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Self-energy of strongly interacting fermions in medium: A holographic approach

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ABSTRACT

We consider the self-energy of strongly interacting fermions in the medium using gauge/gravity duality of $D4/D8$ system. We study the mass generation of the thermal and/or dense medium and the collective excitation called plasmino, by considering the spectral function of fermion and its dispersion relation. Our results are very different from those of the hard thermal loop method: for zero density, there is no thermal mass or plasmino in any phase. Plasmino in the deconfined phase is not allowed in $D4/D8$ setup. In the confined phase, there is a plasmino mode only for a window of density.

1. Introduction

The study of fermion self-energy in medium has a long history due to its fundamental importance in studying electronic as well as nuclear matter system. When the excitations are strongly interacting, perturbative field theory method cannot give a reliable result since the diagrams should be truncated to the ladder or rainbow types, which cannot be justified in strong coupling. Furthermore in the presence of chemical potential, the lattice technique is not much useful due to the sign problem. Therefore it is worthwhile to utilize the gauge/gravity duality for this tantalizing problem. The gauge/gravity duality was used to study the fuzzy Fermi surface [1] and the non-Fermi liquid nature [2–4] of the strongly interacting system, for recent developments, we refer to [5–8].

The weakly interacting field theory (QED or QCD) results for the fermion self-energy in medium can be summarized by the existence of the plasmino mode [9] and thermal mass generation of order gT. Plasmino is a collective mode whose dispersion curve has a minimum at finite momentum. However, the recent study of the thermal field theory [10,11], by solving Schwinger–Dyson equation numerically, showed that the thermal mass is reduced as the coupling grows. It raises an interesting questions what happens to the thermal mass and more generally to the hard and soft momentum scales and magnetic mass scale, T, gT, g^2T respectively, in the strong coupling limit.

In this Letter we study the dispersion relation and thermal mass generation using gauge/gravity duality and report a feature of plasmino in strongly coupled system. We consider $D4/D8$ model [12] and turn on a fermion field on the flavor brane world volume with finite quark/baryon number density [13–15]. In the deconfined phase, the fermions represent the fundamental strings, connecting the horizon of black branes and the probe $D8$. In the zero 't Hooft coupling limit, it is a bi-fundamental representation. In the large 't Hooft coupling limit, $D4$ branes disappear and the color index of the fermions should disappear also. In the confined phase, to describe a baryon we need to introduce Witten's baryon vertex [16]. N_c strings are connecting it to the probe $D8/\overline{D8}$. The dynamics of connecting open strings defines that of the baryon vertex as well as the $D8$. Since each string gives fermionic mode, total vibrational dynamics should be described by the composite operator coming from the product of all N_c fermions. This is the baryon in confined phase which is fermion if N_c is odd. We replace this composite operator by a massive fermion field. So our construction for baryon is rather phenomenological. We consider just one flavor brane case mostly. Previously fermions on a probe brane in adjoint representation, called mesino, were studied in [17,18], which are different from ours.

By solving the Dirac equations in each phase, we obtain dispersion relations for the fermionic excitations in medium. Our results show that for zero density, there is no thermal mass generation and no plasmino in any phase, which is sharply different from weakly coupled field theory result. To get plasmino in the deconfined phase we need to add quark mass as well as quark density. In $D4/D8$ setup, the quark mass is not allowed so plasmino is forbidden in the deconfined phase. In the confined phase, there is a plasmino mode only for a certain window of density.

2. Plasmino in field theory

We begin our discussion by giving a brief discussion of fermionic collective excitations in a plasma. The fermion propagator is written as

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$$G(p) = \frac{1}{\gamma \cdot p - m - \Sigma(p)}, \tag{1}$$

where $\Sigma = \gamma_\mu \Sigma^\mu$ is a self-energy. The gauge invariant result is available in the hard (high temperature) thermal loop approximation (HTL) where fermion mass m can be ignored since it is small compared with T or μ. The most noticeable effect of the medium is the effective mass generation. The HTL result of effective mass is given by [19] $m_f^2 = \frac{1}{8} g^2 C_F (T^2 + \frac{\mu^2}{\pi^2})$, where $C_F = 1$ for electron and $C_F = 4/3$ for quark. There are two branches of dispersion curves $\omega = \omega_\pm(p)$ whose asymptotic forms are given by

$$p \ll m_f: \quad \omega_\pm(p) \simeq m_f \pm \frac{1}{3} p, \tag{2}$$

$$p \gg m_f: \quad \omega_\pm(p) \simeq p. \tag{3}$$

ω_+ is the normal branch and ω_- is the one describing plasmino that has been extensively investigated [9,19–31]. For a review we refer to [32,33]. The presence of plasmino is important since it may enhance the production rate of the light di-lepton [30].

