## Observation of $X(\mathbf{3 8 7 2})$ in $B \rightarrow X(\mathbf{3 8 7 2}) K \pi$ decays

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We report the first observation of $B^{0} \rightarrow X(3872)\left(K^{+} \pi^{-}\right)$and evidence for $B^{+} \rightarrow X(3872)\left(K^{0} \pi^{+}\right)$. We measure the product of branching fractions for the former to be $\mathcal{B}\left(B^{0} \rightarrow X(3872)\left(K^{+} \pi^{-}\right)\right) \times$ $\mathcal{B}\left(X(3872) \rightarrow J / \psi \pi^{+} \pi^{-}\right)=(7.9 \pm 1.3($ stat $) \pm 0.4($ syst $)) \times 10^{-6}$ and find that $B^{0} \rightarrow X(3872) K^{*}(892)^{0}$ does not dominate the $B^{0} \rightarrow X(3872) K^{+} \pi^{-}$decay mode. We also measure $\mathcal{B}\left(B^{+} \rightarrow X(3872)\left(K^{0} \pi^{+}\right)\right) \times$ $\mathcal{B}\left(X(3872) \rightarrow J / \psi \pi^{+} \pi^{-}\right)=(10.6 \pm 3.0($ stat $) \pm 0.9($ syst $)) \times 10^{-6}$. This study is based on the full data sample of $711 \mathrm{fb}^{-1}\left(772 \times 10^{6} B \bar{B}\right.$ pairs) collected at the $\Upsilon(4 S)$ resonance with the Belle detector at the KEKB collider.

About a decade ago, the Belle Collaboration discovered the $X(3872)$ state [1] in the exclusive reconstruction of $B^{+} \rightarrow X(3872)\left(\rightarrow J / \psi \pi^{+} \pi^{-}\right) K^{+}$[2]. Considerable effort by both experimentalists and theorists has been invested to clarify its nature. As a result, we know precisely its mass ( $3871.69 \pm 0.17$ ) $\mathrm{MeV} / c^{2}$ [3], have a stringent limit on its width (less than 1.2 MeV at $90 \%$ confidence level) [4] and have a definitive $J^{P C}$ assignment of $1^{++}$[5]. The $X(3872)$ has been observed to decay to several other final states:
$J / \psi \gamma[6], \psi^{\prime} \gamma[7], J / \psi \pi^{+} \pi^{-} \pi^{0}$ [8] and $D^{0} \bar{D}^{* 0}$ [9,10]. The proximity of its mass to the $D^{0}-\bar{D}^{* 0}$ threshold, along with its measured partial decay rates, suggests that it be a loosely bound "molecule" of $D^{0}$ and $\bar{D}^{* 0}$ mesons [11] or an admixture of $D^{0} \bar{D}^{* 0}$ with a charmonium ( $c \bar{c}$ ) state [11,12]. Some authors have advanced a QCD-tetraquark interpretation for the $X(3872)$, and predict the existence of charged- and $C$-odd partner states that are nearby in mass [13]. Experimental searches for charged-[4,14] and $C$-odd
[15,16] partners report negative results. However, since these searches are restricted to states with narrow total widths, the published limits may not apply if the partner states access more decay channels and are thus broader. More experimental information on the production and decays of the $X(3872)$ will shed additional light on its nature.

In this paper, we present the results of searches for $X(3872)$ production via the $B^{0} \rightarrow X(3872) K^{+} \pi^{-}$and $B^{+} \rightarrow X(3872) K_{S}^{0} \pi^{+}$decay modes, where the $X(3872)$ decays to $J / \psi \pi^{+} \pi^{-}$. The study is based on $711 \mathrm{fb}^{-1}$ of data containing $772 \times 10^{6} B \bar{B}$ events collected with the Belle detector [17] at the KEKB $e^{+} e^{-}$asymmetric-energy collider [18] operating at the $\Upsilon(4 S)$ resonance. In addition to selecting $B \rightarrow X(3872) K \pi$ signal events, the same selection criteria isolate a rather pure sample of $B \rightarrow$ $\psi^{\prime} K \pi$ events that are used for calibration.

