Results in Physics 19 (2020) 103329

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

M-lump, N-soliton solutions, and the collision phenomena for the (2 + 1)-dimensional Date-Jimbo-Kashiwara-Miwa equation

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ARTICLE INFO

Keywords:
N-soliton solutions
M-lump solutions
Collision phenomena
Date-Jimbo-Kashiwara-Miwa equation

ABSTRACT

In this work, N-soliton waves, fusion solutions, mutiple M-lump solutions and the collision phenomena between one-M-lump and one-, two-soliton solutions to the (2 + 1)-dimensional Date-Jimbo-Kashiwara-Miwa equation are successfully revealed. A class of one-, two-, three-soliton, one-, two-fusion solutions are derived via the Hirota bilinear method and 1-M-lump, 2-M-lump solutions are constructed via the long-wave method. Moreover, physical collision phenomenon of 1-M-lump with one-, two-soliton solutions and also, with fusion solutions are successfully presented. The velocity of the 1-M-lump wave in x- and y-direction are also studied.

Introduction

Nonlinear partial differential equations (NPDEs) have been seen as templates for explaining nonlinear physical phenomena resulting through computational chemistry, plasma physics, particle physics, and so on. Much attention has been paid to the study of exact or analytical solutions due to its essential role in the analysis of the physical aspects of the models. Via the analytical methods distinct types of the waves can be constructed, like solitary waves, optical solutions, singular solutions, periodic waves, breather waves, rogue waves, and rational wave solutions. To date, several systematic approaches have been derived to construct the exact solutions of NPDEs, like the Riemann-Hilbert method [1], Wronskian determinant [2], inverse scattering method [3], the Bernoulli sub-equation method [4,5], the modified auxiliary expansion method [6], a $(m + G'/G)$-expansion method [7,8], the Lie symmetry analysis [9], the Darboux transformation method [10], Hirota bilinear method [11], the unified method and its generalized approach [12–15], and many other approaches [16–28].

The KP equation is a NPDEs to describe nonlinear wave motion introduced by Kadomtsev and Petviashvili [29] for the first time. The KP equation is written as:

$$\frac{\partial}{\partial x}(u_t + uu_x + u_{xxx}) + 3u_{yy} = 0. \tag{1}$$

Basic soliton equations, for example, the Kadomtsev-Petviashvili (KP) equation and its Bäcklund transformation formula may well be considered as "atoms" for the construction of different types of soliton equations. Both the unknown equations KP and modified KP hierarchy are fundamental significance not just in the theory of integrable systems, but also play a key role in mathematical physics, too. To investigate that all the KP hierarchy equation are integrable, Dorizzi et al. [30] have shown that the higher equation in the KP hierarch, i.e. (3 + 1)-dimensional Jimbo-Miwa equation, cannot be integrate without conditional sense. The Date-Jimbo-Kashiwara-Miwa (DJKM) equation is one of the KP hierarch equations that pass integrability. The (2 + 1) dimensional Date-Jimbo-Kashiwara-Miwa (DJKM) equation [31] has been written as an integral extension of the KP hierarchy and reads

$$u_{xxxxy} + 4u_{xxy}u_x + 2u_{xxx}u_y + 6u_{xy}u_{xx} - \alpha u_{yyy} - 2\beta u_{xxt} = 0. \tag{2}$$

A lot of studies have been presented to seek the solutions of Eq. (2). Wang and Hu [32] have derived the Grammian solutions of Eq. (2). Guo and Lin [33] have studied interaction solutions between lump and stripe soliton solutions via a quadratic function. Adem et al. [34] have used the extended transformed rational function that depends on the Hirota bilinear form to constructed Complexiton solutions of the DJKM equation. Yuan et al. [35] have studied Wronskian and Grammian solutions to the DJKM equation. Singh and Gupta [36] have investigated

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https://doi.org/10.1016/j.rinp.2020.103329
Received 17 June 2020; Accepted 12 August 2020
Available online 25 August 2020

the Painlevé property of the suggested equation and have revealed some exact solutions to the studied equation by using the Pickering's algorithm. Sajid and Akram [37] have utilized $\exp(-\Phi(\xi))$-expansion method to seek some exact solutions to Eq. (2).

