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On a pair of fuzzy mappings in modular-like metric spaces with applications

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Abstract

The aim of this work is to establish results in fixed point theory for a pair of fuzzy dominated mappings which forms a rational fuzzy dominated V-contraction in modular-like metric spaces. Some results via a partial order and using the graph concept are also developed. We apply our results to ensure the existence of a solution of nonlinear Volterra-type integral equations.

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1 Introduction and preliminaries

Fixed point theory has a basic role in analysis (see [1–51]). Chistyakov [12] developed the idea of modular metric spaces and discussed briefly modular convergence, convex modular, equivalent metrics, abstract convex cones, and metric semigroups. The modular metric spaces generalize classical modulars over linear spaces, like Orlicz, Lebesgue, Musielak–Orlicz, Lorentz, Calderon–Lozanovskii, Orlicz–Lorentz spaces, etc. The main idea behind this new concept is the physical interpretation of the modular. We look at these spaces as the nonlinear version of the classical modular spaces. Padcharoen et al. [29] introduced the concept of α-type F-contractions in modular metric spaces and discussed some results. Further results in such spaces via different directions can be seen in [11, 22, 24, 25].

Nadler [27] presented fixed point theorems for multivalued mappings and generalized the results for single-valued mappings. Fixed point results involving multivalued mappings have applications in engineering, control theory, differential equations, games and economics, see [7, 9]. In this paper, we are concerned with multivalued mappings.

Wardowski [51] introduced the notion of F-contractions to obtain a very practical fixed point result. For more results on this direction, see [2, 4, 23, 26, 43, 47]. Here, we have used a weak family of functions instead of the function F introduced by Wardowski.

Arshad et al. [5] observed that there exist mappings having fixed points, but there were no results to ensure the existence of fixed points of such mappings. They introduced a condition on closed balls to achieve common fixed points for such mappings. For further results on closed balls, see [38, 39, 50]. In this paper, we are using a sequence instead of a closed ball.

Ran and Reurings [37] and Nieto et al. [28] gave results involving fixed point theory in partially ordered sets. For more results in ordered spaces, see [13–15]. Asl et al. [6] gave the idea of α_*-admissible mappings and $\alpha - \psi$ contractive multifunctions (see also [3, 17, 45]) and generalized the restriction of order. Rasham et al. [40] introduced the concept of α_*-dominated mappings to establish a new condition of order and obtained some related fixed point results (see also [41, 42, 49, 50]). They proved that there are mappings that are α_*-dominated, but are not α_*-admissible.

The notion of fuzzy sets was introduced by Zadeh [53] and then a lot of researchers worked in this area. Namely, Weiss [52] and Butnariu [10] firstly discussed the concept of fuzzy mappings and showed many related results. Heilpern [16] gave a result for fuzzy mappings, considered as a generalization of Nadler set-valued result [27]. Due to importance of the Heilpern result, the fixed point theory for fuzzy contractions via a Hausdorff metric becomes much more important, see [32–36, 38, 48].

In this paper, we establish common fixed point theorems for a pair of fuzzy α_*-dominated mappings which form a generalized V-contraction in a generalized setting of modular-like metric spaces. New results can be established in dislocated metric spaces, ordered spaces, partial metric spaces, fuzzy metric spaces and metric spaces as a consequence of our findings. To support our results, applications and examples are discussed. Our theorems generalize the results given in [42, 43, 47, 49, 51]. We give the following preliminary concepts, which will be used in our results.

Definition 1.1 ([44]) Let A be a nonempty set. A function $u : (0,\infty) \times A \times A \to [0,\infty)$ is called a modular-like metric on A, if for all $a, b, c \in A$, $l > 0$ and $u_l(a,b) = u(l,a,b)$, it satisfies:

(i) $u_l(a,b) = u_l(b,a)$ for all $l > 0$;

(ii) $u_l(a,b) = 0$ for all $l > 0$ then $a = b$;

(iii) $u_{l+n}(a,b) \leq u_l(a,c) + u_n(c,b)$ for all $l, n > 0$.

Then (A,u) is called a modular-like metric space. If we replace (ii) by "$u_l(a,b) = 0$ for all $l > 0$ if and only if $a = b$," then (A,u) becomes a modular metric space. If we replace (ii) by "$u_l(a,b) = 0$ for some $l > 0$, then $a = b$," then (A,u) becomes a regular modular-like metric on A. For $e \in A$ and $\varepsilon > 0$, $\overline{B_{u_l}(e,\varepsilon)} = \{p \in A : |u_l(e,p) - u_l(p,p)| \leq \varepsilon\}$ is a closed ball in (A,u). We will use "*m.l.m.* space" instead of "modular like metric space."

Definition 1.2 ([44]) Let (A,u) be an *m.l.m.* space.

(i) The sequence $(a_n)_{n\in\mathbb{N}}$ in A is u-Cauchy for some $l > 0$, iff $\lim_{n,m\to\infty} u_l(a_m, a_n)$ exists and is finite;

(ii) The sequence $(a_n)_{n\in\mathbb{N}}$ in A is called u-convergent to $a \in A$ for some $l > 0$, if and only if $\lim_{n\to+\infty} u_l(a_n, a) = u_l(a,a)$.

(iii) $E \subseteq A$ is called u-complete if for any u-Cauchy sequence $\{a_n\}$ in E is u-convergent to some $a \in E$, so that for some $l > 0$,

$$\lim_{n\to+\infty} u_l(a_n, a) = u_l(a,a) = \lim_{n,m\to+\infty} u_l(a_n, a_m).$$

Definition 1.3 Let (A,u) be an *m.l.m.* space and $E \subseteq A$. An element p_0 belonging to E is said to be a best approximation in E for $e \in A$, if

$$u_l(e,E) = \inf_{p\in E} u_l(e,p) = u_l(e,p_0).$$

If each $e \in A$ has a best approximation in E, then E is known as a proximinal set.

Denote by $P(A)$ the set of compact proximinal subsets in A.

As an example, consider $A = \mathbb{R}^+ \cup \{0\}$ and $u_l(e,p) = \frac{1}{l}(e+p)$ for all $l > 0$. Define a set $E = [4,6]$. Then for each $y \in A$,

$$u_l(y,E) = u_l\bigl(y,[4,6]\bigr) = \inf_{n\in[4,6]} u_l(y,n) = u_l(y,4).$$

Hence, 4 is a best approximation in E for each $y \in A$. Also, $[4,6]$ is a proximinal set.

Definition 1.4 Let (A,u) be an *m.l.m.* space. Consider the Pompieu–Hausdorff map $H_{u_l} : P(A) \times P(A) \to [0,\infty)$ defined by

$$H_{u_l}(N,M) = \max\Bigl\{\sup_{n\in N} u_l(n,M), \sup_{m\in M} u_l(N,m)\Bigr\},$$

for $M,N \in P(A)$.

Again, take $A = \mathbb{R}^+ \cup \{0\}$ endowed with $u_l(e,p) = \frac{1}{l}(e+p)$ for all $l > 0$. If $N = [3,5]$ and $R = [7,8]$, then $H_{u_l}(N,R) = \frac{13}{l}$.

Definition 1.5 ([44]) Let (A,u) be an *m.l.m.* space. Then we will say that u satisfies the $\triangle_M$-condition if $\mathbb{N}\lim_{n,m\to\infty} u_p(e_n,e_m) = 0$ implies $\lim_{n,m\to\infty} u_l(e_n,e_m) = 0$, for some $l > 0$.

Definition 1.6 Let A be a nonempty set, $\xi : A \to P(A)$ be a set-valued mapping, $B \subseteq A$, and $\alpha : A \times A \to [0,+\infty)$. Then ξ is called α_*-admissible on B if $\alpha_*(\xi a,\xi c) = \inf\{\alpha(u,v) : u \in \xi a, v \in \xi c\} \geq 1$, whenever $\alpha(a,c) \geq 1$ for all $a,c \in B$.

Definition 1.7 ([40]) Let A be a nonempty set, $\xi : A \to P(A)$ be a set-valued mapping, $M \subseteq A$, and $\alpha : A \times A \to [0,+\infty)$. Then ξ is called α_*-dominated on M if for all $a \in M$, $\alpha_*(a,\xi a) = \inf\{\alpha(a,l) : l \in \xi a\} \geq 1$.

