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특이섭동이론을 기반으로한 평판모터의 비선형 제어

Simplified Nonlinear Control for Planar Motor based on Singular Perturbation Theory

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Abstract - In this paper, we propose the nonlinear control based on singular perturbation theory for position tracking and yaw regulation of planar motor. Singular perturbation theory is characterized by the existence of slow and fast transients in the system dynamics. The proposed method consists of auxiliary control to decouple error dynamics. We develop model reduction with control input. Also, we derIve decoupled error dynamics with auxiliary input. The controller is designed in order to guarantee the desired position and yaw regulation without current feedback or estimation. Simulation results validate the effect of proposed method.

Key Words : Singular perturbation theory, planar motors, Position feedback, Nonlinear control, Yaw regulation

1. 서 론

Planar motors have been used for manufacturing at especially semiconductor industry. Planar motor is composed of puck and platen. The platen consists of teeth whose intervals are uniform. The puck contains four forcers which are symmetrically placed on each side. The puck is levitated on the platen by air bearings as shown in Fig. 1. X and Y axis motion are composed of resultant forces at each axis. Yaw mothion is generated by interaction of the different forcers.

Several feedback control methods were developed have been studied to improve the position tracking and yaw regulation performance. An adaptive control is used for minimization of ripple and analyzing the control performance [1], [2]. A PD/PID controllers is designed in conjunction with commutation and delay compensation schemes [3], [4]. These methods require the current feedback and position feedback. Position and yaw feedback can be clearly measured by laser interferometer. Alternately, measurements of currents can be corrupted by pulse width modulation switching noise. Thus, low pass filters and current observer are used to reduce the noise [5]. However, the use of the filters cause the phase lag in the current tracking. The use of the current estimation increases complexity of implementation and the computation time. Therefore, using only X, Y position feedback, the control method without current measurement is key work.

In this paper, we propose a simplified nonlinear control for planar motors based on singular perturbation theory to improve the position tracking and yaw regulation performance using only X, Y position and yaw feedback. Since mechanical and electrical dynamics are generally slow and fast in the planar motor, respectively, the singular perturbation theory can be applied to the position tracking control of planar motor. In practically all well designed planar motors, we can put the inductance, L, into the small scalar parameter of singular perturbation system [9]. We design the input voltage, which includes auxiliary input, in order to transform to the three single-input single-output systems at the complex nonlinear multi-input multi-output system. The origin of boundary layer system is globally exponentially stable. The nonlinear controller is designed so as to guarantee stability of the simplified single-input single-output (SISO) system for each motion. We mathematically prove that the origin of tracking error dynamics is globally exponentially stable. Simulation results are performed to evaluate the performance of the proposed method.

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2. Modeling and Controller Design

In this section, we represent planar motor dynamics and tracking error dynamics. we prove the singular perturbation theory in order to make reduced order model. The auxiliary

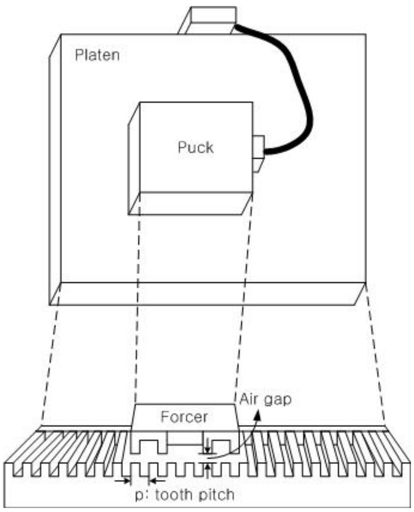


Fig. 1 Planar motor

control input is used to couple the reduced order model. The dynamics of planar motor can be represented in the state space form as follows [8]

