

Received November 10, 2020, accepted November 26, 2020, date of publication December 1, 2020, date of current version December 11, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3041456

Adaptive Nonlinear Output Tracking Control With Rejection of Unmatched Biased Sinusoidal Disturbances for Nonlinear Systems

WONHEE KIM¹, (Member, IEEE), AND JUN MOON², (Senior Member, IEEE)

¹School of Energy Systems Engineering, Chung-Ang University, Seoul 06974, South Korea

²Department of Electrical Engineering, Hanyang University, Seoul 04763, South Korea

Corresponding author: Jun Moon (junmoon@hanyang.ac.kr)


This work was supported in part by the Institute of Information and Communications Technology Planning and Evaluation (IITP) grant funded by the Korea Government (MSIT), Artificial Intelligence Graduate School Program (Hanyang University), under Grant 2020-0-01373, in part by the Korea Institute for Advancement of Technology (KIAT) grant funded by the Korea Government (MOTIE), The Establishment Project of Industry-University Fusion District, under Grant P0011930, and in part by the National Research Foundation of Korea (NRF) grant funded by the Ministry of Science and ICT under Grant NRF-2017R1E1A1A03070936.

ABSTRACT An adaptive nonlinear output tracking control method is proposed to reject unmatched biased sinusoidal disturbances for nonlinear systems that are in the strict feedback form. The proposed method consists of a disturbance observer (DOB) and an adaptive nonlinear controller. The DOB is in the form of a high-pass filter, and it estimates biased sinusoidal disturbances with unknown frequencies. The performance of the DOB is investigated in the time and frequency domains. As the DOB cannot accurately estimate biased sinusoidal disturbances with unknown frequencies, an adaptive nonlinear controller is designed to compensate for the disturbance estimation errors and ensure output tracking. The adaptive nonlinear controller is developed via the backstepping procedure. Thus, matching conditions are not significant in this design. The discontinuous “sgn” function is applied to the controller to compensate for the disturbance estimation errors. An update law of the control gain of the discontinuous “sgn” function is proposed to suppress the disturbance estimation errors without the knowledge of their upper bounds. Low-pass filters are embedded in the controller to smooth the derivative of the discontinuous “sgn” function. Consequently, multiple unmatched biased sinusoidal disturbances are compensated by the proposed method without any knowledge of the disturbances. The proposed method can reject multiple biased sinusoidal disturbances while it guarantees the constraint of the tracking error in steady-state response. Furthermore, the proposed method can reject multiple biased sinusoidal disturbances.

INDEX TERMS Nonlinear system, disturbance observer, disturbance rejection, output tracking.

I. INTRODUCTION

The rejection of sinusoidal disturbances is of particular interest in control systems because these disturbances commonly occur in practice [1]. Sinusoidal disturbances occur in a large number of applications, such as rotating magnetic bearings [2], precise piezoactuated nanopositioning [3], hard disk drives [4], optical disk drives [5], rotating pumps in cryocooler expanders [6], helicopter rotor blades [7], and aircraft landing on oscillating carriers [8].

The associate editor coordinating the review of this manuscript and approving it for publication was Jun Shen .

To address this problem, the internal model principle (IMP) approach was developed to cancel sinusoidal disturbances using disturbance dynamics [9]–[12]. In this approach, a pair of poles that matches the frequency of a sinusoidal disturbance is required to perfectly cancel the disturbance because the sensitivity function from the disturbance to an output is in the form of a notch filter. However, it is difficult to exactly determine the disturbance frequency because it may vary or may be unknown during system operation. Therefore, various adaptive algorithms have been proposed to estimate the disturbance frequency [5], [13], [14]. Even though these algorithms solve general problems, control laws are difficult to implement in practice and may easily destabilize a system

owing to the presence of unmodeled dynamics [15]. Other approaches for canceling sinusoidal disturbances in nonlinear systems are global stabilization and disturbance suppression using output feedback [16]. In addition, a global output feedback compensator was designed to reject unknown sinusoidal disturbances for a class of nonlinear minimum phase systems [17]. These methods effectively cancel sinusoidal disturbances with unknown frequencies when systems are in the normal form or output feedback form and satisfy matching conditions. In practice, a disturbance is not a pure sinusoidal signal but a biased sinusoidal signal owing to the asymmetry of systems. Furthermore, in IMP-based methods, n periodic generators are required to reject n biased sinusoidal disturbances.

Observer based control methods were used to estimate biased sinusoidal disturbances [24]–[29]. A disturbance observers (DOBs)-based controller was designed to cancel disturbances in the frequency domain [24]. A DOB designed in the time domain was generalized to estimate higher-order disturbances in a time-series expansion [25]. DOB-based tracking control algorithms were presented in [26], [27]. The disturbance estimation error cannot converge to zero because a biased sinusoidal disturbance varies with time. Variable structure control methods were applied to tracking controllers to suppress the disturbance estimation error [28], [29]. However, these methods require the upper bound of the disturbance estimation error, which is difficult to determine. Furthermore, the use of the discontinuous function “sgn” may result in the chattering in the control input. Extended observer based control methods were proposed [18], [19]. In [18], only matched disturbance was considered and in [19], it may be difficult to satisfy the constraint of the tracking error in steady-state response.

Recently, fuzzy and neural network based control methods were proposed to compensate for disturbances [20], [21]. These methods can effectively compensate for the disturbance, but, they require the upper bounds of the absolute values of the disturbances. An adaptive asymptotic tracking control was developed for uncertain nonlinear time-delay systems depended on delay estimation information [22]. In this paper, external disturbances were not considered. In [23], an adaptive exact sliding tracking control was proposed for high-order strict-feedback systems with mismatched nonlinearities and external disturbances, but, its control input had ripples.

In this paper, we propose adaptive nonlinear output tracking control to reject unmatched biased sinusoidal disturbances for the nonlinear systems that are in the strict feedback form. The proposed method consists of a DOB and an adaptive nonlinear controller. The DOB is designed to estimate the biased sinusoidal disturbances using the full state feedback and system information. The DOB is in the form of a high-pass filter, and it estimates biased sinusoidal disturbances with unknown frequencies. The performance of the DOB is studied in the time and frequency domains. As the DOB cannot accurately estimate biased sinusoidal

disturbances with unknown frequencies, an adaptive nonlinear controller is designed to compensate for the disturbance estimation error and ensure global output tracking. The adaptive nonlinear controller is developed via the backstepping procedure. Thus, matching conditions are not significant in this design. The discontinuous “sgn” function is applied to the controller to compensate for the disturbance estimation error. An update law of the control gain of the discontinuous “sgn” function is proposed to suppress the disturbance estimation error without the knowledge of its upper bound. Low-pass filters are embedded in the controller to smooth the derivative of the discontinuous “sgn” function. The combination of the backstepping, the adaptive gain, and the discontinuous “sgn” function can reduce the ripple in the control input while it can suppress the disturbance estimation error. Consequently, unmatched biased sinusoidal disturbances are compensated by the proposed method without any knowledge of the disturbances. The proposed method can reject multiple biased sinusoidal disturbances while it guarantees the constraint of the tracking error in steady-state response. Closed-loop stability is mathematically proven. The performance of the proposed method is validated via simulations.

II. PROBLEM FORMULATION

We consider the class of the systems that are in the strict feedback form [30] as follows:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 + d_1 \\ &\vdots \\ \dot{x}_{n-1} &= f_{n-1}(x_1, \dots, x_{n-1}) + g_{n-1}(x_1, \dots, x_{n-1})x_n + d_{n-1} \\ \dot{x}_n &= f_n(x_1, \dots, x_n) + g_n(x_1, \dots, x_n)u + d_n \\ y &= x_1 \end{aligned} \quad (1)$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}$, and $u \in \mathbb{R}$ are the state, output and the input of systems, respectively. d_i for $i \in [1, n]$ denotes biased sinusoidal disturbances with unknown frequencies. f_i and g_i for $i \in [1, n]$ are smooth and always positive and bounded for all x . f_i is zero at the origin for all i and x , and g_i is always positive for all i and x . The disturbance d_i is biased sinusoidal signal given by $d_i = M_{0i} + M_i \sin(\omega_i t + \phi_i)$, where M_{0i} , $M_i > 0$, $\omega_i > 0$, and ϕ_i are unknown. The main goal of the controller design is to make the system output, $y = x_1$, track the desired output, $y_d = x_{1d}$, under the disturbances.