In HTL approximation, fermions can be regarded as massless therefore two branches $\omega_\pm$ can be characterized by chirality and helicity. The ratios of chirality and helicity for $\omega_\pm$ are ± 1 respectively. In our work, we do not neglect the fermion mass so we consider the presence of plasmino as the presence of the minimum of dispersion curve at finite momentum. We will conclude that the plasmino exists if the dispersion relation has a minimum at finite momentum.

Notice that plasmino in HTL approximation exists in high temperature whatever the density is. However, in the strongly coupled limit, we will show that plasmino disappears at zero density in the deconfined phase. And it can survive only for a certain window of chemical potential in confined medium. Especially, at zero chemical potential, there is no plasmino. We find that the constant value $1/3$ in Eq. (2) will be replaced by a function of density. See Fig. 5.

3. Holographic setup

Let us now set up a holographic model to calculate the fermion self-energy, first in the confined phase. We use the Sakai–Sugimoto (SS) model [12] where probe $D8/\overline{D}8$ is embedded in black $D4$ branes background. To introduce finite density, we turn on $U(1)$ gauge field on the probe brane. The sources of the $U(1)$ gauge field are end points of strings which are emanating from horizon in the deconfined phase and from baryon vertex in the confined phase.

3.1. Confined phase

The geometry of confined phase with Euclidean signature is given by

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} \left(\delta_{\mu\nu}\, dx^\mu\, dx^\nu + f(U)\, dx_4^2\right) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2\, d\Omega_4^2\right), \tag{4}$$

where both the time and the Kaluza–Klein direction are periodic: $x^0 \sim x^0 + \delta x^0$, $x_4 \sim x_4 + \delta x_4$ and $f(U) = 1 - \frac{U_{KK}^3}{U^3}$, $U_{KK} = (\frac{4\pi}{3})^2 \frac{R^3}{\delta x_4^2}$. Here following the original Sakai–Sugimoto model, we consider only trivial embedding of the flavor eight brane where $D8$ and $\overline{D8}$ brane are located at the antipodal position in x_4 direction. The dynamics of probe branes with $U(1)$ gauge field on it can be governed by the DBI action as follows

$$S_{D8} = -T_8 \int d^9x\, e^{-\phi} \sqrt{\det(g_{MN} + 2\pi\alpha' F_{MN})} = \mathcal{N} \int dr\, r^4 \left[r^{-3}\left(1/f(r) - a_0'(r)^2\right)\right]^{1/2}, \tag{5}$$

where $\mathcal{N} = T_8 \Omega_4 v_3 \delta x^0 R^5 / g_s$, and Ω_4 is the unit four sphere volume and v_3 is the three Euclidean space volume. Notice that in the case of antipodal configuration of $D8/\overline{D8}$, the embedding becomes trivial, i.e. $x_4'(r) = 0$. For later convenience, we use dimensionless quantities

$$r = U/R, \qquad r_0 = U_{KK}/R, \qquad a_0 = 2\pi\alpha' A_0/R. \tag{6}$$

From the equation of motion for $U(1)$ gauge field, we get conserved dimensionless quantity $\frac{r a_0'(r)}{\sqrt{r^{-3}(1/f(r) - a_0'(r)^2)}} = D$, where D is an integral constant representing the baryon density.

Chemical potential can be defined as the value of $a_0(r)$ at the infinity, once proper IR boundary condition is imposed at $r = r_0$. In the confined case, the baryon vertex which is $D4$ brane wrapping on S^4 with N_C fundamental strings plays the role of source of the $U(1)$ gauge field. The energy of the source should be included in the total free energy of the system and it contributes to the chemical potential also.