Example 1.8 ([40]) Let $B = (-\infty,\infty)$. Define $\gamma : B \times B \to [0,\infty)$ by

$$\gamma(e,r) = \begin{cases} 1 & \text{if } e > r, \\ \frac{1}{4} & \text{if } e \not> r. \end{cases}$$

Define $K, L : B \to P(B)$ by

$$Ku = [-4+u,-3+u] \quad \text{and} \quad Lr = [-2+r,-1+r].$$

Then K and L are not γ_*-admissible, but they are γ_*-dominated.

Definition 1.9 ([51]) Consider a metric space (M, d). A mapping $G : M \to M$ is called an R-contraction if for all $c, k \in M$, there exists $\tau > 0$ such that $d(Ga, Gc) > 0$ implies

$$\tau + R\big(d(Ga, Gc)\big) \leq R\big(d(a, c)\big),$$

where $R : \mathbb{R}_+ \to \mathbb{R}$ is a function satisfying:

(F1) There exists $k \in (0, 1)$ such that $\lim_{\sigma\to 0^+} \sigma^k R(\sigma) = 0$;

(F2) For all $a, c \in \mathbb{R}_+$ such that $a < c$, we have $R(a) < R(c)$, that is, R is strictly increasing;

(F3) $\lim_{n\to+\infty} \sigma_n = 0$ if $\lim_{n\to+\infty} R(\sigma_n) = -\infty$, for each sequence $\{\sigma_n\}_{n=1}^{\infty}$ of positive numbers.

The family of all functions satisfying conditions (F1)–(F3) is denoted by $\digamma$.

A classical result is as follows:

Lemma 1.10 *Let (Q, u) be an m.l.m. space. Let $C, D \in P(Q)$. Then for each $e \in C$, there exists $y_e \in D$ such that $H_{u_l}(C, D) \geq u_l(e, y_e)$.*

Definition 1.11 ([47]) A fuzzy set U is a function from G to $[0, 1]$, $F(G)$ is the family of all fuzzy sets in G. If U is a fuzzy set and $e \in G$, then $U(e)$ is called the grade of membership of e in U. For $\beta \in [0, 1]$, the β-level set of a fuzzy set U is denoted by $[U]_\beta$, and is defined by

$$[U]_\beta = \big\{e : U(e) \geq \beta\big\} \quad \text{where } 0 < \beta \leq 1,$$
$$[U]_0 = \overline{\big\{e : U(e) > 0\big\}}.$$

Now, we select a subset of the family $F(G)$ of all fuzzy sets, a subfamily with stronger properties, i.e., the subfamily of the approximate quantities, denoted by $W(G)$.

Definition 1.12 ([16]) A fuzzy subset U of G is an approximate quantity if and only if its β-level set is a compact convex subset of G for each $\beta \in [0, 1]$ and $\sup_{e\in G} U(e) = 1$.

Definition 1.13 ([16]) Let R be an arbitrary set and G be a metric space. A fuzzy mapping $T : R \to W(G)$ is considered as a fuzzy subset of $R \times G$, $T : R \times G \to [0, 1]$ in the sense that $T(c, y) = T(c)(y)$.

Definition 1.14 ([47]) A point $c \in M$ is called a fuzzy fixed point of a fuzzy mapping $T : M \to W(M)$ if there exists $0 < \beta \leq 1$ such that $c \in [Tc]_\beta$.

Definition 1.15 Let A be a nonempty set, $\xi : A \to W(A)$ be a fuzzy mapping, $M \subseteq A$, and $\alpha : A \times A \to [0, \infty)$. Then ξ is called fuzzy α_*-dominated on M, if for all $a \in M$ and $0 < \beta \leq 1$, we have $\alpha_*(a, [\xi a]_\beta) = \inf\{\alpha(a, l) : l \in [\xi a]_\beta\} \geq 1$.

2 Main results

Let (Δ, u) be an *m.l.m.* space, $\vartheta_0 \in \Delta$, and $S, T : \Delta \to W(\Delta)$ be fuzzy mappings on Δ. Moreover, let $\gamma, \beta : \Delta \to [0, 1]$ be two real functions. Let $\vartheta_1 \in [S\vartheta_0]_{\gamma(\vartheta_0)}$ be an element such that $u_1(\vartheta_0, [S\vartheta_0]_{\gamma(\vartheta_0)}) = u_1(\vartheta_0, \vartheta_1)$. Let $\vartheta_2 \in [T\vartheta_1]_{\beta(\vartheta_1)}$ be such that $u_1(\vartheta_1, [T\vartheta_1]_{\beta(\vartheta_1)}) = u_1(\vartheta_1, \vartheta_2)$. Let $\vartheta_3 \in [S\vartheta_2]_{\gamma(\vartheta_2)}$ be such that $u_1(\vartheta_2, [S\vartheta_2]_{\gamma(\vartheta_2)}) = u_1(\vartheta_2, \vartheta_3)$. Continuing this process, we construct a sequence ϑ_n in Δ such that $\vartheta_{2n+1} \in [S\vartheta_{2n}]_{\gamma(\vartheta_{2n})}$ and $\vartheta_{2n+2} \in [T\vartheta_{2n+1}]_{\beta(\vartheta_{2n+1})}$, where $n = 0, 1, 2, \ldots$ Also,

$$u_1\big(\vartheta_{2n}, [S\vartheta_2]_{\gamma(\vartheta_2)}\big) = u_1(\vartheta_{2n}, \vartheta_{2n+1})$$

and

$$u_1\big(\vartheta_{2n+1}, [T\vartheta_{2n+1}]_{\beta(\vartheta_{2n+1})}\big) = u_1(\vartheta_{2n+1}, \vartheta_{2n+2}).$$

Note that $\{TS(\vartheta_n)\}$ is the notation of this sequence. Then $\{TS(\vartheta_n)\}$ is said to be a sequence in Δ generated by ϑ_0.

Definition 2.1 Let (Δ, u) be a complete *m.l.m.* space. Assume that u is regular and satisfies the $\triangle_M$-condition. Let $\vartheta_0 \in \Delta$, $\alpha : \Delta \times \Delta \to [0, \infty)$, and $S, T : \Delta \to W(\Delta)$ be two fuzzy α_*-dominated mappings on $\{TS(\vartheta_n)\}$. The pair (S, T) is called a rational fuzzy dominated V-contraction, if there exist $\tau > 0$, $\gamma(\vartheta), \beta(g) \in (0, 1]$ and $V \in \digamma$ such that

$$\begin{aligned} &\tau + V\big(H_{u_1}\big([S\vartheta]_{\gamma(\vartheta)}, [Tg]_{\beta(g)}\big)\big) \\ &\quad \le V\left(\max\left\{\begin{matrix} u_1(\vartheta, g), u_1(\vartheta, [S\vartheta]_{\gamma(\vartheta)}), \frac{u_2(\vartheta, [Tg]_{\beta(g)})}{2}, \\ \frac{u_1(\vartheta, [S\vartheta]_{\gamma(\vartheta)}).u_1(g, [Tg]_{\beta(g)})}{1+u_1(\vartheta, g)} \end{matrix}\right\}\right) \end{aligned} \tag{2.1}$$

whenever $\vartheta, g \in \{TS(\vartheta_n)\}$ so that $\alpha(\vartheta, g) \ge 1$, and $H_{u_1}([S\vartheta]_{\gamma(\vartheta)}, [Tg]_{\beta(g)}) > 0$.

