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# Game-Based Lateral and Longitudinal Coupling Control for Autonomous Vehicle Trajectory Tracking

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**ABSTRACT** In this paper, we propose a game-based coupling controller to solve the problem of autonomous vehicles in the trajectory tracking process. The controller ensures path tracking accuracy and shows good tracking performance for the desired speed profile that changes based on navigation while driving. Furthermore, it improves ride comfort and addresses the lateral stability problems that occur when applying classical model predictive control (MPC) controllers. We implemented this controller to plan the trajectory by considering the fundamental interaction between longitudinal and lateral vehicle dynamics, as well as to track the trajectory by using a receding horizon strategy. A comparative study was conducted to compare the proposed controller with the classical MPC controller. Based on CarMaker-Matlab/Simulink co-simulations, this controller outperformed the classical MPC in terms of trajectory tracking performance. Moreover, it exhibits excellent ride comfort and lateral stability that passengers will experience.

**INDEX TERMS** Trajectory tracking, intelligent vehicle, control, motion planning.

## I. INTRODUCTION

The autonomous driving algorithm consists of four stages: perception, localization, decision, and control. Among these, the decision stage determines the target trajectory at which the vehicle will move, and the control stage calculates the input value such that the actual vehicle can follow the determined target path and speed profile. Thus far, in trajectory tracking research for autonomous driving, numerous path tracking studies focusing on tracing the target path have been conducted under the assumption that the longitudinal controller tracks the desired speed profile. Since longitudinal and lateral dynamics, which are in a nonlinearly complex relationship, are difficult to deal with in a coupled form, they are simplified and dealt with separately. And these path tracking studies can be broadly classified into two types: those involving geometric path tracking methods (e.g., the pure pursuit method and the Stanley method [1]–[3]), and those involving the dynamic path tracking methods (e.g., model predictive control (MPC) based on the lateral dynamic model [4], [5]).

Geometric methods are representative path tracking tech-

niques in which the model is simple. For instance, the pure pursuit method is a geometric path tracking method that functions by calculating the curvature for the vehicle to move from the current position to the target point [1]. The key to this algorithm is to select a target point on the path located a certain distance ahead of the vehicle. The distance referred to here the look-ahead-distance (LD) is considerably important because the path tracking method changes depending on the setting. Setting a short LD offers the advantage of allowing the vehicle to return to the route quickly; however, setting it excessively short carries the risk of lateral oscillation. Conversely, setting a long LD allows the vehicle to travel along a path more smoothly than before; however, an excessively long LD can degrade the path tracking performance when encountering large curvatures at corners. Therefore, parameter optimization should be performed according to the driving speed, but the control performance may be rather poor within a specific speed range. Overcoming the disadvantages of pure pursuit involves a method of switching to another algorithm under a specific boundary condition [6]; however, this approach is not appropriate in terms of stability because

the control input can change suddenly and discontinuously during switching.

Another geometric path tracking method, the Stanley method [2] was used by the Stanford University team that won the DARPA Grand Challenge in 2005. Unlike pure pursuit, the target point is set based on the front wheel. In the cross-track error term, the formula is built to ensure smaller steering angles at higher speeds, similar to the pure pursuit method. The Stanley method exhibits superior path tracking performance, as compared to pure pursuit, because it controls the steering by considering both the cross-track error and the heading error. However, similar to pure pursuit, the Stanley method can suffer from poor control performance within a certain speed range. Furthermore, the geometric method does not deal with the various dynamic factors that must be considered in actual vehicle control because the model is simple. For example, the actuator and tire force effects are not considered, which can lead to undesirable riding characteristics.

By contrast, dynamic path tracking methods can reflect the dynamic characteristics of the model well. Among these, MPC, based on a lateral dynamic model, numerically solves the optimization problem at each time step [4], [5]. And this method is characterized in that it can consider various constraints for state variables and steering input. This method considers various constraints for state variables and steering inputs. Therefore, it shows relatively robust performance in terms of system parameter changes [7], [8], which has been considered a problem in geometrical methods. Although the lateral dynamic model is more complicated than the kinematic model, it can represent the actual vehicle behavior even under high-speed driving conditions.

The method described thus far is a decoupling control strategy that performs longitudinal control, followed by lateral control. Although this control strategy is easy to implement and it achieves high tracking accuracy [9], These decoupled design approaches described so far do not take into account the fundamental interaction characteristic of tire forces between longitudinal and lateral motion from a vehicle dynamics point of view. For example, the steering angle directly affects the tire slip angle, which constitutes the tire force. Tire force also affects the vehicle's lateral, longitudinal and yaw rate. However, in the case of longitudinal control, only curvature-based velocity profile information is used and information on steering angle changes is not sufficiently used. This can cause problems near handling limits at high speeds or on slippery roads. This leads to ride comfort and lateral stability issues in autonomous driving.

Therefore, the proposed approach uses an integrated vehicle dynamics model in which the longitudinal and lateral dynamics are not separated. And here we use differential games [10]–[12] to let each controller plan a control strategy that takes into account the effect on each other as game players participating in a simultaneous game. In game theory, differential games provide a framework for dealing with the conflict problems in dynamical systems. Each player in the

game is responsible for one of the controls and aims to minimize their own costs. However, this cost is determined by the player's own control strategy and is also affected by the opponent's control strategy. In this case, even if the opponent's decision is unknown and the opponent's initial state and structure are known, the player can prepare a rational trajectory plan that minimizes its cost, regardless of the opponent's response. When players execute these plans, they are in Nash Equilibrium [10], [13], [14]. Using this differential game approach, the proposed controller aims to effectively address the interaction problem between longitudinal and lateral controllers to improve trajectory tracking accuracy and lateral stability and ride comfort. The related applied studies are [15] and [16]. Subsequently, the proposed coupling controller iteratively revises the initially set trajectory plan through linear approximation. Through the correction, the controller calculates a suitable trajectory for a nonlinear system [15], [17]. Subsequently, the vehicle tracks the target speed and path by using the first input value in the trajectory plan. Here, the receding horizon strategy is applied. The remainder of this paper is organized as follows: Section II describes the Dynamic Bicycle Model, a vehicle model used in the proposed controller to use the differential game, and describes the cost function setting process for each player. And the process of solving the differential game using the set model and cost function is explained. Section III evaluates the system in two scenarios. The first scenario is a scenario in which there is a sharp curve road section at a constant period in a situation where a constant speed profile is given regardless of the road curvature. The second scenario is a scenario in which a straight road section appears again after passing through a curved section in a straight section when the speed profile changes according to the curvature of the road. Finally, conclusions are presented in Section IV.