$$
\begin{aligned}
&\dot{x}=x_v\\
&\dot{x}_v=\frac{-B_x x_v+\kappa(-S_{x_1}i_{a_{x_1}}+C_{x_1}i_{b_{x_1}}-S_{x_2}i_{a_{x_2}}+C_{x_2}i_{b_{x_2}})-d_x}{M}\\
&\dot{y}=y_v\\
&\dot{y}_v=\frac{-B_y y_v+\kappa(-S_{y_1}i_{a_{y_1}}+C_{y_1}i_{b_{y_1}}-S_{y_2}i_{a_{y_2}}+C_{y_2}i_{b_{y_2}})-d_y}{M}\\
&\dot{\theta}=\theta_v\\
&\dot{\theta}_v=\frac{-B_\theta\theta_v+r_x\kappa(-S_{x_1}i_{a_{x_1}}+C_{x_1}i_{b_{x_1}}+S_{x_2}i_{a_{x_2}}-C_{x_2}i_{b_{x_2}})}{J}\\
&\quad+\frac{r_y\kappa(-S_{y_1}i_{a_{y_1}}+C_{y_1}i_{b_{y_1}}+S_{y_2}i_{a_{y_2}}-C_{y_2}i_{b_{y_2}})-d_\theta}{J}\\
&\dot{i}_{a_{x_1}}=\frac{-Ri_{a_{x_1}}+\kappa S_{x_1}(x_v+r_xC_\theta\theta_v)+v_{a_{x_1}}}{L}\\
&\dot{i}_{b_{x_1}}=\frac{-Ri_{b_{x_1}}-\kappa C_{x_1}(x_v+r_xC_\theta\theta_v)+v_{b_{x_1}}}{L}\\
&\dot{i}_{a_{x_2}}=\frac{-Ri_{a_{x_2}}+\kappa S_{x_2}(x_v-r_xC_\theta\theta_v)+v_{a_{x_2}}}{L}\\
&\dot{i}_{b_{x_2}}=\frac{-Ri_{b_{x_2}}-\kappa C_{x_2}(x_v-r_xC_\theta\theta_v)+v_{b_{x_2}}}{L}\\
&\dot{i}_{a_{y_1}}=\frac{-Ri_{a_{y_1}}+\kappa S_{y_1}(y_v+r_yC_\theta\theta_v)+v_{a_{y_1}}}{L}\\
&\dot{i}_{b_{y_1}}=\frac{-Ri_{b_{y_1}}-\kappa C_{y_1}(y_v+r_yC_\theta\theta_v)+v_{b_{y_1}}}{L}\\
&\dot{i}_{a_{y_2}}=\frac{-Ri_{a_{y_2}}+\kappa S_{y_2}(y_v-r_yC_\theta\theta_v)+v_{a_{y_2}}}{L}\\
&\dot{i}_{b_{y_2}}=\frac{-Ri_{b_{y_2}}-\kappa C_{y_2}(y_v-r_yC_\theta\theta_v)+v_{b_{y_2}}}{L}.
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
&S_\theta=\sin(\theta),\ C_\theta=\cos(\theta),\\
&x_1=x+r_xS_\theta,\ x_2=x+r_yS_\theta,\\
&y_1=y+r_yS_\theta,\ y_2=y+r_yS_\theta,\ \gamma=\frac{2\pi}{p},\\
&S_{x_1}=\sin(\gamma x_1),\ C_{x_1}=\cos(\gamma x_1),\\
&S_{x_2}=\sin(\gamma x_2),\ C_{x_2}=\cos(\gamma x_2),\\
&S_{y_1}=\sin(\gamma y_1),\ C_{y_1}=\cos(\gamma y_1),\\
&S_{y_2}=\sin(\gamma y_2),\ C_{y_2}=\cos(\gamma y_2)
\end{aligned}
$$

and x and y are the position [m] of the center of the motor on the platen. xv and yv are the velocity [m/s] of the center of the motor on the platen. θ is the yaw [rad] rotation and θ_v is the yaw rate of the center of the motor on the platen. $i_{a_{x_1}}, i_{b_{x_1}}, i_{a_{x_2}}, i_{b_{x_2}}, i_{a_{y_1}}, i_{b_{y_1}}, i_{a_{y_2}}$ and $i_{b_{y_2}}$ are phase currents [A] in phase A and B of the forcer X_1, X_2, Y_1 and Y_2. $v_{a_{x_1}}, v_{b_{x_1}}, v_{a_{x_2}}, v_{b_{x_2}}, v_{a_{y_1}}, v_{b_{y_1}}, v_{a_{y_2}}$ and $v_{b_{y_2}}$ are input voltages [V] in phase A and B of the forcer X_1, X_2, Y_1 and Y_2, and d_x, d_y and d_θ are unknown constant load forces [N] and torque [N·m]. r_x and r_y are the distance from where x^d and y^d are desired positions of X and Y axes, x_v^d and y_v^d are desired velocities, θ^d and θ_v^d are the desired yaw and yaw rate. We have added the additional integral term of the mechanical errors, e_{zx}, e_{zy} and $e_{z\theta}$, into the mechanical subsystem to decrease of the position errors. The desired currents, $i^d_{a_{x_1}}, \cdots, i^d_{b_{y_2}}$, will be defined.

$$
\begin{aligned}
&e_{zx}=\int_0^t e_x dt,\ e_x=x^d-x,\ e_{x_v}=x_v^d-x_v,\\
&e_{zy}=\int_0^t e_y dt,\ e_y=y^d-y,\ e_{y_v}=y_v^d-y_v,\\
&e_{z\theta}=\int_0^t e_\theta dt,\ e_\theta=\theta^d-\theta,\ e_{\theta_v}=\theta_v^d-\theta_v\\
&e_{a_{x_1}}=i^d_{a_{x_1}}-i_{a_{x_1}},\ e_{b_{x_1}}=i^d_{b_{x_1}}-i_{b_{x_1}},\\
&e_{a_{x_2}}=i^d_{a_{x_2}}-i_{a_{x_2}},\ e_{b_{x_2}}=i^d_{b_{x_2}}-i_{b_{x_2}},\\
&e_{a_{y_1}}=i^d_{a_{y_1}}-i_{a_{y_1}},\ e_{b_{y_1}}=i^d_{b_{y_1}}-i_{b_{y_1}},\\
&e_{a_{y_2}}=i^d_{a_{y_2}}-i_{a_{y_2}},\ e_{b_{y_2}}=i^d_{b_{y_2}}-i_{b_{y_2}}.
\end{aligned}
\tag{2}
$$