Theorem 2.2 *Let (Δ, u) be a complete m.l.m. space. Assume that $S, T : \Delta \to W(\Delta)$ are two fuzzy α_*-dominated mappings on $\{TS(\vartheta_n)\}$. If (S, T) is a rational fuzzy dominated V-contraction, then $\{TS(\vartheta_n)\}$ is a Cauchy sequence in Δ and $\{TS(\vartheta_n)\} \to k \in \Delta$.*

Proof As $S, T : \Delta \to W(\Delta)$ are two fuzzy α_*-dominated mappings on $\{TS(\vartheta_n)\}$, so we have $\alpha_*(\vartheta_{2i}, [S\vartheta_{2i}]_{\gamma(\vartheta_{2i})}) \ge 1$ and $\alpha_*(\vartheta_{2i+1}, [T\vartheta_{2i+1}]_{\beta(\vartheta_{2i+1})}) \ge 1$ for all $i \in \mathbb{N}$. As $\alpha_*(\vartheta_{2i}, [S\vartheta_{2i}]_{\gamma(\vartheta_{2i})}) \ge 1$, this implies that $\inf\{\alpha(\vartheta_{2i}, b) : b \in [S\vartheta_{2i}]_{\gamma(\vartheta_{2i})}\} \ge 1$, and therefore $\alpha(\vartheta_{2i}, \vartheta_{2i+1}) \ge 1$. Now, by using Lemma 1.10 and Definition 2.1, one writes

$$\begin{aligned} &\tau + V\big(u_1(\vartheta_{2i+1}, \vartheta_{2i+2})\big) \\ &\quad \le \tau + V\big(H_{u_1}\big([S\vartheta_{2i}]_{\gamma(\vartheta_{2i})}, [T\vartheta_{2i+1}]_{\beta(\vartheta_{2i+1})}\big)\big) \\ &\quad \le V\left(\max\left\{\begin{matrix} u_1(\vartheta_{2i}, \vartheta_{2i+1}), u_1(\vartheta_{2i}, [S\vartheta_{2i}]_{\gamma(\vartheta_{2i})}), \frac{u_2(\vartheta_{2i}, [T\vartheta_{2i+1}]_{\beta(\vartheta_{2i+1})})}{2}, \\ \frac{u_1(\vartheta_{2i}, [S\vartheta_{2i}]_{\gamma(\vartheta_{2i})}).u_1(\vartheta_{2i+1}, [T\vartheta_{2i+1}]_{\beta(\vartheta_{2i+1})})}{1+u_1(\vartheta_{2i}, \vartheta_{2i+1})} \end{matrix}\right\}\right) \\ &\quad \le V\left(\max\left\{\begin{matrix} u_1(\vartheta_{2i}, \vartheta_{2i+1}), u_1(\vartheta_{2i}, \vartheta_{2i+1}), \frac{u_1(\vartheta_{2i}, \vartheta_{2i+1})+u_1(\vartheta_{2i+1}, \vartheta_{2i+2})}{2}, \\ \frac{u_1(\vartheta_{2i}, \vartheta_{2i+1}).u_1(\vartheta_{2i+1}, \vartheta_{2i+2})}{1+u_1(\vartheta_{2i}, \vartheta_{2i+1})} \end{matrix}\right\}\right) \\ &\quad \le V\big(\max\big\{u_1(\vartheta_{2i}, \vartheta_{2i+1}), u_1(\vartheta_{2i+1}, \vartheta_{2i+2})\big\}\big). \end{aligned}$$

This implies that

$$\tau + V\big(u_1(\vartheta_{2i+1}, \vartheta_{2i+2})\big) \le V\big(\max\big\{u_1(\vartheta_{2i}, \vartheta_{2i+1}), u_1(\vartheta_{2i+1}, \vartheta_{2i+2})\big\}\big). \tag{2.2}$$

If $\max\{u_1(\vartheta_{2i},\vartheta_{2i+1}),u_1(\vartheta_{2i+1},\vartheta_{2i+2})\}) = u_1(\vartheta_{2i+1},\vartheta_{2i+2})$, then from (2.2), we have

$$V\bigl(u_1(\vartheta_{2i+1},\vartheta_{2i+2})\bigr) \le V\bigl(u_1(\vartheta_{2i+1},\vartheta_{2i+2})\bigr) - \tau,$$

a contradiction. Therefore, $\max\{u_1(\vartheta_{2i},\vartheta_{2i+1}),u_1(\vartheta_{2i+1},\vartheta_{2i+2})\}) = u_1(\vartheta_{2i},\vartheta_{2i+1})$, for all $i \in \{0,1,2,\dots\}$. Again, from (2.2), we have

$$V\bigl(u_1(\vartheta_{2i+1},\vartheta_{2i+2})\bigr) \le V\bigl(u_1(\vartheta_{2i},\vartheta_{2i+1})\bigr) - \tau. \tag{2.3}$$

Similarly, we have

$$V\bigl(u_1(\vartheta_{2i},\vartheta_{2i+1})\bigr) \le V\bigl(u_1(\vartheta_{2i-1},\vartheta_{2i})\bigr) - \tau, \tag{2.4}$$

for all $i \in \{0,1,2,\dots\}$. By (2.4) and (2.3), we have

$$V\bigl(u_1(\vartheta_{2i+1},\vartheta_{2i+2})\bigr) \le V\bigl(u_1(\vartheta_{2i-1},\vartheta_{2i})\bigr) - 2\tau.$$

Repeating these steps, we get

$$V\bigl(u_1(\vartheta_{2i+1},\vartheta_{2i+2})\bigr) \le V\bigl(u_1(\vartheta_0,\vartheta_1)\bigr) - (2i+1)\tau. \tag{2.5}$$

Similarly, we have

$$V\bigl(u_1(\vartheta_{2i},\vartheta_{2i+1})\bigr) \le V\bigl(u_1(\vartheta_0,\vartheta_1)\bigr) - 2i\tau. \tag{2.6}$$

Inequalities (2.5) and (2.6) can jointly be written as

$$V\bigl(u_1(\vartheta_n,\vartheta_{n+1})\bigr) \le V\bigl(u_1(\vartheta_0,\vartheta_1)\bigr) - n\tau. \tag{2.7}$$

Taking the limit as $n \to \infty$ in (2.7), we have

$$\lim_{n\to\infty} V\bigl(u_1(\vartheta_n,\vartheta_{n+1})\bigr) = -\infty.$$

Since $F \in \digamma$, one gets

$$\lim_{n\to\infty} u_1(\vartheta_n,\vartheta_{n+1}) = 0. \tag{2.8}$$

Applying the property $(F1)$ of $\digamma$, we have for some $k \in (0,1)$,

$$\lim_{n\to\infty} \bigl(u_1(\vartheta_n,\vartheta_{n+1})\bigr)^k \bigl(V\bigl(u_1(\vartheta_n,\vartheta_{n+1})\bigr) = 0. \tag{2.9}$$

By (2.7), we obtain for all $n \in \mathbb{N}$,

$$\bigl(u_1(\vartheta_n,\vartheta_{n+1})\bigr)^k \bigl(\bigl(V\bigl(u_1(\vartheta_n,\vartheta_{n+1})\bigr) - V\bigl(u_1(\vartheta_0,\vartheta_1)\bigr)\bigr) \le -\bigl(u_1(\vartheta_n,\vartheta_{n+1})\bigr)^k n\tau \le 0. \tag{2.10}$$

Considering (2.8), (2.9) and letting $n \to \infty$ in (2.10), we have

$$\lim_{n\to\infty} \bigl(n\bigl(u_1(\vartheta_n,\vartheta_{n+1})\bigr)^k\bigr) = 0. \tag{2.11}$$

Since (2.11) holds, there exists $n_1 \in \mathbb{N}$ such that $n(u_1(\vartheta_n, \vartheta_{n+1}))^k \leq 1$ for all $n \geq n_1$, or

$$u_1(\vartheta_n, \vartheta_{n+1}) \leq \frac{1}{n^{\frac{1}{k}}} \quad \text{for all } n \geq n_1. \tag{2.12}$$

Take $p > 0$ and $m = n + p$ with $n > n_1$. Then

$$\begin{aligned} u_p(\vartheta_n, \vartheta_m) &\leq u_1(\vartheta_n, \vartheta_{n+1}) + u_1(\vartheta_{n+1}, \vartheta_{n+2}) + \cdots + u_1(\vartheta_{m-1}, \vartheta_m) \\ &\leq \frac{1}{n^{\frac{1}{k}}} + \frac{1}{(n+1)^{\frac{1}{k}}} + \cdots + \frac{1}{(m-1)^{\frac{1}{k}}} \\ &\leq \sum_{i=n}^{\infty} \frac{1}{i^{\frac{1}{k}}}. \end{aligned}$$