## II. DESIGN OF GAME-BASED COUPLING CONTROLLER

In this section, we describe the overall design procedure of the proposed game-based trajectory tracking controller. First, we describe the coupling dynamic model used for controller design. Then, we present a method to combine the longitudinal and lateral dynamics of a vehicle by using a differential game. We also explain how to use an iterative correction strategy [15], [17] for trajectory tracking studies.

### A. MATHEMATICAL MODEL

In the trajectory tracking problem, the vehicle model setting affects the tracking performance [9]. In this study, we used the dynamic bicycle model, shown in Figure 1, to consider the effect on the tire force when the vehicle turns at high speeds. By applying Newton's second law, the dynamic bicycle model is expressed as Eq. (1), which indicates the lateral tire force affecting the lateral, longitudinal, and yaw rates. Eq. (2) shows the lateral tire force to be proportional to the tire slip angle. Furthermore, the tire slip angle is proportional to the steering angle; even if the same steering input is provided, the tire slip angle may increase when the

TABLE 1. Symbols and Definitions in Dynamic Bicycle Model.

Symbol	Definition
$m$	Vehicle weight
$\delta_f$	Front wheel steering angle
$v_x$	Vehicle longitudinal speed in vehicle coordinate systems
$v_y$	Vehicle lateral speed in vehicle coordinate systems
$\dot{\psi}$	Yaw rate
$F_{cf}$	Lateral force on front tires
$F_{cr}$	Lateral force on rear tires
$C_{\alpha f}$	Lateral cornering stiffness for the front wheel
$C_{\alpha r}$	Lateral cornering stiffness for the rear wheel
$\alpha_f$	Front tire slip angle
$\alpha_r$	Rear tire slip angle
$\beta$	Side-slip angle
$I_z$	Yaw inertia moment
$l_f$	Distance from center of mass to front axle
$l_r$	Distance from center of mass to rear axle

longitudinal speed is higher, as compared to that when the longitudinal speed is low.

$$\begin{cases} \dot{X} = v_x \cos \psi - v_y \sin \psi \\ \dot{Y} = v_x \sin \psi + v_y \cos \psi \\ \dot{\psi} = \frac{v_x}{l_f + l_r} \tan \delta_f \\ \dot{v}_x = \dot{\psi} v_y + a_x \\ \dot{v}_y = -\dot{\psi} v_x + \frac{2}{m} (F_{cf} \cos \delta_f + F_{cr}) \\ \ddot{\psi} = \frac{2}{I_z} (l_f F_{cf} - l_r F_{cr}), \end{cases} \quad (1)$$

$$\begin{cases} F_{cf} = C_{\alpha f} \alpha_f \\ F_{cr} = C_{\alpha r} \alpha_r \\ \alpha_f = \delta_f - \beta - \frac{l_f \dot{\psi}}{v_x} \\ \alpha_r = \beta + \frac{l_r \dot{\psi}}{v_x}, \end{cases} \quad (2)$$

With such complex dynamics, decoupling is difficult. However, configuring the controllers by arbitrarily decoupling the longitudinal and lateral dynamics may cause the control performance to become unstable owing to the interaction among variables. Therefore, this study does not decouple the dynamics of complex relationships; instead, a multi-input-multi-output (MIMO) system, in which steering and acceleration inputs are coupled with each other, as shown in Eq. (1), was used for the game-based trajectory tracking research. The state variables and control inputs of the MIMO system are expressed in Eq. (3).

$$x(t) = [X, Y, \psi, v_x, v_y, \dot{\psi}]^T \quad (3a)$$

$$u(t) = [u_\delta, u_{accel}]^T \quad (3b)$$

## B. SETTING UP PLAYER COSTS

A typical optimal control method for a MIMO system involves finding an input that minimizes the cost function,

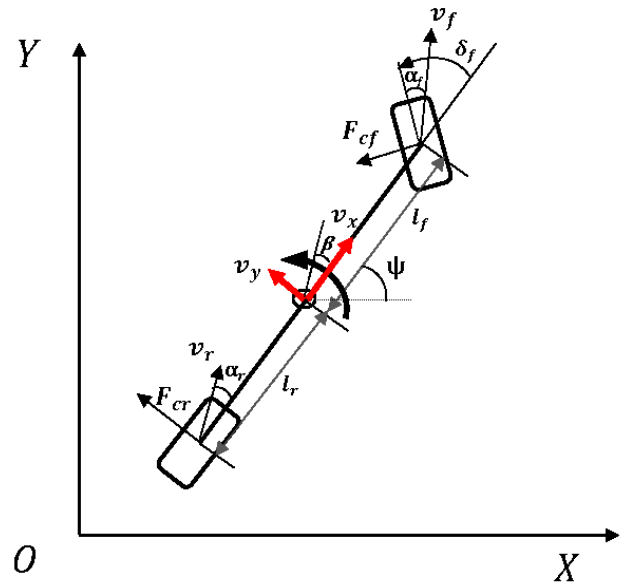


FIGURE 1. Dynamic bicycle model.

defined as the integral of running costs. A typical optimal control method for a MIMO system involves finding an input that minimizes the cost function, defined as the integral of running costs. By contrast, in the case of the game-based trajectory tracking method presented herein, the running cost related to the lateral dynamics of the vehicle and the running cost related to the longitudinal dynamics are set separately, as in Eqs. (4a) and (4b). And it was assumed that the target path (i.e.,  $X_{ref}$ ,  $Y_{ref}$  and  $\psi_{ref}$ ) was generated based on an High Definition Map (HD Map). Since the points on the generated path are dense and expressed with high accuracy, in this paper, the target path was created using curve interpolation rather than curve fitting [18]. The target speed profile (i.e.,  $v_{xref}$ ) was created based on curvature [19].

$$g_{Lat} = g_1 = \frac{1}{2} \left[ (X(t) - X_{ref})^T Q_{11} (X(t) - X_{ref}) + (Y(t) - Y_{ref})^T Q_{22} (Y(t) - Y_{ref}) + (\psi(t) - \psi_{ref})^T Q_{33} (\psi(t) - \psi_{ref}) + u_\delta(t)^T R_{11} u_\delta(t) \right] \quad (4a)$$

$$g_{Long} = g_2 = \frac{1}{2} \left[ (v_x(t) - v_{xref})^T Q_{44} (v_x(t) - v_{xref}) + u_{accel}(t)^T R_{22} u_{accel}(t) \right] \quad (4b)$$

### C. PROBLEM FORMULATION

This study applies the differential game framework reported in [10] such that each player can be in the Nash equilibrium.