From (1) and (2), we can derive the tracking error dynamics of the position and yaw as follows

$$
\begin{aligned}
\dot{e}_{x_v}=&\frac{M\dot{x}_v^d+B_xx_v^d-\kappa(-S_{x_1}i^d_{a_{x_1}}+C_{x_1}i^d_{b_{x_1}}-S_{x_2}i^d_{a_{x_2}}+C_{x_2}i^d_{b_{x_2}})}{M}\\
&+\frac{-B_xe_{x_v}-\kappa(-S_{x_1}e_{a_{x_1}}+C_{x_1}e_{b_{x_1}}-S_{x_2}e_{a_{x_2}}+C_{x_2}e_{b_{x_2}})+d_x}{M}
\end{aligned}
\tag{3}
$$

$$\dot{e}_{y_v} = \frac{M\dot{y}_v^d + B_y y_v^d - \kappa(-S_{y_1} i^d_{a_{y_1}} + C_{y_1} i^d_{b_{y_1}} - S_{y_2} i^d_{a_{y_2}} + C_{y_2} i^d_{b_{y_2}})}{M} + \frac{-B_y e_{y_v} - \kappa(-S_{y_1} e_{a_{y_1}} + C_{y_1} e_{b_{y_1}} - S_{y_2} e_{a_{y_2}} + C_{y_2} e_{b_{y_2}}) + d_y}{M}$$

$$\begin{aligned}\dot{e}_{x_v} =& \frac{J\dot{\theta}_v^d + B_\theta \theta_v^d - r_x\kappa(-S_{x_1} i^d_{a_{x_1}} + C_{x_1} i^d_{b_{x_1}} + S_{x_2} i^d_{a_{x_2}} - C_{x_2} i^d_{b_{x_2}})}{J} \\ &+ \frac{-r_y\kappa(-S_{y_1} i^d_{a_{y_1}} + C_{y_1} i^d_{b_{y_1}} + S_{y_2} i^d_{a_{y_2}} - C_{y_2} i^d_{b_{y_2}})}{J} \\ &+ \frac{-B_\theta e_{\theta_v} + r_x\kappa(-S_{x_1} e_{a_{x_1}} + C_{x_1} e_{b_{x_1}} + S_{x_2} e_{a_{x_2}} - C_{x_2} e_{b_{x_2}})}{M} \\ &+ \frac{r_y\kappa(-S_{y_1} e_{a_{y_1}} + C_{y_1} e_{b_{y_1}} + S_{y_2} e_{a_{y_2}} - C_{y_2} e_{b_{y_2}}) + d_\theta}{M}.\end{aligned}$$

We define the desired currents and desired torque in order to simplify tracking error dynamics of the position and yaw as follows

$$\begin{aligned}
i^d_{a_{x_1}} &= -\frac{S_{x_1}(M\dot{x}_v^d + B_x x_v^d)}{2\kappa} - \frac{S_{x_1}}{4\kappa r_x}\tau^d, \\
i^d_{b_{x_1}} &= -\frac{C_{x_1}(M\dot{x}_v^d + B_x x_v^d)}{2\kappa} + \frac{C_{x_1}}{4\kappa r_x}\tau^d \\
i^d_{a_{x_2}} &= -\frac{S_{x_2}(M\dot{x}_v^d + B_x x_v^d)}{2\kappa} + \frac{S_{x_2}}{4\kappa r_x}\tau^d, \\
i^d_{b_{x_2}} &= -\frac{S_{x_2}(M\dot{x}_v^d + B_x x_v^d)}{2\kappa} - \frac{C_{x_2}}{4\kappa r_x}\tau^d, \\
i^d_{a_{y_1}} &= -\frac{S_{y_1}(M\dot{y}_v^d + B_y y_v^d)}{2\kappa} - \frac{S_{y_1}}{4\kappa r_y}\tau^d, \\
i^d_{b_{y_1}} &= -\frac{C_{y_1}(M\dot{y}_v^d + B_y y_v^d)}{2\kappa} + \frac{C_{y_1}}{4\kappa r_y}\tau^d, \\
i^d_{a_{y_2}} &= -\frac{S_{y_2}(M\dot{y}_v^d + B_y y_v^d)}{2\kappa} + \frac{S_{y_2}}{4\kappa r_y}\tau^d, \\
i^d_{b_{y_2}} &= -\frac{S_{y_2}(M\dot{y}_v^d + B_y y_v^d)}{2\kappa} - \frac{C_{y_2}}{4\kappa r_y}\tau^d, \\
\tau^d &= J\dot{\theta}_v^d + B_\theta \theta_v
\end{aligned} \tag{4}$$

Then, the tracking error dynamics of the position and yaw are rewritten as

$$\begin{aligned}
\dot{e}_{x_v} &= \frac{-B_x e_{x_v} - \kappa(-S_{x_1} e_{a_{x_1}} + C_{x_1} e_{b_{x_1}} - S_{x_2} e_{a_{x_2}} + C_{x_2} e_{b_{x_2}}) + d_x}{M}, \\
\dot{e}_{y_v} &= \frac{-B_y e_{y_v} - \kappa(-S_{y_1} e_{a_{y_1}} + C_{y_1} e_{b_{y_1}} - S_{y_2} e_{a_{y_2}} + C_{y_2} e_{b_{y_2}}) + d_y}{M} \\
\dot{e}_{\theta_v} &= \frac{-B_\theta e_{\theta_v} + r_x\kappa(-S_{x_1} e_{a_{x_1}} + C_{x_1} e_{b_{x_1}} + S_{x_2} e_{a_{x_2}} - C_{x_2} e_{b_{x_2}})}{J} \\
&+ \frac{r_y\kappa(-S_{y_1} e_{a_{y_1}} + C_{y_1} e_{b_{y_1}} + S_{y_2} e_{a_{y_2}} - C_{y_2} e_{b_{y_2}}) + d_\theta}{J}.
\end{aligned} \tag{5}$$

In the following theorem, we propose the nonlinear controller based on singular perturbation theory for position tracking of planar motor and prove the globally exponential stability.