Accordingly we set $a_0(r_0) = m/q$ to include the baryon mass m in the chemical potential μ:

$$\mu = m/q + \mu_0 = m/q + \int_{r_0}^{\infty} a_0'\, dr. \tag{7}$$

In the confined phase, μ_0 is the contribution only from the gauge potential and m is identified as baryon mass, which can be calculated from the DBI action of the baryon vertex. It is related to the 5 dimensional Lagrangian mass m_5 as we will see later. Notice that such identification is not true in bottom up AdS model, where bulk fermion mass should be related to the conformal dimension of an operator.

3.2. Deconfined phase

The deconfined geometry is given by double Wick rotating x_4 and time from the confined one. The DBI action for probe brane can be written as

$$S_{D8} = \mathcal{N}' \int dr\, r^4 \sqrt{r^{-3}\left(1 - a_0'(r)^2\right)}. \tag{8}$$

The gauge field profile on eight brane is solved as before, $\frac{r a_0'(r)}{\sqrt{r^{-3}(1 - a_0'(r)^2)}} = D'$, where D' is a constant. In the deconfined phase the end points of fundamental strings play the role of sources of the $U(1)$ gauge field. Since the probe branes always fall into the black hole horizon, the fermion representing quark is massless in this phase.

The IR boundary condition of a_0 field determined by the regularity condition at the horizon is given by $a_0(r_H) = 0$, where $r_H = U_H/R$. This condition is also consistent with the argument in the previous section. In the deconfined case, mass of source is zero hence it does not contribute to the IR boundary condition for chemical potential.

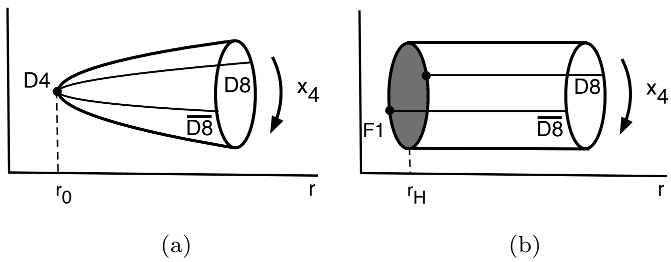

Fig. 1. (a) Embedding of $D8/\overline{D8}$ brane in the confined phase. $D4$ baryon vertices locate at $r = r_0$. (b) Embedding of probe branes in the deconfined phase. The source of $U(1)$ gauge field is located on the black hole horizon.

The schematic configurations of probe branes are drawn in Fig. 1.

4. Fermionic spectrum

Now we study the fermionic spectral function in the holographic dual system. We put a probe fermion field in the world volume of flavor $D8$ brane in the Sakai–Sugimoto model. We ignore the S^4 part and work in the effective 5 dimensional world volume following the original work of Sakai–Sugimoto model. The induced metric is written as $ds_5^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu + g_{rr}\, dr^2$. We use the minimal action

$$S = \int d^5x \sqrt{-g} i\big(\bar\psi \Gamma^M D_M \psi - m_5 \bar\psi \psi\big), \tag{9}$$

where the covariant derivative is $D_M = \partial_M + \frac{1}{4}\omega_{abM}\Gamma^{ab} - iqA_M$. Here M denotes the bulk spacetime index while a, b denote the tangent space index. After a factorizing $\psi = (-gg^{rr})^{-1/4} \times e^{-i\omega t + ik_i x^i}\Psi$, the Dirac equation for Ψ can be given by

$$\sqrt{g_{ii}/g_{rr}}\big(\Gamma^{\underline{r}}\partial_r - m_5\sqrt{g_{rr}}\big)\Psi + iK_\mu \Gamma^{\underline{\mu}}\Psi = 0, \tag{10}$$

where $K_\mu = (-v(r), k_i)$, $v(r) = (\omega + qa_0)\sqrt{-g_{ii}/g_{tt}}$. Following the procedure in [3], we rewrite the Dirac equation (10) in terms of two component spinors

$$\big(\partial_r + m_5\sqrt{g_{rr}}\sigma^3\big)\Phi_\alpha = \sqrt{g_{rr}/g_{ii}}\big(i\sigma^2 v(r) + (-1)^\alpha k\sigma^1\big)\Phi_\alpha, \tag{11}$$

