

# The magnetic effect on a strongly correlated system with two currents near the quantum critical point from holography

Jaeha Lee<sup>1</sup>, Sang-Jin Sin<sup>2</sup>, and Geunho Song<sup>2,\*</sup>

<sup>1</sup>*The Division of Physics, Mathematics and Astronomy, California Institute of Technology, Pasadena, CA 91125, USA*

<sup>2</sup>*Department of Physics, Hanyang University, Seoul 04763, South Korea*

\*E-mail: sgh8774@gmail.com

Received January 18, 2021; Revised February 24, 2021; Accepted February 25, 2021; Published March 2, 2021

.....  
We study the magnetic effect in a strongly interacting system with two conserved currents near the quantum critical point (QCP). For this purpose, we introduce the hyper-scaling violation geometry with the black hole. Considering the perturbation near the background geometry, we compute the transport coefficients using holographic methods. We calculated the magneto-transport for general QCP and discuss the special point  $(z, \theta) = (3/2, 1)$ , where the data of Dirac material have previously been well described.  
.....

Subject Index    B21, I46, I92

## 1. Introduction

For a strongly correlated system, the particle nature is often absent so that theories based on quasi-particles such as Landau–Fermi liquid theory fail. Strong correlation can happen even for weakly interacting system when the Fermi surface can be tuned to be very small, because then sufficient electron–hole pairs, which screen the Coulomb interaction, are not created due to the smallness of the Fermi surface. Therefore, any Dirac fluid can be strongly correlated as far as it has a small Fermi surface, which has already been shown in clean graphene [1,2] and in the surface of a topological insulator with magnetic doping [3–5]. We need to find a new way to describe such a system. We consider the quantum critical point (QCP), where the microscopic details in ultraviolet (UV) are irrelevant and most of the information in UV is apparently lost in a low-energy probe in the sense of coarse graining. This apparent loss of information is very similar to a black hole system, and this similarity between a QCP and a black hole is important motivation for using holography to analyze strongly correlated electron systems. A QCP can be characterized by  $(z, \theta)$ , which is defined by the dispersion relation  $\omega \sim k^z$  and the entropy density  $s \sim T^{(d-\theta)/z}$ . We can use a geometry with the same scaling symmetry with respect to  $(t, r, x) \rightarrow (\lambda^z t, \lambda^{-1} r, \lambda x)$ ,

$$ds^2 = r^{-\theta} \left( -r^{2z} dt^2 + \frac{dr^2}{r^2} + r^2 d\vec{x}^2 \right), \quad (1)$$

which is called hyper-scaling-violation (HSV) geometry.

In our previous works [6–9] we described clean graphene and a topological insulator with magnetic doping in some parameter regions using the holographic method. For the surface of the topological insulator, we introduced just one current with an interaction to encode the magnetic doping [8,9]. We

calculated the magneto-conductivity and investigated the phase transitions from weak localization to weak anti-localization. As in the case of graphene, it turns out that we can have a better fit for  $(z, \theta) = (3/2, 1)$  than  $(1, 0)$ .

For graphene, we need a two-current model [6,7]: when the electron and hole densities fluctuate from their equilibrium states, the system is supposed to reduce the difference by creating or absorbing an electron pair:

$$e^- \leftrightarrow e^- + h^+ + e^-, \quad h^+ \leftrightarrow h^+ + h^+ + e^-. \quad (2)$$

In this process, energy and momentum should be conserved. For graphene, however, the kinematically available states are severely reduced [10] due to the geometry of the Dirac cone, and this constraint makes the two currents  $J_e$  and  $J_h$  independently conserved. Hence, we need two independent currents to describe graphene. In Ref. [7], we analyzed the two-current model for hyperscaling violation geometry (HSV), and found a theory with a QCP at  $(z, \theta) = (3/2, 1)$  rather than  $(1, 0)$  as studied in Ref. [8]. The value  $\theta = 1$  is important because the holographic background with a dual Fermi surface has effective dimension  $d_{\text{eff}} = d - \theta$ , and a system with a Fermi surface should have  $d_{\text{eff}} = 1$  so that  $\theta = 1$  can describe the character of the fermion in this aspect. Indeed, we found that  $(z, \theta) = (3/2, 1)$  can fit the data better and therefore qualified as a more proper critical exponent in both graphene and a topological insulator.