Theorem 1: Consider planar motor dynamics (1). Suppose that the auxiliary control law and control law are given by

$$\begin{aligned}
u_x &= -k_{x_1} e_{zx} - k_{x_2} e_x - k_{x_3} e_{x_v}, \\
u_y &= -k_{y_1} e_{zy} - k_{y_2} e_y - k_{y_3} e_{y_v}, \\
u_\theta &= -k_{\theta_1} e_{z\theta} - k_{\theta_2} e_\theta - k_{\theta_3} e_{\theta_v} \\
u_{a_{x_1}} &= -S_{x_1}\left(\frac{Ru_x}{2\kappa} + \frac{Ru_\theta}{4r_x\kappa}\right), \quad u_{a_{y_1}} = -S_{y_1}\left(\frac{Ru_y}{2\kappa} + \frac{Ru_\theta}{4r_y\kappa}\right), \\
u_{b_{x_1}} &= C_{x_1}\left(\frac{Ru_x}{2\kappa} + \frac{Ru_\theta}{4r_x\kappa}\right), \quad u_{b_{y_1}} = C_{y_1}\left(\frac{Ru_y}{2\kappa} + \frac{Ru_\theta}{4r_y\kappa}\right), \\
u_{a_{x_2}} &= -S_{x_2}\left(\frac{Ru_x}{2\kappa} - \frac{Ru_\theta}{4r_x\kappa}\right), \quad u_{a_{y_2}} = -S_{y_2}\left(\frac{Ru_y}{2\kappa} - \frac{Ru_\theta}{4r_y\kappa}\right), \\
u_{b_{x_2}} &= C_{x_2}\left(\frac{Ru_x}{2\kappa} - \frac{Ru_\theta}{4r_x\kappa}\right), \quad u_{b_{y2}} = C_{y_2}\left(\frac{Ru_y}{2\kappa} - \frac{Ru_\theta}{4r_y\kappa}\right).
\end{aligned} \tag{6}$$

$$\begin{aligned}
v_{a_{x_1}} &= L\dot{i}^d_{a_{x_1}} + Ri^d_{a_{x_1}} - \kappa S_{x_1}(x_v^d + r_x C_\theta \theta_v^d) - u_{a_{x_1}}, \\
v_{b_{x_1}} &= L\dot{i}^d_{b_{x_1}} + Ri^d_{b_{x_1}} + \kappa C_{x_1}(x_v^d + r_x C_\theta \theta_v^d) - u_{b_{x_1}}, \\
v_{a_{x_2}} &= L\dot{i}^d_{a_{x_2}} + Ri^d_{a_{x_2}} - \kappa S_{x_2}(x_v^d - r_x C_\theta \theta_v^d) - u_{a_{x_2}}, \\
v_{b_{x_2}} &= L\dot{i}^d_{b_{x_2}} + Ri^d_{b_{x_2}} + \kappa C_{x_2}(x_v^d - r_x C_\theta \theta_v^d) - u_{b_{x_2}}, \\
v_{a_{y_1}} &= L\dot{i}^d_{a_{y_1}} + Ri^d_{a_{y_1}} - \kappa S_{y_1}(y_v^d + r_y C_\theta \theta_v^d) - u_{a_{y_1}}, \\
v_{b_{y_1}} &= L\dot{i}^d_{b_{y_1}} + Ri^d_{b_{y_1}} + \kappa C_{y_1}(y_v^d + r_y C_\theta \theta_v^d) - u_{b_{y_1}}, \\
v_{a_{y_2}} &= L\dot{i}^d_{a_{y_2}} + Ri^d_{a_{y_2}} - \kappa S_{y_2}(y_v^d - r_y C_\theta \theta_v^d) - u_{a_{y_2}}, \\
v_{b_{y_2}} &= L\dot{i}^d_{b_{y_2}} + Ri^d_{b_{y_2}} + \kappa C_{y_2}(y_v^d - r_y C_\theta \theta_v^d) - u_{b_{y_2}}.
\end{aligned} \tag{7}$$

where $k_{x_1}, k_{x_2}, k_{x_3}, k_{y_1}, k_{y_2}, k_{y_3}, k_{\theta_1}, k_{\theta_2}$ and k_{θ_3} are positive numbers. Then $e_x, e_y, e_\theta, e_{x_v}, e_{y_v}$ and e_{θ_v} are globally exponentially stable.

Proof: Substitute the control laws, $v_{a_{x_1}}, v_{b_{x_1}}, v_{a_{x_2}}, v_{b_{x_2}}, v_{a_{y_1}}, v_{b_{y_1}}, v_{a_{y_2}}$ and $v_{b_{y_2}}$,of (7) into the current error dynamics. In order to make the singular perturbation model of a dynamical system, the derivatives of the current error dynamics are multiplied by a small positive parameter L. Thus, the singular perturbation model becomes

$$\begin{aligned}
L\dot{e}_{a_{x_1}} &= -Re_{a_{x_1}} + \kappa S_{x_1}(e_{x_v} + r_x C_\theta e_{\theta_v}) + u_{a_{x_1}}, \\
L\dot{e}_{b_{x_1}} &= -Re_{b_{x_1}} - \kappa C_{x_1}(e_{x_v} + r_x C_\theta e_{\theta_v}) + u_{b_{x_1}}, \\
L\dot{e}_{a_{x_2}} &= -Re_{a_{x_2}} + \kappa S_{x_2}(e_{x_v} - r_x C_\theta e_{\theta_v}) + u_{a_{x_2}}, \\
L\dot{e}_{b_{x_2}} &= -Re_{b_{x_2}} - \kappa C_{x_2}(e_{x_v} - r_x C_\theta e_{\theta_v}) + u_{b_{x_2}},
\end{aligned} \tag{8}$$

