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Research Article

Multivalued Fixed Point Results for Two Families of Mappings in Modular-Like Metric Spaces with Applications

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The aim of this research work is to find out some results in fixed point theory for a pair of families of multivalued mappings fulfilling a new type of U-contractions in modular-like metric spaces. Some new results in graph theory for multigraph-dominated contractions in modular-like metric spaces are developed. An application has been presented to ensure the uniqueness and existence of a solution of families of nonlinear integral equations.

1. Introduction and Preliminaries

If the image of a point x under two mappings is x itself, then x is called a common fixed point of those mappings. Theory of fixed point has a basic role in analysis (see [1–39]). Chistyakov [7] established the concept of modular metric spaces and showed briefly about modular convergence, convex modular, equivalent metrics, abstract convex cone, and metric semigroup. Padcharoen et al. [16] introduced the concept of α-type F-contractions in modular metric spaces and discussed some related results. Further results on these spaces in different directions can be seen in [6, 13, 14].

In this paper, we establish some common fixed point theorems for two families of set-valued mappings satisfying a generalized contraction on a sequence only in a more generalized setting of modular-like metric spaces. New results can be established in dislocated metric spaces, ordered spaces, partial metric spaces, and metric spaces as a consequence of our findings. To support our results, some applications and examples are discussed. We give the following preliminary concepts which will be used in our results.

Definition 1. (see [16]). Let $A \neq \phi$. A function w: $(0, \infty) \times A \times A \longrightarrow [0, \infty)$ is called a modular-like metric on A if for all $a, b, c \in A$, $l > 0$, and $w_l(a, b) = w(l, a, b)$, it satisfies

(i) $w_l(a, b) = w_l(b, a)$ for all $l > 0$
(ii) $w_l(a, b) = 0$ for all $l > 0$ and then $a = b$
(iii) $w_{l+u}(a, b) \leq w_l(a, c) + w_u(c, b)$ for all $l, u > 0$

If we replace (ii) by $w_l(a, b) = 0$ for all $l > 0$ if and only if $a = b$, then w becomes a modular metric on A. If we replace (ii) by $w_l(a, b) = 0$ for some $l > 0$ and then $a = b$, then w becomes a regular modular metric on A. For $g \in A$ and $\varepsilon > 0$, $\overline{B_{w_l}(g, \varepsilon)} = \{p \in A: w_l(g, p) \leq \varepsilon\}$ is a closed ball in (A, w).

We will use *m.l.m.* space instead of modular-like metric space.

Definition 2. (see [16]). Let (A, w) be an *m.l.m.* space.

(i) $E \subseteq A$ is known as w-complete if for any sequence $(a_n)_{n\in\mathbb{N}}$ in E and for some $l > 0$, $w_l(a_m, a_n) \longrightarrow 0$ as $m, n \longrightarrow \infty$ implies $w_l(a_n, a) \longrightarrow 0$ as $n \longrightarrow \infty$ for some $a \in E$

(ii) The sequence $(a_n)_{n\in\mathbb{N}}$ in A is known as w-Cauchy for some $l > 0$ if $w_l(a_m, a_n) \longrightarrow 0$ as $m, n \longrightarrow \infty$

(iii) The sequence $(a_n)_{n\in\mathbb{N}}$ in A is known as w-convergent to $a \in A$ for some $l > 0$ if and only if $w_l(a_n, a) \longrightarrow 0$ as $n \longrightarrow \infty$

Definition 3. Let (A, w) be an *m.l.m.* space and $E \subseteq A$. A member p_0 which belongs to E is said to be a best approximation in E for $g \in A$ if

$$w_l(g, E) = \inf_{p\in E} w_l(g, p) = w_l(g, p_0). \quad (1)$$

If each $g \in A$ has a best approximation in E, then E is known as a proximinal set. $P(A)$ is equal to the family of proximinal sets in A. Let $A = \mathbb{R}^+ \cup \{0\}$ and $w_l(g, p) = 1/l(g + p)$ for all $l > 0$. Define a set $E = [4, 6]$; then, for each $y \in A$,

$$w_l(y, E) = w_l(y, [4, 6]) = \inf_{u\in[4,6]} w_l(y, u) = w_l(y, 4). \quad (2)$$

Hence, 4 is the best approximation in E for each $y \in A$. Also, $[4, 6]$ is a proximinal set.