The Belle detector is a large solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrellike arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter (ECL) composed of $\mathrm{CsI}(\mathrm{Tl})$ crystals. All these detector components are located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux return located outside the coil is instrumented to detect $K_{L}^{0}$ mesons and to identify muons (KLM). The detector is described in detail elsewhere [17].

Monte Carlo (MC) samples are generated for each decay mode using EvtGen [19], and radiative effects are taken into account using the Рнотоs [20] package. The detector response is simulated using GEANT3 [21].

Charged tracks are required to originate from the interaction point (IP). To identify charged kaons and pions, we use a likelihood ratio $\mathcal{R}_{K / \pi}=\mathcal{L}_{K} /\left(\mathcal{L}_{\pi}+\mathcal{L}_{K}\right)$, where the kaon (pion) likelihood $\mathcal{L}_{K}\left(\mathcal{L}_{\pi}\right)$ is calculated using ACC, TOF and CDC measurements. For the prompt charged kaon (pion), we apply the criterion $\mathcal{R}_{K / \pi}\left(\mathcal{R}_{\pi / K}\right)>$ 0.6 . Here, the kaon (pion) identification efficiency is $93 \%$ ( $95 \%$ ) while the probability of misidentifying a pion as a kaon (kaon as a pion) is $8 \%(7 \%)$. For the pion daughters from $\psi^{\prime}$ or $X(3872)$, we impose $\mathcal{R}_{\pi / K}>0.4$; the corresponding pion identification efficiency is $99 \%$ and the misidentification probability is $8 \%$. Candidates for the $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$decay are formed from pairs of oppositely charged tracks having an invariant mass between 488 and $506 \mathrm{MeV} / c^{2}\left( \pm 4 \sigma\right.$ around the nominal mass of $\left.K_{S}^{0}\right)$. The $K_{S}^{0}$ candidate is also required to satisfy the criteria described in Ref. [22] to ensure that its decay vertex is displaced from the IP. A track is identified as a muon if its muon likelihood ratio is greater than 0.1 , where the muon and hadron likelihoods are calculated by the track penetration depth and hit scatter in the muon detector (KLM).

An electron track is identified with an electron likelihood greater than 0.01 , where the electron likelihood is calculated by combining $d E / d x$ from the CDC, the ratio of the energy deposited in the ECL and the momentum measured by the SVD and the CDC, the shower shape in the ECL, ACC information and the position matching between the shower and the track. With the above selections, the muon (electron) identification efficiency is above $90 \%$ and the hadron fake rate is less than $4 \%(0.5 \%)$.

We reconstruct $J / \psi$ mesons in the $\ell^{+} \ell^{-}$decay channel ( $\ell=e$ or $\mu$ ) and include bremsstrahlung photons that are within 50 mrad of either the $e^{+}$or $e^{-}$tracks [hereinafter denoted as $\left.e^{+} e^{-}(\gamma)\right]$. The invariant mass of the $J / \psi$ candidate is required to satisfy $3.00 \mathrm{GeV} / c^{2}<M_{e^{+} e^{-}(\gamma)}<$ $3.13 \mathrm{GeV} / c^{2}$ or $3.06 \mathrm{GeV} / c^{2}<M_{\mu^{+} \mu^{-}}<3.13 \mathrm{GeV} / c^{2}$ (with a distinct lower value accounting for the residual bremsstrahlung in the electron mode). A mass- and vertexconstrained fit is performed to the selected $J / \psi$ candidate to improve its momentum resolution. The $J / \psi$ candidate is then combined with a $\pi^{+} \pi^{-}$pair to form an $X(3872)\left(\psi^{\prime}\right)$ candidate whose mass must satisfy $3.82 \mathrm{GeV} / c^{2}<M_{J / \psi \pi \pi}<3.92 \mathrm{GeV} / c^{2}\left(3.64 \mathrm{GeV} / c^{2}<\right.$ $\left.M_{J / \psi \pi \pi}<3.74 \mathrm{GeV} / c^{2}\right)$. The dipion mass must also satisfy $\quad M_{\pi \pi}>M_{J / \psi \pi \pi}-\left(m_{J / \psi}+0.2 \mathrm{GeV} / c^{2}\right)$, where $m_{J / \psi}$ is nominal mass. This criterion corresponds to $M_{\pi \pi}>$ $575(389) \mathrm{MeV} / c^{2}$ for the $X(3872)\left(\psi^{\prime}\right)$ mass region and reduces significantly the combinatorial background [4] while also flattening the background shape distribution in $M_{J / \psi \pi \pi}$. To suppress the background from $e^{+} e^{-} \rightarrow q \bar{q}$ (where $q=u, d, s, c$ ) continuum events, we require $R_{2}<0.4$, where $R_{2}$ is the ratio of the second- to zerothorder Fox-Wolfram moments [23].