$$
\begin{aligned}
L\dot{e}_{a_{y_1}} &= -Re_{a_{y_1}} + \kappa S_{y_1}(e_{y_v} + r_y C_\theta e_{\theta_v}) + u_{a_{y_1}},\\
L\dot{e}_{b_{y_1}} &= -Re_{b_{y_1}} - \kappa C_{y_1}(e_{y_v} + r_y C_\theta e_{\theta_v}) + u_{b_{y_1}},\\
L\dot{e}_{a_{y_2}} &= -Re_{a_{y_2}} + \kappa S_{y_2}(e_{y_v} - r_y C_\theta e_{\theta_v}) + u_{a_{y_2}},\\
L\dot{e}_{b_{y_2}} &= -Re_{b_{y_2}} - \kappa C_{y_2}(e_{y_v} - r_y C_\theta e_{\theta_v}) + u_{b_{y_2}}.
\end{aligned}
$$

We can obtain the quasi-steady states at inductance L = 0 as follows

$$
\begin{aligned}
\bar{e}_{a_{x_1}} &= \frac{\kappa S_{x_1}(e_{x_v} + r_x C_\theta e_{\theta_v}) + u_{a_{x_1}}}{R},\\
\bar{e}_{b_{x_1}} &= \frac{-\kappa C_{x_1}(e_{x_v} + r_x C_\theta e_{\theta_v}) + u_{b_{x_1}}}{R},\\
\bar{e}_{a_{x_2}} &= \frac{\kappa S_{x_2}(e_{x_v} - r_x C_\theta e_{\theta_v}) + u_{a_{x_2}}}{R},\\
\bar{e}_{b_{x_2}} &= \frac{-\kappa C_{x_2}(e_{x_v} - r_x C_\theta e_{\theta_v}) + u_{b_{x_2}}}{R},\\
\bar{e}_{a_{y_1}} &= \frac{\kappa S_{y_1}(e_{y_v} + r_y C_\theta e_{\theta_v}) + u_{a_{y_1}}}{R},\\
\bar{e}_{b_{y_1}} &= \frac{-\kappa C_{y_1}(e_{y_v} + r_y C_\theta e_{\theta_v}) + u_{b_{y_1}}}{R},\\
\bar{e}_{a_{y_2}} &= \frac{\kappa S_{y_2}(e_{y_v} - r_y C_\theta e_{\theta_v}) + u_{a_{y_2}}}{R},\\
\bar{e}_{b_{y_2}} &= \frac{-\kappa C_{y_2}(e_{y_v} - r_y C_\theta e_{\theta_v}) + u_{b_{y_2}}}{R}.
\end{aligned} \tag{9}
$$

To move the equilibrium point to zero, let us define $y_{a_{x_1}}, \cdots, y_{b_{y_2}}$ as

$$
\begin{aligned}
y_{a_{x_1}} &= e_{a_{x_1}} - \bar{e}_{a_{x_1}},\ y_{b_{x_1}} = e_{b_{x_1}} - \bar{e}_{b_{x_1}},\\
y_{a_{x_2}} &= e_{a_{x_2}} - \bar{e}_{a_{x_2}},\ y_{b_{x_2}} = e_{b_{x_2}} - \bar{e}_{b_{x_2}},\\
y_{a_{y_1}} &= e_{a_{y_1}} - \bar{e}_{a_{y_1}},\ y_{b_{y_1}} = e_{b_{y_1}} - \bar{e}_{b_{y_1}},\\
y_{a_{y_2}} &= e_{a_{y_2}} - \bar{e}_{a_{y_2}},\ y_{b_{y_2}} = e_{b_{y_2}} - \bar{e}_{b_{y_2}}.
\end{aligned} \tag{10}
$$

Then, the dynamics of $y_{a_{x_1}}, \cdots, y_{b_{y_2}}$ are given by

$$
\begin{aligned}
L\dot{y}_{a_{x_1}} &= -Ry_{a_{x_1}} - L\dot{\bar{e}}_{a_{x_1}},\ L\dot{y}_{b_{x_1}} = -Ry_{b_{x_1}} - L\dot{\bar{e}}_{b_{x_1}},\\
L\dot{y}_{a_{x_2}} &= -Ry_{a_{x_2}} - L\dot{\bar{e}}_{a_{x_2}},\ L\dot{y}_{b_{x_2}} = -Ry_{b_{x_2}} - L\dot{\bar{e}}_{b_{x_2}},\\
L\dot{y}_{a_{y_1}} &= -Ry_{a_{y_1}} - L\dot{\bar{e}}_{a_{y_1}},\ L\dot{y}_{b_{y_1}} = -Ry_{b_{y_1}} - L\dot{\bar{e}}_{b_{y_1}},\\
L\dot{y}_{a_{y_2}} &= -Ry_{a_{y_2}} - L\dot{\bar{e}}_{a_{y_2}},\ L\dot{y}_{b_{y_2}} = -Ry_{b_{y_2}} - L\dot{\bar{e}}_{b_{y_2}}.
\end{aligned} \tag{11}
$$

