (\{0.3, 0.5, 0.8\}, \{0.4, 0.5\}) \end{pmatrix}$

Step 2. All the experts information are benefit type. So in this case, we do not need to normalize the q-ROHFR values.

Step 3. Cumulated the collective information of three professional expert using q-ROHFRWA aggregation operator is evaluated in Table 7 to Table 8:

Table 7. Collective aggregated q-ROHFR information.

	c_1	c_2
A_1	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.2402, 0.5445, 0.7558\},\\ \{0.2572, 0.5100, 0.8403\}\end{array}\right),\\ \left(\begin{array}{c}\{0.3239, 0.6173, 0.7607\},\\ \{0.2748, 0.6263, 0.9597\}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.4280, 0.6443, 0.6630\},\\ \{0.4695, 0.7117, 0.9500\}\end{array}\right),\\ \left(\begin{array}{c}\{0.2706, 0.5993, 0.7281\},\\ \{0.3823, 0.7758, 0.9378\}\end{array}\right)\end{array}\right)$
A_2	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.3141, 0.4201, 0.3675\},\\ \{0.5214, 0.6935, 0.9211\}\end{array}\right),\\ \left(\begin{array}{c}\{0.3880, 0.5004, 0.6121\},\\ \{0.4361, 0.8113, 0.9597\}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.2481, 0.3678, 0.6295\},\\ \{0.4656, 0.8421, 0.9597\}\end{array}\right),\\ \left(\begin{array}{c}\{0.3807, 0.4606, 0.6249\},\\ \{0.2561, 0.6479, 0.8220\}\end{array}\right)\end{array}\right)$
A_3	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.4716, 0.5668, 0.6816\},\\ \{0.3936, 0.5962, 0.9211\}\end{array}\right),\\ \left(\begin{array}{c}\{0.4670, 0.7515, 0.9194\},\\ \{0.2235, 0.5416, 0.8045\}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.2391, 0.3276, 0.6554\},\\ \{0.3213, 0.8005, 0.9608\}\end{array}\right),\\ \left(\begin{array}{c}\{0.4453, 0.7036, 0.8086\},\\ \{0.3975, 0.8123, 0.9608\}\end{array}\right)\end{array}\right)$
A_4	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.3807, 0.5202, 0.6726\},\\ \{0.4446, 0.6484, 0.8891\}\end{array}\right),\\ \left(\begin{array}{c}\{0.2491, 0.4204, 0.5326\},\\ \{0.5627, 0.8376, 0.9761\}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.3880, 0.5532, 0.6396\},\\ \{0.3912, 0.5800, 0.8523\}\end{array}\right),\\ \left(\begin{array}{c}\{0.2762, 0.6417, 0.7634\},\\ \{0.4242, 0.5340, 0.8381\}\end{array}\right)\end{array}\right)$

Table 8. Collective aggregated q-ROHFR information.

	c_3	c_4
A_1	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.2000, 0.3455, 0.6345\},\\ \{0.3661, 0.5100, 0.8016\}\end{array}\right),\\ \left(\begin{array}{c}\{0.3283, 0.5993, 0.6738\},\\ \{0.2195, 0.6437, 0.9597\}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.3228, 0.4068, 0.5780\},\\ \{0.2329, 0.5359, 0.9212\}\end{array}\right),\\ \left(\begin{array}{c}\{0.3921, 0.5873, 0.7607\},\\ \{0.5925, 0.7701, 0.9761\}\end{array}\right)\end{array}\right)$
A_2	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.2950, 0.4448, 0.6998\},\\ \{0.3074, 0.5754, 0.8445\}\end{array}\right),\\ \left(\begin{array}{c}\{0.6105, 0.7212, 0.8155\},\\ \{0.2748, 0.7180, 0.9597\}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.2644, 0.4064, 0.5441\},\\ \{0.2857, 0.5359, 0.8732\}\end{array}\right),\\ \left(\begin{array}{c}\{0.4674, 0.3288, 0.5137\},\\ \{0.1301, 0.2561, 0.6224\}\end{array}\right)\end{array}\right)$
A_3	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.3880, 0.5279, 0.7000\},\\ \{0.4946, 0.6794, 0.9500\}\end{array}\right),\\ \left(\begin{array}{c}\{0.5603, 0.7618, 0.8142\},\\ \{0.1884, 0.3209, 0.4805\}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.2315, 0.7310, 0.8155\},\\ \{0.1884, 0.4145, 0.9500\}\end{array}\right),\\ \left(\begin{array}{c}\{0.1552, 0.3256, 0.5170\},\\ \{0.5069, 0.6115, 0.8732\}\end{array}\right)\end{array}\right)$
A_4	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.4593, 0.5532, 0.7930\},\\ \{0.4581, 0.7006, 0.9761\}\end{array}\right),\\ \left(\begin{array}{c}\{0.4064, 0.5657, 0.8142\},\\ \{0.1790, 0.3779, 0.6253\}\end{array}\right)\end{array}\right)$	$\left(\begin{array}{c}\left(\begin{array}{c}\{0.2481, 0.4030, 0.6345\},\\ \{0.3213, 0.4579, 0.8523\}\end{array}\right),\\ \left(\begin{array}{c}\{0.2706, 0.4212, 0.6774\},\\ \{0.4549, 0.5571, 0.8732\}\end{array}\right)\end{array}\right)$