To reconstruct a neutral (charged) $B$ meson candidate, a $K^{+} \pi^{-}\left(K_{S}^{0} \pi^{+}\right)$candidate is combined with the $X(3872)$ or $\psi^{\prime}$ candidate. We select $B$ candidates using two kinematic variables: the energy difference $\Delta E=E_{B}-E_{\text {beam }}$ and the beam-energy constrained mass $M_{\mathrm{bc}}=$ $\left(\sqrt{E_{\text {beam }}^{2}-p_{B}^{2} c^{2}}\right) / c^{2}$, where $E_{\text {beam }}$ is the beam energy and $E_{B}$ and $p_{B}$ are the energy and magnitude of momentum, respectively, of the candidate $B$ meson, all calculated in the $e^{+} e^{-}$center-of-mass (CM) frame. Only $B$ candidates having $M_{\mathrm{bc}}>5.27 \mathrm{GeV} / c^{2}$ and $|\Delta E|<0.1 \mathrm{GeV}$ are retained for further analysis. After all selection criteria, approximately $35 \%$ of events have multiple $B$ candidates. For an event with more than one $B$ candidate, we select the candidate having the smallest value of

$$
\begin{equation*}
\chi^{2}=\frac{\left(M_{\mathrm{bc}}-5.2792 \mathrm{GeV} / c^{2}\right)^{2}}{\sigma_{M_{\mathrm{bc}}}^{2}}+\frac{\chi_{B}^{2}}{n d f} \tag{1}
\end{equation*}
$$

where $\sigma_{M_{\mathrm{bc}}}$ is the $M_{\mathrm{bc}}$ resolution (estimated to be $2.925 \mathrm{MeV} / c^{2}$ from a fit to $B^{0} \rightarrow \psi^{\prime} K^{+} \pi^{-}$events), $\chi_{B}^{2}$ is the quality of the vertex fit of all charged tracks (excluding
the $K_{S}^{0}$ daughters), $n d f=(2 N-3)$ in this fit and $N$ is the number of fitted tracks. The correct candidate is selected in about $60 \%$ of the $B \rightarrow X(3872) K \pi$ events with multiple entries.

To extract the signal yield of $B \rightarrow X(3872)$ $\left(\rightarrow J / \psi \pi^{+} \pi^{-}\right) K \pi$, we perform a two-dimensional (2D) unbinned extended maximum likelihood fit to the $\Delta E$ and $M_{J / \psi \pi \pi}$ distributions. For the signal, the $\Delta E$ distribution is parametrized by the sum of a Crystal Ball [24] and a Gaussian function while the $M_{J / \psi \pi \pi}$ distribution is modeled using the sum of two Gaussians having a common mean. The 2 D probability distribution function (PDF) is a product of the individual one-dimensional PDFs, as no sizable correlation is found.

The main background contribution in $B \rightarrow\left(J / \psi \pi^{+} \pi^{-}\right) K \pi$ is expected to arise from inclusive $B$ decays to $J / \psi$, which is confirmed by the low background found in the $J / \psi$ mass sidebands in the data. To study this background, we use a large Monte Carlo sample of $B \rightarrow$ $J / \psi X$ events corresponding to 100 times the integrated luminosity of the data sample, and we find that few backgrounds are peaking in the $M_{J / \psi \pi \pi}$ distribution (nonpeaking in the $\Delta E$ distribution) and vice versa. The remaining backgrounds are combinatorial in nature and are flat in both distributions. This background is parametrized by first-order Chebyshev polynomial.