Thus, we can obtain the boundary-layer system as follows

$$
\begin{aligned}
\frac{dy_{a_{x_1}}}{d\xi} &= -Ry_{a_{x_1}},\ \frac{dy_{b_{x_1}}}{d\xi} = -Ry_{b_{x_1}},\\
\frac{dy_{a_{x_2}}}{d\xi} &= -Ry_{a_{x_2}},\ \frac{dy_{b_{x_2}}}{d\xi} = -Ry_{b_{x_2}},\\
\frac{dy_{a_{y_1}}}{d\xi} &= -Ry_{a_{y_1}},\ \frac{dy_{b_{y_1}}}{d\xi} = -Ry_{b_{y_1}},\\
\frac{dy_{a_{y_2}}}{d\xi} &= -Ry_{a_{y_2}},\ \frac{dy_{b_{y_2}}}{d\xi} = -Ry_{b_{y_2}}.
\end{aligned} \tag{12}
$$

where $L\frac{dy}{dt} = \frac{dy}{d\xi}$ is a new time variable. Since R is always positive parameter, the equilibrium point $y_{a_{x_1}}, \cdots, y_{b_{y_2}} = 0$ of the boundary-layer system (12) is globally exponentially stable. From the definition, we can obtain $e_{a_{x_1}} = \bar{e}_{a_{x_1}}, \cdots, e_{b_{y_2}} = \bar{e}_{b_{y_2}}$ as follows

$$
\begin{aligned}
\dot{e}_{x_v} &= \frac{-B_x e_{x_v} - \kappa(-S_{x_1}\bar{e}_{a_{x_1}} + C_{x_1}\bar{e}_{b_{x_1}} - S_{x_2}\bar{e}_{a_{x_2}} + C_{x_2}\bar{e}_{b_{x_2}}) + d_x}{M},\\
\dot{e}_{y_v} &= \frac{-B_y e_{y_v} - \kappa(-S_{y_1}\bar{e}_{a_{y_1}} + C_{y_1}\bar{e}_{b_{y_1}} - S_{y_2}\bar{e}_{a_{y_2}} + C_{y_2}\bar{e}_{b_{y_2}}) + d_y}{M}\\
\dot{e}_{\theta_v} &= \frac{-B_\theta e_{\theta_v} + r_x\kappa(-S_{x_1}\bar{e}_{a_{x_1}} + C_{x_1}\bar{e}_{b_{x_1}} + S_{x_2}\bar{e}_{a_{x_2}} - C_{x_2}\bar{e}_{b_{x_2}})}{J}\\
&+ \frac{r_y\kappa(-S_{y_1}\bar{e}_{a_{y_1}} + C_{y_1}\bar{e}_{b_{y_1}} + S_{y_2}\bar{e}_{a_{y_2}} - C_{y_2}\bar{e}_{b_{y_2}}) + d_\theta}{J}.
\end{aligned} \tag{13}
$$

Substituting quasi-steady-state $\bar{e}_{a_{x_1}}, \ldots, \bar{e}_{b_{y_2}}$ of (9) into the tracking error dynamics of the (13) results in the reduced-order model as

$$
\begin{aligned}
\dot{e}_{x_v} &= \frac{-\left(B_x + \frac{2\kappa^2}{R}\right)e_{x_v}}{M}\\
&+ \frac{\frac{\kappa}{R}(-S_{x_1}u_{a_{x_1}} + C_{x_1}u_{b_{x_1}} - S_{x_2}u_{a_{x_2}} + C_{x_2}u_{b_{x_2}}) + d_x}{M}\\
\dot{e}_{y_v} &= \frac{-\left(B_y + \frac{2\kappa^2}{R}\right)e_{y_v}}{M}\\
&+ \frac{\frac{\kappa}{R}(-S_{y_1}u_{a_{y_1}} + C_{y_1}u_{b_{y_1}} - S_{y_2}u_{a_{y_2}} + C_{y_2}u_{b_{y_2}}) + d_y}{M}\\
\dot{e}_{\theta_v} &= \frac{-\left(B_\theta + \frac{2\kappa^2 C_\theta r_x^2}{R} + \frac{2\kappa^2 C_\theta r_y^2}{R}\right)e_{\theta_v}}{J}\\
&+ \frac{\frac{r_x\kappa}{R}(-S_{x_1}u_{a_{x_1}} + C_{x_1}u_{b_{x_1}} + S_{x_2}u_{a_{x_2}} - C_{x_2}u_{b_{x_2}})}{J}\\
&+ \frac{\frac{r_y\kappa}{R}(-S_{y_1}u_{a_{y_1}} + C_{y_1}u_{b_{y_1}} + S_{y_2}u_{a_{y_2}} - C_{y_2}u_{b_{y_2}}) + d_\theta}{J}.
\end{aligned} \tag{14}
$$

From the auxiliary control law (6), the mechanical tracking error dynamics results in the reduced-order model. The load forces and load torque perturbation, denoted by d_x, d_y and d_θ are assumed to be zero for all analysis purposes as

$$\dot{e}_{x_v} = \frac{-\left(B_x + \frac{2\kappa^2}{R}\right)e_{x_v} + u_x}{M},$$
$$\dot{e}_{y_v} = \frac{-\left(B_y + \frac{2\kappa^2}{R}\right)e_{y_v} + u_y}{M}, \quad (15)$$
$$\dot{e}_{\theta_v} = \frac{-\left(B_\theta + \frac{2\kappa^2 C_\theta r_x^2}{R} + \frac{2\kappa^2 C_\theta r_y^2}{R}\right)e_{\theta_v} + u_\theta}{J}.$$