In this paper, we study the holographic model with two currents and a particular interaction which is shown to describe magnetically doped material [8,9]. We calculate all the transport coefficients and demonstrate some typical behavior of the magneto-transport. Although we have no experimental results for magnetically doped graphene, our work can be considered as predictions for the magnetic effect for graphene or other materials which need two or more currents due to the presence of two layers or two valleys.

## 2. The two-current model with magnetic impurity in hyperscaling violating geometry

We start from a four-dimensional action with asymptotically hyperscaling geometry  $g_{\mu\nu}$ , which includes a dilaton field  $\phi$ , a gauge field  $A_\mu$  to complete the asymptotic hyperscaling violating geometry, two extra gauge fields  $B_\mu^{(a)}$  which are dual to two conserved currents, and the axion fields  $\chi_1, \chi_2$  to break the translational symmetry:

$$\begin{aligned} S &= \int_{\mathcal{M}} d^4x (\mathcal{L}_0 + \mathcal{L}_{\text{int}}), \\ \mathcal{L}_0 &= \sqrt{-g} \left( R + \sum_{i=1}^2 V_i e^{\gamma_i \phi} - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4} Z_A F^2 - \sum_a \frac{1}{4} Z_a G_{(a)}^2 - \frac{1}{2} Y \sum_i (\partial\chi_i)^2 \right), \\ \mathcal{L}_{\text{int}} &= - \sum_{a,i=1,2} \frac{q\chi_a}{16} (\partial\chi_i)^2 \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^{(a)} G_{\rho\sigma}^{(a)}, \end{aligned} \quad (3)$$

where  $F = dA$ ,  $G_{(a)} = dB_a$ . We use the ansatz

$$Z_A = e^{\lambda\phi}, \quad Z_a = \bar{Z}_a e^{\eta\phi}, \quad Y = e^{-\eta\phi}, \quad \chi_i = \beta x_i, \quad (4)$$

where  $\beta$  denotes the strength of momentum relaxation. The equations of motion for gauge fields and gravity are given by

$$\partial_\mu \left( \sqrt{-g} g^{\mu\nu} Y \sum_i \partial_\nu \chi_i \right) + \sum_{a,i=1,2} \frac{q_{\chi_a}}{8} \partial_\mu (\epsilon^{\rho\sigma\lambda\gamma} G_{\rho\sigma}^{(a)} G_{\lambda\gamma}^{(a)} g^{\mu\nu} \partial_\nu \chi_i) = 0, \quad (5)$$

$$\partial_\mu \left( \sqrt{-g} Z_A F^{\mu\nu} \right) = 0, \quad \partial_\mu \left( \sqrt{-g} Z_a G^{(a)\mu\nu} + \frac{q_{\chi_a}}{4} \sum_i (\partial \chi_i)^2 \epsilon^{\alpha\beta\mu\nu} G_{\alpha\beta}^{(a)} \right) = 0, \quad (6)$$

$$R_{\mu\nu} = \frac{1}{2\sqrt{-g}} g_{\mu\nu} \mathcal{L} + \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \frac{Y}{2} \sum_i \partial_\mu \chi_i \partial_\nu \chi_i + \frac{1}{2} Z_A F_\mu^\rho F_{\nu\rho} + \sum_a \frac{1}{2} Z_a G_{(a)\mu}^\rho G_{\nu\rho}^{(a)} + \sum_{a,i=1,2} \frac{q_{\chi_a}}{16} \frac{1}{\sqrt{-g}} (\partial_\mu \chi_i) (\partial_\nu \chi_i) \epsilon^{\rho\sigma\lambda\gamma} G_{\rho\sigma}^{(a)} G_{\lambda\gamma}^{(a)}, \quad (7)$$

$$\square \phi + \sum_i V_i \gamma e^{\gamma\phi} - \frac{1}{4} Z_A'(\phi) F^2 - \frac{1}{4} \sum_a Z_a'(\phi) G_{(a)}^2 - \frac{1}{2} Y'(\phi) \sum_i (\partial \chi_i)^2 = 0. \quad (8)$$

