Journal of the Korean Physical Society, Vol. 67, No. 3, August 2015, pp. 427∼432

Electric-magnetic Duality Implies (Global) Conformal Invariance

Sung-Pil Moon, Sang-Jin Lee, Ji-Hye Lee and Jae-Hyuk Oh*
Department of Physics, Hanyang University, Seoul 133-791, Korea

(Received 14 April 2015, in final form 4 June 2015)

We have examined quantum theories of electric-magnetic duality-invariant vector fields enjoying classical conformal invariance in 4-dimensional flat spacetime. We extend Dirac's argument about "the conditions for a quantum field theory to be relativistic" to "those for a quantum theory to be conformal". We realize that electric-magnetic duality-invariant vector theories, together with classical conformal invariance defined in 4-*d* flat spacetime, are still conformally invariant theories when they are quantized in a way that electric magnetic-duality is manifest.

PACS numbers: 11.25.Hf
Keywords: Electric-magnetic duality, Conformal field theory
DOI: 10.3938/jkps.67.427

I. INTRODUCTION

Electric-magnetic duality is originally observed from Maxwell equations, which describe one of the fundamental forces in nature. Under switching $\vec{E} \to \vec{B}$ and $\vec{B} \to -\vec{E}$, where $\vec{E}$ is the electric field and $\vec{B}$ is the magnetic field (without considering any electric and magnetic sources), the Maxwell equations are invariant [1]. The duality is extended to string theory, various kinds of field theories of free massless fields with various spins, and sometimes to those in curved spacetime, *e.g.*, Maxwell system in de Sitter spacetime [2–4]. It is also applied to non-Abelian vector theories and it turns out that the duality invariance comes into approximately exist upto cubic order in the weak field expansion [5,6].

One of the interesting directions of developing electric-magnetic duality is research on whether electric-magnetic duality ensures that a certain classical stymmetry of a system is retained when the system is quantized (*e.g.*, see [7]). In [7], the authors argue that a classical vector field theory enjoying Lorentz symmetry is still Lorentz invariant in its quantum theory under a condition that electric-magnetic duality is retained when it is quantized. The pioneering argument started from a paper by Dirac [8] in 1962. In his paper, he discussed this issue as follows. It is not manifest if a quantum field theory keeps its classical symmetry (symmetry of the classical Lagrangian and the equations of motion) because of (*e.g.*) the ordering issue of the field variables (due to the second quantization rule on them). Because a state in quantum field theory can be changed to another representation by a unitary transform and its dynamics can be described by a unitary time evolution, acting symmetry generators (spatial translation, rotation and boost, and temporal translation) on that state, then the second quantization being consistent with the algebra of the symmetry generators ensures that the symmetry is retained in its quantum field theory.

More precisely, Dirac introduces a canonical pair of quantum fields as ξ and η satisfying

$$[\xi, \eta'] = \delta, \tag{1}$$

where the prime denotes that the field variable depends on the prime coordinate; *i.e.*, $\eta' = \eta(x')$ and $\delta = \delta^d(x - x')$, being the d-dimensional δ-function so it is an equal-time commutator.[1] ξ may become a field variable in the theory, and η is its canonical conjugate. From them, he constructs a momentum density K_s and introduces an energy density U, which provide the representation of the symmetry generators, where the index s is a spatial index.[2] Such symmetry generators constructed from K_s and U turn out to satisfy Poincare algebra if the energy density satisfies the following commutation relation:

$$[U, U'] = K_{t,t}\delta + 2K_t\delta_{,t}, \tag{2}$$

where $A_{,s} \equiv \frac{\partial A}{\partial x^s}$.

By using this observation, the authors in [7] discovered the following: Suppose a vector field theory in 4-*d* flat spacetime which enjoys electric-magnetic duality and Lorentz symmetry is quantized in a way that electric-magnetic duality is manifest; more precisely, its second

*E-mail: jack.jaehyuk.oh@gmail.com

[1] For further discussion, even if we develop every mathematical equation in d-dimension(spatial), in fact we restrict ourselves to the $d = 3$ case only.

