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Research article

Some theorems in partial metric space using auxiliary functions

Deepak Kumar[1], Sadia Sadat[1], Jung Rye Lee[2,*] and Choonkil Park[3,*]

[1] Department of Mathematics, Lovely Professional University, Jalandhar, 144411, India
[2] Department of Data Science, Daejin University, Kyunggi 11159, Korea
[3] Research Institute for Natural Sciences, Hanyang University, Seoul 04763, Korea

* **Correspondence:** Email: jrlee@daejin.ac.kr, baak@hanyang.ac.kr.

Abstract: In the present manuscript, we establish some theorems for the existence and uniqueness of a fixed point in the framework of partial metric spaces using auxiliary functions. Our results generalize some existing results in the literature. To illustrate our results some examples are provided.

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1. Introduction and preliminaries

Fixed point theory is the most dynamic area of research, with numerous applications both in pure and applied mathematics. The formal theoretic approach of fixed point was originated from the work of Picard. However, it was the polish mathematician Banach [5] who underlined the idea into the abstract framework and provided a constructive tool called Banach construction principle to establish the fixed point of a mapping in complete metric space. Later, many authors attempted to generalize the notion of metric space such as quasimetric space, semimetric spaces etc. In this paper, we consider another generalization of a metric space, so called partial metric space which is a generalization of normal metric space portrayed in 1906 by Fréchet. This notion was introduced by Matthews [26]. The failure of a metric functions in computer science inspired him to introduced the concept of partial metrics. After introducing partial metric functions, Matthews [27] established the partial metric contraction theorem, this makes the partial metric function relevant in fixed point theory. In fact, partial metrics are more adaptable having broader topological properties than that of metrics and create partial orders. Heckmann [16] introduced the concept of weak partial metric function and established some fixed point results. Oltra and Valero [28] generalized the Matthews results in the sense of O'Neil in complete partial metric space. Abdeljawad et al. [1] considered a general form of the weak ϕ-contraction and established some common fixed point results. Karapinar [17] introduced generalized Seghal contraction and obtained a unique common fixed point for a pair of self mappings in complete partial metric space. Karapinar [18] generalized Cristi-Kirk's fixed point theorems using the concept of lower semi-continuous maps. Also, he proved some fixed point theorems in compact partial metric spaces. Karapinar and Erhan [24] established orbitally continuous operator and gave fixed point theorems. Chandok et al. [12] established some results for the existence and uniqueness of fixed point for a certain rational type contraction in partial metric space. Pant et al. [29] presented certain fixed point results for single and multivalued mappings in partial metric spaces. The results presented by Pant et al. [29] cannot be obtained from the corresponding results in metric space. Karapinar et al. [25] introduced rational type contraction and presented new results in partial metric space. To illustrate the usability of the results they provided the supportive example. Aydi et al. [2] established results on fixed point via a control function. Batsari and Kumam [6] established the existence, and uniqueness of globally stable fixed point of terminating mappings in partial metric space with some application in the space of probability density function. Later, many important results in partial metric space were established as an improvement and generalization of the existing results in the literature (see [7–11, 13, 15, 22, 27, 30, 31] and the references cited therein).

Furthermore, another significant area of fixed point theory was brought in light by Karapinar [20], who revisited the well-known fixed point theorem of Kannan under the aspect of interpolation and proposed a new Kannan type contraction to maximize the rate of convergence. Gaba and Karapinar [14] proposed a refinement in the interpolative approach in fixed point theory and gave fixed points and common fixed points for Kannan type contractions. One may have more results in partial metric spaces by using the interpolative theory (see [1, 3, 4, 14, 19, 21, 23] and the references cited therein).

In this manuscript, we establish some theorems for the existence and uniqueness of a fixed point in the framework of partial metric spaces using auxiliary functions. Our results generalize some existing results in the literature. To illustrate our results some examples are provided.

In the sequel we recall the notion of a partial metric space and some of its properties which will be useful in the main section to establish few results.

Definition 1.1. [26] Let X be a nonempty set. A function $p : X \times X \to [0, +\infty)$ is called a partial metric space on X if the following hold:

(i) $p(\rho, \sigma) \geq 0$ for all $\rho, \sigma \in X$ and $p(\rho, \rho) = p(\sigma, \sigma) = p(\rho, \sigma)$ if and only if $\rho = \sigma$;
(ii) $p(\rho, \rho) \leq p(\rho, \sigma)$ for all $\rho, \sigma \in X$;
(iii) $p(\rho, \sigma) = p(\sigma, \rho)$ for all $\rho, \sigma \in X$;
(iv) $p(\rho, \sigma) \leq p(\rho, \xi) + p(\xi, \sigma) - p(\xi, \xi)$ for all $\rho, \sigma, \xi \in X$.

Then the pair (X, p) is called a partial metric space.

It is clear that, if $p(\rho, \sigma) = 0$, then $\rho = \sigma$. But if $\rho = \sigma$, $p(\rho, \sigma)$ may not be 0.

Each partial metric p on X generates a T_0 topology τ_p on X which has as a base the family of open p-balls $\{B_p(\rho, \epsilon) : \rho \in X, \epsilon > 0\}$, where $B_p(\rho, \epsilon) = \{\sigma \in X : p(\rho, \sigma) < p(\rho, \rho) + \epsilon\}$ for all $\rho \in X$ and $\epsilon > 0$.

Similarly, closed p-ball is defined as $B_p[\rho, \epsilon] = \{\sigma \in X : p(\rho, \sigma) \leq p(\rho, \rho) + \epsilon\}$.

Remark 1.2. [12] If p is a partial metric on X, then $d_p : X \times X \to [0, +\infty)$ defined by

$$d_p(\rho, \sigma) = 2p(\rho, \sigma) - p(\rho, \rho) - p(\sigma, \sigma)$$

is a usual metric on X.