If $k \in (0, 1)$, then $\frac{1}{k} > 1$, so the last term is the remainder of a convergent series. Hence, taking the limit as $n, m \to \infty$, we have

$$\lim_{n,m\to\infty} u_p(\vartheta_n, \vartheta_m) = 0. \tag{2.13}$$

Since u satisfies the $\triangle_M$-condition, we have

$$\lim_{n,m\to\infty} u_1(\vartheta_n, \vartheta_m) = 0. \tag{2.14}$$

Hence, $\{TS(\vartheta_n)\}$ is a Cauchy sequence in Δ. Since (Δ, u) is a regular complete modular-like metric space, there exists $k \in \Delta$ such that $\{TS(\vartheta_n)\} \to k$ as $n \to \infty$. □

Theorem 2.3 *Let (Δ, u) be a complete m.l.m. space. Assume that $S, T : \Delta \to W(\Delta)$ are two fuzzy α_*-dominated mappings on $\{TS(\vartheta_n)\}$. Suppose that (S, T) is a rational fuzzy dominated V-contraction and k satisfies (2.1), where k is the limit of the sequence $\{TS(\vartheta_n)\}$. Also, $\alpha(\vartheta_n, k) \geq 1$ and $\alpha(k, \vartheta_n) \geq 1$ for all $n \in \{0, 1, 2, \dots\}$. Then k belongs to both $[Tk]_{\beta(k)}$ and $[Sk]_{\gamma(k)}$.*

Proof As (S, T) is a rational fuzzy dominated V-contraction, then by Theorem 2.2, there exists $k \in \Delta$ such that $\{TS(\vartheta_n)\} \to k$ as $n \to \infty$ and so

$$\lim_{n\to\infty} u_1(\vartheta_n, k) = u_1(k, k) = \lim_{n,m\to\infty} u_1(\vartheta_n, \vartheta_m) = 0. \tag{2.15}$$

Now, by Lemma 1.10, we have

$$\tau + V(u_1(\vartheta_{2n+1}, [Tk]_{\beta(k)}) \leq \tau + V(H_{u_1}([S\vartheta_{2n}]_{(\gamma_{\vartheta_{2n}})}, [Tk]_{\beta(k)}). \tag{2.16}$$

By assumption, $\alpha(\vartheta_n, k) \geq 1$. Assume that $u_1(k, [Tk]_{\beta(k)}) > 0$, then there must be a positive natural number p so that $u_1(\vartheta_{2n+1}, [Tk]_{\beta(k)}) > 0$, for every $n \geq p$. Now $H_{u_1}([S\vartheta_{2n}]_{(\gamma_{\vartheta_{2n}})}, [Tk]_{\beta(k)}) > 0$, so inequality (2.1) implies for every $n \geq p$ that

$$\tau + V(u_1(\vartheta_{2n+1}, [Tk]_{\beta(k)})$$

$$\leq V\left(\max\left\{\begin{array}{c} u_1(\vartheta_{2n},k), u_1(\vartheta_{2n},[S\vartheta_{2n}]_{\gamma(\vartheta_{2n})}), \frac{u_1(\vartheta_{2n},\vartheta_{2n+1})+u_1(\vartheta_{2n+1},[Tk]_{\beta(k)})}{2}, \\ \frac{u_1(\vartheta_{2n},[S\vartheta_{2n}]_{\gamma(\vartheta_{2n})}).u_1(k,[Tk]_{\beta(k)})}{1+u_1(\vartheta_{2n},k)} \end{array}\right\}\right).$$

Letting $n \to \infty$ and using (2.15), we get

$$\tau + V\big(u_1\big(k,[Tk]_{\beta(k)}\big)\big) \leq V\left(\frac{u_1(k,[Tk]_{\beta(k)})}{2}\right) \leq V\big(u_1\big(k,[Tk]_{\beta(k)}\big)\big).$$

Since V is strictly increasing, (2.16) implies

$$u_1\big(k,[Tk]_{\beta(k)}\big) < u_1\big(k,[Tk]_{\beta(k)}\big).$$

This is not true. So our assumption is wrong. Hence, $u_1(k,[Tk]_{\beta(k)}) = 0$ or $k \in [Tk]_{\beta(k)}$. Similarly, by applying Lemma 1.10 and inequality (2.1), we can prove that $u_1(k,[Sk]_{\gamma(k)}) = 0$ or $k \in [Sk]_{\gamma(k)}$. Hence, S and T have a common fuzzy fixed point k in Δ. □

Definition 2.4 Let Δ be a nonempty set, $\preceq$ be a partial order on Δ, and $B \subseteq \Delta$. We say that $a \preceq B$, whenever for all $b \in B$, we have $a \preceq b$. A mapping $S : \Delta \to W(\Delta)$ is said to be fuzzy $\preceq$-dominated on B if $a \preceq [Sa]_\gamma$ for each $a \in \Delta$ and $\gamma \in (0,1]$.

We have the following result for multi-fuzzy $\preceq$-dominated mappings on $\{TS(\vartheta_n)\}$ in an ordered complete *m.l.m.* space.

Theorem 2.5 *Let $(\Delta,\preceq,u)$ be an ordered complete m.l.m. space. Assume that u is regular and satisfies the $\triangle_M$-condition. Let $\vartheta_0 \in \Delta$ and $S,T : \Delta \to W(\Delta)$ be fuzzy dominated mappings on $\{TS(\vartheta_n)\}$. Suppose there exist $\tau > 0$, $\gamma(\vartheta),\beta(g) \in (0,1]$ and $V \in F$ such that the following holds*:

$$\begin{aligned}&\tau + V\big(H_{u_1}\big([S\vartheta]_{\gamma(\vartheta)},[Tg]_{\beta(g)}\big)\big) \\ &\quad \leq V\left(\max\begin{array}{c} u_1(\vartheta,g), u_1(\vartheta,[S\vartheta]_{\gamma(\vartheta)}), \frac{u_2(\vartheta,[Tg]_{\beta(g)})}{2}, \\ \frac{u_1(\vartheta,[S\vartheta]_{\gamma(\vartheta)}).u_1(g,[Tg]_{\beta(g)})}{1+u_1(\vartheta,g)} \end{array}\right)\end{aligned} \tag{2.17}$$

whenever $\vartheta,g \in \{TS(\vartheta_n)\}$, with either $\vartheta \preceq g$ or $g \preceq \vartheta$, and $H_{u_1}([S\vartheta]_{\gamma(\vartheta)},[Tg]_{\beta(g)}) > 0$.

Then $\{TS(\vartheta_n)\} \to k \in \Delta$. Also, if (2.17) holds for k, $\vartheta_n \preceq k$ and $k \preceq \vartheta_n$ for all $n \in \{0,1,2,\dots\}$, then k belongs to both $[Tk]_{\beta(k)}$ and $[Sk]_{\gamma(k)}$.