[2] We will use s, t, r, and u to be spatial indices running from 1 to 3.

quantization rule must be

$$[\mathcal{B}^a_s, \mathcal{B}^b_t] = \epsilon^{ab}\epsilon_{stu}\delta_{,u} \quad , \tag{3}$$

where $a, b = 1, 2$ are $SO(2)$ indices related to electric-magnetic duality rotation, ϵ is a fully anti-symmetric tensor, $\vec{\mathcal{B}}^1 = \vec{E}$ and $\vec{\mathcal{B}}^2 = \vec{B}$. Then one can define the momentum density and the energy density from the fields $\mathcal{B}^a_s$ as

$$K_r = -\frac{1}{2}\mathcal{B}^{as}\mathcal{B}^{bt}\epsilon^{ab}\epsilon_{str} \quad \text{and} \quad U = f(h, v), \tag{4}$$

where

$$h = \frac{1}{2}\mathcal{B}^{as}\mathcal{B}^{bt}\delta^{ab}\delta_{st}, \quad v = K_r K^r \tag{5}$$

and $f(h, v)$ satisfies the condition

$$(f_{,h})^2 + 4hf_{,h}f_{,v} + 4v(f_{,v})^2 = k, \tag{6}$$

for some constant k. The momentum density generates the Lie derivative along a spatial vector field v_i as $\mathcal{L}_v\Phi(\mathcal{B}) = [\Phi, \int d^dx v^s K_s]$ for some field Φ. Such an energy density satisfies the commutation relation that Dirac suggested in his paper. Therefore, one finds that the vector field theory is manifestly Lorentz invariant when it is quantized.

In this paper, we have extended such a discussion to conformal symmetry. Our motivation is that $U(1)$ vector field theory in $4-d$ flat spacetime, whose Lagrangian density is comprised of its kinetic term only, is conformally invariant, because its stress energy tensor vanishes. Thus, one may ask *if a quantum version of such kind of classical field theory is still conformally invariant when its second quantization rule manifestly enjoys electric-magnetic duality transform.*

In fact, we have shown that the theory is still conformal by examining conformal algebra in a manner similar to that ued by Dirac. In Section II, we develop the conditions that the momentum and the energy densities satisfy. The energy density still satisfies Eq. (2); therefore, the momentum density and the energy density that Dirac suggested also satisfy conformal algebra under the condition that **the conformal dimension of the energy density be** $d + 1$. The simplest example for such a case is $U = h$.

In Section III, we conclude that because a specific class of the energy density, Eq. (4), whose conformal dimension is $d + 1$ obtained in [7] satisfies the same commutation relation, Eq. (2), then the conformal symmetry is retained in such a quantum theory of the $U(1)$ vector field, which is manifestly invariant under electric-magnetic duality rotation.

II. CONDITIONS FOR A 4-D QUANTUM FIELD THEORY TO BE CONFORMAL

In this section, we extend Dirac's argument about conditions for a quantum field theory to retain Poincare symmetry to conformal symmetry.

Conformal algebra Conformal algebra in $(d + 1)$-dimensional space time is given by

$$\begin{aligned}
[D, P_\mu] &= -P_\mu, \quad [D, \kappa_\mu] = \kappa_\mu, \quad [\kappa_\mu, P_\nu] = -2(g_{\mu\nu}D + L_{\mu\nu}), \\
[\kappa_\rho, L_{\mu\nu}] &= (g_{\rho\mu}\kappa_\nu - g_{\rho\nu}\kappa_\mu), \quad [P_\rho, L_{\mu\nu}] = g_{\rho\mu}P_\nu - g_{\rho\nu}P_\mu, \\
[L_{\mu\nu}, L_{\rho\sigma}] &= g_{\nu\rho}L_{\mu\sigma} + g_{\mu\sigma}L_{\nu\rho} - g_{\mu\rho}L_{\nu\sigma} - g_{\nu\sigma}L_{\mu\rho}, \quad \text{and the others vanish,}
\end{aligned} \tag{7}$$

where D is the dilatation, κ_μ is the special conformal, P_μ is the translation and $L_{\mu\nu}$ is the rotation and the boost generator.[3]. $g_{\mu\nu}$ is $(d + 1)$-dimensional flat spacetime metric, whose signature is chosen as $g_{\mu\nu} = \text{diag}(+, -, -, ..., -)$.

The symmetry generators are sorted into two different classes. The first class is a set of the generators for which the quantum fields transform in the spatial directions, and the second class is a set of those forcing them to transform in the temporal direction. The former provides a unitary transform of the fields on a given spacelike hypersurface, and the latter provides the dynamics of the fields.