where σ^i are Pauli matrices and $\alpha = 1, 2$ denotes the two up and down spinors respectively. Further decomposing $\Phi_1 = (y_1, z_1)^T$, $\Phi_2 = (y_2, z_2)^T$, we get equations of motion for y_α and z_α. When $\alpha = 2$ we have

$$(\partial_r + m_5\sqrt{g_{rr}})y_2(r) = \sqrt{g_{rr}/g_{ii}}\big(+v(r) + k\big)z_2(r), \tag{12}$$

$$(\partial_r - m_5\sqrt{g_{rr}})z_2(r) = \sqrt{g_{rr}/g_{ii}}\big(-v(r) + k\big)y_2(r). \tag{13}$$

By replacing k by $-k$, we obtain the equations of motion for y_1 and z_1. We would like to define the following new variables $G_1(r) := y_1(r)/z_1(r)$, $G_2(r) := y_2(r)/z_2(r)$. The retarded Green function in boundary field theory is obtained as

$$G^R_{1,2} = \lim_{\epsilon\to 0} e^{8m_5 R r^{1/4}} G_{1,2}(r)\big|_{r=1/\epsilon}, \tag{14}$$

where G_1 and G_2 satisfy the following equations

$$\sqrt{\frac{g_{ii}}{g_{rr}}}\partial_r G_\alpha + 2m_5\sqrt{g_{ii}}G_\alpha = (-1)^\alpha k + v(r) + \big((-1)^{\alpha-1}k + v(r)\big)G_\alpha^2. \tag{15}$$

Now the remaining task is to solve (15) by imposing proper boundary conditions.

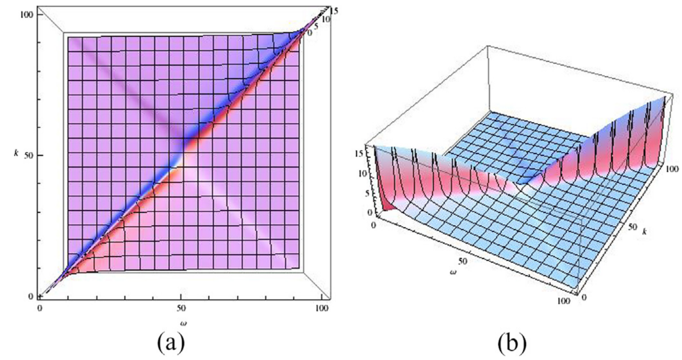

Fig. 2. Spectral functions of G_2^R at zero density: the dispersion curve passes through the origin and thermal mass vanishes. Top view and side view of 3D plot of $\mathrm{Im}\, G_2^R$ with $T = 1$, $m = 0$. True range of both ω and k is $[-5, 5]$.

4.1. Confined phase

In the confined phase of Sakai–Sugimoto model, $v(r)$ function appearing in the Dirac equation is given by $v(r) = \omega + qa_0(r)$, where the $U(1)$ flux solution on flavor brane is obtained from the DBI action as

$$a_0(r) = \mu + \int_\infty^r d\hat r\left(\frac{f(\hat r)^{-1}D^2}{\hat r^5 + D^2}\right)^{1/2}. \tag{16}$$

Notice that $g_{rr} = R^2 r^{-3/2} f(r)^{-1}$ which diverges at r_0. The boundary conditions for flow equation (15) can be found by requiring (15) to be regular at $r = r_0$:

$$G_\alpha(r_0) = \frac{-mR + \sqrt{m^2R^2 + k^2 - \hat\omega^2}}{(-1)^\alpha k - \hat\omega}, \tag{17}$$

where $\hat\omega = \omega + m$. And we define a 4 dimensional vacuum mass $m := m_5\sqrt{g_{ii}}|_{r=r_0}$. Notice that imposing the boundary condition for retarded (advanced) Green function corresponds to $\omega \to \omega + i\epsilon$ ($\omega \to \omega - i\epsilon$).