Example 1.3. [12] Let I denote the set of all intervals $[a, b]$ for any real numbers $a \leq b$. Let $p : I \times I \to [0, \infty)$ be a function such that $p([a, b], [c, d]) = \max\{b, d\} - \min\{a, c\}$. Then (I, p) is a partial metric space.

Example 1.4. [12] Let $X = \mathbb{R}$ and $p(\rho, \sigma) = e^{\max\{\rho,\sigma\}}$ for all $\rho, \sigma \in X$. Then (X, p) is a partial metric space.

Definition 1.5. [12]

(i) A sequence $\{\rho_n\}$ in a partial metric space (X,p) converges to $\rho \in X$ if and only if

$$\lim_{n\to\infty} p(\rho_n, \rho) = p(\rho, \rho).$$

(ii) A sequence $\{\rho_n\}$ in a partial metric space (X,p) is called a Cauchy sequence if and only if

$$\lim_{m,n\to\infty} p(\rho_n, \rho_m)$$

exists and is finite.

(iii) A partial metric space (X, p) is said to be complete if every Cauchy sequence $\{\rho_n\} \in X$ converges to a point $\rho \in X$ such that

$$\lim_{n\to\infty} p(\rho_n, \rho_m) = p(\rho, \rho).$$

The following lemmas in the literature will be useful in the proofs of the main results.

Lemma 1.6. *[12]*

(i) *A sequence $\{\rho_n\}$ is Cauchy in a partial metric space (X, p) if and only if $\{\rho_n\}$ is Cauchy in a metric space (X, d_p) where*

$$d_p(\rho, \sigma) = 2p(\rho, \sigma) - p(\rho, \rho) - p(\sigma, \sigma).$$

(ii) *A partial metric space (X, p) is complete if a metric space (X, d_p) is complete, i.e.,*

$$\lim_{n\to\infty} d_p(\rho, \rho_n) = 0 \Leftrightarrow \lim_{n\to\infty} p(\rho_n, \rho) = p(\rho, \rho) = \lim_{n,m\to\infty} p(\rho_n, \rho_m).$$

Lemma 1.7. *[12] Let (X, p) be a partial metric space.*

(i) *If $p(\rho, \sigma) = 0$, then $\rho = \sigma$.*

(ii) *If $\rho \neq \sigma$, then $p(\rho, \sigma) > 0$.*

Lemma 1.8. *(see [12]). Let $\rho_n \to \xi$ as $n \to \infty$ in a partial metric space (X, p) where $p(\xi, \xi) = 0$. Then* $\lim_{n\to\infty} p(\rho_n, \sigma) = p(\rho, \sigma)$ *for all $\sigma \in X$.*

2. Main results

The following classes of the auxiliary functions will be used later.

1). Let Ψ be the family of continuous and monotone non-decreasing functions $\psi : [0, \infty) \to [0, \infty)$ such that $\psi(t) = 0$ if and only if $t = 0$.

2). Let Φ be the family of lower semi-continuous functions $\phi : [0, \infty) \to [0, \infty)$ such that $\phi(t) = 0$ if and only if $t = 0$.

Theorem 2.1. *Let (X, p) be a complete partial metric space and $T : X \to X$ be a self mapping satisfying*

$$\psi(p(T\rho, T\sigma)) \leq \psi(M(\rho, \sigma)) - \phi(N(\rho, \sigma)) \quad \text{for all} \quad \rho, \sigma \in X, \tag{2.1}$$

where

$$M(\rho, \sigma) = \max\Big\{p(\rho, \sigma), p(\rho, T\rho), p(\sigma, T\sigma), \frac{p(\rho, T\sigma) + p(\sigma, T\rho)}{2}, \frac{p(\sigma, T\sigma)\,(1 + p(\rho, T\rho))}{1 + p(\rho, \sigma)}, \frac{p(\rho, T\rho)\,(1 + p(\rho, T\rho))}{1 + p(\rho, \sigma)}\Big\},$$

$$N(\rho, \sigma) = \max\Big\{p(\rho, \sigma), p(\rho, T\rho), p(\sigma, T\sigma), \frac{p(\sigma, T\sigma)(1 + p(\rho, T\rho))}{1 + p(\rho, \sigma)}, \frac{p(\rho, T\rho)(1 + p(\rho, T\rho))}{1 + p(\rho, \sigma)}\Big\}$$

for all $\psi \in \Psi$ and $\phi \in \Phi$. Then T has a unique fixed point.

Proof. Let $\rho_0 \in X$ be an arbitrary point. Then we construct a sequence $\{\rho_n\} \in X$ as follows:

$$\rho_{n+1} = T\rho_n \text{ for } n \geq 0.$$

If there exists n such that $\rho_{n+1} = \rho_n$ then ρ_n is a fixed point of T and the result is proved. Suppose that $\rho_{n+1} \neq \rho_n$ for all $n \geq 0$. Letting $\rho = \rho_{n-1}, \sigma = \rho_n$, we have

$$\psi(p(T\rho_{n-1}, T\rho_n)) \leq \psi(M(\rho_{n-1}, \rho_n)) - \varphi(N(\rho_{n-1}, \rho_n)), \tag{2.2}$$