For the $B^{0} \rightarrow X(3872) K^{+} \pi^{-}$decay mode, a 2 D fit is performed. The mean and resolution of $M_{J / \psi \pi \pi}$ and $\Delta E$ are fixed for the $X(3872)$ region from signal MC samples after being rescaled from the results of the $B^{0} \rightarrow \psi^{\prime} K^{+} \pi^{-}$decay mode. Further, we correct the mean of a Gaussian function for the $M_{J / \psi \pi \pi}$ distribution because of a difference in the shift of the $\psi^{\prime}$ and $X(3872)$ reconstructed and generated masses as seen in MC samples. The tail parameters, $\alpha$ and $n$ of the Crystal Ball (CB) function, and the ratios of the CB's area and width to the corresponding quantities of the Gaussian component are fixed according to the signal MC simulation. The peaking components can be divided into two categories: the one peaking in $M_{J / \psi \pi \pi}$ but nonpeaking in $\Delta E$ that comes from the $B \rightarrow X(3872) X^{\prime}$ decays where the $X(3872)$ decays in $J / \psi \pi^{+} \pi^{-}$(here $X^{\prime}$ can be any particle), and the other peaking in $\Delta E$ but nonpeaking in $M_{J / \psi \pi \pi}$ which comes from a $B$ with the same final state where $J / \psi \pi^{+} \pi^{-}$is not from a $X(3872)$. The peaking background in $\Delta E\left(M_{J / \psi \pi \pi}\right)$ is found to have the same resolution as that of the signal, so the PDF is chosen to be the same as the signal PDF, while the nonpeaking background in the other dimension is parametrized with a firstorder Chebyshev polynomial. Parameters (slopes) of the background PDFs, which are of nonpeaking or combinatorial nature, are allowed to vary in the fit. The fits are validated on full simulated experiments and no significant bias is seen. Figure 1 (top) shows the signal-enhanced projection plots for the $B^{0} \rightarrow X(3872)\left(K^{+} \pi^{-}\right)$decay


FIG. 1 (color online). Projections of the ( $\Delta E, M_{J / \psi \pi \pi}$ ) fit for the $B^{0} \rightarrow X(3872)\left(\rightarrow J / \psi \pi^{+} \pi^{-}\right) K^{+} \pi^{-}$decay mode (top) and the $B^{0} \rightarrow \psi^{\prime}\left(\rightarrow J / \psi \pi^{+} \pi^{-}\right) K^{+} \pi^{-}$decay mode (bottom): (a) $\Delta E$ distribution for $3.860 \mathrm{GeV} / c^{2}<M_{J / \psi \pi \pi}<3.881 \mathrm{GeV} / c^{2}$, (b) $M_{J / \psi \pi \pi}$ distribution for $-11 \mathrm{MeV}<\Delta E<8 \mathrm{MeV}$, (c) $\Delta E$ distribution for $3.675 \mathrm{GeV} / c^{2}<M_{J / \psi \pi \pi}<3.695 \mathrm{GeV} / c^{2}$, and (d) $M_{J / \psi \pi \pi}$ distribution for $-11 \mathrm{MeV}<\Delta E<8 \mathrm{MeV}$. The curves show the signal (red long-dashed curve) and the background components (black dash-dotted line for the component peaking in $M_{J / \psi \pi \pi}$ but nonpeaking in $\Delta E$, green dashed line for the one peaking in $\Delta E$ but nonpeaking in $M_{J / \psi \pi \pi}$, and magenta long dash-dotted line for combinatorial background) as well as the overall fit (blue solid curve).
mode. The result of the fit and branching fractions derived are listed in Table I. We find a clear signal for $B^{0} \rightarrow$ $X(3872) K^{+} \pi^{-}$of $116 \pm 19$, signal events corresponding to a significance [25] (including systematic uncertainties related to the signal yield as mentioned in Table II) of 7.0 standard deviations $(\sigma)$, and measure the product of branching fractions to be $\mathcal{B}\left(B^{0} \rightarrow X(3872) K^{+} \pi^{-}\right) \times$ $\mathcal{B}\left(X(3872) \rightarrow J / \psi \pi^{+} \pi^{-}\right)=(7.9 \pm 1.3($ stat $) \pm 0.4($ syst $)) \times$ $10^{-6}$. The efficiency used for estimating the branching fraction is calculated from the individual efficiencies and the fractions of the different components obtained in the ( $K^{+} \pi^{-}$) mass, as explained below. The statistical significance is estimated using the value of $\sqrt{-2 \ln \left(\mathcal{L}_{0} / \mathcal{L}_{\text {max }}\right)}$ where $\mathcal{L}_{\text {max }}\left(\mathcal{L}_{0}\right)$ denotes the likelihood value when the yield is allowed to vary (fixed to zero).