Step 4. Aggregation information of the alternative under the given list of attributes are evaluated using

proposed aggregation operators as follows:

Case 1: Aggregation information using q-ROHFRWGA operator presented in Table 9:

Table 9. Aggregated information using q-ROHFRWG.

A_1	$\begin{pmatrix} (\{0.2676, 0.4371, 0.6287\}, \{0.7091, 0.8635, 0.9886\}), \\ (\{0.3119, 0.5836, 0.7148\}, \{0.8360, 0.9404, 0.9981\}) \end{pmatrix}$
A_2	$\begin{pmatrix} (\{0.2598, 0.3920, 0.5514\}, \{0.7420, 0.8994, 0.9859\}), \\ (\{0.4491, 0.4655, 0.6215\}, \{0.6515, 0.8703, 0.9713\}) \end{pmatrix}$
A_3	$\begin{pmatrix} (\{0.2894, 0.5085, 0.7058\}, \{0.7194, 0.8781, 0.9958\}), \\ (\{0.3341, 0.5647, 0.7143\}, \{0.7949, 0.8930, 0.9788\}) \end{pmatrix}$
A_4	$\begin{pmatrix} (\{0.3383, 0.4815, 0.6730\}, \{0.7515, 0.8621, 0.9867\}), \\ (\{0.2847, 0.4909, 0.6940\}, \{0.7927, 0.8770, 0.9804\}) \end{pmatrix}$

Case 2: Aggregation information using q-ROHFROWGA operator presented in Table 10:

Table 10. Aggregated information using q-ROHFROWG.

A_1	$\begin{pmatrix} (\{0.2801, 0.4538, 0.6304\}, \{0.7140, 0.8679, 0.9894\}), \\ (\{0.3083, 0.5836, 0.7181\}, \{0.8370, 0.9418, 0.9980\}) \end{pmatrix}$
A_2	$\begin{pmatrix} (\{0.2610, 0.3868, 0.5116\}, \{0.8134, 0.9279, 0.9910\}), \\ (\{0.4182, 0.4530, 0.6058\}, \{0.7507, 0.9430, 0.9930\}) \end{pmatrix}$
A_3	$\begin{pmatrix} (\{0.2849, 0.4966, 0.7017\}, \{0.7293, 0.9369, 0.9964\}), \\ (\{0.3253, 0.5615, 0.7201\}, \{0.7640, 0.9391, 0.9919\}) \end{pmatrix}$
A_4	$\begin{pmatrix} (\{0.3301, 0.4776, 0.6587\}, \{0.7808, 0.8907, 0.9865\}), \\ (\{0.2672, 0.4723, 0.6567\}, \{0.8336, 0.9462, 0.9954\}) \end{pmatrix}$

Case 3: Aggregation information using q-ROHFRHWG operator with $(0.18, 0.23, 0.28, 0.31)^T$ associated weights information presented in Table 11:

Table 11. Aggregated information using q-ROHFRHWG.