where

$$\begin{aligned} M(\rho_{n-1}, \rho_n) &= \max\Big\{p(\rho_{n-1}, \rho_n), p(\rho_{n-1}, T\rho_{n-1}), p(\rho_n, T\rho_n), \frac{p(\rho_{n-1}, T\rho_n) + p(\rho_n, T\rho_{n-1})}{2}, \\ &\qquad \frac{p(\rho_n, T\rho_n)(1 + p(\rho_{n-1}, T\rho_{n-1}))}{(1 + p(\rho_{n-1}, \rho_n))}, \frac{p(\rho_{n-1}, T\rho_{n-1})(1 + p(\rho_{n-1}, T\rho_{n-1}))}{(1 + p(\rho_{n-1}, \rho_n))}\Big\} \\ &= \max\Big\{p(\rho_{n-1}, \rho_n), p(\rho_{n-1}, \rho_n), p(\rho_n, \rho_{n+1}), \frac{p(\rho_{n-1}, \rho_{n+1}) + p(\rho_n, \rho_n)}{2}, \\ &\qquad \frac{p(\rho_n, \rho_{n+1})(1 + p(\rho_{n-1}, \rho_n))}{(1 + p(\rho_{n-1}, \rho_n))}, \frac{p(\rho_{n-1}, \rho_n)(1 + p(\rho_{n-1}, \rho_n))}{(1 + p(\rho_{n-1}, \rho_n))}\Big\} \\ &= \max\Big\{p(\rho_{n-1}, \rho_n), p(\rho_n, \rho_{n+1}), \frac{p(\rho_{n-1}, \rho_{n+1}) + p(\rho_n, \rho_n)}{2}\Big\}. \end{aligned} \tag{2.3}$$

From the triangular inequality, we have

$$p(\rho_{n-1}, \rho_{n+1}) \leq p(\rho_{n-1}, \rho_n) + p(\rho_n, \rho_{n+1}) - p(\rho_n, \rho_n),$$

or

$$\frac{p(\rho_{n-1}, \rho_{n+1}) + p(\rho_n, \rho_n)}{2} \leq \frac{p(\rho_{n-1}, \rho_n) + p(\rho_n, \rho_{n+1})}{2} \leq \max\Big\{p(\rho_{n-1}, \rho_n), p(\rho_n, \rho_{n+1})\Big\}.$$

By (2.3), we get

$$M(\rho_{n-1},\rho_n) = \max\Big\{p(\rho_{n-1},\rho_n), p(\rho_n,\rho_{n+1})\Big\}, \tag{2.4}$$

$$\begin{aligned} N(\rho_{n-1},\rho_n) &= \max\Big\{p(\rho_{n-1},\rho_n), p(\rho_{n-1},T\rho_{n-1}), p(\rho_n,T\rho_n), \\ &\quad \frac{p(\rho_n,T\rho_n)(1+p(\rho_{n-1},T\rho_{n-1}))}{(1+p(\rho_{n-1},\rho_n))}, \frac{p(\rho_{n-1},T\rho_{n-1})(1+p(\rho_{n-1},T\rho_{n-1}))}{(1+p(\rho_{n-1},\rho_n))}\Big\} \\ &= \max\Big\{p(\rho_{n-1},\rho_n), p(\rho_{n-1},\rho_n), p(\rho_n,\rho_{n+1}), \\ &\quad \frac{p(\rho_n,\rho_{n+1})(1+p(\rho_{n-1},\rho_n))}{(1+p(\rho_{n-1},\rho_n))}, \frac{p(\rho_{n-1},\rho_n)(1+p(\rho_{n-1},\rho_n))}{(1+p(\rho_{n-1},\rho_n))}\Big\} \\ &= \max\Big\{p(\rho_{n-1},\rho_n), p(\rho_n,\rho_{n+1})\Big\}. \end{aligned}$$

By (2.2), we get

$$\psi(p(\rho_n,\rho_{n+1}) \leq \psi\Big(max(p(\rho_n,\rho_{n+1}), p(\rho_{n-1},\rho_n))\Big) - \varphi\Big(max(p(\rho_n,\rho_{n+1}), p(\rho_{n-1},\rho_n))\Big). \tag{2.5}$$

If $p(\rho_n,\rho_{n+1}) > p(\rho_{n-1},\rho_n)$, then from (2.5), we have

$$\psi(p(\rho_n,\rho_{n+1}) \leq \psi(p(\rho_n,\rho_{n+1})) - \varphi(p(\rho_n,\rho_{n+1})) < \psi(p(\rho_n,\rho_{n+1}))$$

which is a contradiction since $p(\rho_n,\rho_{n+1}) > 0$ by Lemma 1.7. So we have $p(\rho_n,\rho_{n+1}) \leq p(\rho_{n-1},\rho_n)$, that is, $p(\rho_n,\rho_{n+1})$ is a non increasing sequence of positive real numbers. Thus there exists $L \geq 0$ such that

$$\lim_{n\to\infty} p(\rho_n,\rho_{n+1}) = L. \tag{2.6}$$

Suppose that $L > 0$. Taking the lower limit in (2.5) as $n \to \infty$ and using (6) and the properties of ψ,φ, we have

$$\psi(L) \leq \psi(L) - \liminf_{n\to\infty} \varphi(p(\rho_{n-1},\rho_n)) \leq \psi(L) - \varphi(L) < \psi(L),$$

which is a contradiction. Therefore

$$\lim_{n\to\infty} p(\rho_n,\rho_{n+1}) = 0. \tag{2.7}$$

Using

$$d_p(\rho_n,\rho_{n+1}) = 2p(\rho,\sigma) - p(\rho,\rho) - p(\sigma,\sigma),$$

we have

$$d_p(\rho_n,\rho_{n+1}) \leq 2p(\rho_n,\rho_{n+1}).$$

This implies

$$d_p(\rho_n,\rho_{n+1}) = 0. \tag{2.8}$$

Now, we shall show that $\lim_{n,m\to\infty} p(\rho_n, \rho_m) = 0$. On the contrary, assume that $\lim_{n,m\to\infty} p(\rho_n, \rho_m) \neq 0$. Then there exists $\epsilon > 0$ for which there exist two subsequences $\{\rho_{m(k)}\}$ and $\{\rho_{n(k)}\}$ of $\{\rho_n\}$ such that $n(k)$ is the smallest index for which