First, we discretized the mathematical model and player cost described in the previous subsection and expressed it as Eq. (5). We used this expression to construct the framework.

$$x(t+1) = A(t)x(t) + \sum_{i=1}^2 B_i(t)u_i(t) \quad (5a)$$

$$J_i(t, x(t)) = \sum_{t=0}^T g_i(t, x(t), u_{1:2}(t)) \quad (5b)$$

According to Bellman's principle of optimality, the relationship between costs can be expressed as Eq. (6a). And assume that the value function is in quadratic form or lower, such as Eq. (6b).

$$\begin{aligned} J_i(t, x(t)) &= \frac{1}{2}(x(t)^T Q_i(t)x(t) + \sum_{j=1}^2 u_j(t)^T R_{ij}(t)u_j(t)) \\ &+ l_i(t)^T x(t) + \sum_{j=1}^2 r_j^T u_j(t) + \zeta_i(t)^T x(t) \\ &+ J_i(t+1, x(t+1)) \end{aligned} \quad (6a)$$

$$\begin{aligned} J_i(t+1, x(t+1)) &= \frac{1}{2}x(t+1)^T Z_i(t+1)x(t+1) \\ &+ \zeta_i(t+1)^T x(t+1), \end{aligned} \quad (6b)$$

Secondly, assume strong convexity and set gradient to zero to find input  $u_i$ . We established the coupled Riccati differential equation expressed in Eq. (7). Subsequently, we solved the coupled Riccati differential equation, expressed in

Eq. (7), to achieve a Nash Equilibrium between the lateral controller (i.e., Player 1) and the longitudinal controller (i.e., Player 2). Here, dynamic programming was used to solve the coupled Riccati differential equation.

$$\begin{aligned} Z_i(t) &= F(t)^T Z_i(t+1)F(t) + \sum_{j=1}^2 P_j(t)^T R_{ij}(t)P_j(t) \\ &+ Q_i(t)^T \end{aligned} \quad (7a)$$

$$\begin{aligned} \zeta_i(t) &= F(t)^T (\zeta_i(t+1) + Z_i(t+1) \sum_{j=1}^2 (B_j(t)\alpha_j(t)) \\ &+ l_i(t) + \sum_{j=1}^2 (P_j(t)^T R_{ij}(t)\alpha_j(t) - P_j(t)^T r_j(t)) \end{aligned} \quad (7b)$$

$$\left\{ \begin{aligned} F(t) &= A(t) - \sum_{j=1}^2 B_j(t)P_j(t) \end{aligned} \right. \quad (7c)$$

After obtaining the  $Z_i$  and  $\zeta_i$  parameter for each player, the input gain of each player can be calculated using Eq. (8):

$$(R_{ii}(t) + B_i(t)^T Z_i(t+1)B_i(t))P_i(t) \quad (8a)$$

$$+ B_i(t)^T Z_i(t+1) \sum_{j \neq i}^2 B_j(t)P_j(t) = B_i(t)^T Z_i(t+1)A(t)$$

$$\begin{aligned} (R_{ii}(t) + B_i(t)^T Z_i(t+1)B_i(t))\alpha_i(t) \\ = B_i(t)^T \zeta_i(t+1) + r_{ii}(t) \end{aligned} \quad (8b)$$

Finally, the optimal input  $u_i^*$  for each player, as shown in Eq. (9), is planned:

$$u_i^*(t) = P_i(t)x(t) + \alpha_i(t) \quad (9)$$

### D. LINEAR APPROXIMATION FOR NONLINEAR SYSTEM

The control strategy described thus far followed linear-quadratic formalism. However, because the actual vehicle is a nonlinear system, a model mismatch problem arises. This may increase the forecast error within the horizon time and degrade the trajectory tracking performance of the vehicle. Therefore, we need to linearly approximate the system model and cost function to fit the actual vehicle control. In this study, we used the iterative linear quadratic game (iLQ game) method, an approximation technique introduced in [15] and [20].

Eqs. (10a) and (10b) indicate the first-order local approximations of the system model. Similarly, Eqs. (10c) and (10d) are the second-order local approximations of the cost function. Thereafter, input  $u_i$  obtained in this approximate environment is input to the coupled dynamic model, which is a non-linear system, to calculate a new  $\hat{x}$ . This process was repeated until the  $\hat{x}$  obtained here converged; finally, the optimal input  $u_i^*$  was determined.

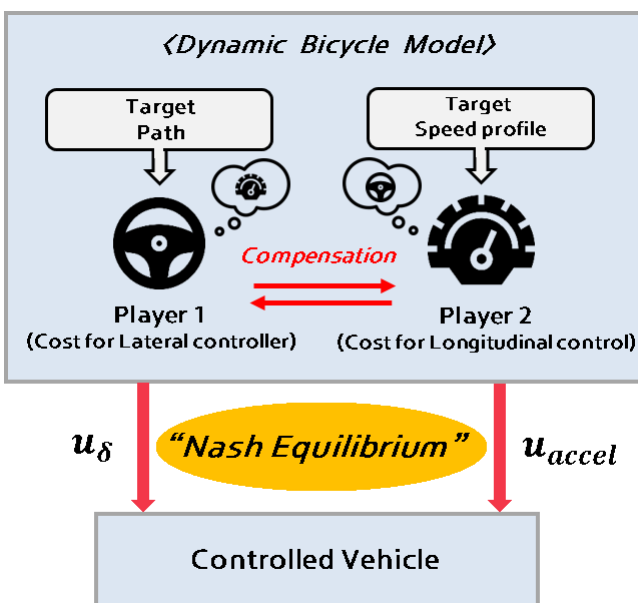


FIGURE 2. Nash equilibrium strategy of game-based coupling control.