Definition 4. The set-valued mapping $H_{w_l}: P(A) \times P(A) \longrightarrow [0, \infty)$, defined by

$$H_{w_l}(N, M) = \max\left\{\sup_{n\in N} w_l(n, M), \sup_{m\in M} w_l(N, m)\right\}, \quad (3)$$

is known as w_l-Hausdorff metric. The pair $(P(A), H_{w_l})$ is called the w_l-Hausdorff metric space. Let $A = \mathbb{R}^+ \cup \{0\}$ and $w_l(g, p) = (1/l)(g + p)$ for all $l > 0$. If $P = [5, 6]$ and $O = [9, 10]$, then $H_{w_l}(P, O) = (13/l)$.

Definition 5. Let (X, w) be a modular-like metric space. Then, we will say that w satisfies the Δ_M-condition if it is the case that $\lim_{n,m\longrightarrow\infty} w_p(x_n, x_m) = 0$, for $p = m - n$ implies $\lim_{n,m\longrightarrow\infty} w_l(x_n, x_m) = 0\, (m, n \in \mathbb{N}, m > n)$ for some $l > 0$.

Definition 6 (see [33]). Let $A \neq \phi$, $\xi: A \longrightarrow P(A)$ be a set-valued mapping, $B \subseteq A$, and $\alpha: A \times A \longrightarrow [0, +\infty)$. Then, ξ is called α_*-admissible on K if $\alpha_*(\xi a, \xi c) = \inf\{\alpha(u, v): u \in \xi a, v \in \xi c\} \geq 1$ whenever $\alpha(a, c) \geq 1$, for all $a, c \in B$.

Definition 7. Let $A \neq \phi$, $\xi: A \longrightarrow P(A)$ be a set-valued mapping, $M \subseteq A$, and $\alpha: A \times A \longrightarrow [0, +\infty)$. Then, ξ is called α_*-dominated on M if for all $a \in M, \alpha_*(a, \xi a) = \inf\{\alpha(a, l): l \in \xi a\} \geq 1$.

Definition 8. (see [39]). Consider a metric space (W, d). A function $H: W \longrightarrow W$ is said A-contraction if for all $c, k \in W$, there exists $\tau > 0$ such that $d(Ha, Hc) > 0$ implies

$$\tau + A(d(Ha, Hc)) \leq A(d(a, c)), \quad (4)$$

where $A: \mathbb{R}_+ \longrightarrow \mathbb{R}$ is a mapping which satisfies the following:

(F1) There exists $k \in (0, 1)$ such that $\lim_{\sigma\longrightarrow 0^+} \sigma^k A(\sigma) = 0$

(F2) For all $a, c \in \mathbb{R}_+$ such that $a < c$, $A(a) < A(c)$, that is, A is strictly increasing

(F3) $\lim_{n\longrightarrow+\infty} \sigma_n = 0$ if and only if $\lim_{n\longrightarrow+\infty} A(\sigma_n) = -\infty$, for each sequence $\{\sigma_n\}_{n=1}^{\infty}$ of positive numbers

The family of all mappings satisfying conditions (F1) to (F3) is denoted by F.

Lemma 1. *Let (A, w) be an m.l.m. space. Let $(P(A), H_{w_l})$ be a Hausdorff w_l-metric-like space. Then, for each $a \in K$ and for all $K, M \in P(A)$, there exists $b_a \in M$ such that $H_{w_l}(K, M) \geq w_l(a, b_a)$.*

Proof. If $H_{w_l}(K, M) = \sup_{a\in K} w_l(a, M)$, then $H_{w_l}(K, M) \geq w_l(a, M)$ for each $a \in K$. As M is the proximinal set, for every $a \in A$, there exists at least one best approximation $b_a \in M$ which satisfies $w_l(a, M) = w_l(a, b_a)$. Now, we have $H_{w_l}(K, M) \geq w_l(a, b_a)$. Now,

$$H_{w_l}(K, M) = \sup_{h\in M} w_l(K, h) \geq \sup_{a\in K} w_l(a, M) \geq w_l(a, b_a), \quad \text{for some } b_a \in M. \quad (5)$$

Hence proved. □

Example 1. (see [21]). Let $A = \mathbb{R}$. Define $B: A \times A \longrightarrow [0, \infty)$ by

$$B(b, t) = \begin{cases} 1 & \text{if } b > t, \\ \dfrac{1}{4} & \text{if } b \not> t. \end{cases} \quad (6)$$

Define $G, M: A \longrightarrow P(A)$ by

$$Gb = [-4 + b, -3 + b], \quad Mt = [-2 + t, -1 + t]. \quad (7)$$

Then, G and M are not α_*-admissible, but they are α_*-dominated.