In order to study the multiple soliton solutions to the DJKM equation, from the relationship of dispersion to the nonlinearity, one can define a logarithmic variable transformation for the suggested equation as

$$u = 2\frac{\partial}{\partial x}(ln(f(x, y, t))). \tag{3}$$

Plugging Eq. (3) to Eq. (2) the result is

$$\begin{aligned} &2\alpha f_y^3 + 4f_x(\beta f_t f_x + f_{xy}f_{xx} - f_x f_{xxy}) \\ &\quad - 2f(f_{xx}(\beta f_t + f_{xxy}) + 2f_x(\beta f_{xt} - f_{xxxy}))+ \\ &f_y(2(f_{xx}{}^2 - 2f_x f_{xxx}) + f(-3\alpha f_{yy} + f_{xxxx})) + f^2(\alpha f_{yyy} + 2\beta f_{xxt} - f_{xxxxy}) \\ &\quad = 0, \end{aligned} \tag{4}$$

and can be rewritten as a Hirota bilinear form

$$D_x((D_x^3 D_y - 3\beta D_x D_t)f.f).f^2 + \frac{1}{2}D_y((D_x^4 - 3\alpha D_y^2)f.f).f^2 = 0. \tag{5}$$

where D_x, D_y, D_t are the Hirota's bilinear operators and defined in Ref. [38–40]. When f is going to solve Eq. (4) or (5), then $u = u(x, y, t)$ in Eq. (3) describe the solution of Eq. (2). In this research paper, we study Eq. (2) to reveal N-soliton waves, fusion solutions, M-lump solutions, and the interactions phenomena that appear between M-lump and N-soliton solutions as well as with fusion solution.

N-soliton solutions

In this section, to ascertain the N-soliton solutions to the DJKM equation in (2 + 1)-dimensions, we use the logarithmic function defined in Eq. (3). First, we use a function $f = f(x, y, t)$ that reads as

$$f \equiv f_N = \sum_{\mu=0,1} \exp\left(\sum_{i<j}^{N} \mu_i\mu_j \log(M_{ij}) + \sum_{i=1}^{N} \mu_i\theta_i\right), \tag{6}$$

where

$$\theta_i = k_i(x + l_i y + \alpha_i + w_i t) + \alpha_i, \tag{7}$$

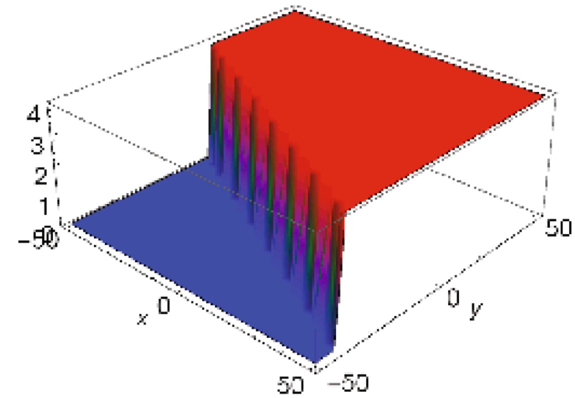
$$M_{ij} = \frac{(k_i - k_j)^2 + (l_i - l_j)^2\alpha}{(k_i + k_j)^2 + (l_i - l_j)^2\alpha}, \tag{8}$$

and the dispersion relation is given by

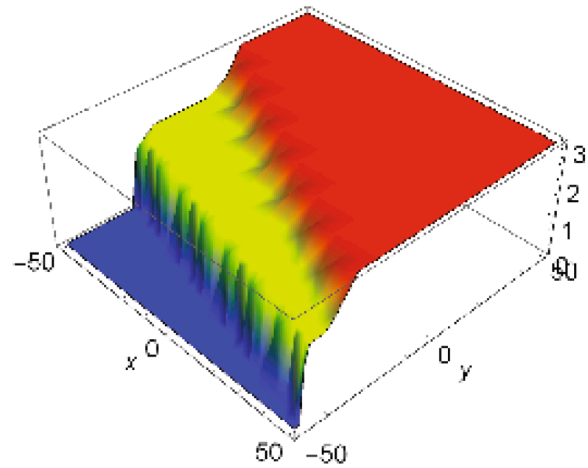
$$w_1 = \frac{k_1{}^2 l_1 - l_1^3\alpha}{2\beta}. \tag{9}$$