A_1	$\begin{pmatrix} (\{0.3002, 0.5011, 0.6787\}, \{0.7771, 0.8959, 0.9860\}), \\ (\{0.3331, 0.6152, 0.7403\}, \{0.7843, 0.9360, 0.9977\}) \end{pmatrix}$
A_2	$\begin{pmatrix} (\{0.2931, 0.4213, 0.5494\}, \{0.8733, 0.9471, 0.9921\}), \\ (\{0.4579, 0.5060, 0.6486\}, \{0.8337, 0.9628, 0.9956\}) \end{pmatrix}$
A_3	$\begin{pmatrix} (\{0.3305, 0.5198, 0.7148\}, \{0.7741, 0.9464, 0.9964\}), \\ (\{0.3820, 0.6231, 0.7763\}, \{0.7753, 0.9466, 0.9925\}) \end{pmatrix}$
A_4	$\begin{pmatrix} (\{0.3730, 0.5197, 0.6860\}, \{0.8456, 0.9236, 0.9899\}), \\ (\{0.3004, 0.5170, 0.6860\}, \{0.8860, 0.9643, 0.9967\}) \end{pmatrix}$

Step 5 & 6: The score values of all alternatives under established aggregation operators are presented in Table 12. The graphical representation of ranking for each alternative is visualized in Figure 1.

Table 12. Ranking of alternative.

Proposed operators	Score values of alternatives				Ranking
	A_1	A_2	A_3	A_4	
q-ROHFRWG	0.3007	0.3016	0.3214	0.3093	$A_3 > A_4 > A_2 > A_1$
q-ROHFROWG	0.3022	0.2681	0.3110	0.2858	$A_3 > A_1 > A_4 > A_2$
q-ROHFRHWG	0.3160	0.2726	0.3263	0.2897	$A_3 > A_1 > A_4 > A_2$

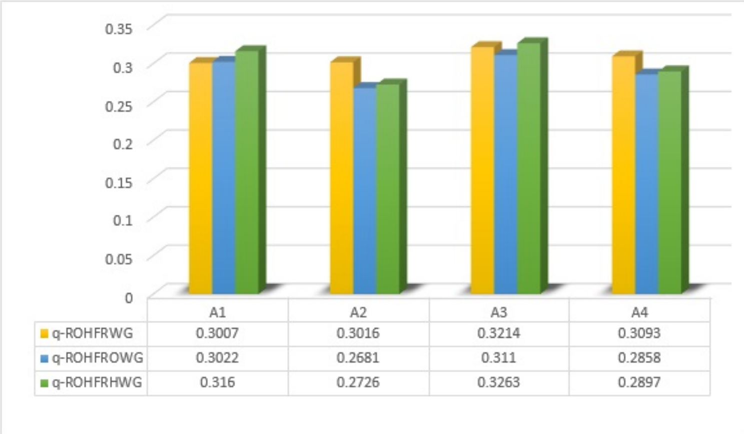

Figure 1. The graphical representation of ranking under proposed operators.

7. Comparison analysis

In this section, we intend to improve the TOPSIS approach for the q-ROHFR environment to account for the MCDM problems.

7.1. The TOPSIS methodology based on q-rung orthopair hesitant fuzzy rough information

Hwang and Yoon [23] proposed the TOPSIS technique for Ideal Solution (TOPSIS), which allows policymakers to compare the Ideal positive solution and Ideal negative solution. The TOPSIS is based on the assumption that the best alternative would be the closest to the ideal and the furthest away from the perfect negative solution [22, 24]. The main parts of the method are as follows:

Step 1. Let $A = \{A_1, A_2, A_3, ..., A_m\}$ be the set of alternatives and $C = \{c_1, c_2, c_3, ..., c_n\}$ be a set of criteria. The decision matrix of the expert is presented as:

$$\begin{aligned} M &= \left[\underline{\mathcal{R}}(\mathcal{U}_{ij}^{\hat{j}}), \overline{\mathcal{R}}(\mathcal{U}_{ij}^{\hat{j}})\right]_{m\times n} \\ &= \begin{bmatrix} \left(\underline{\mathcal{R}}(\mathcal{U}_{11}), \overline{\mathcal{R}}(\mathcal{U}_{11})\right) & \left(\underline{\mathcal{R}}(\mathcal{U}_{12}), \overline{\mathcal{R}}(\mathcal{U}_{12})\right) & \cdots & \left(\underline{\mathcal{R}}(\mathcal{U}_{1j}), \overline{\mathcal{R}}(\mathcal{U}_{1j})\right) \\ \left(\underline{\mathcal{R}}(\mathcal{U}_{21}), \overline{\mathcal{R}}(\mathcal{U}_{21})\right) & \left(\underline{\mathcal{R}}(\mathcal{U}_{22}), \overline{\mathcal{R}}(\mathcal{U}_{22})\right) & \cdots & \left(\underline{\mathcal{R}}(\mathcal{U}_{2j}), \overline{\mathcal{R}}(\mathcal{U}_{2j})\right) \\ \left(\underline{\mathcal{R}}(\mathcal{U}_{31}), \overline{\mathcal{R}}(\mathcal{U}_{31})\right) & \left(\underline{\mathcal{R}}(\mathcal{U}_{32}), \overline{\mathcal{R}}(\mathcal{U}_{32})\right) & \cdots & \left(\underline{\mathcal{R}}(\mathcal{U}_{3j}), \overline{\mathcal{R}}(\mathcal{U}_{3j})\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\underline{\mathcal{R}}(\mathcal{U}_{i1}), \overline{\mathcal{R}}(\mathcal{U}_{i1})\right) & \left(\underline{\mathcal{R}}(\mathcal{U}_{i2}), \overline{\mathcal{R}}(\mathcal{U}_{i2})\right) & \cdots & \left(\underline{\mathcal{R}}(\mathcal{U}_{ij}), \overline{\mathcal{R}}(\mathcal{U}_{ij})\right) \end{bmatrix}, \end{aligned}$$

where

$$\overline{\mathcal{R}}(\mathcal{U}_{ij}) = \left\{\left\langle \jmath, \wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath), \Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath)\right\rangle \mid \jmath \in \mathcal{U}\right\}$$

and

$$\underline{\mathcal{R}}(\mathcal{U}) = \left\{\left\langle \jmath, \wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath), \Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath)\right\rangle \mid \jmath \in \mathcal{U}\right\}$$

such that

$$0 \le \left(\max(\wp_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath))\right)^q + \left(\min(\Im_{h_{\overline{\mathcal{R}}(\mathcal{U})}}(\jmath))\right)^q \le 1 \text{ and } 0 \le \left(\min(\wp_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath)\right)^q + \left(\max(\Im_{h_{\underline{\mathcal{R}}(\mathcal{U})}}(\jmath))\right)^q \le 1$$

are the q-ROHF rough values.

Step 2. First, we collect information from decision makers(DMs) in the form of q-ROPHFR values.

Step 3. Secondly, normalise the data defined by DMs, since the decision matrix may have some benefit and cost criteria all together, as shown below:

$$(H)^{\hat{\jmath}} = \begin{bmatrix} \left(\overline{\mathcal{R}}(\mathcal{U}_{11}^{\hat{\jmath}}), \underline{\mathcal{R}}(\mathcal{U}_{11}^{\hat{\jmath}})\right) & \left(\overline{\mathcal{R}}(\mathcal{U}_{12}^{\hat{\jmath}}), \underline{\mathcal{R}}(\mathcal{U}_{12}^{\hat{\jmath}})\right) & \cdots & \left(\overline{\mathcal{R}}(\mathcal{U}_{1j}^{\hat{\jmath}}), \underline{\mathcal{R}}(\mathcal{U}_{1j}^{\hat{\jmath}})\right) \\ \left(\overline{\mathcal{R}}(\mathcal{U}_{21}^{\hat{\jmath}}), \underline{\mathcal{R}}(\mathcal{U}_{21}^{\hat{\jmath}})\right) & \left(\overline{\mathcal{R}}(\mathcal{U}_{22}^{\hat{\jmath}}), \underline{\mathcal{R}}(\mathcal{U}_{22}^{\hat{\jmath}})\right) & \cdots & \left(\overline{\mathcal{R}}(\mathcal{U}_{2j}^{\hat{\jmath}}), \underline{\mathcal{R}}(\mathcal{U}_{2j}^{\hat{\jmath}})\right) \\ \left(\overline{\mathcal{R}}(\mathcal{U}_{31}^{\hat{\jmath}}), \underline{\mathcal{R}}(\mathcal{U}_{31}^{\hat{\jmath}})\right) & \left(\overline{\mathcal{R}}(\mathcal{U}_{32}^{\hat{\jmath}}), \underline{\mathcal{R}}(\mathcal{U}_{32}^{\hat{\jmath}})\right) & \cdots & \left(\overline{\mathcal{R}}(\mathcal{U}_{3j}^{\hat{\jmath}}), \underline{\mathcal{R}}(\mathcal{U}_{3j}^{\hat{\jmath}})\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\overline{\mathcal{R}}(\mathcal{U}_{i1}^{\hat{\jmath}}), \underline{\mathcal{R}}(\mathcal{U}_{i1}^{\hat{\jmath}})\right) & \left(\overline{\mathcal{R}}(\mathcal{U}_{i2}^{\hat{\jmath}}), \underline{\mathcal{R}}(\mathcal{U}_{i2}^{\hat{\jmath}})\right) & \cdots & \left(\overline{\mathcal{R}}(\mathcal{U}_{ij}^{\hat{\jmath}}), \underline{\mathcal{R}}(\mathcal{U}_{ij}^{\hat{\jmath}})\right) \end{bmatrix}$$

where $\hat{\jmath}$ represents the number of expert.