2. Main Results

Let $(\mathfrak{I}, w)$ be an *m.l.m.* space and $c_0 \in \mathfrak{I}$; let $\{S_\sigma: \sigma \in \Omega\}$ and $\{T_\beta: \beta \in \Phi\}$ be two families of multifunctions from $\mathfrak{I}$ to $P(\mathfrak{I})$. Let $c_1 \in S_a c_0$ be an element such that $w(c_0, S_a c_0) = w(c_0, c_1)$. Let $c_2 \in T_b c_1$ be such that $w(c_1, T_b c_1) = w(c_1, c_2)$. Let $c_3 \in S_c c_2$ such that

$w(c_2, S_c c_2) = w(c_2, c_3)$. In this way, we get a sequence $\{T_\beta S_\sigma(c_n)\}$ in $\mathfrak{I}$, where $c_{2n+1} \in S_i c_{2n}$, $c_{2n+2} \in T_j c_{2n+1}$, $n \in \mathbb{N}$, $i \in \Omega$, and $j \in \Phi$. Also, $w(c_{2n}, S_i c_{2n}) = w(c_{2n}, c_{2n+1})$ and $w(c_{2n+1}, T_j c_{2n+1}) = w(c_{2n+1}, c_{2n+2})$. $\{T_\beta S_\sigma(c_n)\}$ is said to be a sequence in $\mathfrak{I}$ generated by c_0. If $\{S_\sigma\colon \sigma \in \Omega\} = \{T_\beta\colon \beta \in \Phi\}$, then we denote $\{S_\sigma(c_n)\}$ instead of $\{T_\beta S_\sigma(c_n)\}$.

Theorem 1. *Let $(\mathfrak{I}, w)$ be a complete m.l.m. space. Assume that w is regular and satisfies the Δ_M-condition. Let $c_0 \in \mathfrak{I}$, $\alpha\colon \mathfrak{I} \times \mathfrak{I} \longrightarrow [0, \infty)$, $\{T_\beta\colon \beta \in \Phi\}$, and $\{S_\sigma\colon \sigma \in \Omega\}$ be the families of α_*-dominated set-valued functions on w. Suppose there exist $\tau > 0$ and $U \in \digamma$ such that*

$$\tau + U\left(H_{w_1}\left(S_\sigma t, T_\beta g\right)\right) \le U\left(\max\left\{w_1(t, g), w_1(t, S_\sigma t), \frac{w_2\left(t, T_\beta g\right)}{2}, \frac{w_1(t, S_\sigma t).w_1\left(g, T_\beta g\right)}{1 + w_1(t, g)}\right\}\right), \tag{8}$$

whenever $t, g \in \mathfrak{I} \cap \{T_\beta S_\sigma(c_n)\}$, $\alpha(t, g) \ge 1$, $\sigma \in \Omega$, $\beta \in \Phi$, and $H_{w_1}(S_\sigma t, T_\beta g) > 0$.

Then, the sequence $\{T_\beta S_\sigma(c_n)\}$ generated by c_0 converges to $c \in \mathfrak{I}$, and for each $n \in \mathbb{N}$, $\alpha(c_n, c_{n+1}) \ge 1$. Also, if c satisfies (8) and either $\alpha(c_n, \acute{c}) \ge 1$ or $\alpha(c, c_n) \ge 1$ for all $n \in \mathbb{N} \cup \{0\}$, then S_σ and T_β have a common fixed point c in $\mathfrak{I}$ for all $\sigma \in \Omega$ and $\beta \in \Phi$.

Proof. Consider a sequence $\{T_\beta S_\sigma(c_n)\}$. Obviously, $c_n \in \mathfrak{I}$ for each $n \in \mathbb{N}$. If j is odd, then $j = 2‘ + 1$ for some $‘ \in \mathbb{N}$. By the definition of α_*-dominated mappings, we have $\alpha_*(c_{2‘}, S_\sigma c_{2‘}) \ge 1$ and $\alpha_*(c_{2‘+1}, T_\beta c_{2‘+1}) \ge 1$ for all $\sigma \in \Omega$ and $\beta \in \Phi$. As $\alpha_*(c_{2‘}, S_\sigma c_{2‘}) \ge 1$, this implies $\inf\{\alpha(c_{2‘}, b)\colon b \in S_\sigma c_{2‘}\} \ge 1$. Also, $c_{2‘+1} \in S_f c_{2‘}$ for some $f \in \Omega$, so $\alpha(c_{2‘}, c_{2‘+1}) \ge 1$. Also, $c_{2‘+2} \in T_g c_{2‘+1}$ for some $g \in \Phi$. Now, by using Lemma 1, we have