Proof Let $\alpha : \Delta \times \Delta \to [0,+\infty)$ be a mapping defined by $\alpha(\vartheta,g) = 1$ for all $\vartheta \in \Delta$ with $\vartheta \preceq g$, and $\alpha(\vartheta,g) = 0$ for all other elements $\vartheta,g \in \Delta$. Since S and T are the fuzzy prevalent mappings on Δ, $\vartheta \preceq [S\vartheta]_{\gamma(\vartheta)}$ and $\vartheta \preceq [T\vartheta]_{\beta(\vartheta)}$ for all $\vartheta \in \Delta$. It yields that $\vartheta \preceq b$ for all $b \in [S\vartheta]_{\gamma(\vartheta)}$ and $\vartheta \preceq e$ for all $\vartheta \in [T\vartheta]_{\beta(\vartheta)}$. So, $\alpha(\vartheta,b) = 1$ for all $b \in [S\vartheta]_{\gamma(\vartheta)}$ and $\alpha(\vartheta,e) = 1$ for all $\vartheta \in [T\vartheta]_{\beta(\vartheta)}$. This implies that $\inf\{\alpha(\vartheta,g) : g \in [S\vartheta]_{\gamma(\vartheta)}\} = 1$ and $\inf\{\alpha(\vartheta,g) : g \in [T\vartheta]_{\beta(\vartheta)}\} = 1$. Hence, $\alpha_*(\vartheta,[S\vartheta]_{\alpha(\vartheta)}) = 1$, $\alpha_*(\vartheta,[T\vartheta]_{\beta(\vartheta)}) = 1$ for all $\vartheta \in \Delta$. So, $S,T : \Delta \to W(\Delta)$ are α_*-dominated mappings on Δ. Moreover, inequality (2.17) holds and it can be written as

$$\tau + V\big(H_{u_1}\big([S\vartheta]_{\gamma(\vartheta)},[Tg]_{\beta(g)}\big)\big) \leq V\big(u_l(\vartheta,g)\big),$$

for all elements ϑ,g in $\{TS(\vartheta_n)\}$, with either $\alpha(\vartheta,g) \geq 1$ or $\alpha(g,\vartheta) \geq 1$. Then, by Theorem 2.2, $\{TS(\vartheta_n)\}$ is a sequence in Δ and $\{TS(\vartheta_n)\} \to \vartheta^* \in \Delta$. Now, $\vartheta_n,\vartheta^* \in \Delta$ and either

$\vartheta_n \preceq \vartheta^*$, or $\vartheta^* \preceq \vartheta_n$ implies that either $\alpha(\vartheta_n, \vartheta^*) \geq 1$ or $\alpha(\vartheta^*, \vartheta_n) \geq 1$. So, all the requirements of Theorem 2.3 are satisfied. Hence, ϑ^* is the common fuzzy fixed point of both S and T in Δ and $u_l(\vartheta^*, \vartheta^*) = 0$. □

Example 2.6 Let $\Delta = Q^+ \cup \{0\}$ and $u_l(e,\vartheta) = \frac{1}{l}(e+\vartheta)$. Now, $u_2(e,\vartheta) = \frac{1}{2}(e+\vartheta)$ and $u_1(e,\vartheta) = e+\vartheta$ for all $e,\vartheta \in \Delta$. Define $S,T : \Delta \to W(\Delta)$ by

$$(Se)(t) = \begin{cases} \gamma & \text{if } \frac{g}{4} \leq t < \frac{g}{2}, \\ \frac{\gamma}{2} & \text{if } \frac{g}{2} \leq t \leq \frac{3g}{4}, \\ \frac{\gamma}{4} & \text{if } \frac{3g}{4} < t \leq g, \\ 0 & \text{if } g < t < \infty \end{cases}$$

and

$$(T\vartheta)(t) = \begin{cases} \beta & \text{if } \frac{g}{3} \leq t < \frac{g}{2}, \\ \frac{\beta}{4} & \text{if } \frac{g}{2} \leq t \leq \frac{2g}{3}, \\ \frac{\beta}{6} & \text{if } \frac{2g}{3} < t \leq g, \\ 0 & \text{if } g < t < \infty. \end{cases}$$

Now, we consider

$$[Se]_{\frac{\gamma}{2}} = \left[\frac{e}{4}, \frac{3e}{4}\right] \quad \text{and} \quad [T\vartheta]_{\frac{\beta}{4}} = \left[\frac{\vartheta}{3}, \frac{2\vartheta}{3}\right].$$

Taking $e_0 = \frac{1}{2}$, we have $u_1(e_0, [Se_0]_{\frac{\gamma}{2}}) = u_1(\frac{1}{2}, [\frac{1}{8}, \frac{3}{8}]) = u_1(\frac{1}{2}, \frac{1}{8})$. So, we obtain a sequence $\{TS(e_n)\} = \{\frac{1}{2}, \frac{1}{8}, \frac{1}{24}, \frac{1}{96}, \dots\}$ in Δ generated by e_0. Let

$$\alpha(e,\vartheta) = \begin{cases} 1 & \text{if } e,\vartheta \in \Delta, \\ \frac{1}{2} & \text{otherwise.} \end{cases}$$

Now, for all $e,\vartheta \in \{TS(e_n)\}$ with either $\alpha(e,\vartheta) \geq 1$ or $\alpha(\vartheta,e) \geq 1$, we have

$$\begin{aligned} H_{u_1}\left([Se]_{\frac{\gamma}{2}}, [T\vartheta]_{\frac{\beta}{4}}\right) &= \max\left\{\sup_{a\in[Se]_{\frac{\gamma}{2}}} u_1\left(a, [T\vartheta]_{\frac{\beta}{4}}\right), \sup_{b\in[T\vartheta]_{\frac{\beta}{4}}} u_1\left([Se]_{\frac{\gamma}{2}}, b\right)\right\} \\ &= \max\left\{\sup_{a\in[\frac{e}{4},\frac{3e}{4}]} u_1\left(a, \left[\frac{\vartheta}{3}, \frac{2\vartheta}{3}\right]\right), \sup_{b\in[\frac{\vartheta}{3},\frac{2\vartheta}{3}]} u_1\left(\left[\frac{e}{4}, \frac{3e}{4}\right], b\right)\right\} \\ &= \max\left\{u_1\left(\frac{3e}{4}, \left[\frac{\vartheta}{3}, \frac{2\vartheta}{3}\right]\right), u_1\left(\left[\frac{e}{4}, \frac{3e}{4}\right], \frac{2\vartheta}{3}\right)\right\} \\ &= \max\left\{u_1\left(\frac{3e}{4}, \frac{\vartheta}{3}\right), u_1\left(\frac{e}{4}, \frac{2\vartheta}{3}\right)\right\} \\ &= \max\left\{\frac{3e}{4} + \frac{\vartheta}{3}, \frac{e}{4} + \frac{2\vartheta}{3}\right\}. \end{aligned}$$

Now,

$$\max\left\{\begin{matrix} u_1(e,\vartheta), u_1(e,[Se]_{\gamma(e)}), \frac{u_2(e,[T\vartheta]_{\beta(\vartheta)})}{2}, \\ \frac{u_1(e,[Se]_{\gamma(e)}).u_1(\vartheta,[T\vartheta]_{\beta(\vartheta)})}{1+u_1(e,\vartheta)} \end{matrix}\right\}$$

$$= \max\left\{(e+\vartheta), \left(e+\frac{e}{4}\right), \frac{1}{2}\left(e+\frac{\vartheta}{3}\right), \frac{(e+\frac{e}{4}).(\vartheta+\frac{\vartheta}{3})}{1+(e+\vartheta)}\right\}$$

$$= e+\vartheta.$$

Case i. If $\max\{(\frac{3e}{4}+\frac{\vartheta}{3}),(\frac{e}{4}+\frac{2\vartheta}{3})\} = (\frac{3e}{4}+\frac{\vartheta}{3})$ and $\tau = \ln(1.2)$, then we have

$$\frac{9e}{2} + 2\vartheta \leq 5e + 5\vartheta,$$

$$\frac{6}{5}\left(\frac{3e}{4}+\frac{\vartheta}{3}\right) \leq e+\vartheta,$$

$$\ln(1.2) + \ln\left(\frac{3e}{4}+\frac{\vartheta}{3}\right) \leq \ln(e+\vartheta).$$

This implies that

$$\tau + V\left(H_{u_1}\left([Se]_{\frac{\gamma}{2}}, [T\vartheta]_{\frac{\beta}{4}}\right)\right) \leq V\left(\max\left\{\begin{matrix} u_1(e,\vartheta), u_1(e,[Se]_{\gamma(e)}), \frac{u_2(e,[T\vartheta]_{\beta(\vartheta)})}{2}, \\ \frac{u_1(e,[Se]_{\gamma(e)}).u_1(\vartheta,[T\vartheta]_{\beta(\vartheta)})}{1+u_1(e,\vartheta)} \end{matrix}\right\}\right).$$

Case ii. If $\max\{(\frac{3e}{4}+\frac{\vartheta}{3}),(\frac{e}{4}+\frac{2\vartheta}{3})\} = (\frac{e}{4}+\frac{2\vartheta}{3})$ and $\tau = \ln(1.2)$, then we have

$$\frac{3e}{2} + 4\vartheta \leq 5e + 5\vartheta,$$

$$\frac{6}{5}\left(\frac{e}{4}+\frac{2\vartheta}{3}\right) \leq e+\vartheta,$$

$$\ln(1.2) + \ln\left(\frac{e}{4}+\frac{2\vartheta}{3}\right) \leq \ln(e+\vartheta).$$

This implies that

$$\tau + V\left(H_{u_1}\left([Se]_{\frac{\gamma}{2}}, [T\vartheta]_{\frac{\beta}{4}}\right)\right) \leq V\left(\max\left\{\begin{matrix} u_1(e,\vartheta), u_1(e,[Se]_{\gamma(e)}), \frac{u_2(e,[T\vartheta]_{\beta(\vartheta)})}{2}, \\ \frac{u_1(e,[Se]_{\gamma(e)}).u_1(\vartheta,[T\vartheta]_{\beta(\vartheta)})}{1+u_1(e,\vartheta)} \end{matrix}\right\}\right).$$

Hence, all the conditions of Theorem 2.3 are satisfied and so the existence of a common fuzzy fixed point is ensured.