III. DISTURBANCE OBSERVER DESIGN

To estimate multiple biased sinusoidal disturbances, the dynamics of \hat{x} are defined as as

$$\begin{aligned} \dot{\hat{x}}_i &= f_i(x_1, \dots, x_i) + g_i(x_1, \dots, x_i)x_{i+1} + \hat{d}_i, \quad i \in [1, n-1] \\ \dot{\hat{x}}_n &= f_n(x_1, \dots, x_n) + g_n(x_1, \dots, x_n)u + \hat{d}_n \end{aligned} \quad (2)$$

where \hat{x}_i and \hat{d}_i are the estimations of x_i and d_i , respectively. $\hat{x}(0) = x(0)$ and $\hat{d}_i(0) = 0$. \hat{d}_i for $i \in [1, n]$ is defined as

$$\hat{d}_i = k_{p_i}(x_i - \hat{x}_i) + k_{i_i} \int_0^t (x_i - \hat{x}_i) ds, \quad i \in [1, n] \quad (3)$$

where k_{P_i} and k_{I_i} for $i \in [1, n]$ are positive constants to make the polynomial $s^2 + k_{P_i}s + k_{I_i}$ a Hurwitz polynomial, and $\hat{d}_i(0) = 0$ for all i . The disturbance estimation error is defined as

$$\tilde{d}_i = d_i - \hat{d}_i, \quad i \in [1, n]. \quad (4)$$

Theorem 1: Suppose that the DOBs (2) and (3) are applied to the nonlinear system (1). The high pass-filter $H_i(s)$ is

$$H_i(s) = \frac{s^2}{s^2 + k_{P_i}s + k_{I_i}}, \quad i \in [1, n]. \quad (5)$$

If the cutoff frequency ω_{c_i} of $H_i(s)$ is above the maximum frequency $\omega_{i_{\max}}$ of the disturbance, then the disturbance estimation error \tilde{d}_i , $\forall i \in [1, n]$ is bounded and T_i exists such that

$$|\tilde{d}_i| \leq \Gamma_{i_{\max}} = |H(s)_i|_{s=j\omega_{c_i}} \times M_{i_{\max}} \quad (6)$$

for $t > T_i$. \diamond

Proof: The boundedness of the estimation error is proven in both transient and steady-state responses. First, the boundedness of the estimation error is proven in the transient response. Because $\dot{\hat{d}}_i = \dot{d} - \dot{\tilde{d}}_i$, from (1), (2), and (4), the derivative of (3) provides

$$\dot{\hat{d}}_i = \dot{d}_i + k_{P_i}\tilde{d}_i + k_{I_i} \int_0^t \tilde{d}_i ds, \quad i \in [1, n]. \quad (7)$$

\tilde{d}_i is defined as $\int_0^t \tilde{d}_i ds$. Then we obtain

$$\underbrace{\begin{bmatrix} \dot{\tilde{d}}_i \\ \tilde{d}_i \end{bmatrix}}_{\dot{e}_{d_i}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -k_{I_i} & -k_{P_i} \end{bmatrix}}_{A_{d_i}} \underbrace{\begin{bmatrix} \tilde{d}_i \\ \dot{\tilde{d}}_i \end{bmatrix}}_{e_{d_i}} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B_{d_i}} \dot{d}_i. \quad (8)$$

As k_{P_i} and k_{I_i} are positive constants to make the polynomial $s^2 + k_{P_i}s + k_{I_i}$ a Hurwitz polynomial, A_{d_i} is Hurwitz. As $\hat{d}_i(0) = 0$ and $\hat{\dot{d}}_i(0) = 0$, $e_{d_i}(0) = 0$. In (8), e_{d_i} can be obtained [31] as

$$\begin{aligned} e_{d_i}(t) &= \int_0^t (e^{A_{d_i}(t-\tau)} B_{d_i} \dot{d}_i(\tau)) d\tau \\ &= e^{A_{d_i}(t-\tau)} B_{d_i} d_i(\tau) \Big|_0^t - \int_0^t (A_{d_i} e^{A_{d_i}(t-\tau)} B_{d_i} d_i(\tau)) d\tau \\ &= B_{d_i} d_i(t) - \int_0^t (A_{d_i} e^{A_{d_i}(t-\tau)} B_{d_i} d_i(\tau)) d\tau. \end{aligned} \quad (9)$$

As A_{d_i} is Hurwitz, k_i exists such that $\|e^{A_{d_i}(t-\tau)}\|_2 \leq k_i e^{-\lambda_i(t-\tau)}$ where λ_i is the minimum singular value of A_{d_i} . Notes that $\sup_t |d_i(t)| = |M_{0_i}| + |M_{i_{\max}}|$. Thus,

$$\begin{aligned} \|e_{d_i}\|_2 &\leq \|B_{d_i}\|_2 |d_i(t)| + \int_0^t (\|A_{d_i}\|_2 e^{-\lambda_i(t-\tau)} \|B_{d_i}\|_2 |d_i(\tau)|) d\tau \\ &\leq \sup_t |d_i(t)| + \frac{\|A_{d_i}\|_2 k_i}{\lambda_i} \sup_t |d_i(t)| \\ &\leq |M_{0_i}| + |M_{i_{\max}}| + \frac{\|A_{d_i}\|_2 k_i (|M_{0_i}| + |M_{i_{\max}}|)}{\lambda_i}. \end{aligned} \quad (10)$$

As $\tilde{d}_i \leq |\tilde{d}_i| \leq \|e_{d_i}\|_2$,

$$\tilde{d}_i \leq |M_{0_i}| + |M_{i_{\max}}| + \frac{\|A_{d_i}\|_2 k_i (|M_{0_i}| + |M_{i_{\max}}|)}{\lambda_i}. \quad (11)$$

Thus, \tilde{d}_i is bounded in the transient response. The boundedness depends on k_{I_i} and k_{P_i} . \tilde{d}_i is studied in the frequency domain to analyze its upper bound in the steady-state response. Differentiating both sides of (3) twice provides

$$\ddot{\tilde{d}}_i = k_{P_i}(\ddot{x}_i - \ddot{\hat{x}}_i) + k_{I_i}(\dot{x}_i - \dot{\hat{x}}_i). \quad (12)$$

With (1) and (2), (12) becomes

$$\ddot{\tilde{d}}_i = \ddot{d}_i + k_{P_i}\dot{\tilde{d}}_i + k_{I_i}\tilde{d}_i. \quad (13)$$

From (13), we obtain the error transfer functions $H(s)$ as

$$\tilde{d}_i = \underbrace{\frac{s^2}{s^2 + k_{P_i}s + k_{I_i}}}_{H_i(s)} d_i \quad (14)$$

where s is the Laplace operator. In (14), $H_i(s)$ is a typical 2nd order high pass-filter. The disturbances are biased sinusoidal signals, thus they are bounded. In (18), the transfers function from the disturbances to the disturbance estimation errors are in the form of the high pass filter. Consequently, the disturbance estimation errors are also bounded with the bounded disturbances. In $H_i(s)$, the cutoff frequency ω_{c_i} depends on observer parameters k_{P_i} and k_{I_i} . If ω_{c_i} is higher than the maximum frequency $\omega_{i_{\max}}$ of a disturbance, then the estimation error \tilde{d}_i is suppressed by the high pass filter (14). Thus there exists T_{1_i} such that

$$|\tilde{d}_i(t)| \leq \Gamma_{i_{\max}} = |H_i(s)|_{s=j\omega_{c_i}} \times M_{i_{\max}} \quad (15)$$

for $t > T_{1_i}$. \square

In the observer gain tuning, it is recommended that the observer gains are chosen such that ω_{c_i} is higher than the expected frequency of a main component of a disturbance. Even though DOBs can accurately estimate biased sinusoidal disturbances, the disturbance estimation errors cannot converge to zero because the derivatives of biased sinusoidal disturbances are not zero. The next section describes the design of the adaptive nonlinear controller to compensate for the disturbance estimation errors in tracking.