$$n(k) > m(k) > k, \ \ p(\rho_n(k), \rho_m(k)) > \epsilon. \tag{2.9}$$

This implies

$$p(\rho_{n(k)-1}, \rho_{m(k)}) < \epsilon. \tag{2.10}$$

From (2.9) and (2.10), we have

$$\begin{aligned} \epsilon &\leq p(\rho_{n(k)}, \rho_{m(k)}) \leq p(\rho_{n(k)}, \rho_{n(k)-1}) + p(\rho_{n(k)-1}, \rho_{m(k)}) - p(\rho_{n(k)-1}, \rho_{n(k)-1}) \\ &\leq p(\rho_{n(k)}, \rho_{n(k)-1}) + p(\rho_{n(k)-1}, \rho_{m(k)}) < \epsilon + p(\rho_{n(k)}, \rho_{n(k)-1}). \end{aligned}$$

Taking the limit $k \to \infty$ and using (2.10), we get

$$\lim_{k\to\infty} p(\rho_{n(k)}, \rho_{m(k)}) = \epsilon. \tag{2.11}$$

By the triangle inequality, we have

$$\begin{aligned} p(\rho_{n(k)}, \rho_{m(k)}) &\leq p(\rho_{n(k)}, \rho_{n(k)-1}) + p(\rho_{n(k)-1}, \rho_{m(k)}) - p(\rho_{n(k)-1}, \rho_{n(k)-1}) \\ &\leq p(\rho_{n(k)}, \rho_{n(k)-1}) + p(\rho_{n(k)-1}, \rho_{m(k)}) \\ &\leq p(\rho_{n(k)}, \rho_{n(k)-1}) + p(\rho_{n(k)-1}, \rho_{m(k)-1}) \\ &\quad + p(\rho_{m(k)-1}, \rho_{m(k)}) - p(\rho_{m(k)-1}, \rho_{m(k)-1}) \\ &\leq p(\rho_{n(k)}, \rho_{n(k)-1}) + p(\rho_{n(k)-1}, \rho_{m(k)-1}) + p(\rho_{m(k)-1}, \rho_{m(k)}), \end{aligned}$$

$$\begin{aligned} p(\rho_{n(k)-1}, \rho_{m(k)-1}) &\leq p(\rho_{n(k)-1}, \rho_{n(k)}) + p(\rho_{n(k)}, \rho_{m(k)-1}) - p(\rho_{n(k)}, \rho_{n(k)}) \\ &\leq p(\rho_{n(k)-1}, \rho_{n(k)}) + p(\rho_{n(k)}, \rho_{m(k)-1}) \\ &\leq p(\rho_{n(k)-1}, \rho_{n(k)}) + p(\rho_{n(k)}, \rho_{m(k)}) \\ &\quad + p(\rho_{m(k)}, \rho_{m(k)-1}) - p(\rho_{m(k)}, \rho_{m(k)}) \\ &\leq p(\rho_{n(k)-1}, \rho_{n(k)}) + p(\rho_{n(k)}, \rho_{m(k)}) + p(\rho_{m(k)}, \rho_{m(k)-1}). \end{aligned}$$

Taking the limit $k \to \infty$ in the above two inequalities and using (2.7) and (2.11), we get

$$\lim_{k\to\infty} p(\rho_{n(k)-1}, \rho_{m(k)-1}) = \epsilon. \tag{2.12}$$

Now from (2.1), we have

$$\begin{aligned} \psi(p(\rho_{m(k)}, \rho_{n(k)})) &= \psi(p(T\rho_{m(k)-1}, T\rho_{n(k)-1})) \\ &\leq \psi(M(\rho_{m(k)-1}, \rho_{n(k)-1})) - \varphi(N(\rho_{m(k)-1}, \rho_{n(k)-1})), \end{aligned} \tag{2.13}$$

where

$$M(\rho_{m(k)-1}, \rho_{n(k)-1}) = \max\Big\{ p(\rho_{m(k)-1}, \rho_{n(k)-1}), p(\rho_{m(k)-1}, T\rho_{m(k)-1}), p(\rho_{n(k)-1}, T\rho_{n(k)-1}),$$

$$\frac{p(\rho_{m(k)-1},T\rho_{n(k)-1})+p(\rho_{n(k)-1},T\rho_{m(k)-1})}{2},$$
$$\frac{p(\rho_{m(k)-1},T\rho_{m(k)-1})(1+p(\rho_{m(k)-1},T\rho_{m(k)-1}))}{1+p(\rho_{m(k)-1},\rho_{n(k)-1})},$$
$$\frac{p(\rho_{n(k)-1},T\rho_{n(k)-1})(1+p(\rho_{m(k)-1},T\rho_{m(k)-1}))}{1+p(\rho_{m(k)-1},\rho_{n(k)-1})}\Big\} \tag{2.14}$$
$$= \max\Big\{p(\rho_{m(k)-1},\rho_{n(k)-1}),p(\rho_{m(k)-1},\rho_{m(k)}),p(\rho_{n(k)-1},\rho_{n(k)}),$$
$$\frac{p(\rho_{m(k)-1},\rho_{n(k)})+p(\rho_{n(k)-1},\rho_{m(k)})}{2},\frac{p(\rho_{m(k)-1},\rho_{m(k)})(1+p(\rho_{m(k)-1},\rho_{m(k)}))}{1+p(\rho_{m(k)-1},\rho_{n(k)-1})},$$
$$\frac{p(\rho_{n(k)-1},\rho_{n(k)})(1+p(\rho_{m(k)-1},\rho_{m(k)}))}{1+p(\rho_{m(k)-1},\rho_{n(k)-1})}\Big\}.$$