The notation $\sum_{\mu=0,1}$ States summation over all possible combinations. Let $N = 1$ in Eq. (6) and substituting a result with Eqs. (7)–(9) into Eq. (3), a 1-soliton wave solution can be derived as below

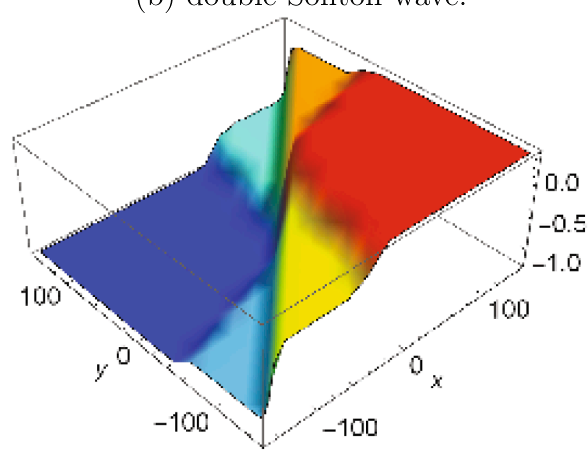
$$u = \frac{2k_1 e^{k_1\left(x + l_1 y + \frac{(k_1{}^2 l_1 - l_1^3\alpha)}{2\beta}t\right)}}{1 + e^{k_1\left(x + l_1 y + \frac{(k_1{}^2 l_1 - l_1^3\alpha)}{2\beta}t\right)}}. \tag{10}$$



(a) single-Soliton wave.



(b) double-Soliton wave.



(c) triple-Soliton wave.

Fig. 1. 3D surface solutions of Eqs. (11)–(13) plotted when (a) $t = 4$, $k_1 = 2$, $l_1 = 2$, $\alpha_1 = 0$, $\alpha = -1$, $\beta = 1$, (b) $t = 4$, $k_1 = 0.5$, $k_2 = 1$, $l_1 = 2$, $l_2 = 4$, $\alpha_1 = 1$, $\alpha_2 = 1$, $\alpha = -1$, $\beta = 1$, (c) $t = 4$, $k_1 = -0.4$, $k_2 = -0.2$, $k_3 = 0.1$, $l_1 = -1$, $l_2 = -0.3$, $l_3 = -3$, $\alpha_1 = 1$, $\alpha_2 = 1$, $\alpha_3 = 1$, $\alpha = -1$, $\beta = 1$.

When $N = 2, 3$, in the same way, we can construct 2- and 3-soliton solutions of (2 + 1)-dimensional DJKM equation as follows

$$u = \frac{2\left(\sum_{i=1}^{2} k_i e^{k_i\left(x+l_i y+\alpha_i+\frac{l_i(k_i^2-l_i^2\alpha)}{2\beta}t\right)} + (k_1+k_2)M_{12} e^{\sum_{i=1}^{2} k_i\left(x+l_i y+\alpha_i+\frac{l_i(k_i^2-l_i^2\alpha)}{2\beta}t\right)}\right)}{1+\sum_{i=1}^{2} k_i e^{k_i\left(x+l_i y+\alpha_i+\frac{l_i(k_i^2-l_i^2\alpha)}{2\beta}t\right)} + (k_1+k_2)M_{12} e^{\sum_{i=1}^{2} k_i\left(x+l_i y+\alpha_i+\frac{l_i(k_i^2-l_i^2\alpha)}{2\beta}t\right)}}, \tag{11}$$

$$u = \frac{2\left(\sum_{i=1}^{3} k_i e^{\theta_i} + \sum_{i<j}^{(3)} (k_i+k_j)M_{ij} e^{\theta_i+\theta_j} + M_{ijk} e^{\sum_{i=1}^{3}\theta_i} \sum_{i=1}^{3} k_i\right)}{1+\sum_{i=1}^{3} k_i e^{\theta_i} + \sum_{i<j}^{(3)} (k_i+k_j)M_{ij} e^{\theta_i+\theta_j} + M_{ijk} e^{\sum_{i=1}^{3}\theta_i} \sum_{i=1}^{3} k_i}, \tag{12}$$