IV. ADAPTIVE NONLINEAR CONTROLLER DESIGN

The output tracking error is defined as $e_1 = x_1 - x_{1_d}$. Then tracking errors $e = [e_1 \ e_2 \ \dots \ e_n]^T$ can be written as

$$e_i = x_i - x_{i_d}, \quad i \in [1, n]. \quad (16)$$

The tracking error dynamics are

$$\begin{aligned} \dot{e}_i &= \dot{x}_i - \dot{x}_{i_d} = f_i + g_i x_{i+1} + d_i - \dot{x}_{i_d}, \quad i \in [1, n-1] \\ \dot{e}_n &= \dot{x}_n - \dot{x}_{n_d} = f_n + g_n u + d_n - \dot{x}_{n_d}. \end{aligned} \quad (17)$$

As $e_i = x_i - x_{i_d}$, (17) may be written as

$$\begin{aligned} \dot{e}_i &= f_i + g_i e_{i+1} + g_i x_{i+1_d} + d_i - \dot{x}_{i_d}, \quad \forall i \in [1, n-1] \\ \dot{e}_n &= f_n + g_n u + d_n - \dot{x}_{n_d}. \end{aligned} \quad (18)$$

The adaptive nonlinear controller is designed via the backstepping procedure as

$$\begin{aligned} x_{2_d} &= \frac{1}{g_1}(-f_1 + \dot{x}_{1_d} - k_1 e_1 - \hat{d}_1 - \hat{\Gamma}_1 \text{sgn}(e_1)) \\ x_{i+1_d} &= \frac{1}{g_i}(-f_i + \dot{x}_{i_d} - g_{i-1}e_{i-1} - k_i e_i - \hat{d}_i - \hat{\Gamma}_i \text{sgn}(e_i)), \\ & \quad i \in [2, n-1] \\ u &= \frac{1}{g_n}(-f_n + \dot{x}_{n_d} - g_{n-1}e_{n-1} - k_n e_n - \hat{d}_n - \hat{\Gamma}_n \text{sgn}(e_n)) \end{aligned} \quad (19)$$

where control gains $k_i > 0, \forall i \in [1, n]$. $\hat{\Gamma}_i \forall i \in [1, n]$ is the adaptive gain to suppress the disturbance estimation error in the steady-state response. $\Gamma_{i_{\max}}$ should be known to sufficiently suppress the disturbance estimation error in the steady-state response. However, it is difficult to determine $\Gamma_{i_{\max}}$ even though the disturbance estimation errors are bounded. We define the desirably adaptive gain as $\Gamma_{i_d} > \Gamma_{i_{\max}}$. Note that Γ_{i_d} is also unknown. To make $\hat{\Gamma}_i$ converge to Γ_{i_d} , the update law for $\hat{\Gamma}_i \forall i \in [1, n]$ is designed as

$$\dot{\hat{\Gamma}}_i = c_i |e_i| \quad (20)$$

where $\hat{\Gamma}_i(0) > 0$ and $c_i > 1$. $\hat{\Gamma}_i$ increases with nonzero e_i . Then, $\hat{\Gamma}_i \geq \Gamma_{i_{\max}}$ in finite time T_{2_i} where T_{2_i} is yet to be defined. e_i becomes zero after $t \geq T_{2_i}$. This yields that $\hat{\Gamma}_i$ admits a bounded value, which yields that there always exists a positive constant $\Gamma_{i_d} > \Gamma_{i_{\max}}$ for $\forall t$ and $\Gamma_{i_d} > \Gamma_{i_{\max}}$. The convergence of e_i to zero is proven later.

Theorem 2: Suppose that the DOBs (2) (3), nonlinear controller (19), and adaptive gain update law (20) are applied to the tracking error dynamics (18). Then, the origin of the tracking error dynamics (18) is globally exponentially stable for $t > T_{\max_i}$, where $T_{\max_i} = \max_i T_{2_i}$ and $\hat{\Gamma}_i \forall i \in [1, n]$ converges to Γ_{i_d} . \diamond

Proof: In Theorem 1, it was proven that the disturbance estimation errors are bounded. With the DOBs (2) (3), backstepping controller (19), and the adaptive gain update law (20), the tracking error dynamics can be derived by

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 + g_1 e_2 + (\tilde{d}_1 - \hat{\Gamma}_1 \text{sgn}(e_1)) \\ \dot{e}_i &= -k_i e_i - g_{i-1}e_{i-1} + g_i e_{i+1} + (\tilde{d}_i - \hat{\Gamma}_i \text{sgn}(e_i)), \\ & \quad \forall i \in [2, n-1] \\ \dot{e}_n &= -k_n e_n - g_{n-1}e_{n-1} + (\tilde{d}_n - \hat{\Gamma}_n \text{sgn}(e_n)). \end{aligned} \quad (21)$$

The tracking error dynamics (21) are input-to-state stable (ISS). In the adaptive gain update law (20), the derivative of $\hat{\Gamma}_i \forall i \in [1, n]$ is always positive and nonzero. Thus, $\hat{\Gamma}_i$ increases with nonzero e_i . Then $\hat{\Gamma}_i = \Gamma_{i_{\max}}$ in finite time T_{2_i} . After $t > T_{2_i}$, $\hat{\Gamma}_i > \Gamma_{i_{\max}}$. Hence, the term $-\hat{\Gamma}_i \text{sgn}(e_i)$ can sufficiently suppress the disturbance estimation error \tilde{d}_i . A Lyapunov candidate function V_e is defined as

$$V_e = \frac{1}{2} e^2. \quad (22)$$

Then we obtain

$$\dot{V}_e = -Ke^T e - \Delta^T e \quad (23)$$

where $K = \text{diag}(k_1, \dots, k_n)$, $\Delta = [\hat{\Gamma}_1 \text{sgn}(e_1) - \tilde{d}_1, \dots, \hat{\Gamma}_n \text{sgn}(e_n) - \tilde{d}_n]^T$, and $\hat{\Gamma}_i \text{sgn}(e_i) - \tilde{d}_i \geq 0, \forall i \in [1, n]$. Thus, after $t > T_{2_{\max}}$ where $T_{2_{\max}} = \max_i T_{2_i}$, e converges to zero in finite time T_3 . After $t > T_3$, $\hat{\Gamma}_i$ admits a bounded value Γ_{i_d} . A Lyapunov candidate function, V , is defined for the stability analysis of the controller (19) and adaptive gain update law (20) as follows:

$$V = \sum_{i=1}^n e_i^2 + \left(\hat{\Gamma}_i - \Gamma_{i_d} \right)^2. \quad (24)$$

Then we obtain

$$\dot{V} = \sum_{i=1}^n -k_i e_i^2 + e_i \left(\tilde{d}_i - \hat{\Gamma}_i \text{sgn}(e_i) \right) + \left(\hat{\Gamma}_i - \Gamma_{i_d} \right) \dot{\hat{\Gamma}}_i. \quad (25)$$