$$\begin{aligned}
\tau + U(w_1(c_{2‘+1}, c_{2‘+2})) &\le \tau + U\left(H_{w_1}\left(S_f c_{2‘}, T_g c_{2‘+1}\right)\right) \\
&\le U\left(\max\left\{w_1(c_{2‘}, c_{2‘+1}), w_1\left(c_{2‘}, S_f c_{2‘}\right), \frac{w_2\left(c_{2‘}, T_g c_{2‘+1}\right)}{2}, \frac{w_1\left(c_{2‘}, S_f c_{2‘}\right).w_1\left(c_{2‘+1}, T_g c_{2‘+1}\right)}{1 + w_1(c_{2‘}, c_{2‘+1})}\right\}\right) \\
&\le U\left(\max\left\{w_1(c_{2‘}, c_{2‘+1}), w_1(c_{2‘}, c_{2‘+1}), \frac{w_1(c_{2‘}, c_{2‘+1}) + w_1(c_{2‘+1}, c_{2‘+2})}{2}, \frac{w_1(c_{2‘}, c_{2‘+1}).w_1(c_{2‘+1}, c_{2‘+2})}{1 + w_1(c_{2‘}, c_{2‘+1})}\right\}\right) \\
&\le U(\max\{w_1(c_{2‘}, c_{2‘+1}), w_1(c_{2‘+1}, c_{2‘+2})\}).
\end{aligned} \tag{9}$$

This implies

$$\tau + U(w_1(c_{2‘+1}, c_{2‘+2})) \le U(\max\{w_1(c_{2‘}, c_{2‘+1}), w_1(c_{2‘+1}, c_{2‘+2})\}). \tag{10}$$

Now, if

$$\max\{w_1(c_{2‘}, c_{2‘+1}), w_1(c_{2‘+1}, c_{2‘+2})\} = w_1(c_{2‘+1}, c_{2‘+2}), \tag{11}$$

then from (10), we have

$$U(w_1(c_{2i+1}, c_{2i+2})) \le U(w_1(c_{2i+1}, c_{2i+2})) - \tau, \tag{12}$$

a contradiction. Therefore,

$$\max\{w_1(c_{2‘}, c_{2‘+1}), w_1(c_{2‘+1}, c_{2‘+2})\} = w_1(c_{2‘}, c_{2‘+1}), \tag{13}$$

for all $i \in \mathbb{N} \cup \{0\}$. Hence, from (10), we have

$$U(w_1(c_{2i+1}, c_{2i+2})) \le U(w_1(c_{2i}, c_{2i+1})) - \tau. \tag{14}$$

Similarly, we have

$$U(w_1(c_{2i}, c_{2i+1})) \le U(w_1(c_{2i-1}, c_{2i})) - \tau, \tag{15}$$

for all $i \in \mathbb{N} \cup \{0\}$. By (14) and (15), we have

$$U(w_1(c_{2i+1}, c_{2i+2})) \le U(w_1(c_{2i-1}, c_{2i})) - 2\tau. \tag{16}$$

Repeating these steps, we get

$$U(w_1(c_{2i+1}, c_{2i+2})) \le U(w_1(c_0, c_1)) - (2i + 1)\tau. \tag{17}$$

Similarly, we have

$$U(w_1(c_{2i}, c_{2i+1})) \le U(w_1(c_0, c_1)) - 2i\tau. \tag{18}$$

Inequalities (17) and (18) can jointly be written as

$$U(w_1(c_n, c_{n+1})) \le U(w_1(c_0, c_1)) - n\tau. \tag{19}$$

Taking the limit as $n \longrightarrow \infty$ in (19), we have

$$\lim_{n \longrightarrow \infty} U(w_1(c_n, c_{n+1})) = -\infty. \tag{20}$$

Since $U \in \digamma$,

$$\lim_{n \longrightarrow \infty} w_1(c_n, c_{n+1}) = 0. \tag{21}$$

Applying the property (F1) of $\digamma$, there exists $k \in (0, 1)$ such that

$$\lim_{n\longrightarrow\infty}(w_1(c_n,c_{n+1}))^k(U(w_1(c_n,c_{n+1})))=0. \quad (22)$$

By (19), for all $n\in\mathbb{N}$, we obtain

$$(w_1(c_n,c_{n+1}))^k(U(w_1(c_n,c_{n+1})))-U(w_1(c_0,c_1))\le-(w_1(c_n,c_{n+1}))^k n\tau\le 0. \quad (23)$$