Momentum density We first examine the generators having fields that transform in spatial directions. For this, we decompose these generators into spatial and temporal parts as

$$\begin{aligned}
P_\mu &\rightarrow P_s, P_0, \quad L_{\mu\nu} \rightarrow L_{st}, L_{0t}, \\
\kappa_\mu &\rightarrow \kappa_s, \kappa_0 \quad \text{and} \quad D \rightarrow D^{(s)} + D^{(t)},
\end{aligned} \tag{9}$$

where we have defined the spatial parts of the symmetry

[3] The generators are given by

$$D = x^\mu P_\mu, \quad L_{\mu\nu} = x_\mu P_\nu - x_\nu P_\mu, \quad \kappa_\mu = 2x_\mu x^\nu P_\nu - x^\nu x_\nu P_\mu \tag{8}$$

in terms of the translation generator, P_μ.

generators in terms of a momentum density K_s as

$$\begin{aligned} P_t &= \int K_t d^d x, \\ L_{rs} &= \int (x_r K_s - x_s K_r) d^d x, \\ D^{(s)} &= -\int x_s K_s d^d x, \\ \kappa_t &= \int (-2x_t x_r K_r + x_r x_r K_t) d^d x. \end{aligned} \quad (10)$$

To specify the field variables $V_t^{(1)}$ and $V_t^{(2)}$ and the momentum density in our vector theory, we introduce the variables ξ_s and η_s as

$$V_t^{(1)} = \eta_t,\ V_t^{(2)} = \xi_t,\ \text{and } K_t = \eta_u \xi_{u,t} - (\eta_u \xi_t)_{,u}, \quad (11)$$

where η_s and ξ_s form a canonical pair as

$$[\xi_t, \eta_s'] = \delta_{ts}\delta \ , \quad (12)$$

with δ_{ts} being Kronecker's delta and δ being d-dimensional delta function.

By using the canonical commutation relation of ξ_s and η_s, we obtain the transformation rules for the field variables as

$$\begin{aligned} [V_t^{(1)}, P_r] &= V_{t,r}^{(1)}, \\ [V_t^{(1)}, L_{rs}] &= x_r V_{t,s}^{(1)} - x_s V_{t,r}^{(1)} + (-\delta_{rt} V_s^{(1)} + \delta_{st} V_r^{(1)}), \\ [V_t^{(1)}, D^{(s)}] &= -x_s V_{t,s}^{(1)} + (\Delta_1 V_t^{(1)}), \\ [V_t^{(1)}, \kappa_s] &= -2x_s x_r V_{t,r}^{(1)} + x_r x_r V_{t,s}^{(1)} + (2\delta_{ts} x_r V_r^{(1)} - 2x_t V_s^{(1)} + 2\Delta_1 x_s V_t^{(1)}) \end{aligned} \quad (13)$$

and

$$\begin{aligned} [V_t^{(2)}, P_r] &= V_{t,r}^{(1)}, \\ [V_t^{(2)}, L_{rs}] &= x_r V_{t,s}^{(1)} - x_s V_{t,r}^{(1)} + (-\delta_{rt} V_s^{(1)} + \delta_{st} V_r^{(1)}), \\ [V_t^{(2)}, D^{(s)}] &= -x_s V_{t,s}^{(1)} + (\Delta_2 V_t^{(1)}), \\ [V_t^{(2)}, \kappa_s] &= -2x_s x_r V_{t,r}^{(2)} + x_r x_r V_{t,s}^{(2)} + (2\delta_{ts} x_r V_r^{(2)} - 2x_t V_s^{(2)} + 2\Delta_2 x_s V_t^{(2)}), \end{aligned} \quad (14)$$

where $\Delta_1 = d-1$ and $\Delta_2 = 1$, which are conformal dimensions of the field variables $V_t^{(1)}$ and $V_t^{(2)}$, respectively. From these, we can obtain the following relations:

$$\begin{aligned} [K_t, P_r] &= K_{t,r}, \\ [K_t, L_{rs}] &= x_r K_{t,s} - x_s K_{t,r} - \delta_{rt} K_s + \delta_{st} K_r, \\ [K_t, D^{(s)}] &= -x_r K_{t,r} + (\Delta_1 + \Delta_2 + 1) K_t \\ [K_t, \kappa_s] &= -2x_s x_r K_{t,r} + x_r x_r K_{t,s} + 2\delta_{st} x_r K_r + 2(\Delta_1 + \Delta_2 + 1) x_s K_t - 2x_t K_s, \end{aligned} \quad (15)$$

which are the commutation relations of the conformal algebra for the spacetime indices μ and ν to be restricted to $\mu, \nu = 1, 2..., d$.