By the triangular inequality, we have

$$p(\rho_{m(k)-1},\rho_{n(k)})\leq p(\rho_{m(k)-1},\rho_{m(k)})+p(\rho_{m(k)},\rho_{n(k)})-p(\rho_{m(k)},\rho_{m(k)}), \tag{2.15}$$

$$p(\rho_{n(k)-1},\rho_{m(k)})\leq p(\rho_{n(k)-1},\rho_{n(k)})+p(\rho_{n(k)},\rho_{m(k)})-p(\rho_{n(k)},\rho_{n(k)}). \tag{2.16}$$

From (2.15) and (2.16), we have

$$p(\rho_{n(k)-1},\rho_{m(k)})+p(\rho_{m(k)-1},\rho_{n(k)})\leq p(\rho_{m(k)-1},\rho_{m(k)})+p(\rho_{n(k)-1},\rho_{n(k)})$$
$$+2p(\rho_{n(k)},\rho_{m(k)})-p(\rho_{m(k)},\rho_{m(k)})-p(\rho_{n(k)},\rho_{n(k)}). \tag{2.17}$$

Using (2.17) and (2.14), we get

$$M(\rho_{m(k)-1},\rho_{n(k)-1})=\max\Big\{p(\rho_{m(k)-1},\rho_{n(k)-1}),p(\rho_{m(k)-1},\rho_{m(k)}),p(\rho_{n(k)-1},\rho_{n(k)}), \tag{2.18}$$
$$\frac{p(\rho_{m(k)-1},\rho_{m(k)})+p(\rho_{n(k-1)},\rho_{n(k)})+2p(\rho_{n(k)},\rho_{m(k)})-p(\rho_{m(k)},\rho_{m(k)})-p(\rho_{n(k)},\rho_{n(k)})}{2},$$
$$\frac{p(\rho_{m(k)-1},\rho_{m(k)})(1+p(\rho_{m(k)-1},\rho_{m(k)}))}{1+p(\rho_{m(k)-1},\rho_{n(k)-1})},\frac{p(\rho_{n(k)-1},\rho_{n(k)})(1+p(\rho_{m(k)-1},\rho_{m(k)}))}{1+p(\rho_{m(k)-1},\rho_{n(k)-1})}\Big\}.$$

Taking the limit as $k\to\infty$ and using (2.6), (2.10) and (2.11), we have

$$\lim_{k\to\infty}M(\rho_{m(k)-1},\rho_{n(k)-1})=\max\{0,\epsilon\}=\epsilon, \tag{2.19}$$

$$N(\rho_{m(k)-1},\rho_{n(k)-1})=\max\Big\{p(\rho_{m(k)-1},\rho_{n(k)-1}),p(\rho_{m(k)-1},\rho_{m(k)}),p(\rho_{n(k)-1},\rho_{n(k)}),$$
$$\frac{p(\rho_{n(k)-1},\rho_{n(k)})(1+p(\rho_{m(k)-1},\rho_{m(k)}))}{1+p(\rho_{m(k)-1},\rho_{n(k)-1})},\frac{p(\rho_{m(k)-1},\rho_{m(k)})(1+p(\rho_{m(k)-1},\rho_{m(k)}))}{1+p(\rho_{m(k)-1},\rho_{n(k)-1})}\Big\}.$$

Taking the limit as $k\to\infty$ and using (2.6), (2.10) and (2.11), we have

$$\lim_{k\to\infty}N(\rho_{m(k)-1},\rho_{n(k)-1})=\epsilon. \tag{2.20}$$

Now taking the lower limit when $k \to \infty$ in (2.13) and using (2.10) and (2.12), we have

$$\begin{aligned}\psi(\epsilon) &\leq \psi(\epsilon) - \liminf_{k\to\infty} \varphi(N(\rho_{m(k)-1}, \rho_{n(k)-1}))\\ &\leq \psi(\epsilon) - \varphi(\epsilon) < \psi(\epsilon),\end{aligned}$$

which is a contradiction. So we have

$$\lim_{n,m\to\infty} p(\rho_n, \rho_m) = 0.$$

Since $\lim_{n,m\to\infty} p(\rho_n, \rho_m)$ exists and is finite, we conclude that ρ_n is a Cauchy sequence in (X, p). Using Remark 1.2, we have

$$d_p(\rho_n, \rho_m) \leq 2p(\rho_n, \rho_m).$$

Therefore,

$$\lim_{n,m\to\infty} d_p(\rho_n, \rho_m) = 0. \tag{2.21}$$

Thus by Lemma 1.6, $\{\rho_n\}$ is a Cauchy sequence in both (X, d_p) and (X, p). Since (X, p) is a complete partial metric space, there exists $\rho \in X$ such that

$$\lim_{n\to\infty} p(\rho_n, \rho) = p(\rho, \rho).$$

Since $\lim_{n,m\to\infty} p(\rho_n, \rho_m) = 0$, by Lemma 1.6, we have $p(\rho, \rho) = 0$. Now, we shall prove that ρ is a fixed point of T. Suppose that $T\rho \neq \rho$. From (2.1) and using Lemma 1.8, we have

$$\begin{aligned}\psi(p(\rho_n, T\rho)) &= \psi(p(T\rho_{n-1}, T\rho)\\ &\leq \psi\Big(\max\Big\{p(\rho_{n-1}, \rho), p(\rho_{n-1}, T\rho_{n-1}), p(\rho, T\rho), \frac{p(\rho_{n-1}, T\rho_{n-1})(1 + p(\rho_{n-1}, T\rho_{n-1}))}{1 + p(\rho_{n-1}, \rho)},\\ &\quad \frac{p(\rho, T\rho)(1 + p(\rho_{n-1}, T\rho_{n-1})))}{1 + p(\rho_{n-1}, \rho)}, \frac{p(\rho_{n-1}, T\rho) + p(\rho, T\rho_{n-1})}{2}\Big\}\Big)\\ &\quad -\varphi(\max\Big\{p(\rho_{n-1}, \rho), p(\rho_{n-1}, T\rho_{n-1}), p(\rho, T\rho),\\ &\quad \frac{p(\rho, T\rho)(1 + p(\rho_{n-1}, T\rho_{n-1}))}{1 + p(\rho_{n-1}, \rho)}, \frac{p(\rho_{n-1}, T\rho_{n-1})(1 + p(\rho_{n-1}, T\rho_{n-1}))}{1 + p(\rho_{n-1}, \rho)}\Big\}\Big).\end{aligned} \tag{2.22}$$