Considering (21) and (22) and letting $n\longrightarrow\infty$ in (23), we have

$$\lim_{n\longrightarrow\infty}n(w_1(c_n,c_{n+1}))^k=0. \quad (24)$$

Since (24) holds, there exists $n_1\in\mathbb{N}$ such that $n(w_1(c_n,c_{n+1}))^k\le 1$ for all $n\ge n_1$ or

$$w_1(c_n,c_{n+1})\le\frac{1}{n^{(1/k)}}\quad\text{for all } n\ge n_1. \quad (25)$$

Take $p>0$ and $m=n+p>n>n_1$; then,

$$w_p(c_n,c_m)\le w_1(c_n,c_{n+1})+w_1(c_{n+1},c_{n+2})+\cdots+w_1(c_m,c_{m+1})\le\frac{1}{n^{(1/k)}}+\frac{1}{(n+1)^{(1/k)}}+\cdots+\frac{1}{m^{(1/k)}}. \quad (26)$$

Applying the limit as $n,m\longrightarrow\infty$ on both sides, we have

$$\lim_{n,m\longrightarrow\infty}w_p(c_n,c_m)\le\lim_{n,m\longrightarrow\infty}\frac{1}{n^{(1/k)}}+\lim_{n,m\longrightarrow\infty}\frac{1}{(n+1)^{(1/k)}}+\cdots+\lim_{n,m\longrightarrow\infty}\frac{1}{m^{(1/k)}}. \quad (27)$$

As $k\in(0,1)$, then $(1/k)>1$, and so,

$$\lim_{n,m\longrightarrow\infty}\frac{1}{n^{(1/k)}}=\lim_{n,m\longrightarrow\infty}\frac{1}{(n+1)^{(1/k)}}=\lim_{n,m\longrightarrow\infty}\frac{1}{m^{(1/k)}}=0. \quad (28)$$

As $w\colon(0,\infty)\times\mathfrak{I}\times\mathfrak{I}\longrightarrow[0,\infty)$, then

$$\lim_{n,m\longrightarrow\infty}w_p(c_n,c_m)=0. \quad (29)$$

Since w satisfies the Δ_M-condition, we have

$$\lim_{n,m\longrightarrow\infty}w_1(c_n,c_m)=0. \quad (30)$$

Hence, $\{T_\beta S_\sigma(c_n)\}$ is a Cauchy sequence in w. Since $(\mathfrak{I},w)$ is a regular complete modular-like metric space, there exists $\acute{c}\in\mathfrak{I}$ such that $\{T_\beta S_\sigma(c_n)\}\longrightarrow\acute{c}$ as $n\longrightarrow\infty$, and so,

$$\lim_{n\longrightarrow\infty}w_1\left(c_n,\acute{c}\right)=0. \quad (31)$$

Now, by Lemma 1, we have

$$\tau+U\left(w_1\left(c_{2n+1},T_\beta\acute{c}\right)\right)\le\tau+U\left(H_{w_1}\left(S_e c_{2n},T_\beta\acute{c}\right)\right), \quad (32)$$

for some $\beta\in\Phi$ and some $e\in\Omega$. Now, there exists $c_{2n+1}\in S_e c_{2n}$ such that $w_1(c_{2n},S_e c_{2n})=w_1(c_{2n},c_{2n+1})$. From the assumption, $\alpha(c_n,c)\ge 1$. Assume that $w_1(c,T_\beta c)>0$; then, there must be a positive real number p such that $w_1(c_{2n+1},T_\beta c)>0$ for $n\ge p$. Now, $H_{w_1}(S_e c_{2n},T_\beta c)>0$; then, by using inequality (8), we have

$$\tau+U\left(w_1\left(c_{2n+1},T_\beta\acute{c}\right)\right)\le U\left(\max\left\{w_1\left(c_{2n},\acute{c}\right),w_1\left(c_{2n},\acute{c}\right),\frac{w_1(c_{2n},c_{2n+1})+w_1\left(c_{2n+1},T_\beta\acute{c}\right)}{2},\frac{w_1(c_{2n},S_e c_{2n})\cdot w_1\left(f,T_\beta\acute{c}\right)}{1+w_1\left(c_{2n},\acute{c}\right)}\right\}\right). \quad (33)$$

Letting $n\longrightarrow\infty$ and using (31), we get

$$\tau+U\left(w_1\left(\acute{c},T_\beta\acute{c}\right)\right)\le U\left(w_1\left(\acute{c},T_\beta\acute{c}\right)\right). \quad (34)$$

Since U is strictly increasing, (32) implies

$$w_1\left(\acute{c},T_\beta\acute{c}\right)<w_1\left(\acute{c},T_\beta\acute{c}\right). \quad (35)$$