Energy density To complete the conformal algebra, Eq. (7), we need to examine the temporal parts of the generators. To do this, we define a local quantity, the "energy density" U, and use it to express these generators as

$$\begin{aligned} P_0 &= \int U d^d x, \quad L_{t0} = \int x_t U d^d x, \\ D^{(t)} &= 0, \quad \kappa_0^{(t)} = \int x_s x_s U d^d x. \end{aligned} \quad (16)$$

This energy density is scalar under the spatial parts of the symmetry transforms, and we suppose that it has a conformal dimension Δ_E, so it might transform as

$$\begin{aligned} &[U, P_t] = U_{,t}, \quad [U, L_{st}] = x_s U_{,t} - x_t U_{,s}, \\ &[U, D^{(s)}] = -x_s U_{,s} + \Delta_E U, \\ &[U, \kappa_s] = -2x_s x_r U_{,r} + x_r x_r U_{,s} + 2\Delta_E x_s U. \end{aligned} \quad (17)$$

Such energy density commutation relations lead to

$$\begin{aligned} &[P_0, P_t] = 0, \quad [P_0, L_{st}] = 0, \quad [P_s, L_{t0}] = -\delta_{st} P_0, \\ &[L_{t0}, L_{rs}] = \delta_{ts} L_{r0} - \delta_{tr} L_{s0}, \quad [D^{(s)}, P_0] = -P_0, \\ &[D^{(s)}, L_{0t}] = 0, \quad [\kappa_r, L_{0t}] = \delta_{rt} \kappa_0, \quad [\kappa_0, L_{st}] = 0, \\ &[D^{(s)}, \kappa_0] = \kappa_0^{(t)}, \quad [\kappa_0, P_t] = -2L_{0t}, \\ &[\kappa_s, P_0] = -2L_{s0}, \quad [\kappa_0, \kappa_s] = 0, \end{aligned} \quad (18)$$

under the condition that **the conformal dimension of**

the energy density be given by

$$\Delta_E = d + 1. \quad (19)$$

These are the commutation relations between the temporal and the spatial parts of the generators.

Finally, we need the commutation relations between the temporal parts of the generators to complete our discussion. They are given by

$$\begin{aligned} [P_0, P_0] &= 0, \quad [L_{t0}, L_{s0}] = L_{st}, \quad [P_0, L_{0t}] = P_t, \quad (20) \\ [\kappa_0, L_{0t}] &= \kappa_t, \quad [\kappa_0, P_0] = -2D^{(s)}, \quad [\kappa_0, \kappa_0] = 0, \end{aligned}$$

These are translated to the following equations by using Eq.(16):

$$\int\int [U, U'] d^d x d^d x' = 0, \quad (21)$$

$$\int\int x_t x'_s [U, U'] d^d x d^d x' = \int (x_s K_t - x_t K_s) d^d x, \quad (22)$$

$$\int\int x_t [U, U'] d^d x d^d x' = \int K_t d^d x, \quad (23)$$

$$\int\int x_s x_s x'_t [U, U'] d^d x d^d x' = \int (2x_t x_s K_s - x_s x_s K_t) d^d x, \quad (24)$$

$$\int\int x_s x_s [U, U'] d^d x d^d x' = 2\int x_s K_s d^d x, \quad (25)$$

$$\int\int x_u x_u x'_s x'_s [U, U'] d^d x d^d x' = 0. \quad (26)$$

The remaining task is to find the commutation relation between the energy densities satisfying the above relations. We start with the most general form of the energy density commutation relation as Dirac suggested [8]. It is

$$[U, U'] = a\delta + b_r \delta_{,r} + c_{rs}\delta_{,rs} + d_{rst}\delta_{,rst} + ..., \quad (27)$$

where the coefficients in front of the δ-functions are functions of x_s only. If we switch U and U', based on their anti commuting natures, we have

$$\begin{aligned} [U', U] &= a\delta - b'_r \delta_{,r} + c'_{rs}\delta_{,rs} - d'_{rst}\delta_{,rst} + ... \quad (28) \\ &= a\delta - (b_r\delta)_{,r} + (c_{rs}\delta)_{,rs} - (d_{rst}\delta)_{,rst} + ... \\ &= \delta(a - b_{r,r} + c_{rs,rs} - d_{rst,rst} + ...) \\ &+ \delta_{,r}(-b_r + 2c_{ru,u} - 3d_{rsu,su} + ...) \\ &+ \delta_{,rs}(c_{rs} - 3d_{rsu,u} + ...). \end{aligned}$$