With the adaptive gain update law (20), (25) becomes

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^n -k_i e_i^2 + \Gamma_{i_d} |e_i| - \hat{\Gamma}_i |e_i| + \left(\hat{\Gamma}_i - \Gamma_{i_d} \right) c_i |e_i| \\ &\leq \sum_{i=1}^n -k_i e_i^2 + \Gamma_{i_d} |e_i| - \hat{\Gamma}_i |e_i| + \left(\hat{\Gamma}_i - \Gamma_{i_d} \right) c_i |e_i| \\ & \quad + \Gamma_{i_d} |e_i| - \Gamma_{i_d} |e_i| \\ &\leq \sum_{i=1}^n -k_i e_i^2 - (c_i - 1) |e_i| \left(\hat{\Gamma}_i - \Gamma_{i_d} \right). \end{aligned} \quad (26)$$

As $(c_i - 1) > 0$, the origin of the tracking error dynamics (18) is globally exponentially stable for $t > T_{\max_i}$, where $T_{\max_i} = \max_i T_{2_i}$ and $\hat{\Gamma}_i \forall i \in [1, n]$ converges to Γ_{i_d} . \square

The backstepping controller (19) can ensure exponential stability. However, $\hat{\Gamma}_i$ continuously increases with nonzero errors; thus, adaptive gain may become extremely large in a few cases. To prevent this problem, the update law of $\hat{\Gamma}_i$ (20) is modified as

$$\dot{\hat{\Gamma}}_i = \begin{cases} c_i |e_i|, & |e_i| > \varepsilon_i \\ 0, & |e_i| \leq \varepsilon_i \end{cases} \quad (27)$$

where ε_i is the constraint for e_i . With the modified update law (27), the backstepping controller (19) ensures the boundedness of the tracking errors because the tracking error dynamics (21) are ISS.

The backstepping controller (19) involves the derivatives of the discontinuous ‘‘sgn’’ function, which is bounded in the digital implementation. This problem has been resolved in an ad hoc manner via numerical differentiation, i.e., in other words,

$$\dot{x}_{i+1_d}(n) \approx \frac{x_{i+1_d}(n) - x_{i+1_d}(n-1)}{\Delta T}, \quad i \in [2, n] \quad (28)$$

where ΔT is the sampling time. This ad hoc approach has worked well in numerous experimental applications [34], [35]. However, even though \dot{x}_{i+1_d} becomes bounded using of the ad hoc approach, this approach may face the problem of the explosion of terms. To overcome this problem, low pass filters are designed to smooth the signal \dot{x}_{i+1} as follows:

$$\tau_i \dot{\bar{x}}_{i_d} + \bar{x}_{i_d} = x_{i_d}, \quad i \in [2, n] \quad (29)$$

With the use of \bar{x}_{i_d} instead of x_{i_d} , the nonlinear controller (19) becomes

$$\begin{aligned} x_{2_d} &= \frac{1}{g_1}(-f_1 + \dot{x}_{1_d} - k_1 e_1 - \hat{d}_1 - \hat{\Gamma}_1 \text{sgn}(e_1)) \\ x_{i+1_d} &= \frac{1}{g_i}(-f_i + \dot{x}_{i_d} - g_{i-1}e_{i-1} - k_i e_i - \hat{d}_i \\ &\quad - \hat{\Gamma}_i \text{sgn}(e_i)) \quad i \in [2, n-1] \\ \tau_i \dot{\bar{x}}_{i_d} + \bar{x}_{i_d} &= x_{i_d}, \quad i \in [2, n] \\ u &= \frac{1}{g_n}(-f_n + \dot{x}_{n_d} - g_{n-1}e_{n-1} - k_n e_n - \hat{d}_n \\ &\quad - \hat{\Gamma}_n \text{sgn}(e_n)) \end{aligned} \quad (30)$$

where $e_1 = x_1 - x_{1_d}$ and $e_i = x_i - \bar{x}_{i_d}$, $i \in [2, n]$ are the tracking errors. With the DOBs (2) (3), the backstepping controller (30), and the adaptive gain update law (20), the tracking error dynamics can be derived as

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 + g_1 e_2 + (\tilde{d}_1 - \hat{\Gamma}_1 \text{sgn}(e_1)) \\ &\quad + g_1(\bar{x}_{2_d} - x_{2_d}) \\ \dot{e}_i &= -k_i e_i - g_{i-1}e_{i-1} + g_i e_{i+1} + (\tilde{d}_i - \hat{\Gamma}_i \text{sgn}(e_i)) \\ &\quad + g_2(\bar{x}_{3_d} - x_{3_d}) \quad \forall i \in [2, n-1] \\ \dot{e}_n &= -k_n e_n - g_{n-1}e_{n-1} + (\tilde{d}_n - \hat{\Gamma}_n \text{sgn}(e_n)). \end{aligned} \quad (31)$$

Theorem 3: Suppose that the DOBs (2) (3), the nonlinear controller (30), and the adaptive gain update law (20) are applied to the tracking error dynamics (18). Then, e_1 is ultimately uniformly bounded. \diamond

Proof: The tracking error dynamics (31) can be rewritten by

$$\dot{e} = A(e)e + B(e)\tilde{x}_d \quad (32)$$

where $\tilde{x}_d = \bar{x}_d - x_d$, $\bar{x}_d = [x_{1_d} \bar{x}_{2_d} \dots \bar{x}_{2_n}]^T$, $x_d = [x_{1_d} x_{2_d} \dots x_{2_n}]^T$,

$$A(e) = \begin{bmatrix} -k_1 & g_1 & 0 & \dots & 0 & 0 \\ -g_1 & -k_2 & g_2 & \dots & 0 & 0 \\ 0 & -g_2 & -k_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & \dots & -k_{n-1} & g_{n-1} \\ 0 & 0 & 0 & \dots & -g_{n-1} & -k_n \end{bmatrix} \in \mathbb{R}^{n \times n},$$

$$B(e) = \begin{bmatrix} 0 & g_1 & 0 & \dots & 0 & 0 \\ 0 & 0 & g_2 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & \dots & 0 & g_{n-1} \\ 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \in \mathbb{R}^{n \times n}.$$

In Theorem 1, it was proven that the tracking error dynamics (21) are exponentially stable. Thus, the tracking error dynamics (32) are finite-gain L_p stable for each $p \in [1, \infty]$ [30]. From (29), \tilde{x}_d exponentially enters the bounded ball, thus the dynamics of \tilde{x}_d are finite-gain L_p stable for each $p \in [1, \infty]$ [30]. Therefore, there exists τ_i that satisfies the

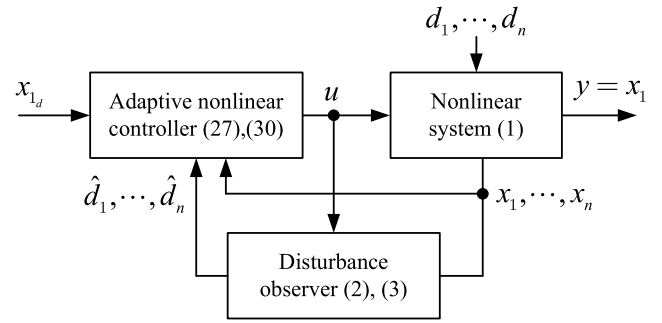


FIGURE 1. Block diagram of the proposed control method.

small-gain theorem [30] for the closed-loop system. Consequently, e_1 is ultimately uniformly bounded. \square

Remark 1: The proposed method can be easily extended to the case of multiple biased sinusoidal disturbances. Disturbances can be expressed as the sums of biased sinusoidal signals as follows:

$$d_i = \sum_{k=1}^m d_{i_k} = \sum_{k=1}^m (M_{0_{i_k}} + M_{i_k} \sin(\omega_{i_k} t + \phi_{i_k})) \text{ for } i \in [1, n]. \quad (33)$$

These multiple biased sinusoidal disturbances (33) can be regarded as one biased sinusoidal disturbance. Consequently, as the proposed method does not require the frequencies of the disturbances, it can compensate multiple biased sinusoidal disturbances. In contrast, n periodic generators, such as the IMP and compensators, are requirblack to reject n biased sinusoidal disturbances. This is the main advantage of the proposed method. \diamond

The block diagram of the control method is shown in Fig. 1. The DOBs (2) and (3) estimate the disturbances using the state x . Then, adaptive gains are updated using the tracking error e by (31). Based on the updated adaptive gains, the control input is generated using the state x , the tracking error e , and the estimated disturbances \hat{d}_i by (34).