where $M_{ijk} = M_{ij}M_{ik}M_{jk}\ (1 \leqslant m < n < k \leqslant N)$. Eqs. (10)–(12) are drawn to more understand its physical phenomena as shown in Fig. 1. To study and finding the fusion solutions, we have to choose $M_{ij} = 0$ in Eq. (11) and Eq. (12). To construct 1-fusion solution, we choose $k_1 = -3, k_2 = -2, l_1 = 1, l_2 = 2, \alpha = -1$, and for 2-fusion solution, we let $k_1 = -3, k_2 = -2, k_3 = -2, l_1 = 1, l_2 = 2, l_3 = 2, \alpha = -1$.

From N-soliton solutions to rational solutions

Lump solutions are a specific type of rational function solutions, observed in all aspects of natural processes, contrary to soliton solutions. In this portion of work, we seek M-lumps solutions by using a long-wave method. When and taking the limit $k_m \to 0$, $\frac{k_1}{k_2} = O(1)$ and $\alpha_m = -1\ (m = 1, 2)$ in Eq. (6), we have

$$f_2 = \eta_1\eta_2 + V_{12}, \tag{13}$$

where
$\eta_1 = x + l_1 y + w_1 t,\ \eta_2 = x + l_2 y + w_2 t,\ w_1 = -\frac{\alpha}{2\beta}l_1^3,\ V_{12} = -\frac{4}{(l_1-l_2)^2\alpha}$ and $l_2 = l_1^*$. Putting Eq. (13) into Eq. (3), we obtain a 1-M-lump solution as shown in Fig. 2 and read as follows

$$u = 2\frac{\partial}{\partial x}\log\left((x' + ay')^2 + b^2y'^2 + \frac{1}{b^2\alpha}\right) = \frac{4(x' + ay')}{(x' + ay')^2 + b^2y'^2 + \frac{1}{b^2\alpha}}, \tag{14}$$

$$x' = x + \frac{\alpha a}{\beta}\left(\frac{a^2}{2} + b^2\right)t,\ \ y' = y + \frac{\alpha}{2\beta}(b^2 - 3a^2)t \tag{15}$$

The rational solution (14) is a permanent lump solution that decays as $O\left(\frac{1}{x^2}, \frac{1}{y^2}\right)$ for $|x|, |y| \to \infty$and to move with the velocity

$$v_x = -\frac{\alpha a}{\beta}\left(\frac{a^2}{2} + b^2\right),$$
$$v_y = \frac{\alpha}{2\beta}(3a^2 - b^2). \tag{16}$$

From 2-soliton solutions, we can derive a 2-M-lump solution by taking a limit $k_m \to 0$ and $\alpha_m = -1\ (m = 1, 2, 3, 4)$, then f_4 is equivalent to

$$f_4 = \eta_1\eta_2\eta_3\eta_4 + V_{12}\eta_3\eta_4 + V_{13}\eta_2\eta_4 + V_{14}\eta_2\eta_3 + V_{23}\eta_1\eta_4 + V_{24}\eta_1\eta_3 + V_{34}\eta_1\eta_2 + V_{12}V_{34} + V_{13}V_{24} + V_{14}V_{23}, \tag{17}$$

where

$$\eta_i = x + l_i y + w_i t, \tag{18}$$

$$w_i = -\frac{\alpha}{2\beta}l_i^3, \tag{19}$$

$$V_{ij} = -\frac{4}{(l_i - l_j)^2\alpha}. \quad (i < j) \tag{20}$$

Eq. (17) gives us a singular solution at some position, to find a 2-M-lump solution we choose $l_{\frac{N}{2}+i} = l_i^*$, $\left(i = 1, 2, .., \frac{N}{2}\right)$. Substituting Eq. (17) with Eqs. (18)–(20), lead a two-M-lump solution to the studied equation as shown in Fig. 3.