The above fit is validated on the calibration mode $B^{0} \rightarrow \psi^{\prime} K^{+} \pi^{-}$. In contrast to the $X(3872)$ mass region, the mean and resolution in both distributions $\left(M_{J / \psi \pi \pi}\right.$ and $\Delta E$ ) are allowed to vary in the fit. Figure 1 (bottom) shows the signal-enhanced projection plots for the $B^{0} \rightarrow$ $\psi^{\prime}\left(K^{+} \pi^{-}\right)$decay mode. We measure the branching fraction to be $\mathcal{B}\left(B^{0} \rightarrow \psi^{\prime} K^{+} \pi^{-}\right)=(5.79 \pm 0.14($ stat $)) \times 10^{-4}$, consistent with an independent Belle result based on a Dalitz-plot analysis [26].

TABLE I. Signal yield (Y) from the fit, weighted efficiency $(\epsilon)$ after particle-identification correction, significance $(\Sigma)$ and measured $\mathcal{B}$ for $B^{0} \rightarrow X(3872) K^{+} \pi^{-}$and $B^{+} \rightarrow X(3872) K^{0} \pi^{+}$. The first (second) uncertainty represents a statistical (systematic) contribution.

| Decay mode | Y | $\epsilon(\%)$ | $\Sigma(\sigma)$ | $\mathcal{B}(B \rightarrow X(3872) K \pi) \times \mathcal{B}\left(X(3872) \rightarrow J / \psi \pi^{+} \pi^{-}\right)$ |
| :--- | :---: | :---: | :---: | :---: |
| $B^{0} \rightarrow X(3872) K^{+} \pi^{-}$ | $116 \pm 19$ | 15.99 | 7.0 | $(7.9 \pm 1.3 \pm 0.4) \times 10^{-6}$ |
| $B^{+} \rightarrow X(3872) K^{0} \pi^{+}$ | $35 \pm 10$ | 10.31 | 3.7 | $(10.6 \pm 3.0 \pm 0.9) \times 10^{-6}$ |

Further, to determine the contribution of the $K^{*}(892)$ and other intermediate states, we perform a 2 D fit to $\Delta E$ and $M_{J / \psi \pi \pi}$ in each bin of $M_{K \pi}\left(0.1 \mathrm{GeV} / c^{2}\right.$ wide bins of $M_{K \pi}$ in the range $[0.62,1.42] \mathrm{GeV} / c^{2}$ ), which provides a back-ground-subtracted $M_{K \pi}$ signal distribution. All parameters of the signal PDFs for $M_{J / \psi \pi \pi}$ and $\Delta E$ distributions are fixed from the previous 2D fit to all events. We perform a $\chi^{2}$ fit to the $M_{K \pi}$ distribution using $K^{*}(892)^{0}$ and $\left(K^{+} \pi^{-}\right)_{\mathrm{NR}}$ components, which are histogram PDFs obtained from MC samples. Note that the $B^{0} \rightarrow X(3872) K_{2}{ }^{*}(1430)^{0}$ decay is kinematically suppressed. We do not consider the interference between the $K^{*}(892)$ and nonresonant component since the number of candidates is not large enough to make a full amplitude analysis. The resulting fit result is shown in Fig. 2(a). We obtain $38 \pm 14(82 \pm 21)$ signal events for the $B^{0} \rightarrow X(3872) K^{*}(892)^{0}\left(B^{0} \rightarrow X(3872)\left(K^{+} \pi^{-}\right)_{\mathrm{NR}}\right)$ decay mode, whose sum is consistent with the total yield obtained from the global fit. This corresponds to a $3.0 \sigma$ significance (including systematic uncertainties related to the signal yield) for the $B^{0} \rightarrow X(3872)\left(\rightarrow J / \psi \pi^{+} \pi^{-}\right) K^{*}(892)^{0}$ decay mode, and a product of branching fractions of $\mathcal{B}\left(B^{0} \rightarrow\right.$ $\left.X(3872) K^{*}(892)^{0}\right) \times \mathcal{B}\left(X(3872) \rightarrow J / \psi \pi^{+} \pi^{-}\right)=(4.0 \pm$ 1.5 (stat) $\pm 0.3($ syst $)) \times 10^{-6}$. The ratio of branching fractions is