If we take $S = T$ in Theorem 2.3, we obtain the following result.

Corollary 2.7 *Let (Δ, u) be a complete m.l.m. space. Assume that u is regular and satisfies the $\triangle_M$-condition. Let $\vartheta_0 \in \Delta$, $\alpha : \Delta \times \Delta \to [0,\infty)$, and $S : \Delta \to W(\Delta)$ be a fuzzy α_*-dominated mapping on $\{SS(\vartheta_n)\}$. Suppose there exist $\tau > 0$, $\gamma(\vartheta), \beta(g) \in (0,1]$, and $V \in F$ such that*

$$\tau + V\left(H_{u_1}\left([S\vartheta]_{\gamma(\vartheta)}, [Sg]_{\beta(g)}\right)\right)$$

$$\leq V\left(\max\left\{\begin{array}{c} u_1(\vartheta,g), u_1(\vartheta,[S\vartheta]_{\gamma(\vartheta)}), \frac{u_2(e,[Sg]_{\beta(g)})}{2}, \\ \frac{u_1(\vartheta,[S\vartheta]_{\gamma(\vartheta)}).u_1(g,[Sg]_{\beta(g)})}{1+u_1(\vartheta,g)} \end{array}\right\}\right) \tag{2.18}$$

whenever $\vartheta, g \in \{SS(\vartheta_n)\}$, $\alpha(\vartheta,g) \geq 1$, *and* $H_{u_1}([S\vartheta]_{\gamma(\vartheta)}, [Sg]_{\beta(g)}) > 0$.

Then, $\alpha(\vartheta_n,\vartheta_{n+1}) \geq 1$ *for all* $n \in \{0,1,2,\dots\}$ *and* $\{SS(\vartheta_n)\} \to k \in \Delta$. *Also, if* k *satisfies* (2.18) *and either* $\alpha(\vartheta_n,k) \geq 1$ *or* $\alpha(k,\vartheta_n) \geq 1$ *for all* $n \in \{0,1,2,\dots\}$, *then* $k \in [k]_{\gamma(k)}$.

If we take in Theorem 2.3, multivalued α_**-dominated mappings from a ground set* Δ *to the proximinal subsets of* Δ *instead of fuzzy* α_**-dominated mappings from* Δ *to the approximate quantities* $W(\Delta)$, *we obtain the following result.*

Corollary 2.8 *Let* (Δ,u) *be a complete m.l.m. space. Assume that* u *is regular and satisfies the* $\triangle_M$*-condition. Let* $\vartheta_0 \in \Delta$, $\alpha : \Delta \times \Delta \to [0,\infty)$ *and* $S,T : \Delta \to W(\Delta)$ *are two multivalued* α_**-dominated mappings on* $\{TS(\vartheta_n)\}$. *Suppose there exist* $\tau > 0$ *and* $V \in F$ *such that*

$$\tau + V\big(H_{u_1}(S\vartheta,Tg)\big) \leq V\left(\max\left\{\begin{array}{c} u_1(\vartheta,g), u_1(\vartheta,S\vartheta), \frac{u_2(\vartheta,Tg)}{2}, \\ \frac{u_1(\vartheta,S\vartheta).u_1(g,Tg)}{1+u_1(\vartheta,g)} \end{array}\right\}\right) \tag{2.19}$$

whenever $\vartheta, g \in \{TS(\vartheta_n)\}$, $\alpha(\vartheta,g) \geq 1$, *and* $H_{u_1}(S\vartheta,Tg) > 0$.

Then, $\alpha(\vartheta_n,\vartheta_{n+1}) \geq 1$ *for all* $n \in \{0,1,2,\dots\}$ *and* $\{TS(\vartheta_n)\} \to k \in \Delta$. *Also, if* k *satisfies* (2.19) *and either* $\alpha(\vartheta_n,k) \geq 1$ *or* $\alpha(k,\vartheta_n) \geq 1$ *for all* $n \in \{0,1,2,\dots\}$, *then* k *belongs to both* Tk *and* Sk.

If we take $S = T$ in Corollary 2.8, we obtain the following result.

Corollary 2.9 *Let* (Δ,u) *be a complete m.l.m. space. Assume that* u *is regular and satisfies the* $\triangle_M$*-condition. Let* $\vartheta_0 \in \Delta$, $\alpha : \Delta \times \Delta \to [0,\infty)$ *and* $S : \Delta \to W(\Delta)$ *be a multivalued* α_**-dominated mapping on* $\{SS(\vartheta_n)\}$. *Suppose there exist* $\tau > 0$ *and* $V \in F$ *such that*

$$\tau + V\big(H_{u_1}(S\vartheta,Sg)\big) \leq V\left(\max\left\{\begin{array}{c} u_1(\vartheta,g), u_1(\vartheta,S\vartheta), \frac{u_2(\vartheta,Sg)}{2}, \\ \frac{u_1(\vartheta,S\vartheta).u_1(g,Sg)}{1+u_1(\vartheta,g)} \end{array}\right\}\right) \tag{2.20}$$

whenever $\vartheta, g \in \{(\vartheta_n)\}$, $\alpha(\vartheta,g) \geq 1$, *and* $H_{u_1}(S\vartheta,Sg) > 0$.

Then, $\alpha(\vartheta_n,\vartheta_{n+1}) \geq 1$ *for all* $n \in \{0,1,2,\dots\}$ *and* $\{S(\vartheta_n)\} \to k \in \Delta$. *Also, if* k *satisfies* (2.20) *and either* $\alpha(\vartheta_n,k) \geq 1$ *or* $\alpha(k,\vartheta_n) \geq 1$ *for all* $n \in \{0,1,2,\dots\}$, *then* k *belongs to* Sk.

3 Applications in graph theory

Jachymski [21] developed a relation between fixed point theory and graph theory by introducing graphic contractions. Hussain et al. [19] established some results for a new type of contraction endowed with a graph. Let A be a nonempty set, $V(Y)$ and $L(Y)$ denote the set of vertices and the set of edges containing all loops, respectively, for a graph Y.

Definition 3.1 Let A be a nonempty set and $Y = (V(Y), L(Y))$ be a graph with $V(Y) = A$. A fuzzy mapping F from A to $W(A)$ is known as a fuzzy-graph dominated mapping on A if $(a,b) \in L(Y)$, whenever $a \in A$, $b \in [Fa]_\beta$ and $0 < \beta \leq 1$.

Theorem 3.2 *Let* (Δ,u) *be a complete m.l.m. space endowed with a graph* Y, $\vartheta_0 \in \Delta$, *and the following hold*:

(i) $S, T : \Delta \to W(\Delta)$ *are fuzzy-graph dominated functions on* $\{TS(\vartheta_n)\}$.