Interactions phenomena between 1-M-lump and soliton solutions

The interaction physical phenomena between M-lump wave and 1-, 2-soliton solutions as well as with fusion solution will construct in this section of the work. To study the interaction between M-lump and 1-soliton solution, we let $N = 3$ in Eq. (6), taking the limit $k_m \to 0$, $(m = 1, 2)$ and $\frac{k_1}{k_2} = O(1)$, then f_3 can set up as

$$f_3 = \eta_1\eta_2 + V_{12} + \Omega_1 e^{\theta_3}, \tag{21}$$

where

$$\Omega_1 = \eta_1\eta_2 + V_{12} + C_{23}\eta_1 + C_{13}\eta_2 + C_{13}C_{23}. \tag{22}$$

where θ_3 is stated in Eq. (7), $\eta_i(i = 1, 2)$ are stated in Eq. (18), V_{12} is given in Eq. (20). The constants $C_{m3}(r = 1, 2)$ are defined as below

$$C_{m3} = -\frac{4k_3}{k_3^2 + l_m^2\alpha - 2l_m l_3\alpha + l_3^2\alpha}. \quad m = 1, 2 \tag{23}$$

As a result of substituting Eq. (21) together with Eq. 22,23 into Eq. (3), we obtain an equation that presents a collision between a 1-M-lump solution and a 1-soliton solution (see Fig. 4). The collision physical phenomena between single-M-lump and 2-soliton solution can be

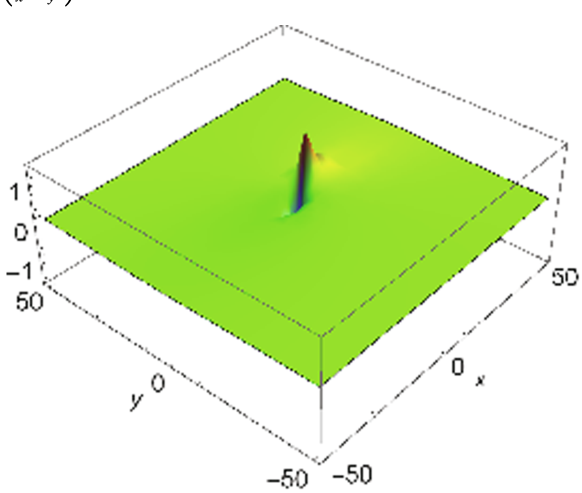


Fig. 2. 3D surface solution of Eq. (15) plotted when $t = 4$, $a = 1/2$, $b = 2$, $\alpha = -1$, $\beta = 1$.

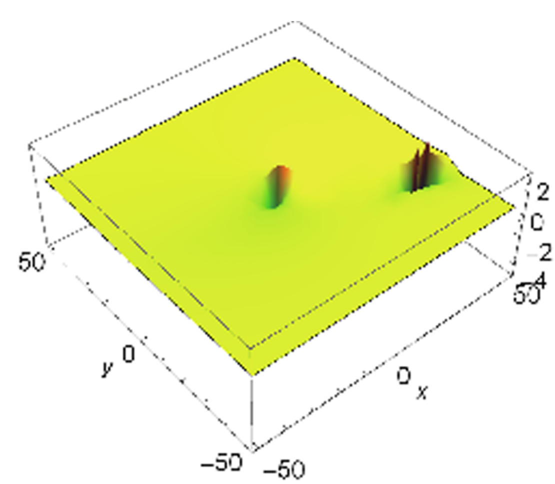
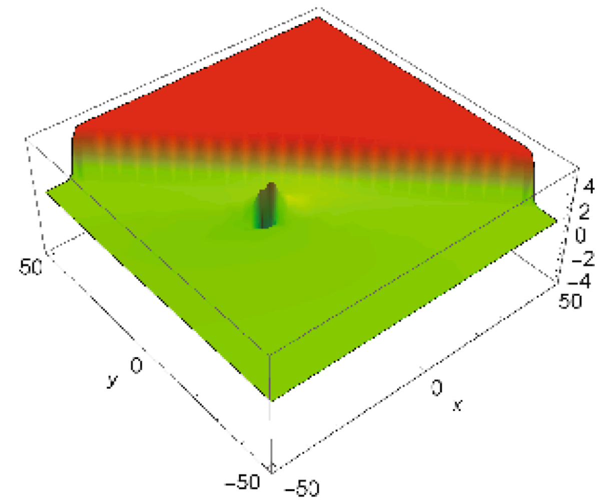
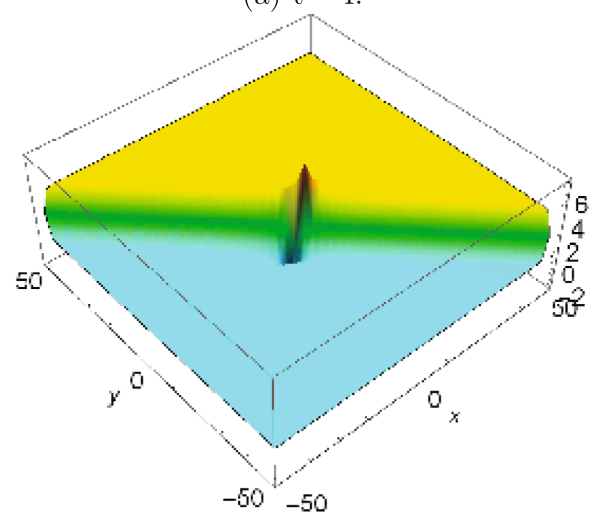