Because Eq. (27) and (28) add to zero, from that condition, we have

$$\begin{aligned} 0 &= 2a - b_{r,r} + c_{rs,rs} - d_{rst,rst} + ..., \quad (29) \\ 0 &= 2c_{rs,s} - 3d_{rst,st} + ..., \quad (30) \\ 0 &= 2c_{rs} - 3d_{rsu,u} + ... \quad (31) \\ &... \end{aligned}$$

Eq. (29) gives a solution for a as

$$a = \alpha_{r,r}, \quad \text{where} \quad 2\alpha_r = b_r - c_{rs,s} + d_{rst,st} - ..., \quad (32)$$

and Eq.(30) implies that $c_{ru,u}$ is, indeed, a second derivative. Then,

$$\begin{aligned} &\int (2\alpha_r - b_r) d^d x = 0 \quad \text{and} \quad (33) \\ &\int x_s(2\alpha_r - b_r) d^d x = 0, \\ &\text{because} \quad 2\alpha_r - b_r = -c_{rs,s} + d_{rst,st} - ... \\ &\rightarrow \text{(second derivative and higher)} \end{aligned}$$

By using these, we derive more useful relations as

$$\begin{aligned} \int [U, U'] d^d x' &= \alpha_{r,r}, \quad (34) \\ \int x'_s [U, U'] d^d x &= x_s \alpha_{r,r} - b_s. \quad (35) \end{aligned}$$

After all, we plug Eq. (27) into Eq. (22)-Eq. (24) to fix the coefficients of the δ-functions(and their derivatives) on the right-hand side of Eq. (27). The relation on Eq. (34) directly solves Eq. (21). Eq. (23) gives

$$\int K_t d^d x = \int x_t \alpha_{r,r} d^d x = \int \alpha_t d^d x = \frac{1}{2}\int b_t d^d x, \quad (36)$$

where we have used Eq. (34). From this, we get the most general form of the solutions α_r and β_r as

$$\alpha_t = K_t + \beta_{tr,r} + \zeta_{,t} \quad \text{and} \quad b_t = 2K_t + \bar{\beta}_{tr,r} + \bar{\zeta}_{,t}, \quad (37)$$

where β_t, $\bar{\beta}_t$, ζ and $\bar{\zeta}$ are arbitrary functions of x_s. Eq. (22) provides

$$\begin{aligned} &\int (x_s K_t - x_t K_s) d^d x = \int x_t(x_s \alpha_{u,u} - b_s) = \frac{1}{2}\int d^d x (x_s b_t - x_t b_s) \quad (38) \\ &= \frac{1}{2}\int d^d x (2x_s K_t - 2x_t K_s + x_s \bar{\beta}_{tr,r} - x_t \bar{\beta}_{sr,r} + x_s \bar{\zeta}_{,t} - x_t \bar{\zeta}_{,s}). \end{aligned}$$

This relation restricts $\bar{\beta}_{st}$ to be

$$\int (\bar{\beta}_{ts} - \bar{\beta}_{st}) d^d x = 0, \tag{39}$$

and similarly

$$\int (\beta_{ts} - \beta_{st}) d^d x = 0. \tag{40}$$

Next, we consider Eq. (24), which is given by

$$2\int x_s K_s d^d x = \int x_s x_s \alpha_{t,t} d^d x \tag{41}$$
$$= \int 2x_t (K_t + \beta_{tr,r} + \zeta_{,t}),$$

which provides the following:

$$\int (\beta_{tt} + d\zeta) d^d x = 0. \tag{42}$$

Moreover, Eq. (24) becomes

$$\int (2x_t x_s K_s - x_s x_s K_t) d^d x = \int x_s x_s (x_t \alpha_{r,r} - b_t) \tag{43}$$
$$= \int (2x_t x_s K_s - x_s x_s K_t) d^d x + \int \{x_t(2\beta_{rr} + 2(d+2)\zeta - 2\bar{\zeta}) + 2x_s(\beta_{st} + \beta_{ts} - \bar{\beta}_{ts})\} d^d x.$$