Letting the limit $n \to \infty$ in the above inequality and using the property of φ, ψ, we have

$$\begin{aligned}\psi(p(\rho, T\rho)) &\leq \psi\Big(\max\Big\{p(\rho, \rho), p(\rho, T\rho), \frac{p(\rho, T\rho)(1 + p(\rho, \rho))}{1 + p(\rho, \rho)}, \frac{p(\rho, T\rho) + p(\rho, \rho)}{2}\Big\}\Big)\\ &- \varphi(\max\Big\{p(\rho, \rho), p(\rho, T\rho)\Big\}\Big) \leq \psi(p(\rho, T\rho)) - \varphi(p(\rho, T\rho)) < \psi(p(\rho, T\rho)),\end{aligned}$$

which is a contradiction. Thus $T\rho = \rho$, i.e., ρ is a fixed point of T. Finally to prove uniqueness, suppose that σ is another fixed point of T such that $\rho \neq \sigma$. From (2.1), we have

$$\psi(p(\rho, \sigma)) = \psi(p(T\rho, T\sigma)) \leq \psi(M(\rho, \sigma)) - \varphi(N(\rho, \sigma)), \tag{2.23}$$

where

$$M(\rho,\sigma) = \max\left\{p(\rho,\rho), p(\rho,\sigma), p(\sigma,\sigma), \frac{p(\rho,\rho)(1+p(\rho,\sigma))}{1+p(\rho,\sigma)}, \frac{p(\rho,\sigma)+p(\sigma,\rho)}{2}\right\}$$
$$= p(\rho,\sigma). \tag{2.24}$$

Similarly

$$N(\rho,\sigma) = p(\rho,\sigma). \tag{2.25}$$

Using (2.24), (2.25) and (2.23), we have

$$\psi(p(\rho,\sigma)) \le \psi(p(\rho,\sigma)) - \varphi(p(\rho,\sigma)) < \psi(p(\rho,\sigma)),$$

which is a contradiction since $p(\rho,\sigma) > 0$. Hence $\rho = \sigma$. □

Corollary 2.2. *Let (X, p) be a complete partial metric space and $T : X \to X$ be a self mapping satisfying*

$$\psi(p(T\rho, T\sigma)) \le \psi(M(\rho,\sigma)) - \phi(M(\rho,\sigma)) \quad \text{for all} \quad \rho,\sigma \in X,$$

where

$$M(\rho,\sigma) = \max\left\{p(\rho,\sigma), p(\rho,T\rho), p(\sigma,T\sigma), \frac{p(\rho,T\sigma)+p(\sigma,T\rho)}{2}, \frac{p(\sigma,T\sigma)\,(1+p(\rho,T\rho))}{1+p(\rho,\sigma)}, \frac{p(\rho,T\rho)\,(1+p(\rho,T\rho))}{1+p(\rho,\sigma)}\right\}$$

for all $\psi \in \Psi$ and $\phi \in \Phi$. Then T has a unique fixed point.

Corollary 2.3. *Let (X, p) be a complete partial metric space and $T : X \to X$ be a self mapping satisfying*

$$\psi(p(T\rho, T\sigma)) \le \psi(N(\rho,\sigma)) - \phi(N(\rho,\sigma)) \quad \text{for all} \quad \rho,\sigma \in X,$$

where

$$N(\rho,\sigma) = \max\left\{p(\rho,\sigma), p(\rho,T\rho), p(\sigma,T\sigma), \frac{p(\sigma,T\sigma)(1+p(\rho,T\rho))}{1+p(\rho,\sigma)}, \frac{p(\rho,T\rho)(1+p(\rho,T\rho))}{1+p(\rho,\sigma)}\right\}$$

for all $\psi \in \Psi$ and $\phi \in \Phi$. Then T has a unique fixed point.

Taking ψ to an identity mapping and $\phi(s) = (1-k)s$ for all $s \ge 0$, where $k \in (0,1)$, we obtain the following results.

Corollary 2.4. *Let (X, p) be a complete partial metric space and $T : X \to X$ be a self mapping satisfying*

$$p(T\rho, T\sigma) \le k \max\left\{p(\rho,\sigma), p(\rho,T\rho), p(\sigma,T\sigma), \frac{p(\rho,T\sigma)+p(\sigma,T\rho)}{2}, \frac{p(\sigma,T\sigma)\,(1+p(\rho,T\rho))}{1+p(\rho,\sigma)}, \frac{p(\rho,T\rho)\,(1+p(\rho,T\rho))}{1+p(\rho,\sigma)}\right\}$$

for all $\rho,\sigma \in X$ and $k \in (0,1)$. Then T has a unique fixed point.

Corollary 2.5. *Let (X, p) be a complete partial metric space and $T : X \to X$ be a self mapping satisfying*

$$\begin{aligned} p(T\rho, T\sigma) &\leq k \max\Big\{p(\rho,\sigma), p(\rho, T\rho), p(\sigma, T\sigma), \\ &\quad \frac{p(\sigma, T\sigma)(1 + p(\rho, T\rho))}{1 + p(\rho,\sigma)}, \frac{p(\rho, T\rho)(1 + p(\rho, T\rho))}{1 + p(\rho,\sigma)}\Big\} \end{aligned}$$

for all $\rho, \sigma \in X$ and $k \in (0, 1)$. Then T has a unique fixed point.