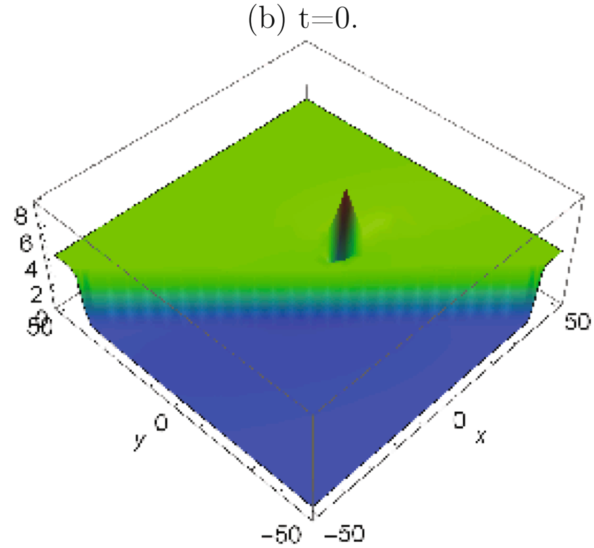
Fig. 3. 3D surface solution of Eq. (18) plotted when $t = 4$, $a = 2$, $b = 1$, $d = 1$, $c = 1/4$, $\alpha = -1$, $\beta = 1$.



(a) t=-4.

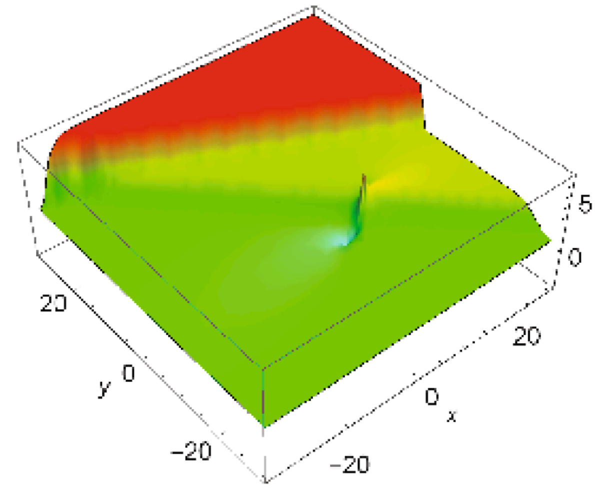


(b) t=0.