(ii) *There exist* $\tau > 0$, $\gamma(\vartheta), \beta(y) \in (0,1]$, *and* $V \in F$ *such that*

$$\tau + V\big(H_{u_1}([S\vartheta]_{\gamma(\vartheta)}, [Ty]_{\beta(y)})\big) \leq V\left(\max\left\{\begin{matrix} u_1(\vartheta, y), u_1(\vartheta, [S\vartheta]_{\gamma(\vartheta)}), \frac{u_2(\vartheta, [Ty]_{\beta(y)})}{2}, \\ \frac{u_1(\vartheta, [S\vartheta]_{\gamma(\vartheta)}).u_1(y, [Ty]_{\beta(y)})}{1+u_1(\vartheta, y)} \end{matrix}\right\}\right), \tag{3.1}$$

whenever $t, y \in \{TS(\vartheta_n)\}$, $(\vartheta, y) \in L(Y)$, *and* $H_{u_1}([S\vartheta]_{\gamma(\vartheta)}, [Ty]_{\beta(y)}) > 0$.

Assume that Δ *is regular and satisfies the* $\triangle_M$*-condition. Then* $(\vartheta_n, \vartheta_{n+1}) \in L(Y)$ *and* $\{TS(\vartheta_n)\} \to k^*$. *Also, if* k^* *satisfies* (3.1) *and* $(\vartheta_n, k^*) \in L(Y)$ *or* $(k^*, \vartheta_n) \in L(Y)$ *for each* $n \in \{0, 1, 2, \dots\}$, *then* k^* *belongs to both* $[Tk^*]_{\beta(k^*)}$ *and* $k \in [Sk^*]_{\gamma(k^*)}$.

Proof Define $\alpha : \Delta \times \Delta \to [0, \infty)$ by $\alpha(\vartheta, y) = 1$, if $\vartheta \in \Delta$ and $(\vartheta, y) \in L(Y)$. Otherwise, set $\alpha(\vartheta, y) = 0$. By definition of graph domination on Δ, we have $(\vartheta, y) \in L(Y)$ for all $y \in [S\vartheta]_{\gamma(\vartheta)}$ and $(\vartheta, y) \in L(Y)$ for each $y \in [Ty]_{\beta(y)}$. So, $\alpha(\vartheta, y) = 1$ for all $y \in [S\vartheta]_{\gamma(\vartheta)}$ and $\alpha(\vartheta, y) = 1$ for every $y \in [Ty]_{\beta(y)}$. This means that $\inf\{\alpha(\vartheta, y) : y \in [S\vartheta]_{\gamma(\vartheta)}\} = 1$ and $\inf\{\alpha(\vartheta, y) : y \in [Ty]_{\beta(y)}\} = 1$. Hence, $\alpha_*(\vartheta, [S\vartheta]_{\gamma(\vartheta)}) = 1$, $\alpha_*(\vartheta, [Ty]_{\beta(y)}) = 1$ for every $\vartheta \in \Delta$. So, the pair of mappings are α_*-dominated on Δ. Furthermore, inequality (3.1) can be expressed as

$$\tau + V\big(H_{u_1}([S\vartheta]_{\gamma(\vartheta)}, [Ty]_{\beta(y)})\big) \leq V\left(\max\left\{\begin{matrix} u_1(\vartheta, y), u_1(\vartheta, [S\vartheta]_{\gamma(\vartheta)}), \frac{u_2(\vartheta, [Ty]_{\beta(y)})}{2}, \\ \frac{u_1(\vartheta, [S\vartheta]_{\gamma(\vartheta)}).u_1(y, [Ty]_{\beta(y)})}{1+u_1(\vartheta, y)} \end{matrix}\right\}\right),$$

whenever $\vartheta, y \in \{TS(\vartheta_n)\}$ with $\alpha(\vartheta, y) \geq 1$ and $H_{u_1}([S\vartheta]_{\gamma(\vartheta)}, [Ty]_{\beta(y)}) > 0$. Also, (ii) holds. Then, by Theorem 2.2, $\{TS(\vartheta_n)\}$ is a sequence in Δ and $\{TS(\vartheta_n)\} \to k^* \in \Delta$. Now, $\vartheta_n, k^* \in \Delta$ and either $(\vartheta_n, k^*) \in L(Y)$ or $(k^*, \vartheta_n) \in L(Y)$ implies that either $\alpha(\vartheta_n, k^*) \geq 1$ or $\alpha(k^*, \vartheta_n) \geq 1$. So, all the requirements of Theorem 2.2 are satisfied. Hence, k^* belongs to both $[Tk^*]_{\beta(k^*)}$ and $k \in [Sk^*]_{\gamma(k^*)}$. □

4 Results for single-valued mappings

In this section, some consequences of our results related to single-valued mappings in *m.l.m.* spaces are discussed. Let (Δ, u) be an *m.l.m.* space, $g_0 \in \Delta$, and $S, T : \Delta \to \Delta$ be a pair of multivalued mappings. Let $g_1 = Sg_0$, $g_2 = Tg_1$, and $g_3 = Sg_2$. Similarly, we make a sequence g_n in Δ so that $g_{2n+1} = Sg_{2n}$ and $g_{2n+2} = Tg_{2n+1}$, where $n = 0, 1, 2, \dots$ We represent this kind of iterative sequence by $\{TS(g_n)\}$. We say that $\{TS(g_n)\}$ is a sequence in Δ generated by g_0.

Theorem 4.1 *Let* (Δ, u) *be a complete m.l.m. space. Assume that u is regular and satisfies the* $\triangle_M$*-condition. Let* $r > 0$, $g_0 \in \Delta$, $\alpha : \Delta \times \Delta \to [0, \infty)$, *and* $S, T : \Delta \to \Delta$ *be* α*-dominated functions on* $\{TS(g_n)\}$. *Suppose that there exist* $\tau > 0$ *and* $V \in F$ *such that*

$$\tau + V\big(u_1(St, Tg)\big) \leq V\left(\max\left\{\begin{matrix} u_1(t, g), u_1(t, St), \frac{u_2(t, Tg)}{2}, \\ \frac{u_1(t, St).u_1(g, Tg)}{1+u_1(t, g)} \end{matrix}\right\}\right), \tag{4.1}$$

whenever $t, g \in \{TS(g_n)\}$ *with* $\alpha(t, g) \geq 1$ *and* $u_1(St, Tg) > 0$.

Then $\alpha(g_n, g_{n+1}) \geq 1$ for each $n \in \mathbb{N} \cup \{0\}$ and $\{TS(g_n)\} \to h \in \Delta$. Also, if h satisfies (4.1) and either $\alpha(g_n, h) \geq 1$ or $\alpha(h, g_n) \geq 1$ for all $n \in \mathbb{N} \cup \{0\}$, then S and T have a common fixed point h in Δ.

If we take $S = T$ in Theorem 4.1, then we get the following result.

Corollary 4.2 *Let (Δ, u) be a complete m.l.m. space. Assume that u is regular and satisfies the $\triangle_M$-condition. Let $g_0 \in \Delta$, $\alpha : \Delta \times \Delta \to [0, \infty)$ and $S : \Delta \to \Delta$ be an α-dominated function on $\{SS(g_n)\}$. Suppose that there exist $\tau > 0$ and $V \in \digamma$ such that*

$$\begin{aligned} &\tau + V\bigl(u_1(St, Sg)\bigr) \\ &\quad \leq V\left(\max\left\{\begin{array}{c} u_1(t, g), u_1(t, St), \frac{u_2(t,Sg)}{2}, \\ \frac{u_1(t,St).u_1(g,Sg)}{1+u_1(t,g)} \end{array}\right\}\right), \end{aligned} \tag{4.2}$$

whenever $t, g \in \{SS(g_n)\}$ with $\alpha(t, g) \geq 1$ and $u_1(St, Sg) > 0$. Then $\alpha(g_n, g_{n+1}) \geq 1$ for each $n \in \mathbb{N} \cup \{0\}$ and $\{SS(g_n)\} \to h \in \Delta$. Also, if h satisfies (4.2) and either $\alpha(g_n, h) \geq 1$ or $\alpha(h, g_n) \geq 1$ for each $n \in \mathbb{N} \cup \{0\}$, then h is the fixed point of S.