Since each error dynamics are decoupling, We can put the reduced tracking error dynamics into the matrix form respectively as follows

$$\dot{e}_1 = A_x e_1 + B_1 u_x,$$
$$\dot{e}_2 = A_y e_2 + B_2 u_y, \quad (16)$$
$$\dot{e}_3 = A_\theta e_3 + B_3 u_\theta.$$

where

$$A_x = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{\left(B_x + \frac{2\kappa^2}{R}\right)}{M} \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \end{bmatrix},$$
$$A_y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{\left(B_y + \frac{2\kappa^2}{R}\right)}{M} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \end{bmatrix}$$
$$A_\theta = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \frac{-\left(B_\theta + \frac{2\kappa^2 C_\theta r_x^2 e_{\theta_v}}{R} + \frac{2\kappa^2 C_\theta r_y^2 e_{\theta_v}}{R}\right)}{J} \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{J} \end{bmatrix},$$
$$e_1 = \begin{bmatrix} e_{zx} \\ e_x \\ e_{x_v} \end{bmatrix}, e_2 = \begin{bmatrix} e_{zy} \\ e_y \\ e_{y_v} \end{bmatrix}, e_3 = \begin{bmatrix} e_{z\theta} \\ e_\theta \\ e_{\theta_v} \end{bmatrix}, e_m = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

Decoupled error dynamics are linear state equations. The controllability matrix of the error dynamics is given by

$$C_x = \left[B_1 \; A_x B_1 \; A_x^2 B_1\right],$$
$$C_y = \left[B_2 \; A_y B_2 \; A_y^2 B_2\right], \quad (17)$$
$$C_\theta = \left[B_3 \; A_\theta B_3 \; A_\theta^2 B_3\right].$$

Since each controllability matrix is full rank, we design control law in order to move left half plane at all eigenvalues of tracking error dynamics as follows

$$u_x = -k_{x_1} e_{zx} - k_{x_2} e_x - k_{x_3} e_{x_v},$$
$$u_y = -k_{y_1} e_{zy} - k_{y_2} e_y - k_{y_3} e_{y_v}, \quad (18)$$
$$u_\theta = -k_{\theta_1} e_{z\theta} - k_{\theta_2} e_\theta - k_{\theta_3} e_{\theta_v}.$$

Then, the dynamics of e_{x_v}, e_{y_v} and e_{θ_v} are given by

$$\dot{e}_{x_v} = \frac{-k_{x_1} e_{zx} - k_{x_2} e_x - \left(k_{x_3} + B_x + \frac{2\kappa^2}{R}\right)e_{x_v}}{M},$$
$$\dot{e}_{y_v} = \frac{-k_{y_1} e_{zy} - k_{y_2} e_y - \left(k_{y_3} + B_y + \frac{2\kappa^2}{R}\right)e_{y_v}}{M}, \quad (19)$$
$$\dot{e}_{\theta_v} = \frac{-k_{\theta_1} e_{z\theta} - k_{\theta_2} e_\theta}{J} + \frac{-\left(k_{\theta_3} + B_\theta + \frac{2\kappa^2 r_x C_\theta e_{\theta_v}}{R} + \frac{2\kappa^2 r_y C_\theta e_{\theta_v}}{R}\right)e_{\theta_v}}{J}.$$

Therefore, The origin of reduced order error dynamics, $e_m = 0$, is globally exponentially stable.

Remark 1: If the unknown disturbances, d_x, d_y and d_θ, are non-vanishing perturbation, the proposed method guarantees uniformly ultimately boundedness of the tracking error dynamics.

3. Simulation Results

Table 1 Planar motor Parameters and Control gains

	Parameter	Value	Unit
Parameters	M	1.8	kg
	J	2.2×10^{-3}	$kg \cdot m^2$
	B_x, B_y	1×10^{-5}	$N \cdot s/m$
	B_θ	1×10^{-5}	$N \cdot m \cdot s/$
	d_x, d_y	7.5	N
	d_θ	1	$N \cdot m$
	p	6.4×10^{-4}	m
	R	2	Ω
	L	7×10^{-4}	H
	κ	17	N/A
Gains	k_{x_1}, k_{y_1}	2.0×10^6	
	k_{x_2}, k_{y_2}	1.8×10^5	
	k_{x_3}, k_{y_3}	54	
	k_{θ_1}	2200	
	k_{θ_2}	220	
	k_{θ_3}	22	

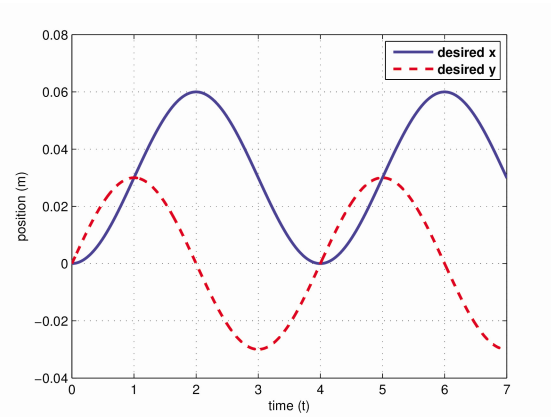
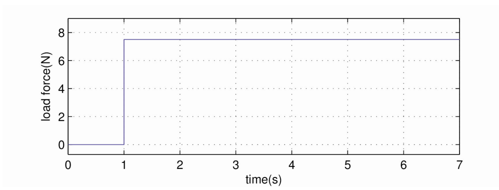
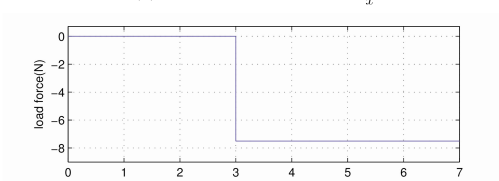