The solution for the dilaton field is given by

$$\phi(r) = \nu \ln r, \quad \text{with } \nu = \sqrt{(2-\theta)(2z-2-\theta)}. \quad (9)$$

By solving the equations of motion, we can get the gauge couplings and dilaton coupling  $Z_A$ ,  $Z_a$ , and  $Y$  as follows:

$$Z_A(\phi) = e^{\lambda\phi} = r^{\theta-4}, \quad Z_a(\phi) = \bar{Z}_a e^{\eta\phi} = \bar{Z}_a r^{2z-\theta-2}, \quad Y(\phi) = e^{-\eta\phi}, \quad (10)$$

where  $\lambda = (\theta - 4)/\nu$ ,  $\eta = \nu/(2 - \theta)$ .

Other exponents and potentials are given by

$$\gamma_1 = \frac{\theta}{\nu}, \quad \gamma_2 = \frac{\theta + 2z - 6}{\nu}, \quad V_1 = \frac{z - \theta + 1}{2(z - 1)} q_A^2, \quad V_2 = \frac{H^2(2z - \theta - 2)}{4(z - 2)}, \quad (11)$$

where  $H$  is a constant magnetic field and  $q_A = \sqrt{2(-1+z)(2+z-\theta)}$ . Finally, we have the following background solutions:

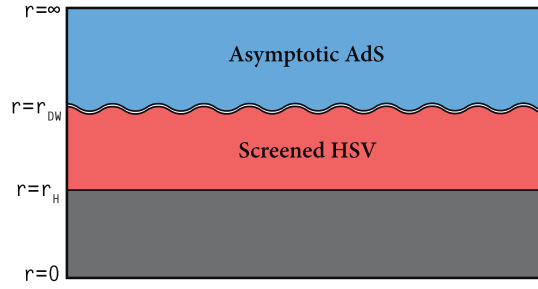
$$A = a(r)dt, \quad B_a = b_a(r)dt - \frac{1}{2}Hydx + \frac{1}{2}Hxdy, \quad (12)$$

$$\chi = (\beta x, \beta y), \quad (13)$$

$$ds^2 = r^{-\theta} \left( -r^{2z} f(r) dt^2 + \frac{dr^2}{r^{2z} f(r)} + r^2(dx^2 + dy^2) \right), \quad (14)$$

$$f(r) = 1 - mr^{\theta-z-2} - \frac{\beta^2}{(\theta-2)(z-2)} r^{\theta-2z} + \frac{(\bar{Z}_1 q_1^2 + \bar{Z}_2 q_2^2)(\theta-z)r^{2\theta-2z-2}}{2(\theta-2)} + \frac{(\bar{Z}_1 + \bar{Z}_2)H^2 r^{2z-6}}{4(z-2)(3z-\theta-4)} - \frac{c_2 \beta^2 H (q_{\chi_1} q_1 + q_{\chi_2} q_2)}{r^{4+2z-3\theta}} + \frac{c_3 \beta^4 H^2 \left( \frac{q_{\chi_1}}{\bar{Z}_1} + \frac{q_{\chi_2}}{\bar{Z}_2} \right)}{r^{6+2z-4\theta}}, \quad (15)$$

$$a(r) = \frac{-q_A}{2+z-\theta} (r_{\text{H}}^{2+z-\theta} - r^{2+z-\theta}), \quad b_a(r) = \mu_a - q_a r^{\theta-z} - \frac{c_4 q_{\chi_a} \beta^2 H}{\bar{Z}_a r^{z-2\theta+2}},$$



(a) The scheme of embedding HSV to AdS

**Fig. 1.** Schematic figure of our geometry. The region inside the black hole is colored gray, and there is a domain wall ( $r = r_{\text{DW}}$ ) somewhere between the AdS boundary and the black hole horizon at  $r_{\text{H}}$ .

where  $a = 1, 2$  and  $c_2, c_3,$  and  $c_4$  are given by

$$c_2 = \frac{(z - \theta)}{(\theta - 2)(2\theta - z - 2)}, \quad c_3 = \frac{1}{2(2 - \theta)(4 + z - 3\theta)}, \quad c_4 = \frac{1}{2\theta - z - 2}. \quad (16)$$

This HSV solution should be embedded into asymptotically anti-de Sitter (AdS) spacetime so that it is just the infrared part of the total domain-wall solution. Here, we only conceptually embedded but did not write down the explicit solution in the entire region, since it is not important for the computation of the DC transports [11]; see Fig. 1.