$$
\begin{align*}
& \frac{\mathcal{B}\left(B^{0} \rightarrow X(3872) K^{*}(892)^{0}\right) \times \mathcal{B}\left(K^{*}(892)^{0} \rightarrow K^{+} \pi^{-}\right)}{\mathcal{B}\left(B^{0} \rightarrow X(3872) K^{+} \pi^{-}\right)} \\
& \quad=0.34 \pm 0.09(\text { stat }) \pm 0.02(\text { syst }) . \tag{2}
\end{align*}
$$

TABLE II. Summary of the systematic uncertainties in percent.

|  | $X(3872)$ | $X(3872)$ |
| :--- | :---: | :---: |
| Source | $K^{+} \pi^{-}$ | $K^{0} \pi^{+}$ |
| Lepton ID | 3.4 | 3.4 |
| Kaon ID | 1.1 | $\ldots$ |
| Pion ID | 2.5 | 3.2 |
| PDF modeling | +1.8 | +4.2 |
| Tracking efficiency | -1.1 | 2.9 |
| $K_{S}^{0}$ reconstruction | $\ldots$ | 0.7 |
| $N_{B \bar{B}}$ | 1.4 | 1.4 |
| Secondary $\mathcal{B}$ | 0.4 | 0.4 |
| Efficiency | 0.6 | 1.0 |
| Fit bias | 0.6 | 3.1 |
| Total | 5.4 | 8.0 |

In the above ratio, all systematic uncertainties cancel except those from the PDF model, fit bias and efficiency variation over the Dalitz distribution. We generate pseudoexperiments to estimate the significance of the $\chi^{2}$ fit.

The $B^{0} \rightarrow \psi^{\prime} K^{+} \pi^{-}$mode is analyzed with the same procedure, with $0.051 \mathrm{GeV} / c^{2}$ wide bins, due to the copious yield, and in the $M_{K \pi}$ range $[0.600$, $1.569] \mathrm{GeV} / c^{2}$. We perform a $\chi^{2}$ fit to the obtained $M_{К \pi}$ signal distribution again to extract the contributions of the $K \pi$ nonresonant and resonant components. For this purpose, we use histogram PDFs obtained from MC samples of several possible components of the $\left(K^{+} \pi^{-}\right)$system: $K^{*}(892)^{0}, K_{2}^{*}(1430)^{0}$ and nonresonant $K^{+} \pi^{-}\left(\left(K^{+} \pi^{-}\right)_{\mathrm{NR}}\right)$; in the last case, $B^{0} \rightarrow \psi^{\prime}\left(K^{+} \pi^{-}\right)_{\mathrm{NR}}$ is generated uniformly in phase space. The fit result is shown in Fig. 2(b). The $K^{*}(892)$ dominates and we measure $\mathcal{B}\left(B^{0} \rightarrow\right.$ $\left.\psi^{\prime} K^{*}(892)^{0}\right)=(5.88 \pm 0.18($ stat $)) \times 10^{-4}$, which is consistent with the world average [3].

In contrast to $B^{0} \rightarrow \psi^{\prime}\left(K^{+} \pi^{-}\right)$[where the ratio of branching fractions is $0.68 \pm 0.01$ (stat) $], B^{0} \rightarrow X(3872) K^{*}(892)^{0}$ does not dominate in the $B^{0} \rightarrow X(3872) K^{+} \pi^{-}$.