V. NUMERICAL EXAMPLE

The performance of the proposed method was evaluated by conducting simulations using MATLAB/Simulink. We used the following nonlinear system:

$$\begin{aligned} \dot{x}_1 &= x_1 + (2 + \sin(x_1))x_2 + d_1 \\ \dot{x}_2 &= x_1 x_2 + u + d_2. \end{aligned} \quad (34)$$

The desirblack output was $x_{1_d} = (-e^{-10t} + 1) \sin(2\pi t)$. To estimate d_1 and d_2 , the DOBs were designed such that

$$\begin{aligned} \dot{\hat{x}}_1 &= x_1 + (2 + \sin(x_1))x_2 + \hat{d}_1 \\ \dot{\hat{x}}_2 &= x_1 x_2 + u + \hat{d}_2 \\ \dot{\hat{d}}_1 &= k_{P_1}(x_1 - \hat{x}_1) + k_{I_1} \int_0^t (x_1 - \hat{x}_1) d\tau \\ \dot{\hat{d}}_2 &= k_{P_2}(x_2 - \hat{x}_2) + k_{I_2} \int_0^t (x_2 - \hat{x}_2) d\tau. \end{aligned} \quad (35)$$

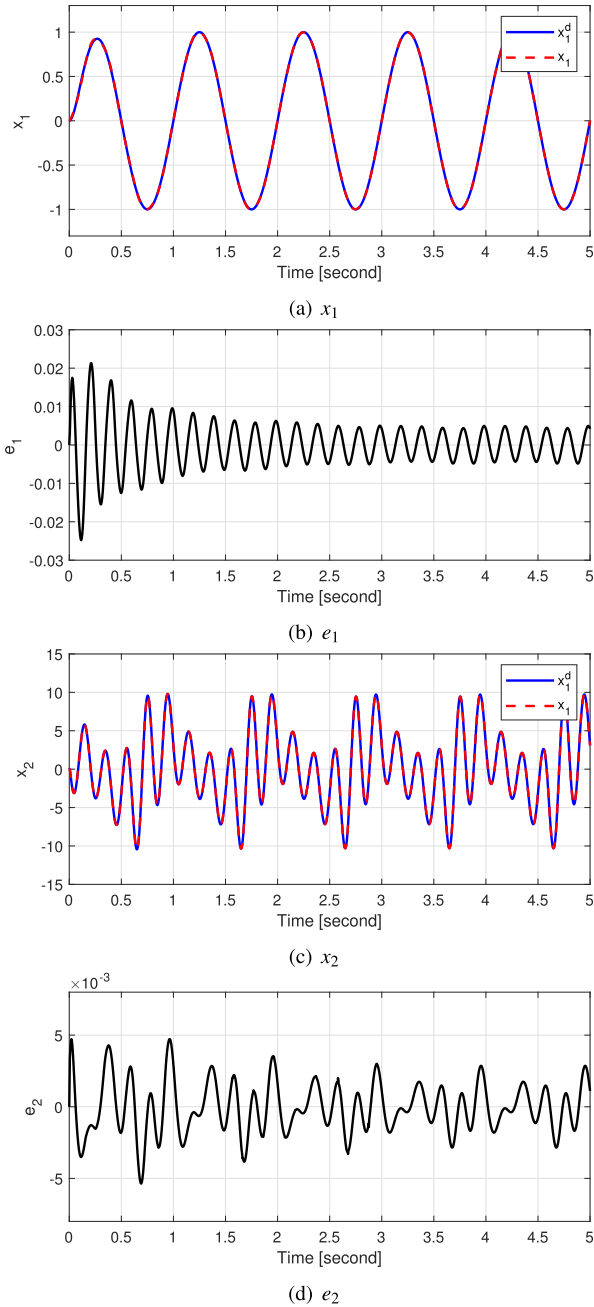


FIGURE 2. Tracking performance of the proposed method for Case 1.

where $k_{p1} = 20$, $k_{I1} = (10 \times 2\pi)^2$, $k_{p2} = 20 \times 2\pi$ and $k_{I2} = (10 \times 2\pi)^2$. The nonlinear controller was designed such that

$$\begin{aligned}
 x_{1d} &= \sin(2\pi t) \\
 x_{2d} &= \frac{1}{2 + \sin(x_1)}(-x_1 + \dot{x}_{1d} - k_1 e_1 - \hat{d}_1 - \hat{\Gamma}_1 \text{sgn}(e_1)) \\
 &\quad \tau_2 \ddot{x}_{2d} + \dot{x}_{2d} \\
 &= x_{2d} \\
 u &= -x_1 x_2 + \dot{x}_{2d} - (2 + \sin(x_1))e_1 - k_2 e_2 - \hat{d}_2 \\
 &\quad - \hat{\Gamma}_2 \text{sgn}(e_2)
 \end{aligned}$$

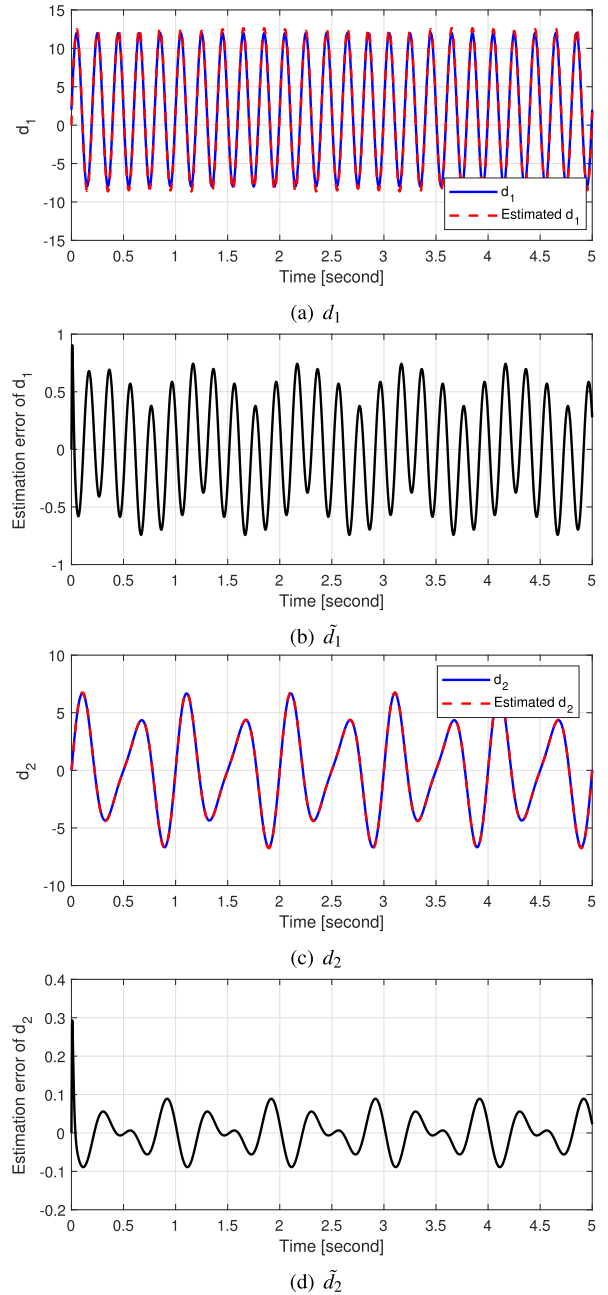


FIGURE 3. Estimation performance of the proposed method for Case 1.

$$\begin{aligned}
 \dot{\hat{\Gamma}}_1 &= \begin{cases} c_1 |e_1|, & |e_1| > \varepsilon_1 \\ 0, & |e_1| \leq \varepsilon_1 \end{cases} \\
 \dot{\hat{\Gamma}}_2 &= \begin{cases} c_2 |e_2|, & |e_2| > \varepsilon_2 \\ 0, & |e_2| \leq \varepsilon_2 \end{cases} \quad (36)
 \end{aligned}$$

where $e_1 = x_1 - x_{1d}$ and $e_2 = x_2 - \bar{x}_{2d}$, $k_1 = 1$, $k_2 = 1$, $c_1 = 100$, $c_2 = 100$, $\hat{\Gamma}_1(0) = 0.1$, $\hat{\Gamma}_2(0) = 0.1$, $\varepsilon_1 = 0.005$, and $\varepsilon_2 = 0.003$.