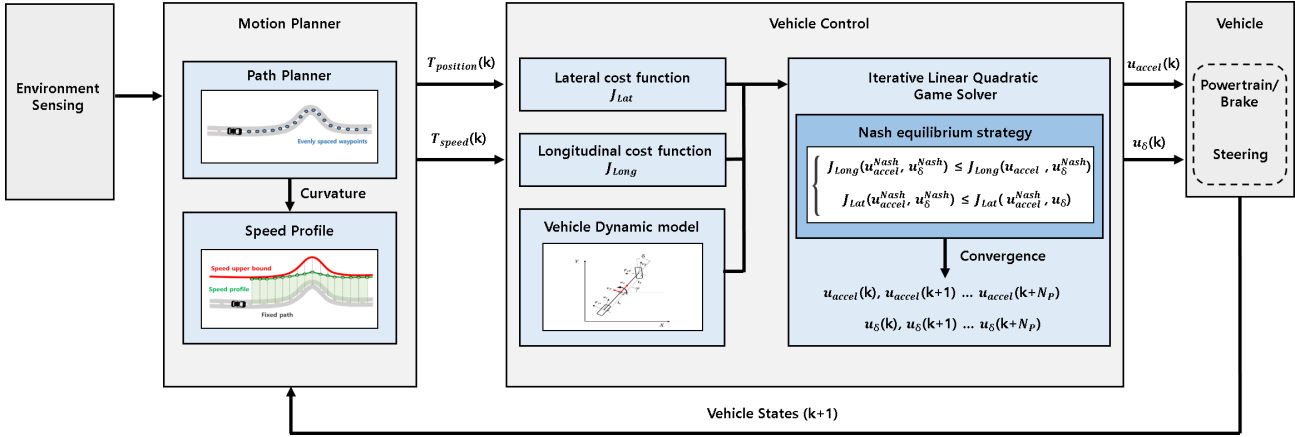


FIGURE 3. Overall procedure of game-based coupling control scheme for autonomous vehicle trajectory tracking.

$$\dot{x} = f(t, x, u_{1:2})$$

$$\approx f(t, \hat{x}, \hat{u}_{1:2}) + \nabla_{x(t), u_{1:2}(t)} f(t, \hat{x}, \hat{u}_{1:2}) \begin{bmatrix} \delta x(t) \\ \delta u_1(t) \\ \delta u_2(t) \end{bmatrix} \quad (10a)$$

$$\delta \dot{x} \approx A(t) \delta x(t) + \sum_{i=1}^2 B_i(t) \delta u_i(t) \quad (10b)$$

$$g_i(t, x, u_{1:2}) \approx g_i(t, \hat{x}, \hat{u}_{1:2}) + \begin{bmatrix} \delta x(t) \\ \delta u_1(t) \\ \delta u_2(t) \end{bmatrix}^T \nabla_{x(t), u_i(t)} g_i(t, \hat{x}, \hat{u}_{1:2}) \quad (10c)$$

$$+ \frac{1}{2} \begin{bmatrix} \delta x(t) \\ \delta u_1(t) \\ \delta u_2(t) \end{bmatrix}^T \nabla_{x(t), u_i(t)}^2 g_i(t, \hat{x}, \hat{u}_{1:2}) \begin{bmatrix} \delta x(t) \\ \delta u_1(t) \\ \delta u_2(t) \end{bmatrix} \\ \approx g_i(t, \hat{x}, \hat{u}_{1:2}) + \frac{1}{2} \delta x(t)^T (Q_i(t) \delta x(t) + 2l_i(t)) \\ + \frac{1}{2} \sum_{j=1}^2 \delta u_j(t)^T (R_{ij}(t) \delta u_j(t) + 2r_{ij}(t)) \quad (10d)$$

$$\begin{cases} \delta x(t) \triangleq x(t) - \hat{x}(t), & \delta u_i(t) \triangleq u_i(t) - \hat{u}_i(t) \end{cases} \quad (10e)$$

### E. MOTION PLANNING AND RECEDING HORIZON CONTROL

As shown in Figure 3, this motion planner transmits information regarding the target path and speed profile to the vehicle control level. With the received information, the game-based coupling controller solves the finite-horizon optimal control problem through the iLQ game. Among the optimal control inputs, the vehicle control uses only the input corresponding to the first step; at the next sampling time, solving the finite-horizon optimal control problem in the same manner yields the control input.

### III. SYSTEM EVALUATION

To evaluate the proposed game-based coupling controller, we conducted a series of simulation experiments using CarMaker-Matlab/Simulink co-simulations. Figure 3 shows the structure of the simulation. We divided the scenarios into two types according to the following reference path types: sinusoidal and curved. To prove the validity of the tracking method proposed herein, we compared the results when using the classical MPC controller based on the coupled dynamic model shown in Eq. (1) for trajectory tracking. Table 2 lists the vehicle model parameters, and we tested both controllers with a prediction horizon of the same length. The constraints of the vehicle in each scenario indicate the maximum steering angle at  $30^\circ$  and the acceleration command at  $-5$  to  $2 \text{ m/s}^2$ .

#### A. SCENARIO 1: SINUSOIDAL PATH TRACKING

Under this scenario, we checked the path and speed tracking performance under a constant target speed ( $54 \text{ km/h}$ ) input to the controller. Figure 4 shows the road used under Scenario 1. The minimum turning radius of the road in this scenario was  $52 \text{ m}$ . A characteristic of this road is that the turning radius changes regularly and rapidly. For the vehicle to maintain a constant speed along this path, an acceleration input considering the lateral acceleration is required. The experimental results were as follows.

In Figures 4 and 5, when the MPC controller is used, it can be seen that the vehicle does not follow the path properly in the straight section, unlike when the proposed controller is used. And rather, in the curved section, the MPC controller performed control to be closer to the path than the proposed controller. When evaluated with these results alone, the MPC controller may show better control performance in the curved section than the proposed controller. Even in

TABLE 2. Vehicle model parameters.

m (kg)	$I_z (\text{kg} \cdot \text{m}^2)$	$l_f$ (m)	$l_r$ (m)
1350	4000	1.016	1.562

the results of lateral error analysis for quantitative evaluation shown in Table 3, the performance difference between the two controllers is not large. However, as shown in Figure 6, When using the MPC controller, the vehicle has excessive side slip. As a result, it is expected to be pushed outward from the curve section and closer to the center of the path. On the other hand, the proposed controller shows good performance in terms of lateral stability due to less side slippage in Figure 6. And in Figure 7, the proposed controller, unlike the MPC

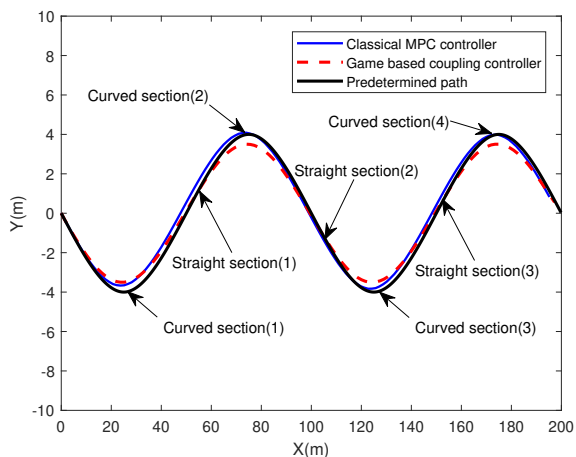


FIGURE 4. Pre-determined path and tracking path under Scenario 1.