We also investigate the decays $B^{+} \rightarrow X(3872)$ $\left(\rightarrow J / \psi \pi^{+} \pi^{-}\right)\left(K^{0} \pi^{+}\right)$. The PDFs of $\Delta E$ and $M_{J / \psi \pi \pi}$ are the same as those for the neutral $B$ meson decay mode. The projections of the 2D fit for $B^{+} \rightarrow X(3872)(\rightarrow$ $\left.J / \psi \pi^{+} \pi^{-}\right)\left(K^{0} \pi^{+}\right)$in the signal-enhanced regions are


FIG. 2 (color online). Fit to the background-subtracted $M_{K \pi}$ distribution: (a) for the $B^{0} \rightarrow X(3872)\left(K^{+} \pi^{-}\right)$decay mode, the curves show the $B^{0} \rightarrow X(3872) K^{*}(892)^{0}$ (red long-dashed lines), $B^{0} \rightarrow X(3872)\left(K^{+} \pi^{-}\right)_{\mathrm{NR}}$ (green dot-dashed lines), as well as the overall fit (blue solid lines). (b) for the $B^{0} \rightarrow \psi^{\prime}\left(K^{+} \pi^{-}\right)$decay mode, the curves show the $B^{0} \rightarrow \psi^{\prime} K^{*}(892)^{0}$ (red long-dashed lines), $B^{0} \rightarrow \psi^{\prime}\left(K^{+} \pi^{-}\right)_{N R} \quad$ (green dot-dashed lines), $B^{0} \rightarrow$ $\psi^{\prime} K_{2}^{*}(1430)^{0}$ (magenta dashed lines) as well as the overall fit (blue solid lines).


FIG. 3 (color online). Projections of the $\left(\Delta E, M_{J / \psi \pi \pi}\right)$ fit for the $B^{ \pm} \rightarrow X(3872)\left(\rightarrow J / \psi \pi^{+} \pi^{-}\right) K_{S}^{0} \pi^{ \pm}$decay mode (top) and for the $B^{ \pm} \rightarrow \psi^{\prime}\left(\rightarrow J / \psi \pi^{+} \pi^{-}\right) K_{S}^{0} \pi^{ \pm}$decay mode (bottom): (a) $\Delta E$ distribution for $3.859 \mathrm{GeV} / c^{2}<M_{J / \psi \pi \pi}<3.882 \mathrm{GeV} / c^{2}$, (b) $M_{J / \psi \pi \pi}$ distribution for $-11 \mathrm{MeV}<\Delta E<9 \mathrm{MeV}$, (c) $\Delta E$ distribution for $3.675 \mathrm{GeV} / c^{2}<M_{J / \psi \pi \pi}<3.695 \mathrm{GeV} / c^{2}$, and
(d) $M_{J / \psi \pi \pi}$ distribution for $-11 \mathrm{MeV}<\Delta E<9 \mathrm{MeV}$. The curves show the signal (red long-dashed curves: and the background components (black dash-dotted curves for the component peaking in $M_{J / \psi \pi \pi}$ but nonpeaking in $\Delta E$, green dashed lines for the one peaking in $\Delta E$ but nonpeaking in $M_{J / \psi \pi \pi}$, and magenta long dash-dotted lines for combinatorial background) as well as the overall fit (blue solid curves).
shown in Figs. 3(a) and 3(b). We find $35 \pm 10$ events for the $B^{+} \rightarrow X(3872)\left(\rightarrow J / \psi \pi^{+} \pi^{-}\right)\left(K^{0} \pi^{+}\right)$decay mode, corresponding to a $3.7 \sigma$ significance (including systematic uncertainties). The product of branching fractions is $\mathcal{B}\left(B^{+} \rightarrow X(3872) K^{0} \pi^{+}\right) \times \mathcal{B}\left(X(3872) \rightarrow J / \psi \pi^{+} \pi^{-}\right)=$ $(10.6 \pm 3.0($ stat $) \pm 0.9($ syst $)) \times 10^{-6}$. The above fit is validated for the $\psi^{\prime}$ mass region. The projections of the 2D fit for $B^{+} \rightarrow \psi^{\prime}\left(\rightarrow J / \psi \pi^{+} \pi^{-}\right)\left(K^{0} \pi^{+}\right)$in the signalenhanced regions are shown in Figs. 3(c) and 3(d). The branching fraction for $B^{+} \rightarrow \psi^{\prime}\left(\rightarrow J / \psi \pi^{+} \pi^{-}\right)\left(K^{0} \pi^{+}\right)$is $(6.00 \pm 0.28($ stat $)) \times 10^{-4}$, while the world average of this quantity is $(5.88 \pm 0.34) \times 10^{-4}$.