This is not true, So, our assumption is wrong. Hence, $w_1(c,T_\beta c)=0$ or $c\in T_\beta c$ for each $\beta\in\Phi$. Similarly, by proceeding with Lemma 1 and inequality (8), we can prove that $w_1(c,S_\sigma c)=0$ or $c\in S_\sigma c$ for all $\sigma\in\Omega$. Hence, c is a common fixed point of both the mappings S_σ and T_β in $\mathfrak{I}$ for all $\sigma \in \Omega$ and $\beta \in \Phi$. □

Example 2. Let $\mathfrak{I} = \mathbb{R}_+ \cup \{0\}$. Take $w_2(q,g) = (q+g)$ and $w_1(q,g) = 1/2(q+g)$ for all $q,g \in \mathfrak{I}$. Suppose that $S_\sigma, T_\beta\colon \mathfrak{I}\times\mathfrak{I} \longrightarrow P(\mathfrak{I})$ are two families of multivalued mappings defined by

$$S_m v = \left[\frac{v}{3m}, \frac{2v}{3m}\right] \quad \text{if } v \in \mathfrak{I}, \tag{36}$$

where $m = 1,2,3,\ldots$, and

$$T_n v = \left[\frac{v}{4n}, \frac{3v}{4n}\right] \quad \text{if } v \in \mathfrak{I}, \tag{37}$$

where $n = 1,2,3,\ldots$. Suppose that $v_0 = 1$ and $w_1(v_0, S_1 v_0) = w_1(1, S_1 1) = w_1(1,(1/3))$. So, $v_1 = (1/3)$. Now, $w_1(v_1, T_1 v_1) = w_1((1/3), T_1(1/3)) = w_1((1/3),(1/12))$. So, $v_2 = (1/12)$. Now, $w_1(v_2, S_2 v_2) = w_1((1/12), S_2(1/12)) = w_1((1/12),(1/72))$. So, $v_3 = (1/72)$. Continuing in this way, we have $\{T_n S_m(v_n)\} = \{1,(1/3),(1/12),(1/72),\ldots\}$. Consider the mapping $\alpha\colon \mathfrak{I}\times\mathfrak{I} \longrightarrow [0,\infty)$ defined by

$$\alpha(r,t) = \begin{cases} 1, & \text{if } r > t, \\ \dfrac{1}{2}, & \text{otherwise.} \end{cases} \tag{38}$$

Now, if $v, y \in \mathfrak{I} \cap \{T_\beta S_\sigma(v_n)\}$ with $\alpha(v,y) \geq 1$, we have

$$\begin{aligned} H_{w_1}(S_m v, T_n y) &= \max\left\{\sup_{a\in S_m x} w_1(a, T_n y), \sup_{b\in T_n y} w_1(S_m v, b)\right\} \\ &= \max\left\{\begin{array}{l} w_1\left(\dfrac{2v}{3m}, \left[\dfrac{y}{4n}, \dfrac{3y}{4n}\right]\right), \\ w_1\left(\left[\dfrac{v}{3m}, \dfrac{2v}{3m}\right], \dfrac{3y}{4n}\right) \end{array}\right\} \\ &= \max\left\{w_1\left(\frac{2v}{3m}, \frac{y}{4n}\right), w_1\left(\frac{v}{3m}, \frac{3y}{4n}\right)\right\} = \frac{1}{2}\max\left\{\frac{2v}{3m} + \frac{y}{4n}, \frac{v}{3m} + \frac{3y}{4n}\right\}, \\ W(x,y) &= \max\left(\frac{1}{2}\left\{\begin{array}{c} v+y, v + \dfrac{v}{3m}, \\ v + \dfrac{y}{4n}, \dfrac{(v+(v/3m))\cdot(y+(y/4n))}{\{1+v+y\}} \end{array}\right\}\right). \end{aligned} \tag{39}$$

Case (i). If $\max\{(2v/3m) + (y/4n), (y/3m) + (3y/4n)\} = (2v/3m) + (y/4n)$ and $\tau = \ln(1.2)$, then we have

$$\begin{aligned} &8vn + 3my \leq 10mnx + 10mny, \\ &48vn + 18my \leq 60mnx + 60mny, \\ &\frac{6}{5}\left(\frac{2v}{3m} + \frac{y}{4n}\right) \leq x + y, \\ &\ln(1.2) + \ln\left(\frac{2v}{3m} + \frac{y}{4n}\right) \leq \ln(x+y). \end{aligned} \tag{40}$$