4.2. Deconfined phase

In the deconfined phase, $v(r)$ function is given by $v(r) = \frac{\omega + qa_0(r)}{\sqrt{f}}$ and the electric flux is given by

$$a_0(r) = \mu + \int_\infty^r d\hat r\left(\frac{D'^2}{\hat r^5 + D'^2}\right)^{1/2}. \tag{18}$$

Due to the black hole horizon, we impose the in-falling boundary condition at $r = r_H$. The boundary condition at $r = r_H$ is obtained as

$$G_{1,2}(r_H) = i. \tag{19}$$

The fermion dispersion relation can be obtained by searching for poles of spectral function, which is the imaginary part of retarded Green function. We solve (15) numerically with IR boundary conditions (17) in the confined phase and (19) in the deconfined phase.

5. Numerical results

Now we discuss the results of fermionic spectral function. First in the deconfined phase, the self-energy term gets some imaginary

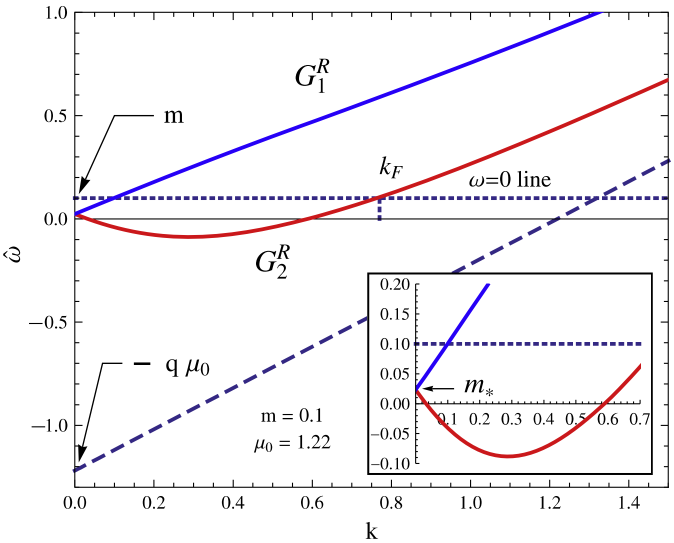

Fig. 3. Dispersion relations in the confined phase. The upper and lower branches describe the normal fermion ω_+ and plasmino ω_- respectively. Dotted line denotes the light cone.

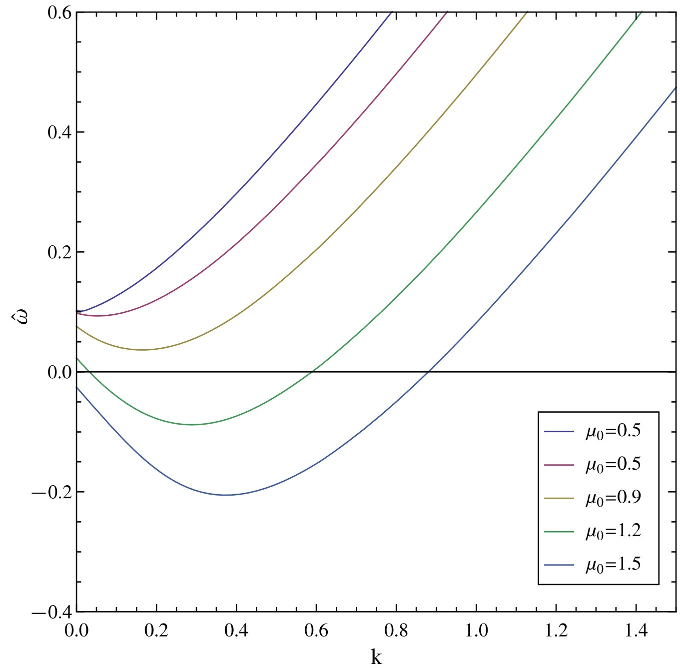

Fig. 4. Dispersion relations of G_2^R in the confined phase. As chemical potential increases, dispersion curve moves down. Parameter $m = 0.1$.