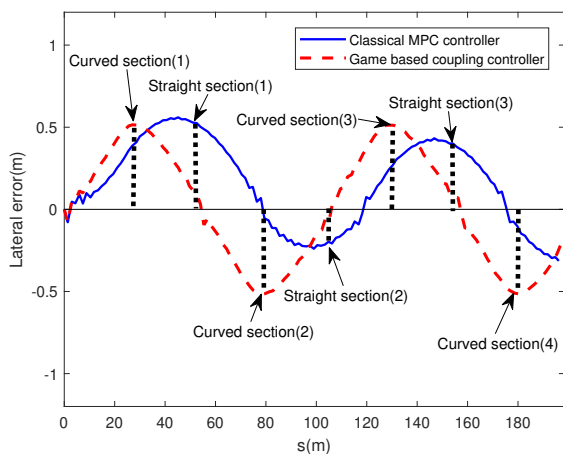


FIGURE 5. Lateral tracking error under Scenario 1.

TABLE 3. Analysis of lateral deviation under Scenario 1.

	Game-based coupling controller	Classical MPC controller
Mean absolute error(m)	0.31	0.26
Root mean square error(m)	0.34	0.30
Maximum absolute error(m)	0.52	0.56

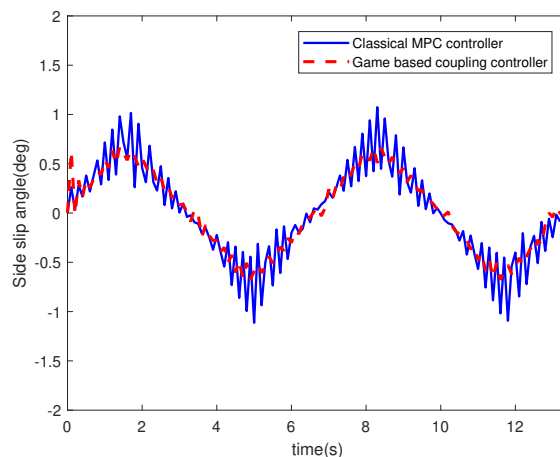


FIGURE 6. Side-slip angle under Scenario 1.

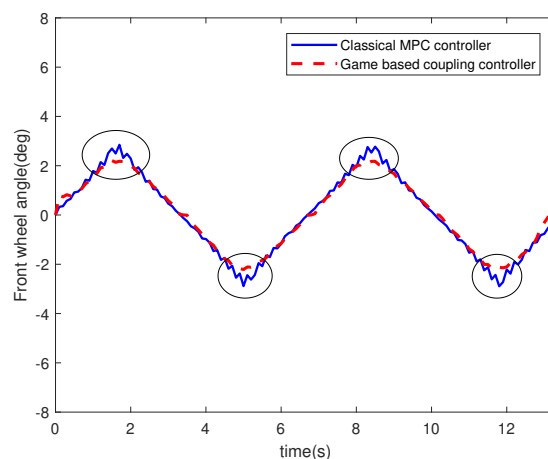


FIGURE 7. Front steering angles under Scenario 1.

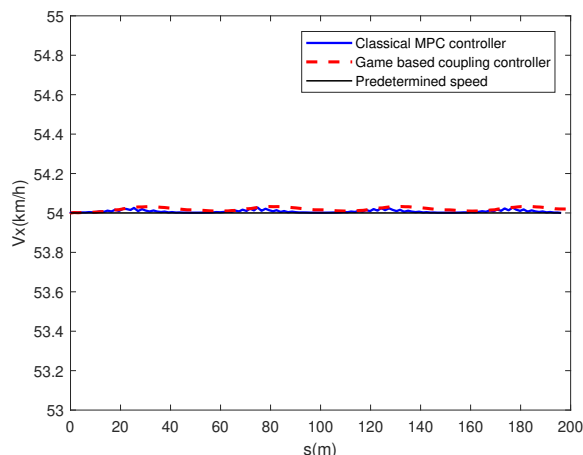


FIGURE 8. Pre-determined speed profile and tracking speed under Scenario 1.

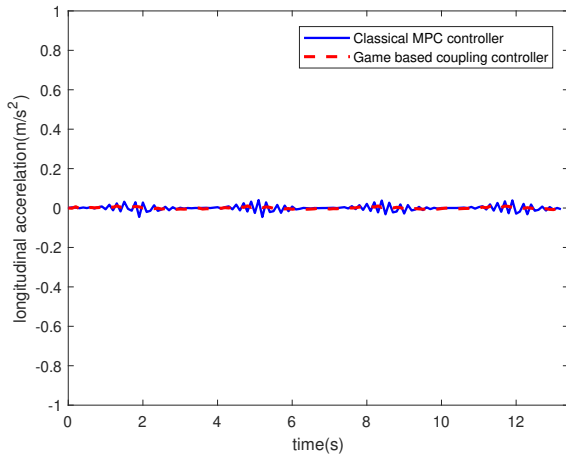


FIGURE 9. Longitudinal acceleration under Scenario 1.