This implies that

$$\tau + U\left(H_{w_1}(S_m v, T_n y)\right) \leq U(W(x,y)). \tag{41}$$

Case (ii). If $\max\{2v/3m + y/4n, y/3m + 3y/4n\} = v/3m + 3y/4n$ and $\tau = \ln(1.2)$, then we have

$$\begin{aligned} &4vn + 9my \leq 10mnx + 10mny, \\ &24vn + 54my \leq 60mnx + 60mny, \\ &\frac{6}{5}\left(\frac{v}{3m} + \frac{3y}{4n}\right) \leq x + y, \\ &\ln(1.2) + \ln\left(\frac{v}{3m} + \frac{3y}{4n}\right) \leq \ln(x+y). \end{aligned} \tag{42}$$

This implies that

$$\tau + U\left(H_{w_1}(S_m v, T_n y)\right) \leq U(W(x,y)). \tag{43}$$

Hence, all the requirements of Theorem 1 are satisfied. So, the families $\{S_m\}$ and $\{T_n\}$ have a common fixed point.

If we take $\{S_\sigma\colon \sigma \in \Omega\} = \{T_\beta\colon \beta \in \Phi\}$ in Theorem 1, then we obtain the above result.

Corollary 1. *Let $(\mathfrak{I}, w)$ be a complete m.l.m. space. Assume that w is regular and satisfies the Δ_M-condition. Let $c_0 \in \mathfrak{I}$, $\alpha\colon \mathfrak{I}\times\mathfrak{I} \longrightarrow [0,\infty)$, and $\{S_\sigma\colon \sigma \in \Omega\}$ be a family of*

α_*-dominated set-valued functions on $\mathfrak{I}$. If there exist $\tau > 0$ and $U \in \digamma$ such that

$$\tau + U\left(H_{w_l}\left(S_\sigma t, S_\beta g\right)\right) \leq U\left(\max\left\{w_1(t,g), w_1(t,S_\sigma t), w_2\left(t,S_\beta g\right), \frac{w_1(t,S_\sigma t)\cdot w_1\left(g,S_\beta g\right)}{1+w_1(t,g)}\right\}\right), \tag{44}$$

whenever $t,g \in \mathfrak{I}\cap\{S_\sigma(c_n)\}$, $\alpha(t,g)\geq 1$, $\sigma,\beta\in\Omega$, and $H_{w_1}(S_\sigma t, S_\beta g) > 0$, then the sequence $\{\mathfrak{I}S_\sigma(c_n)\}$ generated by c_0 converges to $c\in\mathfrak{I}$, and for each $n\in\mathbb{N}$, $\alpha(c_n,c_{n+1})\geq 1$. Also, if c satisfies (44) and either $\alpha(c_n,c)\geq 1$ or $\alpha(c,c_n)\geq 1$ for every $n\in\mathbb{N}\cup\{0\}$, then $\{S_\sigma\colon \sigma\in\Omega\}$ has a common fixed point c in $\mathfrak{I}$.

3. Applications in Graph Theory

Jachymski [11] developed a relation between fixed point theory and graph theory by the induction of graphic contractions. Hussain et al. [9] established some results for the new type of contractions endowed with a graph and also showed an application. Further useful results on the graph can be seen in [34, 35, 40].

Definition 9. Let A be a nonempty set and $Y = (V(Y), L(Y))$ be a graph with $V(Y) = A$. A mapping F from A to $P(A)$ is known as multigraph-dominated on A if $(a,b)\in L(Y)$, whenever $a\in A$ and $b\in Fa$.

Theorem 2. *Let $(\mathfrak{I}, w)$ be a complete m.l.m. space endowed with a graph Y, $c_0\in\mathfrak{I}$, and the following hold:*

(i) $\left\{T_\beta\colon \beta\in\Phi\right\}$ and $\{S_\sigma\colon \sigma\in\Omega\}$ are sets of multigraph-dominated functions on $\mathfrak{I}\cap\left\{T_\beta S_\sigma(c_n)\right\}$.

(ii) There exist $\tau>0$ and $U\in\digamma$ such that

$$\tau + U\left(H_{w_1}\left(S_\sigma t, T_\beta y\right)\right) \leq U\left(\max\left\{w_1(t,y), w_1(t,S_\sigma t), \frac{w_2\left(t,T_\beta y\right)}{2}, \frac{w_1(t,S_\sigma t)\cdot w_1\left(y,T_\beta y\right)}{1+w_1(t,y)}\right\}\right), \tag{45}$$

whenever $t,y\in\mathfrak{I}\cap\left\{T_\beta S_\sigma(c_n)\right\}$, $(t,y)\in L(Y)$, $\sigma\in\Omega$, $\beta\in\Phi$, and $H_{w_1}(S_\sigma t, T_\beta y)>0$.