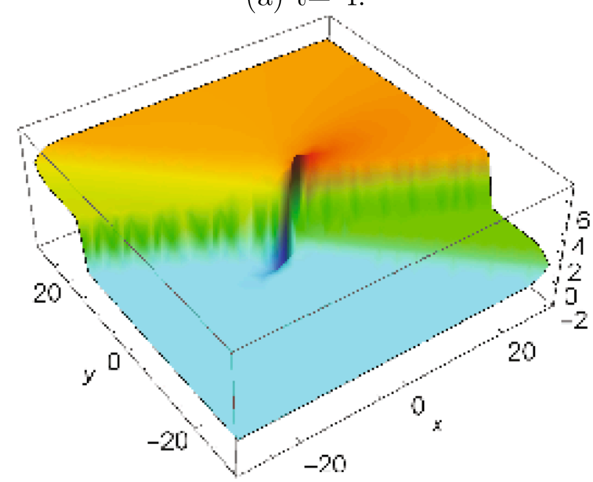


(c) t=4.

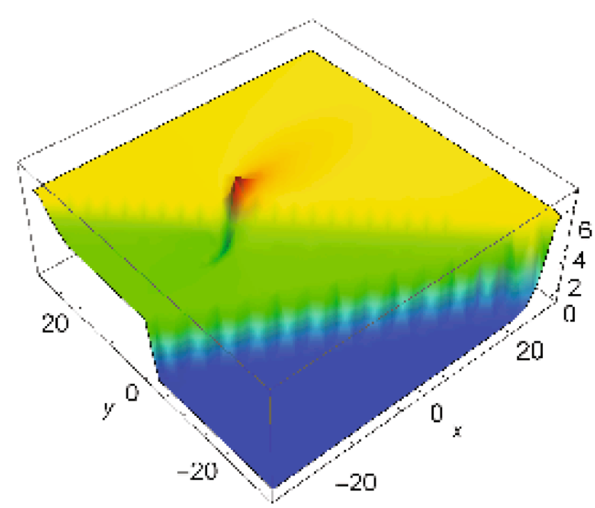
Fig. 4. 3D surface solution of Eq. (22) plotted for different value of time (t) when $a = 1$, $b = 1$, $k_3 = 2$, $l_3 = 1$, $\alpha = -1$, $\beta = 1$.



(a) t=-4.



(b) t=0.



(c) t=4.

Fig. 5. 3D surface solution of Eq. (25) plotted for different value of time (t) when $a = -1/2$, $b = 2$, $k_3 = 1$, $k_4 = 2$, $l_3 = 1$, $l_4 = 2$, $\alpha = -1$, $\beta = 1$.