Example 2.6. Let $X = [0, 1]$. Define $T : X \to X$ by $T\rho = \frac{\rho}{3}$ and $p : X \times X \to [0, \infty)$ by $p(\rho, \sigma) = \max\{\rho, \sigma\}$, then (X, p) is a complete partial metric space and

$$\begin{aligned} p(T\rho, T\sigma) &\leq \frac{1}{3} \max\Big\{p(\rho,\sigma), p(\rho, T\rho), p(\sigma, T\sigma), \\ &\quad \frac{p(\rho, T\sigma) + p(\sigma, T\rho)}{2}, \frac{p(\sigma, T\sigma)\,(1 + p(\rho, T\rho))}{1 + p(\rho,\sigma)}, \frac{p(\rho, T\rho)\,(1 + p(\rho, T\rho))}{1 + p(\rho,\sigma)}\Big\} \end{aligned}$$

Thus by Corollary 2.4, T has a unique fixed point. Here 0 is the unique fixed point of T.

Example 2.7. Let $X = [0, 1]$. Define $T : X \to X$ by $T\rho = \frac{\rho}{2}$ and $p : X \times X \to [0, \infty)$ by $p(\rho, \sigma) = \max\{\rho, \sigma\}$, then (X, p) is a complete partial metric space and

$$\begin{aligned} p(T\rho, T\sigma) &\leq \frac{1}{2} \max\Big\{p(\rho,\sigma), p(\rho, T\rho), p(\sigma, T\sigma), \\ &\quad \frac{p(\sigma, T\sigma)(1 + p(\rho, T\rho))}{1 + p(\rho,\sigma)}, \frac{p(\rho, T\rho)(1 + p(\rho, T\rho))}{1 + p(\rho,\sigma)}\Big\} \end{aligned}$$

Thus by Corollary 2.5, T has a unique fixed point. Here 0 is the unique fixed point of T.

Example 2.8. Let $X = [0, \infty)$ and $p(\rho, \sigma) = \max\{\rho, \sigma\}$. Then (X, p) is a complete partial metric space. Consider the mapping $T : X \to X$ defined by

$$T(\rho) = \begin{cases} 0 & \text{if } 0 \leq \rho < 1; \\ \dfrac{\rho^2}{\rho + 1} & \text{if } \rho \geq 1. \end{cases} \tag{2.26}$$

and $\varphi(t), \psi(t) : [0, \infty) \to [0, \infty)$, $\varphi(t) = \dfrac{t}{1 + t}$ and $\psi(t) = t$.
We have the following cases.
Case (i) If $\rho, \sigma \in [0, 1)$ and assume that $\rho \geq \sigma$, we have

$$p(T\rho, T\sigma) = 0,$$

and

$$\begin{aligned} M(\rho,\sigma) &= \max\Big\{p(\rho,\sigma), p(\rho, T\rho), p(\sigma, T\sigma), \frac{p(\rho, T\sigma) + p(\sigma, T\rho)}{2}, \\ &\quad \frac{p(\sigma, T\sigma)(1 + p(\rho, T\rho))}{1 + p(\rho,\sigma)}, \frac{p(\rho, T\rho)(1 + p(\rho, T\rho))}{1 + p(\rho,\sigma)}\Big\} \end{aligned}$$

$$
\begin{aligned}
&= \max\Big\{\rho, \rho, \sigma, \frac{\rho+\sigma}{2}, \frac{\rho(1+\sigma)}{1+\rho}, \frac{\sigma(1+\sigma)}{1+\rho}\Big\}\\
&= \max\{\sigma, \rho, \rho\} = \rho.
\end{aligned}
$$

On the same lines

$$N(\rho, \sigma) = \rho.$$

Therefore

$$\psi(p(T\rho, T\sigma)) = 0, \tag{2.27}$$

and

$$\psi(M(\rho, \sigma)) - \varphi(N(\rho, \sigma)) = \rho - \frac{\rho}{1+\rho} = \frac{\rho^2}{1+\rho}. \tag{2.28}$$

From (2.27) and (2.28), we have $\psi(p(T\rho, T\sigma)) \leq \psi(M(\rho, \sigma)) - \varphi(N(\rho, \sigma))$.
Case (ii) If $\sigma \in [0, 1)$ and $\rho \geq 1$, we have

$$p(T\rho, T\sigma) = \max\left\{\frac{\rho^2}{1+\rho}, 0\right\} = \frac{\rho^2}{1+\rho},$$

and

$$
\begin{aligned}
M(\rho, \sigma) &= \max\Big\{p(\rho, \sigma), p(\rho, T\rho), p(\sigma, T\sigma), \frac{p(\rho, T\sigma) + p(\sigma, T\rho)}{2},\\
&\quad \frac{p(\sigma, T\sigma)(1 + p(\rho, T\rho))}{1 + p(\rho, \sigma)}, \frac{p(\rho, T\rho)(1 + p(\rho, T\rho))}{1 + p(\rho, \sigma)}\Big\}\\
&= \max\Big\{\rho, \rho, \sigma, \frac{2\rho^2 + \sigma}{2(\rho+1)}, \frac{\rho(1+\sigma)}{1+\rho}, \frac{\sigma(1+\sigma)}{1+\rho}\Big\}\\
&= \rho.
\end{aligned}
$$