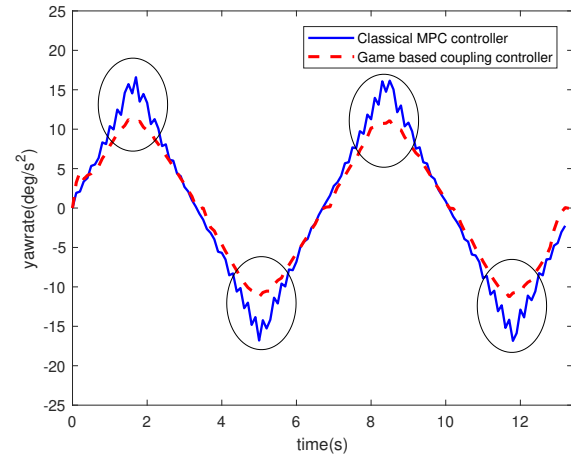


FIGURE 11. Yaw rate under Scenario 1.

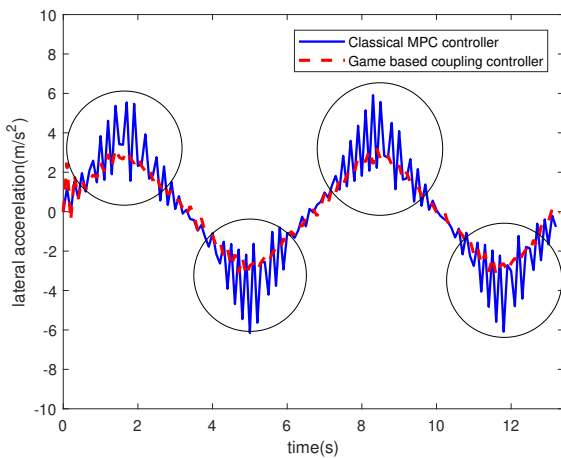


FIGURE 10. Lateral acceleration under Scenario 1.

controller, is smooth in the curve section and does not change the steering in a short time. This means that the proposed controller controlled the vehicle to move along the inner course of the path by considering the longitudinal and lateral interactions.

As shown in Figure 8, both controllers showed good speed tracking performance under a constant target speed. However, when comparing the results of the lateral acceleration shown in Figure 10, under Scenario 1, the game-based controller shows less change in the lateral acceleration within the curved section than the classical MPC controller. By contrast, in the case of MPC, the lateral acceleration value was large and the change was significant. This can be inconvenient for the driver in terms of the ride comfort performance.

On comparing the lateral stability performance with Figure 11, the yaw rate change in the curved section was small when using a game-based controller. Conversely, the conventional MPC controller exhibited high-frequency results. Therefore,

the proposed controller is better in terms of lateral stability.

### B. SCENARIO 2: CURVED PATH TRACKING

Unlike Scenario 1, where the target speed was constant, Scenario 2 involved different target speeds for each section based on curvature information, as shown in Figure 12. Figure 13 shows the road selected for the simulation, with the minimum turning radius of 165 m. We assumed that the curvature information of the road was obtained from the navigation system. In this scenario, the vehicle decelerates for safe driving when entering the circular curved section from the transition curve section of the road. When entering a straight road from the circular curve section through the transition curve section, returning to the original set speed requires acceleration.

The simulation results are as follows. First, as shown in Figure 14, the MPC controller and the game-based controller showed excellent performance in tracking the rapidly changing target speed. However, as shown in Figure 15, the conventional MPC controller affords high-frequency acceleration results. Unlike the game-based controller, which considers the Nash balance between longitudinal and lateral motions, the MPC controller blindly pursues speed tracking accuracy. Furthermore, as shown in Figure 16, the game-based controller shows less change in the lateral acceleration than the existing MPC, similar to the results for Scenario 1. Thus, the game-based coupling controller enables a better

TABLE 4. Analysis of lateral deviation under Scenario 2.

	Game-based coupling controller	Classical MPC controller
Mean absolute error(m)	0.09	0.20
Root mean square error(m)	0.12	0.23
Maximum absolute error(m)	0.25	0.45

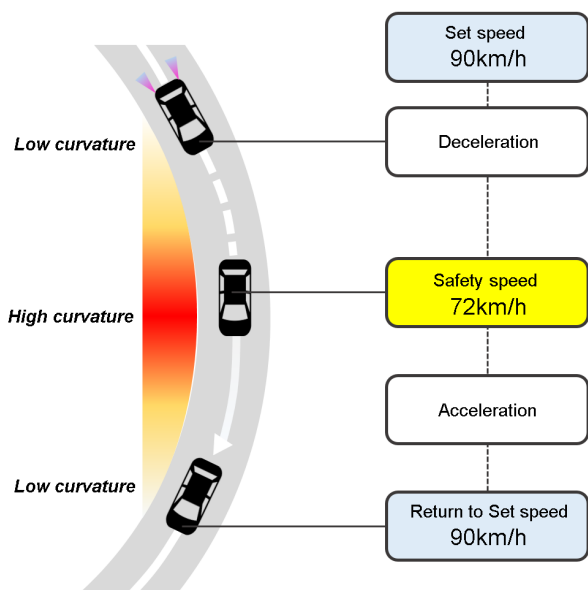


FIGURE 12. Target speed setting strategy under Scenario 2.

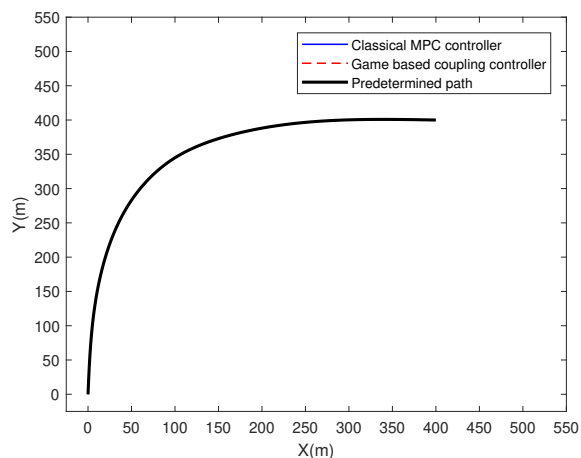


FIGURE 13. Pre-determined path and tracking path under Scenario 2.

ride.