Step 4. Evaluate the normalized experts matrices $(N)^{\hat{\jmath}}$, as

$$(N)^{\hat{\jmath}} = \begin{cases} \mathcal{R}(\mathcal{U}_{ij}) = \left(\underline{\mathcal{R}}\left(\mathcal{U}_{ij}\right), \overline{\mathcal{R}}\left(\mathcal{U}_{ij}\right)\right) & \text{if} \quad \text{for benefit} \\ \left(\mathcal{R}(\mathcal{U}_{ij})\right)^c = \left(\left(\underline{\mathcal{R}}\left(\mathcal{U}_{ij}\right)\right)^c, \left(\overline{\mathcal{R}}\left(\mathcal{U}_{ij}\right)\right)^c\right) & \text{if} \quad \text{for cost} \end{cases}$$

Step 5. Based on the score value, we determine the positive ideal solution and the negative ideal solution. Herein, the positive ideal solutions and negative ideal solution are denoted as $\daleth^+ = \left(\gimel_1^+, \gimel_2^+, \gimel_3^+, ..., \gimel_n^+\right)$ and $\daleth^- = \left(\gimel_1^-, \gimel_2^-, \gimel_3^-, ..., \gimel_n^-\right)$ respectively. For positive ideal solution $\daleth^+$, it can be computed by the formula:

$$\begin{aligned} \daleth^+ &= (\gimel_1^+, \gimel_2^+, \gimel_3^+, ..., \gimel_n^+) \\ &= \left(\max_i score(\gimel_{i1}), \max_i score\gimel_{i2}, \max_i score\gimel_{i3}, ..., \max_i score\gimel_{in}.\right) \end{aligned}$$

Likewise, the negative ideal solution calculated by the formula as follows:

$$\begin{aligned} \daleth^- &= (\gimel_1^-, \gimel_2^-, \gimel_3^-, ..., \gimel_n^-) \\ &= \left(\min_i score\gimel_{i1}, \min_i score\gimel_{i2}, \min_i score\gimel_{i3}, ..., \min_i score\gimel_{in}.\right) \end{aligned}$$

Afterward, find the geometric distance between all the alternatives and positive ideal $\daleth^+$ as follows:

$$\begin{aligned} d(\alpha_{ij}, \daleth^+) &= \frac{1}{8}\left(\begin{array}{c} \left(\begin{array}{c} \frac{1}{\#h}\sum_{s=1}^{\#h} \left| \left(\underline{\mu}_{ij(s)}\right)^2 - \left(\underline{\mu}_i^+\right)^2 \right| \\ + \left| \left(\overline{\mu}_{ij(s)}\right)^2 - \left(\overline{\mu}_{i(s)}^+\right)^2 \right| \end{array} \right) \\ + \left(\begin{array}{c} \frac{1}{\#g}\sum_{s=1}^{\#g} \left| \left(\underline{\nu}_{ij(s)}\right)^2 - \left(\underline{\nu}_{i(s)}^+\right)^2 \right| \\ + \left| \left(\overline{\nu_{h_{ij}}}\right)^2 - \left(\overline{\nu_{h_i}^+}\right)^2 \right| \end{array} \right) \end{array} \right), \\ \text{where } i &= 1, 2, 3, ..., n, \text{and } j = 1, 2, 3, ..., m. \end{aligned}$$