We can define the conserved charge from the equations of motion for the gauge fields  $B_a$  as the constants of integration,

$$\begin{aligned} Q_a &= \sqrt{-g} Z_a G_{(a)}^{tr} + \frac{q_{\chi_a}}{4} \sum_i (\partial \chi_i)^2 \epsilon^{\alpha\beta tr} G_{\alpha\beta}^{(a)} \\ &= \bar{Z}_a q_a (z - \theta) = (z - \theta) \left( \mu_a \bar{Z}_a r_{\text{H}}^{z-\theta} + \frac{q_{\chi_a} \beta^2 r_{\text{H}}^{-2+\theta}}{2 + z - 2\theta} \right). \end{aligned} \quad (17)$$

The entropy density and the Hawking temperature are given by

$$s = 4\pi r_{\text{H}}^{2-\theta}, \quad (18)$$

$$4\pi T = (z + 2 - \theta) r_{\text{H}}^z - \frac{\beta^2 r_{\text{H}}^{\theta-z}}{2 - \theta} - \frac{r_{\text{H}}^{2\theta-2-z}}{2(2 - \theta)} \sum_{a=1,2} \frac{1}{\bar{Z}_a} (\Theta_a H - Q_a)^2 - \bar{Z} \frac{H^2 r_{\text{H}}^{3z-6}}{4(2 - z)}, \quad (19)$$

where  $\Theta_a = q_{\chi_a} \beta^2 r_{\text{H}}^{\theta-2}$  and  $\bar{Z} = \sum_{a=1,2} \bar{Z}_a$ .

### 3. Conserved currents and DC transports

We consider the following perturbations to compute the transport coefficients based on the idea of linear response theory [11]:

$$\delta g_{ti} = h_{ti}(r) + t f_{3i}(r), \quad \delta g_{ri} = h_{ri}(r), \quad \delta B_{ai} = \tilde{b}_{ai} - t f_{ai}, \quad \delta \chi_i = \varphi_i(r). \quad (20)$$

We take the functions  $f_i(r)$  as

$$f_{1i} = -E_{1i} + \zeta_i b_1(r), \quad f_{2i} = -E_{2i} + \zeta_i b_2(r), \quad f_{3i} = -\zeta_i U(r) \quad (21)$$

to make the linearized Einstein equations time independent. Here,  $E_{ai}$  are the external electric fields and  $\zeta_i$  is thermal gradient, defined as  $\zeta_i = -(\nabla_i T/T)$ . In the final expression for each conserved

current, we will set  $E_{1i} = E_{2i} = E_i$ . Since all the transports can be computed at the event horizon, we need to find the regularity condition at the horizon. We take the Eddington–Finkelstein coordinates  $(v, r)$  where the background metric is regular at the horizon,

$$ds^2 = -Udt^2 - 2\sqrt{UV}dvdr + Wd\vec{x}^2, \quad (22)$$

where  $v = t + \int dr \sqrt{V/U}$ . In these coordinates, the metric perturbation is given by

$$\delta g_{\mu\nu} dx^\mu dx^\nu = h_{tx} dv dx + \left( h_{rx} - \sqrt{\frac{V}{U}} h_{tx} \right) dr dx. \quad (23)$$

To guarantee the regularity of the metric with perturbation at the horizon, we require the last term to vanish at the horizon so that

$$h_{ri} \sim \sqrt{\frac{V}{U}} h_{ti}. \quad (24)$$

The gauge fields can be reexpressed in the Eddington–Finkelstein coordinates to get the regularity condition at the event horizon:

$$\delta B_{ai} \sim \tilde{b}_{ai} + E_{ai} v - E_{ai} \int dr \sqrt{\frac{V}{U}}. \quad (25)$$