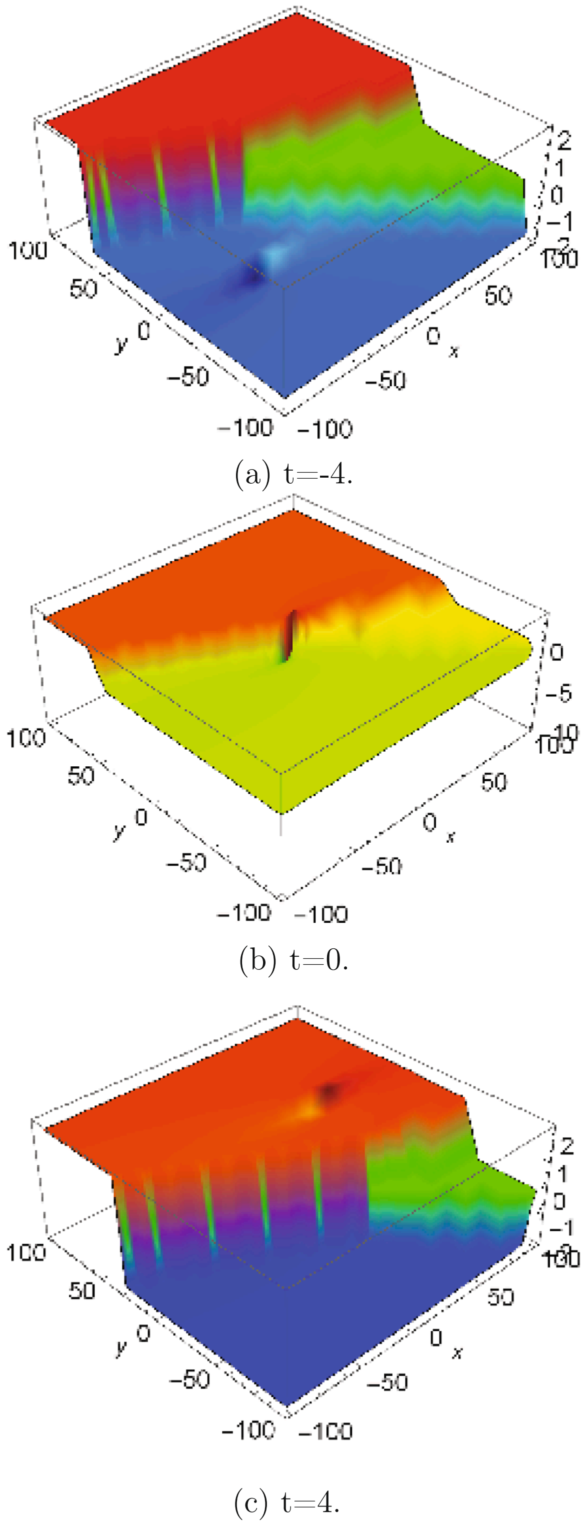


Fig. 6. 3D surface solution of Eq. (28) plotted for different value of time (t) when $a = 1$, $b = 3$, $k_3 = -1$, $k_4 = 1$, $l_3 = 1$, $l_4 = 3$, $\alpha = -1$, $\beta = 1$.

derived by choosing $N = 4$ in Eq. (6), and taking the limit $k_m \to 0$, $\alpha_m = -1\,(m = 1, 2)$ and letting $\frac{k_1}{k_2} = O(1)$, f_4 will be equivalent to

$$f_4 = \eta_1\eta_2 + V_{12} + \Omega_1 e^{\theta_3} + \Omega_2 e^{\theta_4} + M_{34} e^{\theta_3+\theta_4}(\Omega_1 + \Omega_2 - \eta_1\eta_2 - V_{12} + C_{13}C_{24} + C_{14}C_{23}), \quad (24)$$

$$\Omega_2 = \eta_1\eta_2 + V_{12} + C_{24}\eta_1 + C_{14}\eta_2 + C_{14}C_{24}. \quad (25)$$

Here θ_3, θ_4 are stated in Eq. (7), η_1, η_2 are stated in Eq. (18), V_{12} is given from Eq. (20). The $C_{r4}\,(r = 1, 2)$ are constants and defined as

$$C_{m4} = -\frac{4k_4}{k_4^2 + l_m^2\alpha - 2l_m l_4\alpha + l_4^2\alpha}. \qquad m = 1, 2 \quad (26)$$

As a result of replacing Eq. (25) with Eq. 26,27 into Eq. (3), an equation that describes a collision phenomenon between single-M-lump and a two-soliton solution is revealed (see Fig. 5).

In the case, the value of $M_{34} = 0$ in Eq. (24), f_4 will be rewritten as

$$f_4 = \eta_1\eta_2 + V_{12} + \Omega_1 e^{\theta_3} + \Omega_2 e^{\theta_4}. \quad (27)$$

This equation describes interaction phenomena that appear between 1-M-lump solution and 1-fusion solution as shown in Fig. 6.

Conclusions

In this study, the (2 + 1)-dimensional Date-Jimbo-Kashiwara-Miwa equation is investigated by using a Hirota method. First, the studied equation is formulated in the form of Hirota bilinear form by using the logarithmic variable transformation, then the N-soliton solutions, rational solutions, and the interaction phenomena between 1-m-lump with 1-soliton and 2-soliton solutions are presented. the velocity in x- and y-direction for a single-M-lump solution is also studied. The 3-dimensional figures with corresponding z-axis surfaces are drawn to more understand the physical phenomena for the gained solutions.

CRediT authorship contribution statement

Hajar F. Ismael: Data curation, Formal analysis, Investigation, Methodology, Resources, Software, Validation, Writing - original draft, Writing - review & editing. **Hasan Bulut:** Conceptualization, Formal analysis, Methodology, Project administration, Resources, Supervision, Writing - original draft. **Choonkil Park:** Funding acquisition, Methodology, Resources, Software, Validation, Writing - review & editing. **M.S. Osman:** Conceptualization, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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