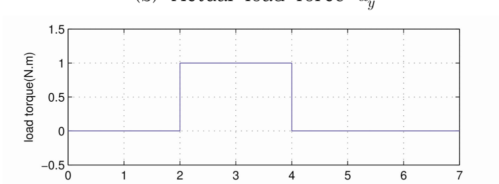
Fig. 2 Reference profiles



(a) Actual load force d_x

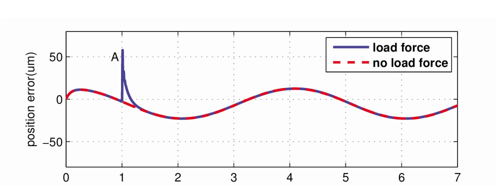


(b) Actual load force d_y

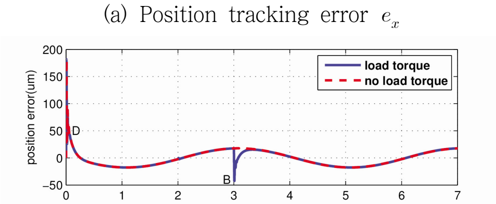


(c) Actual load torque d_θ

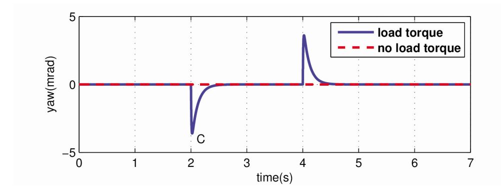
Fig. 3 Load force and load torque



(a) Position tracking error e_x



(b) Position tracking error e_y



(c) Actual θ

Fig. 4 Position tracking error and yaw regulation performance at load and no load

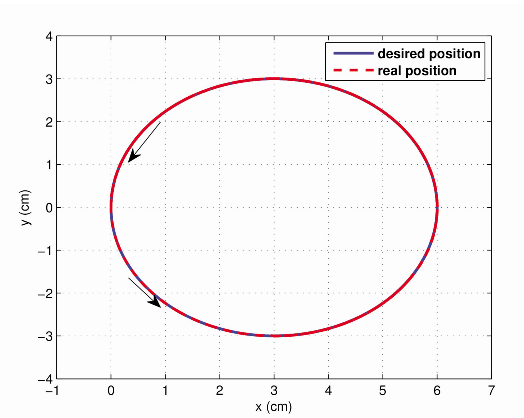


Fig. 5 Desired and real positions of x and y

To evaluate the position tracking and yaw regulation performances of the proposed method, simulations were performed using MATLAB/Simulink. The control frequency is 5 kHz. The eigenvalues of closed loop are $\lambda_x = [-11.37 \quad -101.59+295.58i \quad -101.59-295.58i]$,$\lambda_y = [-11.37 \quad -101.59+295.58i - 101.59-295.58i]$, $\lambda_\theta = [-9990 \quad -5.0+8.7i \quad -5.0-8.7i]$. The parameters of planar motor and proposed controller gains were listed in Table 1. Sine wave profile was shown in Fig. 2. Sine wave profile was used to evaluate position tracking and yaw regulation of circular motion. Since the load force and torque are unknown disturbance, step function was represented for analysis. The position tracking error and yaw regulation with and without load were shown in Fig. 4. Desired and real position of planar motor was shown in Fig. 5. Error position of x, y and θ was used to identify the effect of load force and torque shown in Fig. 6. The position tracking error of x and y was the peak and valley. We propose arbitrary load torque and force in order to identify unknown constant effect. Since the load force of x and y were added at 1.0 sec. and 3.0 sec. as step function, there were the peaks of the position tracking error of x and y respectively. However, the position tracking error of x and y

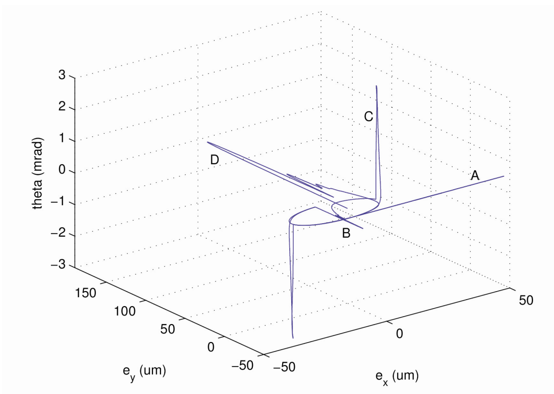


Fig. 6 Error position of x, y and θ

also asymptotically converged to zero. When the load torque of yaw were added from 2.0 [sec] to 4.0 [sec], the yaw regulation was the valley and peak respectively at each time. However, the yaw well regulated.

3. Conclusion

In this paper, we propose the position and yaw control method for the planar motor. The proposed controller was designed with the nonlinear control in order to improve position tracking error and yaw regulation. The proposed singular perturbation theory was used to make the reduced order model. Since the electrical dynamics was neglected using singular perturbation theory, this model was not required both a nonlinear observer and a current feedback. The simulation results showed that the position tracking and yaw regulation performance were improved by the proposed method. We observed that the position x, y and yaw were decoupled.

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