Then, the full gauge field has the regular form of  $\delta B_{ai} \sim E_{ai} v + \dots$  in the Eddington–Finkelstein coordinates by requiring

$$\tilde{b}'_{ai} \sim \sqrt{\frac{V}{U}} E_{ai}. \quad (26)$$

We can define the radially conserved currents by

$$J_a^\mu = \sqrt{-g} Z_a G^{\mu r}_{(a)} + \frac{q\chi_a}{4} \sum_i (\partial \chi_i)^2 \varepsilon^{\alpha\beta\mu r} G_{\alpha\beta}^{(a)},$$

$$Q_i = \frac{U^2}{\sqrt{UV}} \left( \frac{h_{ti}}{U} \right)' - \sum_{a=1,2} b_a J_{ai}, \quad (27)$$

where the index  $a = 1, 2$  denotes the two currents which are dual to the two gauge fields  $B_a$ , and  $i = x, y$  is the index of directions. Since  $J_a$  and  $Q_i$  are the conserved quantities along the radial direction, they can be evaluated at arbitrary values. Hence, it is enough to compute them at the horizon [11].

Finally, we can express the boundary current in terms of the external sources and transport coefficients:

$$J_{ai} = \sum_{bj} (\sigma_{ab})_{ij} E_{bj} + \sum_j (\alpha_a)_{ij} T \zeta_j,$$

$$Q_i = \sum_{aj} (\tilde{\alpha}_a)_{ij} T E_{aj} + \sum_j \tilde{\kappa}_{ij} T \zeta_j. \quad (28)$$

Before we express the transport coefficients explicitly, it will be useful to define the following functions to simplify the expressions:

$$\mathcal{F} = WY\beta^2 + (Z_1 + Z_2)H^2 - \sum_{a=1,2} \frac{1}{Z_a} (Q_a \Theta_a H - \Theta_a^2 H^2),$$

$$\mathcal{G} = \sum_{a=1,2} (Q_a - \Theta_a H), \quad (29)$$

where  $\Theta_a = q_{\chi_a} \beta^2 / W$ . One can define the total electric current as  $J_i = \sum_a J_{ai}$  and identify the external electric field as  $E_{ai} = E_i$ . Then, each transport coefficient based on this total current and electric field is given by

$$\begin{aligned} \sigma_{ij} &= \frac{\partial J_i}{\partial E_j} = \sum_{ab} (\sigma_{ab})_{ij} \\ &= \delta_{ij} \frac{Z(\mathcal{F} + \mathcal{G}^2/Z)(\mathcal{F} - ZH^2)}{\mathcal{F}^2 + H^2\mathcal{G}^2} + \epsilon_{ij} \left( \Theta + \frac{ZH\mathcal{G}(2\mathcal{F} + \mathcal{G}^2/Z - ZH^2)}{\mathcal{F}^2 + H^2\mathcal{G}^2} \right), \end{aligned} \quad (30)$$

$$\begin{aligned} \alpha_{ij} &= \frac{1}{T} \frac{\partial J_i}{\partial \zeta_j} = \sum_a (\alpha_a)_{ij} \\ &= \delta_{ij} \frac{s\mathcal{G}(\mathcal{F} - ZH^2)}{\mathcal{F}^2 + H^2\mathcal{G}^2} + \epsilon_{ij} \frac{sH(\mathcal{G}^2 + Z\mathcal{F})}{\mathcal{F}^2 + H^2\mathcal{G}^2}, \end{aligned} \quad (31)$$

$$\bar{\kappa}_{ij} = \delta_{ij} \frac{s^2 T \mathcal{F}}{\mathcal{F}^2 + H^2 \mathcal{G}^2} + \epsilon_{ij} \frac{s^2 T H \mathcal{G}}{\mathcal{F}^2 + H^2 \mathcal{G}^2}, \quad (32)$$