As depicted in Figure 17, when each controller is used, the side-slip change does not occur excessively in all sections. Therefore, unlike Scenario 1, where quantitative evaluation was difficult, Scenario 2 performed a quantitative evaluation of path tracking accuracy using Table 4. The maximum lateral error of the proposed controller is 0.25 m. This value is less than the maximum error (0.45 m) when using the conventional MPC controller. The RMSE (0.12 m) and MAE (0.09 m) were also lower than the RMSE (0.23 m) and MAE (0.20 m) of the MPC controller, respectively. In other words, when using the game-based coupling controller, the lateral error was lower than that under MPC and distributed evenly,

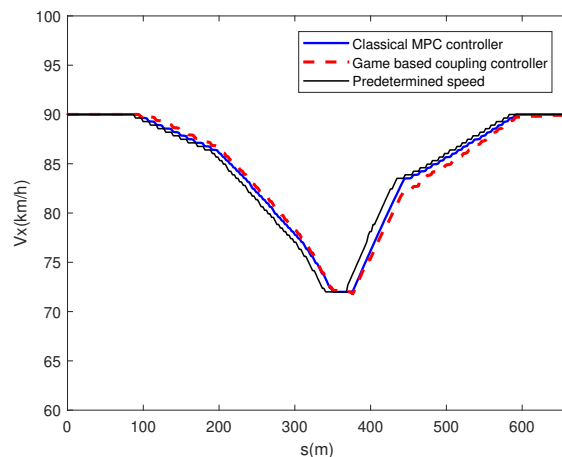


FIGURE 14. Pre-determined speed profile and tracking speed under Scenario 2.

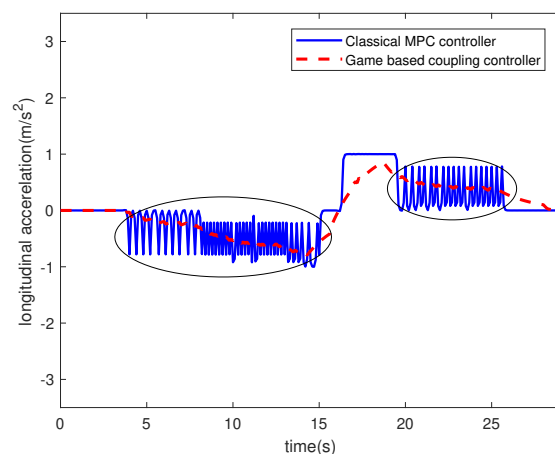


FIGURE 15. Longitudinal acceleration under Scenario 2.

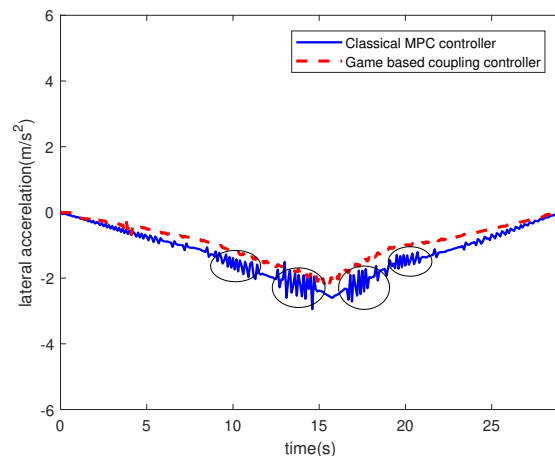


FIGURE 16. Lateral acceleration under Scenario 2.



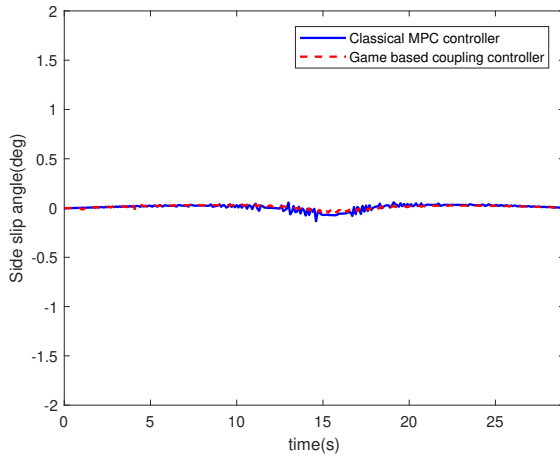


FIGURE 17. Side-slip angle under Scenario 2.

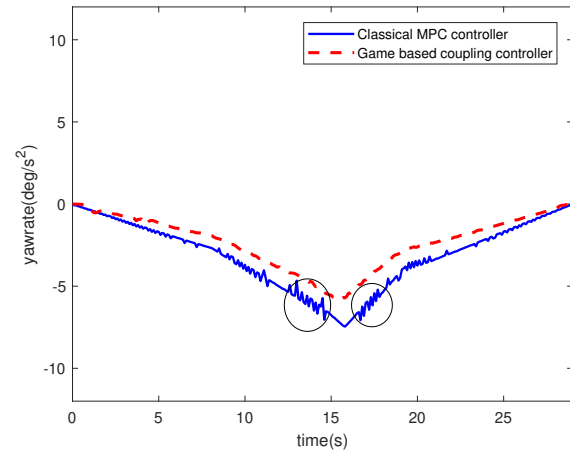


FIGURE 20. Yaw rate under Scenario 2.

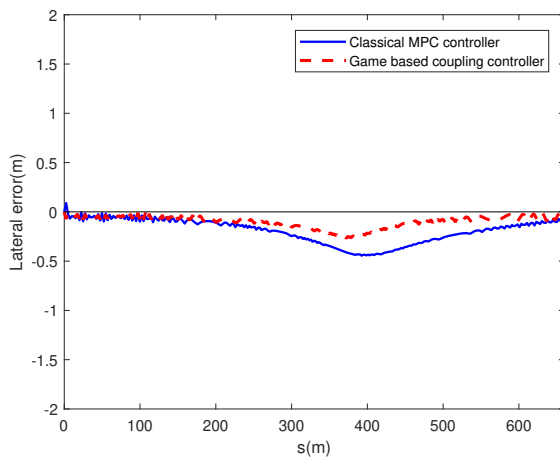


FIGURE 18. Lateral tracking error under Scenario 2.

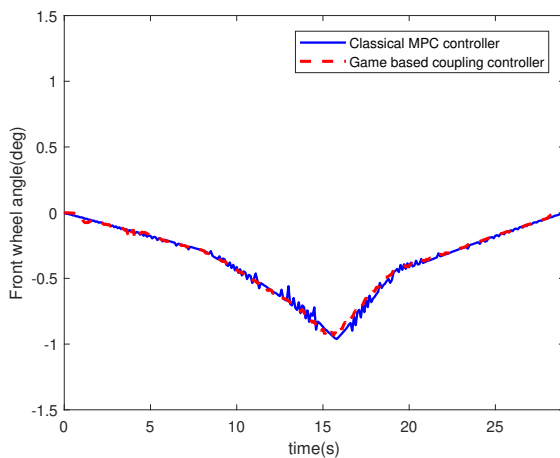


FIGURE 19. Front steering angles under Scenario 2.