Corollary 4.3 *Let (Δ, u) be a complete m.l.m. space. Assume that u is regular and satisfies the $\triangle_M$-condition. Let $r > 0$, $g_0 \in \Delta$, $\alpha : \Delta \times \Delta \to [0, \infty)$, and $S, T : \Delta \to \Delta$ be α-dominated functions on $\{TS(g_n)\}$. Suppose that there exists $k \in (0, 1)$ such that*

$$u_1(St, Tg) \leq k u_1(t, g), \tag{4.3}$$

whenever $t, g \in \{TS(g_n)\}$, $\alpha(t, g) \geq 1$, and $u_1(St, Tg) > 0$.

Then $\alpha(g_n, g_{n+1}) \geq 1$ for each $n \in \mathbb{N} \cup \{0\}$ and $\{TS(g_n)\} \to h \in \Delta$. Also, if h satisfies (4.3) and either $\alpha(g_n, h) \geq 1$ or $\alpha(h, g_n) \geq 1$ for all $n \in \mathbb{N} \cup \{0\}$, then S and T have a common fixed point h in Δ.

Remark 4.4 If we impose the Banach condition

$$w(St, Tg) \leq k u_1(t, g) \quad \text{for all } t, g \in \Delta,$$

for a pair $S, T : \Delta \to \Delta$ of mappings on a regular modular metric space (Δ, w), then it follows that $Sg = Tg$, for all $g \in \Delta$ (that is, S and T are equal). Therefore, the above condition fails to find common fixed points of S and T. However, the same condition in *m.l.m.* spaces does not assert that $S = T$.

5 Application on nonlinear Volterra-type integral equations

In this section, we discuss the application of our work to integral equations. First of all, we present our main result without α_*-dominated functions for self-mappings and then apply it to attain an application on integral equations.

Theorem 5.1 *Let (Δ, u) be a complete m.l.m. space. Assume that u is regular and satisfies the $\triangle_M$-condition. Let $g_0 \in \Delta$ and $S, T : \Delta \to \Delta$ be self-mappings. If there exist $\tau > 0$ and $V \in \digamma$ such that*

$$\tau + V\bigl(u_1(St, Tg)\bigr)$$

$$\leq V\left(\max\left\{\begin{array}{c} u_1(t,g), u_1(t,St), \frac{u_2(t,Tg)}{2}, \\ \frac{u_1(t,St).u_1(g,Tg)}{1+u_1(t,g)} \end{array}\right\}\right), \tag{5.1}$$

whenever $t,g \in \{TS(g_n)\}$ *and* $u_1(St,Tg) > 0$, *then* $\{TS(g_n)\} \to h \in \Delta$. *Also, if inequality* (5.1) *holds for* $t,g \in \{h\}$, *then* S *and* T *have a common fixed point* h *in* Δ.

Let $X = C([0,1],\mathbb{R}_+)$ *be the set of all continuous nonnegative functions on* $[0,1]$. *Consider the nonlinear Volterra-type integral equations*

$$u(k) = \int_0^k H\bigl(k,h,u(h)\bigr)\,dh, \tag{5.2}$$

$$g(k) = \int_0^k G\bigl(k,h,g(h)\bigr)\,dh, \tag{5.3}$$

for all $k \in [0,1]$, *and suppose* H,G *are the functions from* $[0,1]\times[0,1]\times X$ *to* $\mathbb{R}$. *For* $g \in C([0,1],\mathbb{R}_+)$, *define the supremum norm as* $\|g\|_\tau = \sup_{k\in[0,1]}\{|g(k)|e^{-\eta k}\}$, *where* $\eta > 0$ *is arbitrarily taken. Define*

$$\begin{aligned} u_l(g,p) &= \frac{1}{l+1}\sup_{k\in[0,1]}\bigl\{\bigl|g(k)+p(k)\bigr|\bigr\}e^{-\tau k} \\ &= \frac{1}{l+1}\|g+p\|_\tau \end{aligned}$$

for all $g,p \in C([0,1],\mathbb{R}_+)$. *With these settings,* $(C([0,1],\mathbb{R}_+),d_\tau)$ *becomes a complete m.l.m. space.*

Now, we prove the following theorem to ensure the existence and uniqueness of a solution of families of the nonlinear integral equations (5.2) and (5.3).

Theorem 5.2 *Assume that the following conditions are satisfied*:

(i) *H and G are two functions from* $[0,1]\times[0,1]\times C([0,1],\mathbb{R}_+)$ *to* $\mathbb{R}$;

(ii) *Define*

$$\begin{aligned} (Su)(k) &= \int_0^k H\bigl(k,h,u(h)\bigr)\,dh, \\ (Tg)(k) &= \int_0^k G\bigl(k,h,g(h)\bigr)\,dh. \end{aligned}$$

Suppose there exists $\tau > 0$ *such that*

$$\bigl|H(k,h,u)+G(k,h,g)\bigr| \leq \frac{2\tau e^{\tau h}E(u,g)}{(\tau\sqrt{E(u,g)}+1)^2},$$

for all $k,h \in [0,1]$ *and* $u,g \in C([0,1],\mathbb{R}^+)$, *where*

$$E(u,g) = \max\left\{\begin{array}{c} \frac{1}{2}\|u+g\|_\tau, \frac{1}{2}\|u+Su\|_\tau, \\ \frac{1}{3}\|u+Tg\|_\tau, \\ \frac{1}{4}\frac{\|u+Su\|_\tau.\|g+Tg\|_\tau}{1+\frac{1}{2}\|u+g\|_\tau} \end{array}\right\}.$$

Then the integral equations (5.2) *and* (5.3) *have a unique solution.*

Proof By assumption (ii),

$$\begin{aligned}
|Su+Tg| &= \int_0^k \bigl|H(k,h,u)+G(k,h,g)\bigr|\,dh \\
&\le \int_0^k \frac{2\tau e^{\tau h}E(u,g)}{(\tau\sqrt{E(u,g)}+1)^2}\,dh \\
&\le \frac{2\tau E(u,g)}{(\tau\sqrt{E(u,g)}+1)^2}\int_0^k e^{\tau h}\,dh \\
&\le \frac{2E(u,g)}{(\tau\sqrt{E(u,g)}+1)^2}e^{\tau k}.
\end{aligned}$$

This implies

$$\begin{aligned}
&|Su+Tg|e^{-\tau k} \le \frac{2E(u,g)}{(\tau\sqrt{E(u,g)}+1)^2}, \\
&\|Su+Tg\|_\tau \le \frac{2E(u,g)}{(\tau\sqrt{E(u,g)}+1)^2}, \\
&\sqrt{\|Su+Tg\|_\tau} \le \frac{\sqrt{2E(u,g)}}{\tau\sqrt{E(u,g)}+1}, \\
&\frac{\tau\sqrt{E(u,g)}+1}{\sqrt{E(u,g)}} \le \sqrt{\frac{2}{\|Su+Tg\|_\tau}}, \\
&\tau+\sqrt{\frac{1}{E(u,g)}} \le \sqrt{\frac{2}{\|Su+Tg\|_\tau}},
\end{aligned}$$

which further implies

$$\begin{aligned}
&\tau-\sqrt{\frac{2}{\|Su(k)+Tg(k)\|_\tau}} \le -\sqrt{\frac{1}{E(u,g)}}, \\
&\tau+V\left(\frac{1}{2}\bigl\|Su(k)+Tg(k)\bigr\|_\tau\right) \le V\bigl(E(u,g)\bigr).
\end{aligned}$$

So, all the requirements of Theorem 5.1 are satisfied for $V(f)=\frac{-1}{\sqrt{f}}$, $f>0$, and $u_l(f,g)=\frac{1}{l+1}\|f+g\|_\tau$. Hence, the integral equations (5.2) and (5.3) have a common solution. □

6 Conclusion

In this article, we have achieved some new results for a pair of fuzzy α_*-dominated mappings, which are generalizations of Wardowski's contraction. Further results in ordered spaces and graph theory are presented. Results for multivalued and single-valued mappings are also discussed. Moreover, we investigate our results in new generalized modular-like metric spaces. An application is presented to ensure the existence of a solution of nonlinear Volterra-type integral equations. Many consequences of our results in dislocated metric spaces, dislocated fuzzy metric spaces, fuzzy metric spaces, ordered spaces, metric spaces, and partial metric spaces can be easily established.

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