A. CASE 1

In Case 1, $d_1 = 10 \sin(5 \times 2\pi t) + 2$ and $d_2 = 5 \sin(2 \times 2\pi t) + 2 \sin(3 \times 2\pi t)$ were used as the disturbances.

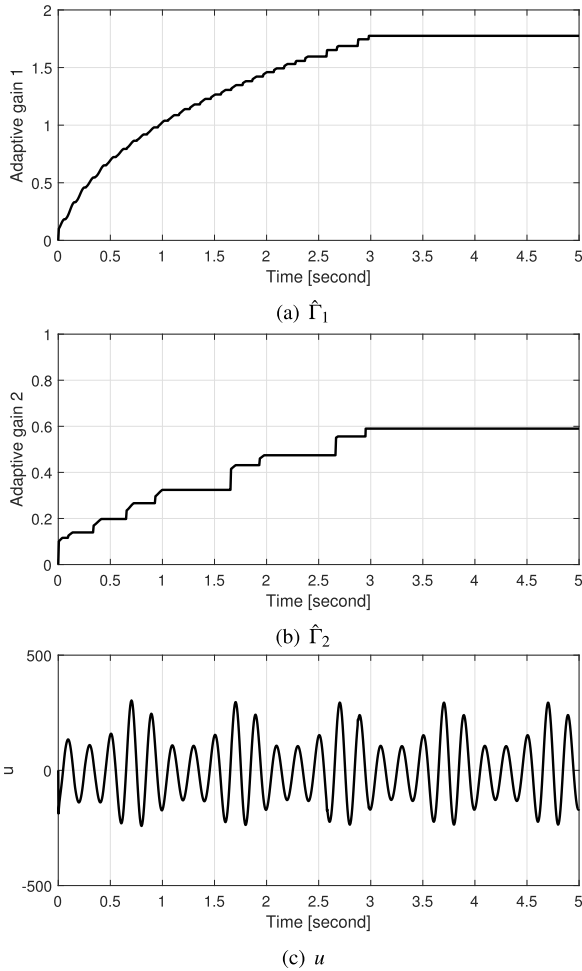


FIGURE 4. Adaptive gains and control input for Case 1.

The simulation results of the proposed method are shown in Figs. 2, 3 and 4. The adaptive nonlinear controller accurately tracked the desirables states. The proposed DOB accurately estimated the disturbances. However, as the bandwidth of the DOB was not sufficiently larger than the disturbance frequencies, the estimation errors of d_1 and d_2 were 5% and 2%, respectively. The disturbance estimation errors caused the tracking errors, as shown in Figs. 2 and 3. To suppress these tracking errors, adaptive gains $\hat{\Gamma}_1$ and $\hat{\Gamma}_2$ increased until $|e_1| = 0.005$ and $|e_1| = 0.003$, as shown in Fig. 4. After $|e_1|$ and $|e_2|$ settled below 0.005 and 0.003, respectively, $\hat{\Gamma}_1$ and $\hat{\Gamma}_2$ did not increase. As shown in Fig. 4(c), the control input was smooth because the low-pass filter was embedded in the adaptive nonlinear controller. In the control (36) of the proposed method, “sgn” function that is the function of e_2 was used. As shown in Fig. 2 (d), the sign of e_2 did not vary frequently. Thus, the chattering was not a problem in the proposed method.

B. CASE 2

In Case 2, to evaluate the performance of the proposed method with the disturbances those are suddenly changed,

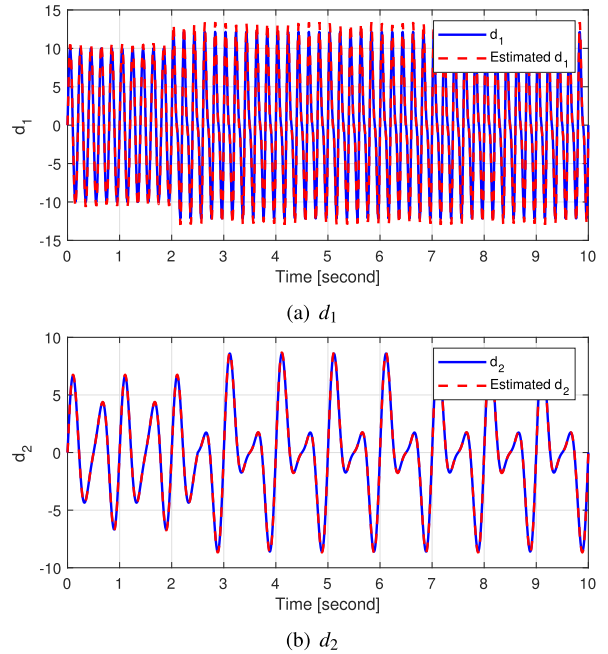


FIGURE 5. Estimation performance of the proposed method for Case 1.

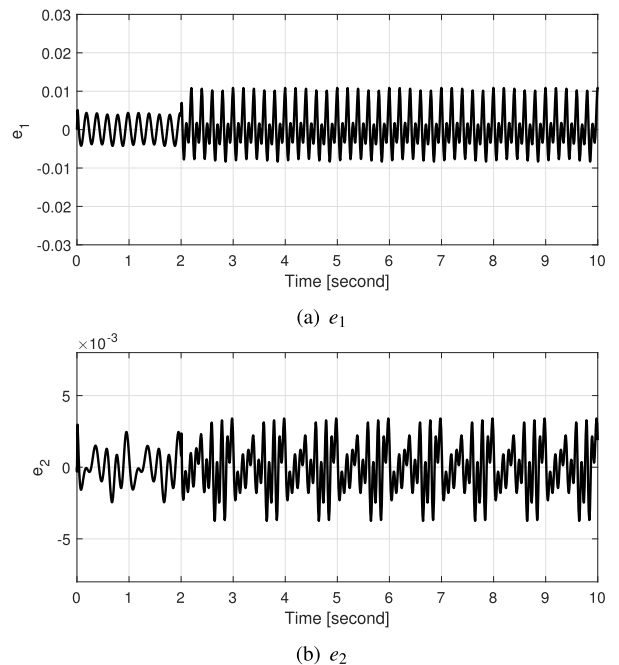


FIGURE 6. Tracking errors of the backstepping with the DOB (38) for Case 2.

d_1 and d_2 was used as follows:

$$d_1 = \begin{cases} 10 \sin(5 \times 2\pi t), & 0 \leq t < 2 \\ 10 \sin(5 \times 2\pi t) + 5 \sin(5 \times 4\pi t), & t \geq 2 \end{cases}$$

$$d_2 = \begin{cases} 5 \sin(2 \times 2\pi t) + 2 \sin(3 \times 2\pi t), & 0 \leq t < 2 \\ 5 \sin(2 \times 2\pi t) + 2 \sin(3 \times 2\pi t) + 3 \sin(2\pi t), & t \geq 2. \end{cases} \quad (37)$$