TABLE 5. Performance analysis of two controllers.

	Game-based coupling controller	Classical MPC controller
Speed tracking accuracy	High	High
Path tracking accuracy	High	Mid
Ride comfort	Good	Bad
Lateral stability	Good	Bad

indicating good performance in terms of the path tracking accuracy.

Finally, as shown in Figure 20, the MPC controller showed a high-frequency yaw rate within the turning section; however, when using the proposed controller, the change in the yaw rate was smooth and a better lateral error result was obtained simultaneously. Thus, both lateral stability and path tracking performance are guaranteed simultaneously. Table 5 presents the performance analysis results obtained through the two scenarios described herein.

#### IV. CONCLUSION

In this paper, we proposed a game-based coupling controller to improve speed tracking and path tracking performance. In the proposed controller, each player in charge of the longitudinal and lateral dynamics plans a control strategy to reach the Nash equilibrium in game theory for control by considering the fundamental interaction of dynamics. Subsequently, we iteratively modified the initially planned strategy to realize control suitable for the actual vehicle (a nonlinear system). We used this final revised plan to track the target velocity and path through a receding horizon strategy.

The effectiveness of the proposed controller was verified by CarMaker-Matlab/Simulink co-simulations. The game based coupling control algorithm was implemented in MATLAB/Simulink, and the actual plant used in the simulations was a CarMaker vehicle model. The conclusions drawn from the simulation results are as follows.

- The game-based coupling controller exhibits high accuracy in following the target path and is stable in terms of the lateral stability performance.

- When following a rapidly changing target speed, the accuracy of speed tracking is good. In addition, control is performed in consideration of the acceleration change; hence, it achieves excellent control performance in terms of ride comfort.

To summarize, we expect the proposed controller to enable safe trajectory tracking on highway entrances/exits or sharp curves based on navigation information and simultaneously provide considerable stability for the driver.

## REFERENCES

- [1] R. C. Coulter, "Implementation of the pure pursuit path tracking algorithm," Carnegie-Mellon UNIV Pittsburgh PA Robotics INST, Tech. Rep., 1992.
- [2] S. Thrun, M. Montemerlo, H. Dahlkamp, D. Stavens, A. Aron, J. Diebel, P. Fong, J. Gale, M. Halpenny, G. Hoffmann et al., "Stanley: The robot that won the darpa grand challenge," *Journal of field Robotics*, vol. 23, no. 9, pp. 661–692, 2006.
- [3] J. M. Snider et al., "Automatic steering methods for autonomous automobile path tracking," *Robotics Institute, Pittsburgh, PA, Tech. Rep. CMU-RITR-09-08*, 2009.
- [4] F. Borrelli, A. Bemporad, and M. Morari, *Predictive control for linear and hybrid systems*. Cambridge University Press, 2017.
- [5] R. Rajamani, *Vehicle dynamics and control*. Springer Science & Business Media, 2011.
- [6] M. Cibooglu, U. Karapinar, and M. T. Söylemez, "Hybrid controller approach for an autonomous ground vehicle path tracking problem," in *2017 25th Mediterranean Conference on Control and Automation (MED)*. IEEE, 2017, pp. 583–588.
- [7] A. Carvalho, Y. Gao, A. Gray, H. E. Tseng, and F. Borrelli, "Predictive control of an autonomous ground vehicle using an iterative linearization approach," in *16th International IEEE Conference on Intelligent Transportation Systems (ITSC 2013)*, 2013, pp. 2335–2340.
- [8] C. E. Beal and J. C. Gerdes, "Model predictive control for vehicle stabilization at the limits of handling," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 4, pp. 1258–1269, 2013.
- [9] C. Olsson, "Model complexity and coupling of longitudinal and lateral control in autonomous vehicles using model predictive control," 2015.
- [10] T. Başar and G. J. Olsder, *Dynamic noncooperative game theory*. SIAM, 1998.
- [11] X. Na and D. J. Cole, "Linear quadratic game and non-cooperative predictive methods for potential application to modelling driver-afs interactive steering control," *Vehicle system dynamics*, vol. 51, no. 2, pp. 165–198, 2013.
- [12] R. N. Banavar and J. L. Speyer, "A linear-quadratic game approach to estimation and smoothing," in *1991 American control conference*. IEEE, 1991, pp. 2818–2822.
- [13] T. Basar, "On the uniqueness of the nash solution in linear-quadratic differential games," *International Journal of Game Theory*, vol. 5, no. 2, pp. 65–90, 1976.
- [14] J. Cruz and C. Chen, "Series nash solution of two-person, nonzero-sum, linear-quadratic differential games," *Journal of Optimization Theory and Applications*, vol. 7, no. 4, pp. 240–257, 1971.
- [15] D. Fridovich-Keil, E. Ratner, L. Peters, A. D. Dragan, and C. J. Tomlin, "Efficient iterative linear-quadratic approximations for nonlinear multi-player general-sum differential games," in *2020 IEEE International Conference on Robotics and Automation (ICRA)*, 2020, pp. 1475–1481.
- [16] B. Ma, Y. Liu, X. Na, Y. Liu, and Y. Yang, "A shared steering controller design based on steer-by-wire system considering human-machine goal consistency," *Journal of the Franklin Institute*, vol. 356, no. 8, pp. 4397–4419, 2019.
- [17] W. Li and E. Todorov, "Iterative linear quadratic regulator design for nonlinear biological movement systems," in *ICINCO (1)*. Citeseer, 2004, pp. 222–229.
- [18] H. Wang, J. Kearney, and K. Atkinson, "Arc-length parameterized spline curves for real-time simulation," in *Proc. 5th International Conference on Curves and Surfaces*, vol. 387396, 2002.
- [19] K. Chu, M. Lee, and M. Sunwoo, "Local path planning for off-road autonomous driving with avoidance of static obstacles," *IEEE Transactions on Intelligent Transportation Systems*, vol. 13, no. 4, pp. 1599–1616, 2012.
- [20] B. Gravell, K. Ganapathy, and T. Summers, "Policy iteration for linear quadratic games with stochastic parameters," *IEEE Control Systems Letters*, vol. 5, no. 1, pp. 307–312, 2020.



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