where  $Z = Z_1 + Z_2$  and  $\Theta = \Theta_1 + \Theta_2$ . Notice that  $\bar{\alpha}_{ij} = \alpha_{ij}$ . The resistivity is defined as the inverse of the conductivity matrix:

$$\begin{aligned} \rho_{ii} &= \frac{\sigma_{ii}}{\sigma_{ii}^2 + \sigma_{ij}^2} = \mathcal{R}_{ii}/\mathcal{D}, \\ \rho_{ij} &= \frac{\sigma_{ij}}{\sigma_{ii}^2 + \sigma_{ij}^2} = \mathcal{R}_{ij}/\mathcal{D}, \end{aligned} \quad (33)$$

where

$$\begin{aligned} \mathcal{R}_{ii} &= (\mathcal{G}^2 + Z\mathcal{F})(\mathcal{F} - ZH^2), \\ \mathcal{R}_{ij} &= (\mathcal{F}^2 + H^2\mathcal{G}^2)\Theta + H\mathcal{G}(\mathcal{G}^2 + 2Z\mathcal{F} - Z^2H^2), \\ \mathcal{D} &= (Z\mathcal{F} + \mathcal{G}^2)^2 + (\mathcal{F}^2 + \mathcal{G}^2H^2)\Theta^2 + H(Z^2H - 2\mathcal{G}\Theta)(Z^2H^2 - 2Z\mathcal{F} - \mathcal{G}^2). \end{aligned} \quad (34)$$

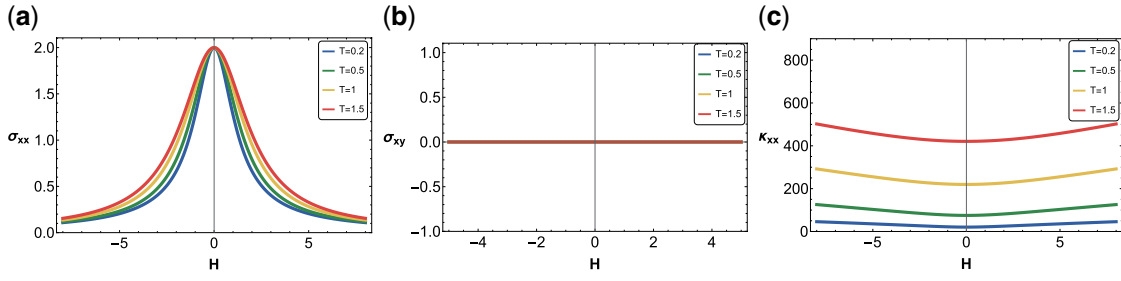
The thermal conductivity  $\kappa$  is defined by the response of the temperature gradient  $T\zeta_i$  to the heat current  $Q_i$  in the absence of the electric currents  $J_{ai}$ . Setting  $J_{ai} = 0$  in Eq. (28), we can write  $E_{bj}$  in terms of  $\zeta_j$  to substitute into the expression for the heat current in Eq. (28). Then, we can get

$$\kappa = \bar{\kappa} - T(\bar{\alpha}_1(\alpha_1\sigma_{22} - \alpha_2\delta) + \bar{\alpha}_2(\alpha_2\sigma_{11} - \alpha_1\delta))(\sigma_{11}\sigma_{22} - \delta^2)^{-1}, \quad (35)$$

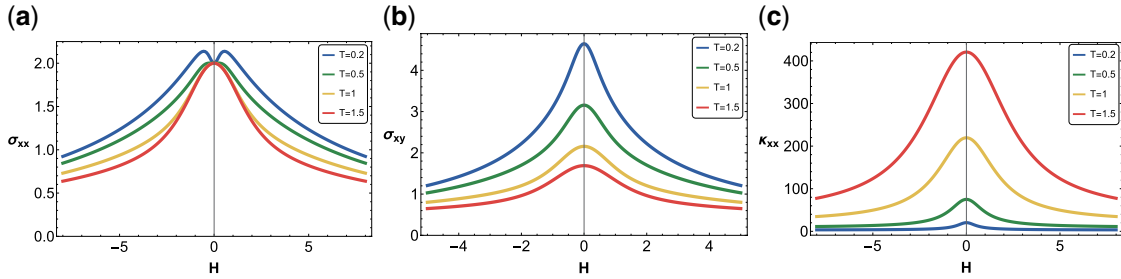
where  $\delta = \sigma_{12} = \sigma_{21}$ . Notice that this expression is very similar to that in Ref. [6], but it is  $2 \times 2$  matrix multiplication, which is different from the simple scalar multiplication in Ref. [6].