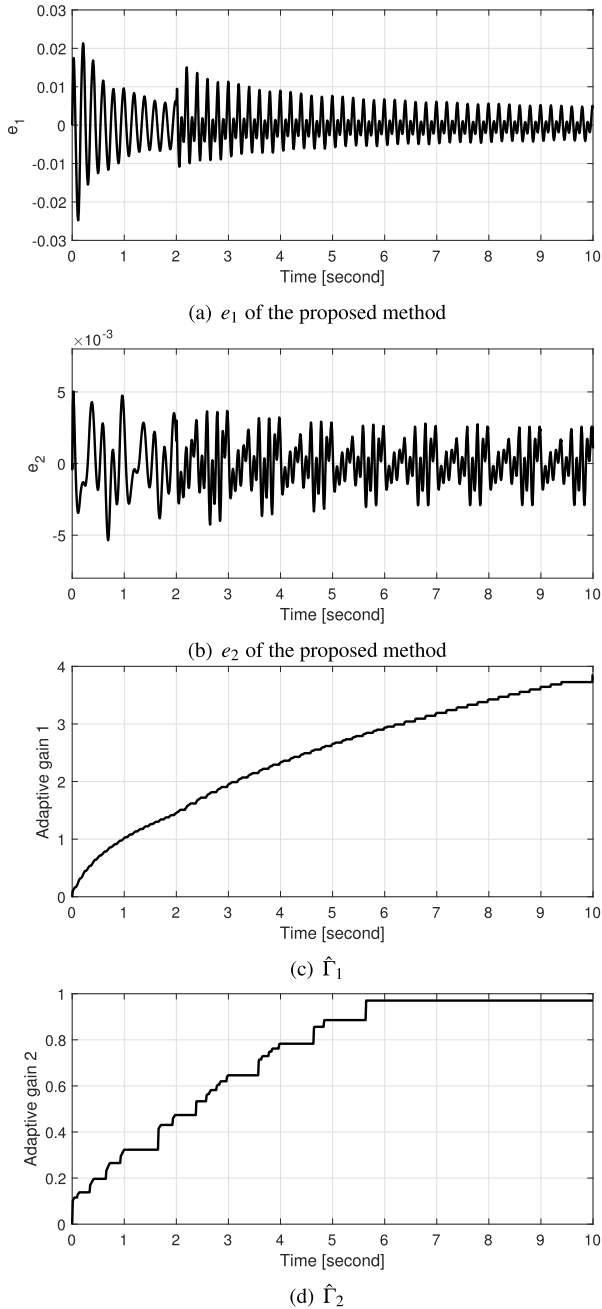


FIGURE 7. Tracking errors and adaptive gains of the proposed method for Case 2.

The proposed method was compared to the backstepping with the DOB as follows:

$$\begin{aligned}
 x_{1d} &= \sin(2\pi t) \\
 x_{2d} &= \frac{1}{2 + \sin(x_1)}(-x_1 + \dot{x}_{1d} - k_1 e_1 - \hat{d}_1 - \rho_1 \text{sgn}(e_1)) \\
 \tau_2 \ddot{x}_{2d} + \dot{x}_{2d} &= x_{2d} \\
 u &= -x_1 x_2 + \dot{x}_2^d - (2 + \sin(x_1))e_1 - k_2 e_2 - \hat{d}_2 \\
 &\quad - \rho_2 \text{sgn}(e_2)
 \end{aligned} \tag{38}$$

where $e_1 = x_1 - x_{1d}$ and $e_2 = x_2 - \bar{x}_{2d}$, $k_1 = 1$, $k_2 = 1$, $\rho_1 = 2$, $\rho_2 = 0.7$. In (38), the disturbance estimated by the DOB 35 was used. The main difference between the proposed method (36) and the backstepping with the DOB (38) was the control gain of the discontinuous function “sgn”. In the backstepping with the DOB (38), the constant gains ρ_1 and ρ_2 were used. For the backstepping with the DOB (38), we considered the case where the change of the disturbances were not expected. From the simulation results of Case 1, the sufficiently large gains $\rho_1 = 2$ and $\rho_2 = 0.7$ were used in the backstepping with the DOB (38). The simulation results of the proposed method are shown in Figs. 5, 6, and 7. The estimated disturbances were shown in Fig. 5. The disturbances suddenly changed in $t = 2$. The tracking errors of the backstepping with the DOB (38) were smaller than those of the proposed method for $t < 2$ because the large gains were used. After the disturbances got larger in $t = 2$, the tracking errors of the backstepping with the DOB (38) increased due to the increased disturbances. In the proposed method, the tracking errors increased due to the increased disturbances in $t = 2$. However, the tracking errors were again blackuced until $|e_1| = 0.005$ and $|e_1| = 0.003$ because the adaptive gains $\hat{\Gamma}_1$ and $\hat{\Gamma}_2$ increased until $|e_1| = 0.005$ and $|e_1| = 0.003$ as shown in Figs. 7 (c) and (d).

VI. CONCLUSION

Nonlinear output tracking control was developed to reject unmatched biased sinusoidal disturbances for nonlinear systems. A DOB was proposed to estimate biased sinusoidal disturbances with unknown frequencies. An adaptive nonlinear controller was designed via the backstepping procedure to compensate for the disturbance estimation errors and ensure global output tracking. The discontinuous “sgn” function with adaptive gain was applied to the controller to compensate for the disturbance estimation errors. Low-pass filters were embedded in the controller to smooth the derivative of the discontinuous “sgn” function. Through simulations, it was shown that the proposed method was effective for output tracking under multiple biased sinusoidal disturbances. Furthermore, it was observed that the proposed method was robust against the suddenly changed disturbances. In future work, we will design the controller to cancel general disturbances without the information of the nonlinear terms of the system and full state feedback.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

REFERENCES

- [1] M. Bodson, J. S. Jensen, and S. C. Douglas, “Active noise control for periodic disturbances,” *IEEE Trans. Control Syst. Technol.*, vol. 9, no. 1, pp. 200–205, Jan. 2001.
- [2] S. Beals, B. Shafai, and E. Cusson, “Adaptive force balancing for magnetic bearing control systems,” in *Proc. Conf. Decis. Control*, 1992, pp. 3535–3539.
- [3] A. Esbrook, X. Tan, and H. K. Khalil, “An indirect adaptive servocompensator for signals of unknown frequencies with application to nanopositioning,” *Automatica*, vol. 49, no. 7, pp. 2006–2016, Jul. 2013.