Then, from this, we get

$$\int \{x_t(2\beta_{rr} + 2(d+2)\zeta - 2\bar{\zeta}) + 2x_s(\beta_{st} + \beta_{ts} - \bar{\beta}_{ts})\} d^d x = 0. \tag{44}$$

Finally we examine Eq. (26). Eq. (37) satisfies Eq. (26) under the condition that

$$\int \{2x_t x_u (2\beta_{ut} - \bar{\beta}_{ut}) + x_s x_s (2\beta_{tt} - \bar{\beta}_{tt} + 2(2+d)\zeta - (2+d)\bar{\zeta} + c_{uu})\} = 0. \tag{45}$$

Minimal solutions for the coefficients in front of the δ-functions on the right-hand side of Eq. (27) are given by

$$2\alpha_t = b_t = 2K_t \quad \text{and} \quad \beta_{st} = \bar{\beta}_{st} = \zeta = \bar{\zeta} = c_{rs\cdots} = 0. \tag{46}$$

Therefore, the minimal solution of the commutation relation between the energy densities that satisfies the conformal algebra becomes

$$[U, U'] = K_{t,t}\delta + 2K_t \delta_{,t}. \tag{47}$$

III. CONFORMAL INVARIANCE AND 4-D VECTOR THEORIES

The main result of the last section is Eq. (47). Once we quantize our vector field theory as in Eq. (3) and define the momentum and the energy densities as in Eq. (4), then

$$[U, U'] = -\varepsilon \delta^{st}(K_s + K'_s)\delta_{,t} \tag{48}$$

is satisfied where $\varepsilon = 0$ or -1 [7]. The conformal algebra is consistently constructed from the energy density only when the conformal dimension of the energy density is $\Delta_E = 4$ in 4-dimensional spacetime. The simplest candidate for this is $U = h$, because $\mathcal{B}^a_s$ has conformal dimension 2.

The way of choosing the energy density is to find $U = f(h, v)$ as a solution to Eq.(6) under a constraint that its conformal dimension be 4. The field strength $\mathcal{B}^a_s$ has conformal dimension 2, then, h has conformal dimension 4, and v has conformal dimension 8.

The simplest solution is $U_1 = h = \frac{1}{2}(\vec{B}^2 + \vec{E}^2)$, which is nothing but free Maxwell theory. Another thoery satisfying Eq. (6) is $U_2 = \sqrt{v} = \sqrt{(\vec{B} \times \vec{E}) \cdot (\vec{B} \times \vec{E})}$. The two different theories can be interpreted as certain limits of Born-Infeld electromagnetic theory (BI-theory). In [7], the author obtained BI-theory as $U_3 = \sqrt{1 + 2h + v} - 1$. This theory is not conformal at all, but its strong/weak field limits are conformal. A well-known fact is that free Maxwell theory is the weak field limit of BI-theory. The opposite limit, the large amplitude limit of the BI-theory leads to U_2. These theories correspond to certain IR and UV fixed points of the BI-thoery, respectively.

ACKNOWLEDGMENTS

We would like to thank Alfred D. Shapere for the useful discussion. J.H.O thanks his $\mathcal{W}.J$. This work is supported only by the research fund of Hanyang University (HY-2013).

REFERENCES

[1] S. Deser and C. Teitelboim, Phys. Rev. D **13**, 1592 (1976); S. Deser, J. Phys. A **15**, 1053 (1982).
[2] M. Henneaux and C. Teitelboim, Phys. Rev. D **71**, 024018 (2005) [gr-qc/0408101].
[3] S. Deser and D. Seminara, Phys. Lett. B **607**, 317 (2005) [hep-th/0411169].
[4] S. Deser and A. Waldron, Phys. Rev. D **87**, 087702 (2013) [arXiv:1301.2238 [hep-th]].
[5] S. Deser and D. Seminara, Phys. Rev. D **71**, 081502 (2005) [hep-th/0503030].
[6] D. P. Jatkar and J-H. Oh, JHEP **1208**, 077 (2012) [arXiv:1203.2106 [hep-th]].
[7] C. Bunster and M. Henneaux, Phys. Rev. Lett. **110**, 011603 (2013) [arXiv:1208.6302 [hep-th]].
[8] P. A. M. Dirac, Rev. Mod. Phy **34**, 592 (1962).