The Seebeck coefficient  $S$  and the Nernst signal  $N$  are given by

$$\begin{aligned} S &= -(\Sigma^{-1} \cdot \mathcal{A})_{xx}, \\ N &= -(\Sigma^{-1} \cdot \mathcal{A})_{yx}, \end{aligned} \quad (36)$$



**Fig. 2.** Magnetotransport for  $z = 3/2$ ,  $\theta = 1$  without magnetic impurities. We choose the parameters as  $\bar{Z}_1 = \bar{Z}_2 = 1$ ,  $q_{\chi_1} = 0$ ,  $q_{\chi_2} = 0$ ,  $g_n = 3$ , and  $\beta = 1.5$ .



**Fig. 3.** Magnetotransport for  $z = 3/2$ ,  $\theta = 1$  with magnetic impurities. We choose the parameters as  $\bar{Z}_1 = \bar{Z}_2 = 1$ ,  $q_{\chi_1} = 1$ ,  $q_{\chi_2} = 2$ ,  $g_n = 3$ , and  $\beta = 1.5$ .

where

$$\Sigma = \begin{pmatrix} \sigma_{11} & \delta \\ \delta & \sigma_{22} \end{pmatrix}, \quad \mathcal{A} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}. \quad (37)$$

Here,  $\Sigma$  and  $\mathcal{A}$  are  $4 \times 4$  and  $4 \times 2$  matrices respectively.

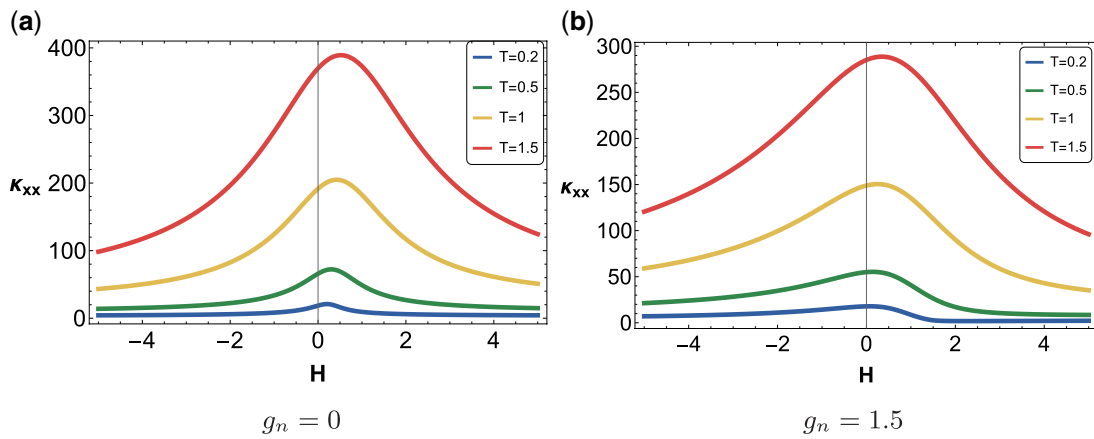
As discussed in the introduction, we need the two currents as independently conserved currents, identified with  $J_e$  and  $J_h$ . The total electric current  $\vec{J}$  and the total number current  $\vec{J}_n$  are defined by  $\vec{J} = \sum_a \vec{J}_a \equiv \vec{J}_e + \vec{J}_h$  and  $\vec{J}_n = \vec{J}_e - \vec{J}_h$  respectively. Their corresponding densities are related by  $Q_1 = q_e n_1$  and  $Q_2 = -q_e n_2$ , with the charge of the electron  $q_e = -1$ . The total electric charge density and the total number density are defined by  $Q = Q_1 + Q_2$  and  $Q_n = -Q_1 + Q_2$ , and we can connect each density with the proportional constant  $g_n$  where  $Q_n = g_n Q$ . We have simple expressions for the case in the absence of an external magnetic field [7]:

$$\sigma_{xx} = Z \left( 1 + \frac{Q^2}{Q_0^2} \right), \quad \kappa_{xx} = \frac{\bar{\kappa}_{xx}}{1 + (1 + g_n^2) \left( \frac{Q^2}{Q_0^2} \right)}, \quad (38)$$

where  $Q_0^2 = WYZ\beta^2$ .