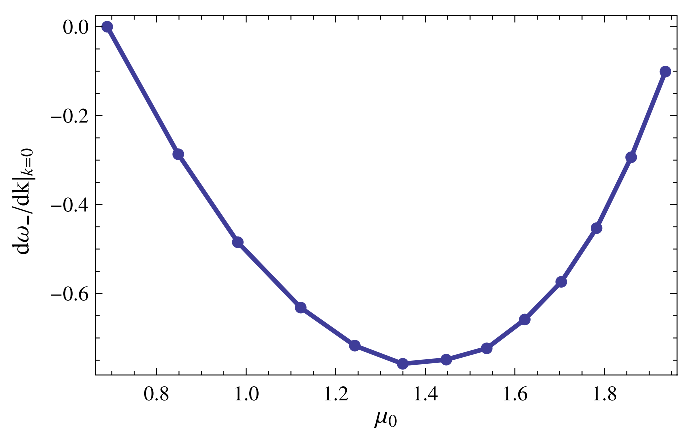

Fig. 5. μ_0 dependence of $\alpha = \frac{d\omega}{dk}$ at $k = 0$. The curve is plotted only in the density window $\mu_1 \leqslant \mu_0 \leqslant \mu_2$ where there is plasmino. Parameter $m = 0$. Notice that $\alpha = \frac{d\omega}{dk}$ at $k = 0$ has a constant value $-1/3$ for the weakly coupled field theory.

part due to the in-falling boundary condition. If the imaginary part is not large we can read off the dispersion relation from the positions of the maxima of spectral function. The 3D plot of spectral function with *zero density* is shown in Fig. 2. Since the fermions here are deconfined quarks, we set $m = 0$. Our main question here is whether thermal mass can be generated by finite temperature effect in strong coupling limit. In Fig. 2 the dispersion curve passes through the origin and this feature is independent of temperature although it is illustrated for $T = 1$. As a result, no thermal mass is generated and there is no plasmino in the deconfined phase with zero density. The absence of thermal mass is actually one of the most drastic differences compared with the weakly coupled field theory result. Namely

$$m_T = \frac{1}{\sqrt{6}} gT \quad \text{in weak coupling},$$
$$m_T = 0 \quad \text{in strong coupling}. \tag{20}$$

The result of vanishing thermal mass is actually consistent with a recent claim made in [11] by the numerical study of Dyson–Schwinger equation in the strong coupling region.

If we turn on finite density in the system, density effect can generate effective mass. However for massless fermion we have an exact Green function $G_R = i$ at $k = 0$, so it is impossible to observe peak structure near $k = 0$. We cannot observe plasmino mode in finite density for the massless fermion.

What would happen if we added a finite bulk fermion mass for curiosity? We find that density effect can generate effective mass as well as plasmino mode for large enough chemical potential.

Now we turn to the confined phase. In this phase, all the peaks of spectral function are delta function-like peaks. In general the Fermi momentum is defined by $\omega(k_F) = 0$. We plot dispersion relations $\omega = \omega_\pm(k)$ in Fig. 3.

5.1. *Observation of plasmino*

We observe a plasmino dispersion relation characterized by the presence of the minimal energy at finite momentum. As we change chemical potential, the slope of ω_- at $k = 0$ changes. We plot the slope $\alpha(\mu_0)$ as a function of density in Fig. 5. In HTL approximation, the value of slope at $k = 0$ is $-1/3$, independent of density or temperature.

The high density behavior of the dispersion curve is complex and we will report it elsewhere. We restrict ourselves to the density range where traditional plasmino mode exists. We could observe plasmino only in a chemical potential window $\mu_1 < \mu_0 < \mu_2$. When $m = 0.1$, the window is given by $\mu_1 = 0.69$, $\mu_2 = 1.94$. As m increases, this window gets wider. Inside the window, as density becomes larger, dispersion curve moves down and bends more and more as shown in Fig. 4. This may be compared with the field theory result in weak coupling, where effective mass and plasmino are generated for any density.

We can read off baryon mass in medium defined by $m_* := \hat{\omega}(0)$. We will show that m_* is a monotonically increasing function of chemical potential. The mass shift in medium is defined as $\delta m := m_* - m$. The normalized mass shift quantity $\chi(\mu_0) := \delta m / m$ is plotted in Fig. 6. Notice that for the massless case, there is no mass correction at all.