Analogously, the geometric distance between all the alternatives and negative ideal $\daleth^{-}$ as follows:

$$
\begin{aligned}
d(\alpha_{ij}, \daleth^{-}) &= \frac{1}{8}\left(\begin{array}{c} \left(\begin{array}{c} \frac{1}{\#h}\sum_{s=1}^{\#h} \left| \left(\underline{\mu}_{ij(s)}\right)^{2} - \left(\underline{\mu}_{i(s)}^{-}\right)^{2} \right| \\ + \left| \left(\overline{\mu}_{ij(s)}\right)^{2} - \left(\overline{\mu}_{i(s)}^{-}\right)^{2} \right| \end{array} \right) \\ + \left(\begin{array}{c} \frac{1}{\#g}\sum_{s=1}^{\#g} \left| \left(\underline{\nu}_{ij(s)}\right)^{2} - \left(\underline{\nu}_{i(s)}^{-}\right)^{2} \right| \\ + \left| \left(\overline{\nu_{h_{ij}}}\right)^{2} - \left(\overline{\nu_{h_{i}}^{-}}\right)^{2} \right| \end{array} \right) \end{array} \right), \\
\text{where } i &= 1, 2, 3, ..., n, \text{ and } \ j = 1, 2, 3, ..., m.
\end{aligned}
$$

Step 6. The relative closeness indices for all DMe of the alternatives are calculated as follows:

$$
RC(\alpha_{ij}) = \frac{d(\alpha_{ij}, \daleth^{+})}{d(\alpha_{ij}, \daleth^{-}) + d(\alpha_{ij}, \daleth^{+})}
$$

Step 7. The ranking orders of alternatives can be determined and choose the most desirable alternative having minimum distance.

7.2. Numerical example of q-ROHFR-TOPSIS methodology

A numerical example relevant to "site selection for a wind power plant" is given below to demonstrate the validity of our approach:

Step 1. The decision maker information in the form of q-ROHFRNs is given in Tables 1–6.

Step 2. Positive and negative ideal solution are computed in Table 13, as follows:

Table 13. Ideal solutions.

Criteria	$\daleth^{+}$	$\daleth^{-}$
c_1	$\left(\begin{array}{c} \left(\begin{array}{c} \{0.3807, 0.5202, 0.6726\}, \\ \{0.4446, 0.6484, 0.8891\} \end{array} \right), \\ \left(\begin{array}{c} \{0.2491, 0.4204, 0.5326\}, \\ \{0.5627, 0.8376, 0.9761\} \end{array} \right) \end{array} \right)$	$\left(\begin{array}{c} \left(\begin{array}{c} \{0.3141, 0.4201, 0.3675\}, \\ \{0.5214, 0.6935, 0.9211\} \end{array} \right), \\ \left(\begin{array}{c} \{0.3880, 0.5004, 0.6121\}, \\ \{0.4361, 0.8113, 0.9597\} \end{array} \right) \end{array} \right)$
c_2	$\left(\begin{array}{c} \left(\begin{array}{c} \{0.2481, 0.3678, 0.6295\}, \\ \{0.4656, 0.8421, 0.9597\} \end{array} \right), \\ \left(\begin{array}{c} \{0.3807, 0.4606, 0.6249\}, \\ \{0.2561, 0.6479, 0.8220\} \end{array} \right) \end{array} \right)$	$\left(\begin{array}{c} \left(\begin{array}{c} \{0.2391, 0.3276, 0.6554\}, \\ \{0.3213, 0.8005, 0.9608\} \end{array} \right), \\ \left(\begin{array}{c} \{0.4453, 0.7036, 0.8086\}, \\ \{0.3975, 0.8123, 0.9608\} \end{array} \right) \end{array} \right)$
c_3	$\left(\begin{array}{c} \left(\begin{array}{c} \{0.4593, 0.5532, 0.7930\}, \\ \{0.4581, 0.7006, 0.9761\} \end{array} \right), \\ \left(\begin{array}{c} \{0.4064, 0.5657, 0.8142\}, \\ \{0.1790, 0.3779, 0.6253\} \end{array} \right) \end{array} \right)$	$\left(\begin{array}{c} \left(\begin{array}{c} \{0.2950, 0.4448, 0.6998\}, \\ \{0.3074, 0.5754, 0.8445\} \end{array} \right), \\ \left(\begin{array}{c} \{0.6105, 0.7212, 0.8155\}, \\ \{0.2748, 0.7180, 0.9597\} \end{array} \right) \end{array} \right)$
c_4	$\left(\begin{array}{c} \left(\begin{array}{c} \{0.2644, 0.4064, 0.5441\}, \\ \{0.2857, 0.5359, 0.8732\} \end{array} \right), \\ \left(\begin{array}{c} \{0.4674, 0.3288, 0.5137\}, \\ \{0.1301, 0.2561, 0.6224\} \end{array} \right) \end{array} \right)$	$\left(\begin{array}{c} \left(\begin{array}{c} \{0.3228, 0.4068, 0.5780\}, \\ \{0.2329, 0.5359, 0.9212\} \end{array} \right), \\ \left(\begin{array}{c} \{0.3921, 0.5873, 0.7607\}, \\ \{0.5925, 0.7701, 0.9761\} \end{array} \right) \end{array} \right)$