In Figs. 2 and 3, we show the typical behaviors of each magnetotransport when  $z = 1.5$ ,  $\theta = 1$ , and  $Q_1 = Q_2 = 0$ , our region of interest. Notice that  $\kappa_{xy} = 0$  when there is no conserved electric charge.

As one can see from the figures, there is no significant difference between a single-current model and a two-current model in the qualitative sense compared to the results in Ref. [9]. But, as in Refs. [6,7], the two-current model can give a physical implication if there is experimental data to



**Fig. 4.** Magnetotransport for  $z = 3/2$ ,  $\theta = 1$  with magnetic impurities. We choose the parameters as  $\bar{Z}_1 = \bar{Z}_2 = 1$ ,  $q_{x1} = 1$ ,  $q_{x2} = 2$ ,  $Q = 2$ , and  $\beta = 1.5$ .

compare with this model. Unfortunately, there are no relevant experiments to be conducted, so we leave our results as a qualitative prediction for experiments with graphene with magnetic doping.

Finally, we set  $Q \neq 0$  to see the effect of  $g_n$  which corresponds to the two-current effect (see Fig. 4). Due to the presence of the charge, we have the asymmetric feature in  $\kappa_{xx}$ , and this asymmetry is enhanced by  $g_n$ . For larger  $g_n$ , we have the non-analytic behavior that comes from Eq. (19). But, we would like to leave the analysis of this non-analyticity to future work.

#### 4. Conclusion

We have investigated the two-current model with magnetic doping in the presence of a magnetic field, based on Refs. [6–9]. From this model, we calculated all the transport coefficients. Although we do not expect a qualitative difference between the single-current and two-current models, the presence of two currents is definitely necessary to describe the quantitative data fitting for materials involving a system of two independent electrons that are very weakly coupled, like graphene [6,7] or other multi-valley or multi-layer systems, which will be studied in a future.

#### Acknowledgements

This work is supported by the Mid-career Researcher Program through the National Research Foundation of Korea grant No. NRF-2016R1A2B3007687.

#### Funding

Open Access funding: SCOAP<sup>3</sup>.

#### References

- [1] A. Lucas, J. Crossno, K. C. Fong, P. Kim, and S. Sachdev, Phys. Rev. B **93**, 075426 (2016).
- [2] J. Crossno et al., Science, **351**, 1058 (2016).
- [3] M. Liu et al., Phys. Rev. Lett. **108**, 036805 (2012).
- [4] D. Zhang et al., Phys. Rev. B **86**, 205127 (2012).
- [5] L. Bao, W. Wang, N. Meyer, Y. Liu, C. Zhang, K. Wang, P. Ai, and F. Xiu, Sci. Rep. **3**, 2391 (2013).
- [6] Y. Seo, G. Song, P. Kim, S. Sachdev, and S.-J. Sin, Phys. Rev. Lett. **118**, 036601 (2017).
- [7] G. Song, Y. Seo, and S.-J. Sin, Phys. Rev. D **102**, 126023 (2020) [arXiv:2007.08143 [hep-th]] [Search INSPIRE].
- [8] Y. Seo, G. Song, and S.-J. Sin, Phys. Rev. B **96**, 041104(R) (2017).
- [9] X.-H. Ge, Y. Seo, S.-J. Sin, and G. Song, J. High Energy Phys. **2006**, 128 (2020).
- [10] M. S. Foster and I. L. Aleiner, Phys. Rev. B **79**, 085415 (2009).
- [11] A. Donos and J. P. Gauntlett, J. High Energy Phys. **1411**, 081 (2014).