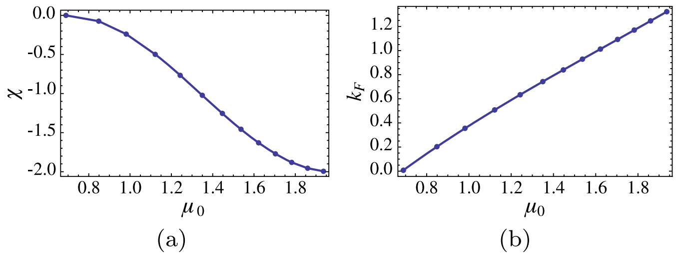

Fig. 6. (a) μ_0 dependence of χ. (b) μ_0 dependence of k_F. Parameter $m = 0.1$.

If we turn off both charge and bulk mass in the confined phase, we have exact solution $G_2(r) = \sqrt{\frac{k+\omega}{k-\omega}}$, which is independent of radial direction.

5.2. Fitting dispersion relation

Now we try to fit the dispersion curve of plasmino. We write the Green function as

$$G_2^R(\omega, k) = \frac{Z}{\omega - \delta m - \frac{k(k-C)}{k+B}}. \tag{21}$$

Although the numerator Z is some undetermined function, the pole information is assumed to be contained only in the denominator. Parameters B and C can be written in terms of $\delta m = m_* - m$, k_F and v_F as follows

$$B = \frac{k_F^2(1 - v_F)}{\delta m + k_F v_F}, \qquad C = \frac{\delta m^2 + 2\delta m k_F + k_F^2 v_F}{\delta m + k_F v_F}, \tag{22}$$

where δm is a mass shift, k_F is the Fermi momentum and v_F is the Fermi velocity. v_F is defined as the slope at $k = k_F$. Expanding (21) near the Fermi momentum, we get

$$G_2^R(\omega, k) \sim \frac{Z}{\omega - v_F(k - k_F) - \Sigma}, \tag{23}$$

where the self-energy near the Fermi momentum is obtained as

$$\Sigma = \delta m + \frac{(1 - v_F)(\delta m + k_F v_F)}{k_F(\delta m + k_F)} \cdot (k - k_F)^2 + \mathcal{O}\big((k - k_F)^3\big). \tag{24}$$

The vanishing mass shift limit $\delta m = 0$ corresponds to the massless fermion in the confined phase. In this limit, self-energy becomes

$$\Sigma = \frac{(1 - v_F)v_F}{k_F} \cdot (k - k_F)^2 + \mathcal{O}\big((k - k_F)^3\big). \tag{25}$$

6. Conclusion and discussion

In this Letter we discuss our observations on the characters of fermion's self-energy in the dense medium. By using gauge/gravity dual, we recover normal and plasmino branches. In the deconfined phase, we showed that for zero density, there is no thermal mass generation. In the weakly coupled thermal gauge theory, T, gT and g^2T provide three well separated scales, with physical interpretations: hard momentum, thermal mass for fermion (or electric screening mass for gluon), magnetic mass respectively. These masses play the role of the infrared regulator of different scales. Such separation does not happen for large coupling $\sim O(1)$, and we believe that we cannot define any such physical scale for strong coupling either. In the confined phase, plasmino excitations are present only for a window of the density. The group velocity of the plasmino mode at zero momentum turns out to be density dependent rather than a constant, $-1/3$, showing the deviation from the HTL approximation. It is also worthwhile to notice that without medium effect, there is no mass correction. We found a simple empirical formula for plasmino dispersion relations. We expect that the phenomena we discovered here is common in many holographic models, so that it is a universal character.

There is an issue whether m_5 can be calculated in a way following [18], where one starts with massless fermion in 10 dimension and performs the KK reduction from 10 dim. to 5 dim. The answer is negative because $D4$ background does not have a direct product structure. To see this, let us consider

$$(\gamma.D_x + \gamma.D_y - m_{10})\Psi(x, y) = 0, \tag{26}$$

where x represents the non-compact directions and y does the compact directions. For $D3$ background, where manifold is direct product $AdS_5 \times S^5$, the $\gamma.D_y$ term produces $\sim (l+2)/R_{ads}$ which can be interpreted as a mass term. While in our model, $D4$ background is not a direct product. Therefore the KK reduction in our case generates term $\sim (l+2)/r$ which is a potential rather than a mass. One may call such potential term as a "mass function" which depends on the radius. We will study the effect of mass function in future work.

Note added in proof

In the closing period of this work, an observation on two branches of dispersion relation appeared in [34] with emphasis on the spin physics.

Acknowledgements

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