Step 3. Compute the distance measure of Positive ideal solution (PIS) and negative ideal solution (NIS)

0.2606	0.1296	0.3446	0.1838

and

0.1652	0.1429	0.4767	0.2486

Step 4. The relative closeness indices for all decision makers of the alternatives are calculated as follows:

0.6120	0.4756	0.4196	0.4251

Step 5. From ranking of alternative it could be seen that A_3 has the minimum distance. Hence A_3 is the best alternative. The graphical representation of ranking under TOPSIS-method is presented in Figure 2.

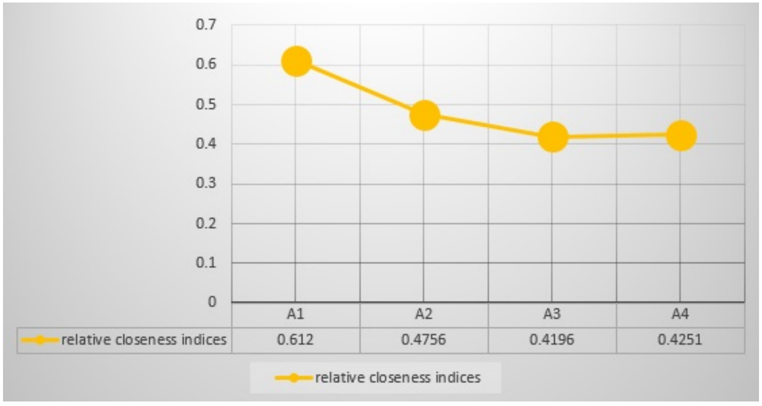

Figure 2. The graphical representation of ranking under TOPSIS-method.

8. Conclusions

Wind energy is considered one of the greatest clean energies due to its abundance, minimum environmental impact, and high cost-effectiveness. One of the most significant challenges in the utilization of wind energy is the proper planning of optimal wind energy extraction locations to generate electricity. The appropriate site selection for wind power plants is based on the principles and criteria of sustainable environmental advancement, which results in a cheap and renewable energy source in addition to cost-effectiveness and employment generation. In this study, a novel fuzzy set extension called q-rung orthopair hesitant fuzzy rough set is introduced as a new hybrid structure of q-rung orthopair fuzzy set, hesitant fuzzy set and rough set. This concept provide a more versatile and efficient basis for fuzzy system modeling and decision making under uncertainty, due to the implementation of the concept of the rough set theory. Based on the developed concept, a list of aggregation operators such as q-ROHFR weighted geometric operators are presented on the basis of algebraic t-norm and t-conorm. Furthermore, the basic characteristics of developed operators are discussed in detail. A decision-making algorithm is developed to tackle the uncertain and incomplete information in real word decision making problems. On the other hand, a case study in the evaluating of wind power plant site schemes is used to demonstrate the applicability and rationality of the suggested approach. The empirical results suggest that using proposed technique to evaluate the final site selection scheme based on the ranking findings is reasonable and appropriate. Meanwhile, the comparison analysis performed by the developed models with the q-ROHFR-TOPSIS technique further justifies the feasibility, superiority, and reliability of the presented approaches. The findings of the comparison reveal that this strategy has considerable advantages of evaluating and analyzing˜the impact of evaluation information value. This study assists wind industry researchers and stakeholder in identifying locations for projects with considerable technical performance, cheap installation cost, and little environmental effect. It also serves as a comprehensive information road map for future study to support in the identification of optimum wind farm locations.

Hypothetically, the objective of this study may be expanded to establish the Hamacher and Dombi t-norm and t-conorm based on the generalized aggregation information under q-ROHFRSs. Besides these theoretical aspects, these notions can be extended to solve numerous real world problems and decision making under uncertainty in various fields comprising computational intelligence, cognitive sciences, commerce, business, sociology, econometrics, cleaner production, human resource management, robotics, agri-farming and medical diagnosis. We hope that this article will serve as a foundation stone for the researchers working in these fields.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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