- [4] F. Hong, C. Du, K. P. Tee, and S. S. Ge, "Adaptive disturbance rejection in the presence of uncertain resonance mode in hard disk drives," *Trans. Inst. Meas. Control*, vol. 32, no. 2, pp. 99–119, Apr. 2010.
- [5] W. Kim, H. Kim, C. C. Chung, and M. Tomizuka, "Adaptive output regulation for the rejection of a periodic disturbance with an unknown frequency," *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 5, pp. 1296–1304, Sep. 2011.
- [6] Y. W. A. Wu, "Demonstration of active vibration control of the hughes cryocooler testbed," in *Proc. 31st IEEE Conf. Decis. Control*, Dec. 1992, pp. 2580–2585.
- [7] S. Bittanti and L. Moiraghi, "Active control of vibrations in helicopters via pole assignment techniques," *IEEE Trans. Control Syst. Technol.*, vol. 2, no. 4, pp. 343–351, Dec. 1994.
- [8] L. Marconi, A. Isidori, and A. Serrani, "Autonomous vertical landing on an oscillating platform: An internal-model based approach," *Automatica*, vol. 38, no. 1, pp. 21–32, Jan. 2002.
- [9] B. A. Francis and W. M. Wonham, "The internal model principle of control theory," *Automatica*, vol. 12, no. 5, pp. 457–465, Sep. 1976.
- [10] C. Kempf, W. Messner, M. Tomizuka, and M. Horowitz, "Comparison of four discrete-time repetitive control algorithms," *IEEE Control Syst. Mag.*, vol. 13, no. 6, pp. 48–54, Dec. 1993.
- [11] M. Bodson, A. Sacks, and P. Khosla, "Harmonic generation in adaptive feedforward cancellation schemes," *IEEE Trans. Autom. Control*, vol. 39, no. 9, pp. 1939–1944, Sep. 1994.
- [12] P. Wieland, R. Sepulchre, and F. Allgöwer, "An internal model principle is necessary and sufficient for linear output synchronization," *Automatica*, vol. 47, no. 5, pp. 1068–1074, May 2011.
- [13] L. J. Brown and Q. Zhang, "Identification of periodic signals with uncertain frequency," *IEEE Trans. Signal Process.*, vol. 51, no. 6, pp. 1538–1545, Jun. 2003.
- [14] R. Marino and P. Tomei, "Output regulation for linear systems via adaptive internal model," *IEEE Trans. Autom. Control*, vol. 48, no. 12, pp. 2199–2202, Dec. 2003.
- [15] M. Bodson, "Rejection of periodic disturbances of unknown and time-varying frequency," *Int. J. Adapt. Control Signal Process.*, vol. 19, nos. 2–3, pp. 67–88, Mar. 2005.
- [16] Z. Ding, "Global stabilization and disturbance suppression of a class of nonlinear systems with uncertain internal model," *Automatica*, vol. 39, no. 3, pp. 471–479, Mar. 2003.
- [17] R. Marino and G. L. Santosuosso, "Global compensation of unknown sinusoidal disturbances for a class of nonlinear nonminimum phase systems," *IEEE Trans. Autom. Control*, vol. 50, no. 11, pp. 1816–1822, Nov. 2005.
- [18] W. Deng, J. Yao, and D. Ma, "Time-varying input delay compensation for nonlinear systems with additive disturbance: An output feedback approach," *Int. J. Robust Nonlinear Control*, vol. 28, no. 1, pp. 31–52, Jan. 2018.
- [19] W. Deng and J. Yao, "Extended-State-Observer-Based adaptive control of electrohydraulic servomechanisms without velocity measurement," *IEEE/ASME Trans. Mechatronics*, vol. 25, no. 3, pp. 1151–1161, Jun. 2020.
- [20] S. Sui, C. L. P. Chen, and S. Tong, "Fuzzy adaptive finite-time control design for nontriangular stochastic nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 1, pp. 172–184, Jan. 2019.
- [21] S. Sui, C. L. P. Chen, and S. Tong, "Neural network filtering control design for nontriangular structure switched nonlinear systems in finite time," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 7, pp. 2153–2162, Jul. 2019.
- [22] C. Xi and J. Dong, "Adaptive asymptotic tracking control of uncertain nonlinear time-delay systems depended on delay estimation information," *Appl. Math. Comput.*, vol. 391, Feb. 2021, Art. no. 125662.
- [23] C. Xi and J. Dong, "Adaptive exact sliding tracking control of high-order strict-feedback systems with mismatched nonlinearities and external disturbances," *Int. J. Robust Nonlinear Control*, vol. 30, no. 18, pp. 8228–8243, Dec. 2020, doi: 10.1002/rnc.5231.
- [24] J. Ishikawa and M. Tomizuka, "Pivot friction compensation using an accelerometer and a disturbance observer for hard disk drives," *IEEE/ASME Trans. Mechatronics*, vol. 3, no. 3, pp. 194–200, Sep. 1998.
- [25] K.-S. Kim, K.-H. Rew, and S. Kim, "Disturbance observer for estimating higher order disturbances in time series expansion," *IEEE Trans. Autom. Control*, vol. 55, no. 8, pp. 1905–1911, Aug. 2010.
- [26] C.-S. Liu and H. Peng, "Disturbance observer based tracking control," *J. Dyn. Syst., Meas., Control*, vol. 122, no. 2, pp. 332–335, Jun. 2000.
- [27] H. Li, X. Ning, and B. Han, "Composite decoupling control of gimbal servo system in double-gimbaled variable speed CMG via disturbance observer," *IEEE/ASME Trans. Mechatronics*, vol. 22, no. 1, pp. 312–320, Feb. 2017.
- [28] W. Kim, D. Shin, and C. C. Chung, "Microstepping using a disturbance observer and a variable structure controller for permanent-magnet stepper motors," *IEEE Trans. Ind. Electron.*, vol. 60, no. 7, pp. 2689–2699, Jul. 2013.
- [29] W. Kim, D. Shin, D. Won, and C. C. Chung, "Disturbance-observer-based position tracking controller in the presence of biased sinusoidal disturbance for electrohydraulic actuators," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 6, pp. 2290–2298, Nov. 2013.
- [30] H. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.
- [31] C. T. Chen, *Linear System Theory and Design*, 3rd ed. New York, NY, USA: Oxford Univ. Press, 1999.
- [32] B. Yao, F. P. Bu, and G. T. C. Chiu, "Nonlinear adaptive robust control of electro-hydraulic servo system with discontinuous projection," in *Proc. IEEE Conf. Decis. Control*, Dec. 1998, pp. 2265–2270.
- [33] K. D. Do and J. Pan, "Boundary control of transverse motion of marine risers with actuator dynamics," *J. Sound Vib.*, vol. 318, nos. 4–5, pp. 768–791, Dec. 2008.
- [34] S.-B. Choi, M. C. Won, and J. K. Hedrick, "Fuel injection control of SI engines," in *Proc. Conf. Dec. Control*, 1996, pp. 1609–1614.
- [35] A. Alleyne and J. K. Hedrick, "Nonlinear adaptive control of active suspensions," *IEEE Trans. Control Syst. Technol.*, vol. 3, no. 1, pp. 94–105, Mar. 1995.



WONHEE KIM (Member, IEEE) received the B.S. and M.S. degrees in electrical and computer engineering and the Ph.D. degree in electrical engineering from Hanyang University, Seoul, South Korea, in 2003, 2005, and 2012, respectively. From 2005 to 2007, he was with Samsung Electronics Company, Suwon, South Korea. In 2012, he was with the Power and Industrial Systems Research and Development Center, Hyosung Corp., Seoul. In 2013, he was a Postdoctoral Researcher with the Institute of Nano Science and Technology, Hanyang University, and a Visiting Scholar with the Department of Mechanical Engineering, University of California, Berkeley, CA, USA. From 2014 to 2016, he was with the Department of Electrical Engineering, Dong-A University, Busan, South Korea. He is currently an Associate Professor with the School of Energy Systems Engineering, Chung-Ang University, Seoul. His current research interests include nonlinear control and nonlinear observers, as well as their industrial applications. He served as an Associate Editor for IEEE ACCESS and the *Journal of Electrical Engineering and Technology*.



JUN MOON (Senior Member, IEEE) received the B.S. degree in electrical and computer engineering and the M.S. degree in electrical engineering from Hanyang University, Seoul, South Korea, in 2006 and 2008, respectively, and the Ph.D. degree in electrical and computer engineering from the University of Illinois at Urbana-Champaign, USA, in 2015. From 2008 to 2011, he was a Researcher with the Agency for Defense Development (ADD), South Korea. From 2016 to 2019, he was with the School of Electrical and Computer Engineering, Ulsan National Institute of Science and Technology (UNIST), South Korea, as an Assistant Professor. From 2019 to 2020, he was with the School of Electrical and Computer Engineering, University of Seoul, South Korea, as an Associate Professor. He is currently an Associate Professor with the Department of Electrical Engineering, Hanyang University. His research interests include stochastic optimal control and filtering, distributed control, networked control systems, and mean field games. He was a recipient of the Fulbright Graduate Study Award in 2011.

• • •