On the same lines

$$N(\rho, \sigma) = \rho.$$

Therefore

$$\psi(p(T\rho, T\sigma) = \frac{\rho^2}{1+\rho}, \tag{2.29}$$

and

$$\psi(M(\rho, \sigma)) - \varphi(N(\rho, \sigma)) = \frac{\rho^2}{1+\rho}. \tag{2.30}$$

From (2.29) and (2.30), we have $\psi(p(T\rho, T\sigma)) = \psi(M(\rho, \sigma)) - \varphi(N(\rho, \sigma))$.
Case (iii) If $\rho \geq \sigma \geq 1$, we have

$$p(T\rho, T\sigma) = \max\left\{\frac{\rho^2}{1+\rho}, \frac{\sigma^2}{1+\sigma}\right\} = \frac{\rho^2}{1+\rho},$$

and

$$M(\rho, \sigma) = \max\Big\{p(\rho, \sigma), p(\rho, T\rho), p(\sigma, T\sigma), \frac{p(\rho, T\sigma) + p(\sigma, T\rho)}{2},$$

$$
\begin{aligned}
& \frac{p(\sigma, T\sigma)(1 + p(\rho, T\rho))}{1 + p(\rho, \sigma)}, \frac{p(\rho, T\rho)(1 + p(\rho, T\rho))}{1 + p(\rho, \sigma)}\bigg\} \\
= \ & \max\bigg\{\rho, \rho, \sigma, \frac{2\rho^2 + \sigma}{2(\rho + 1)}, \frac{\rho(1 + \sigma)}{1 + \rho}, \frac{\sigma(1 + \sigma)}{1 + \rho}\bigg\} \\
= \ & \rho.
\end{aligned}
$$

On the same lines

$$N(\rho, \sigma) = \rho.$$

Therefore

$$\psi(p(T\rho, T\sigma) = \frac{\rho^2}{1 + \rho}, \tag{2.31}$$

and

$$\psi(M(\rho, \sigma)) - \varphi(N(\rho, \sigma)) = \frac{\rho^2}{1 + \rho}. \tag{2.32}$$

From (2.31) and (2.32), we have $\psi(p(T\rho, T\sigma)) = \psi(M(\rho, \sigma)) - \varphi(N(\rho, \sigma))$.

Thus it satisfies all the conditions of Theorem 2.1. Hence T has a unique fixed point, indeed, $\rho = 0$ is the required point. However, the inequality 2.1 is not satisfied when the partial p is replaced by the usual metric. Indeed, Take $\rho = 2$ and $\sigma = 2.5$, then

$$\psi(d(T\rho, T\sigma)) = 19/42 \ \ \& \ \ \psi(M(\rho, \sigma)) - \phi(N(\rho, \sigma)) = 1/6.$$

Hence, inequality 2.1 is not satisfied.

Example 2.9. Let $X = [0, 1/2]$ and $p(\rho, \sigma) = \max\{\rho, \sigma\}$. Then (X, p) is a complete partial metric space. Consider the mapping $T : X \to X$ defined by $T\rho = \rho^3 - \frac{3}{2}\rho^2 + \frac{31}{36}\rho + \frac{23}{72}$ for all $\rho \in X$ and $\varphi(t), \psi(t) : [0, \infty) \to [0, \infty)$, $\varphi(t) = \dfrac{t}{100000 + t}$ and $\psi(t) = t$.

Without loss of generality, assume that $\rho \geq \sigma$, we have

$$
\begin{aligned}
p(T\rho, T\sigma) &= \max\left\{\rho^3 - \frac{3}{2}\rho^2 + \frac{31}{36}\rho + \frac{23}{72}, \sigma^3 - \frac{3}{2}\sigma^2 + \frac{31}{36}\sigma + \frac{23}{72}\right\} \\
&= \rho^3 - \frac{3}{2}\rho^2 + \frac{31}{36}\rho + \frac{23}{72},
\end{aligned}
$$

and

$$
\begin{aligned}
M(\rho, \sigma) &= \max\bigg\{p(\rho, \sigma), p(\rho, T\rho), p(\sigma, T\sigma), \frac{p(\rho, T\sigma) + p(\sigma, T\rho)}{2}, \\
&\quad \frac{p(\sigma, T\sigma)(1 + p(\rho, T\rho))}{1 + p(\rho, \sigma)}, \frac{p(\rho, T\rho)(1 + p(\rho, T\rho))}{1 + p(\rho, \sigma)}\bigg\} \\
&= \frac{\left(\rho^3 - \frac{3}{2}\rho^2 + \frac{31}{36}\rho + \frac{23}{72}\right)\left(\rho^3 - \frac{3}{2}\rho^2 + \frac{31}{36}\rho + \frac{95}{72}\right)}{1 + \rho}.
\end{aligned}
$$

On the same lines

$$N(\rho, \sigma) = \frac{\left(\rho^3 - \frac{3}{2}\rho^2 + \frac{31}{36}\rho + \frac{23}{72}\right)\left(\rho^3 - \frac{3}{2}\rho^2 + \frac{31}{36}\rho + \frac{95}{72}\right)}{1 + \rho}.$$

One can easily verify, that $\psi(p(T\rho, T\sigma)) \leq \psi(M(\rho, \sigma)) - \varphi(N(\rho, \sigma))$.

Thus it satisfies all the conditions of Theorem 2.1. Hence T has a unique fixed point in X, indeed, $\rho = 1/2$ is the required point in $X = [0, 1/2]$.

3. Conclusions

Chandok et al. [8] established some results on fixed point for rational type of contraction in the framework of metric space endowed with a partial order. In this paper, we have extended the results of Chandok et al. [8] in a space having non-zero self distance, that is, partial metric space and established some theorems for the existence and uniqueness of a fixed point using auxiliary functions. Our results generalize some existing results in the literature. To illustrate our results some examples have been provided.

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Conflict of interest

The authors declare that they have no competing interests.

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