Equal production of neutral and charged $B$ meson pairs in the $\Upsilon(4 S)$ decay is assumed. Secondary branching fractions used for calculation of $\mathcal{B}$ are taken from Ref. [3]. Systematic uncertainties are summarized in Tables II and III. A correction for small differences in the signal detection efficiency between signal MC events and data due to lepton, kaon and pion identification differences is applied; samples of $J / \psi \rightarrow \ell^{+} \ell^{-}$and $D^{*+} \rightarrow$ $D^{0}\left(\rightarrow K^{-} \pi^{+}\right) \pi^{+}$decays are used to estimate this correction. The uncertainties on these corrections are included in the systematic error. The uncertainty due to the fitting model is obtained by varying all fixed parameters by $\pm 1 \sigma$

TABLE III. Summary of systematic uncertainties (in percent) used for the $M_{K \pi}$ background-subtracted fit in $B^{0} \rightarrow$ $X(3872) K^{+} \pi^{-}$.

| Source | $X(3872) K^{*}(892)^{0}$ |
| :--- | :---: |
| Lepton ID | 3.4 |
| Kaon ID | 1.1 |
| Pion ID | 2.6 |
| PDF modeling | -1.5 |
| Tracking efficiency | 2.1 |
| $N_{B \bar{B}}^{+1.4}$ | 1.4 |
| Secondary $\mathcal{B}$ | 0.4 |
| MC statistics | 0.2 |
| Fit bias | 4.6 |
| Total | 7.0 |

and considering the corresponding change in the yield as the systematic error. The uncertainties due to tracking efficiency, $K_{S}^{0}$ reconstruction and $N_{B \bar{B}}$ are estimated to be $0.35 \%$ per track, $0.7 \%$ and $1.4 \%$, respectively. The systematic uncertainty due to secondary branching fractions is included. The systematic uncertainty for the efficiency arises from the limited MC statistics, and the efficiency variation over the Dalitz distribution is also considered. Small biases in the fitting procedure, estimated in the ensemble study, are also considered as a source of systematic uncertainty. For this study we perform a fit to 100 pseudoexperiments after embedding signal events obtained from MC samples to each inclusive MC sample. All the above stated systematic uncertainties are added in quadrature and result in a total systematic uncertainty of $5.4 \%, 8.0 \%, 7.0 \%$ for $B^{0} \rightarrow X(3872) K^{+} \pi^{-}, B^{+} \rightarrow$ $X(3872) K_{S}^{0} \pi^{+}$and $B^{0} \rightarrow X(3872) K^{*}(892)^{0}$, respectively.

In summary, we report the first observation of the $X(3872)$ in the decay $B^{0} \rightarrow X(3872) K^{+} \pi^{-}, X(3872) \rightarrow$ $J / \psi \pi^{+} \pi^{-}$. The result for the $X(3872)$, where $B^{0} \rightarrow$ $X(3872) K^{*}(892)^{0}$ does not dominate the $B^{0} \rightarrow X(3872)$ ( $K^{+} \pi^{-}$) decay, is in marked contrast to the $\psi^{\prime}$ case. We have checked for a structure in the $X(3872) \pi$ and $X(3872) K$ invariant masses and found no evident peaks. We measure $\mathcal{B}\left(B^{0} \rightarrow X(3872)\left(K^{+} \pi^{-}\right)\right) \times \mathcal{B}(X(3872) \rightarrow$ $\left.J / \psi \pi^{+} \pi^{-}\right)=(7.9 \pm 1.3($ stat $) \pm 0.4($ syst $)) \times 10^{-6} \quad$ and $\mathcal{B}\left(B^{+} \rightarrow X(3872) K^{0} \pi^{+}\right) \times \mathcal{B}\left(X(3872) \rightarrow J / \psi \pi^{+} \pi^{-}\right)=$ $(10.6 \pm 3.0($ stat $) \pm 0.9($ syst $)) \times 10^{-6}$.

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