Assume that $\mathfrak{I}$ is regular and satisfies the Δ_M-condition. Then, $\left\{T_\beta S_\sigma(c_n)\right\}$ is a sequence in $\mathfrak{I}$, $(c_n,c_{n+1})\in L(Y)$, and $\left\{T_\beta S_\sigma(c_n)\right\}\longrightarrow g^*$. Also, if g^* satisfies (45) and $(c_n,g^*)\in L(Y)$ or $(g^*,c_n)\in L(Y)$ for each $n\in\mathbb{N}\cup\{0\}$, then S_σ and T_β have common fixed point g^* in $\mathfrak{I}$ for all $\sigma\in\Omega$ and $\beta\in\Phi$.

Proof. Define $\alpha\colon \mathfrak{I}\times\mathfrak{I}\longrightarrow[0,\infty)$ by $\alpha(t,y)=1$ if $t\in\mathfrak{I}$ and $(t,y)\in L(Y)$. Otherwise, $\alpha(t,y)=0$. By the definition of the graph dominated on $\mathfrak{I}$, we have $(t,y)\in L(Y)$ for all $y\in S_\sigma t$ and $(t,y)\in L(Y)$ for each $y\in T_\beta t$. So, $\alpha(t,y)=1$ for all $y\in S_\sigma t$ and $\alpha(t,y)=1$ for every $y\in T_\beta t$. This means that $\inf\{\alpha(t,y)\colon y\in S_\sigma t\}=1$ and $\inf\left\{\alpha(t,y)\colon y\in T_\beta t\right\}=1$. Hence, $\alpha_*(t,S_\sigma t)=1$ and $\alpha_*(t,T_\beta t)=1$ for every $t\in\mathfrak{I}$. So, the families of mappings are α_*-dominated on $\mathfrak{I}$. Furthermore, inequality (45) can be expressed as

$$\tau + U\left(H_{w_1}\left(S_\sigma t, T_\beta y\right)\right) \leq U\left(\max\left\{w_1(t,y), w_1(t,S_\sigma t), \frac{w_2\left(t,T_\beta y\right)}{2}, \frac{w_1(t,S_\sigma t)\cdot w_1\left(y,T_\beta y\right)}{1+w_1(t,y)}\right\}\right), \tag{46}$$

whenever $t,y\in\mathfrak{I}\cap\left\{T_\beta S_\sigma(c_n)\right\}$, $\alpha(t,y)\geq 1$, and $H_{w_1}(S_\sigma t, T_\beta y)>0$. Also, (ii) holds. Then, by Theorem 1, we have $\left\{T_\beta S_\sigma(c_n)\right\}$ is a sequence in $\mathfrak{I}$, and $\left\{T_\beta S_\sigma(c_n)\right\}\longrightarrow g^*\in\mathfrak{I}$. Now, $c_n, g^*\in\mathfrak{I}$, and either $(c_n,g^*)\in L(Y)$ or $(g^*,c_n)\in L(Y)$ implies that either $\alpha(c_n,g^*)\geq 1$ or $\alpha(g^*,c_n)\geq 1$. So, all the requirements of Theorem 1 are satisfied. Hence, from Theorem 1, S_σ and T_β have a common fixed point g^* in $\mathfrak{I}$, and $w_1(g^*,g^*)=0$. □

4. Results on Single-Valued Mappings

In this section, some consequences of our results related to single-valued mappings in *m.l.m.* spaces have been discussed. Let $(\mathfrak{I}, w)$ be an *m.l.m.* space, $c_0\in\mathfrak{I}$, and $S_\sigma, T_\beta\colon \mathfrak{I}\longrightarrow\mathfrak{I}$ be two families of mappings. Let $c_1 = S_\sigma c_0$,

$c_2 = T_\beta c_1$, and $c_3 = S_\sigma c_2$. Adopting this way, we make a sequence c_n in $\mathfrak{I}$ so that $c_{2n+1} = S_\sigma c_{2n}$ and $c_{2n+2} = T_\beta c_{2n+1}$, where $n = 0, 1, 2, \ldots$. We represent this kind of iterative sequence by $\{T_\beta S_\sigma(c_n)\}$. We say that $\{T_\beta S_\sigma(c_n)\}$ is a sequence in $\mathfrak{I}$ generated by c_0. If $\{S_\sigma\colon \sigma \in \Omega\} = \{T_\beta\colon \beta \in \Phi\}$, then we denote $\{\mathfrak{I}S_\sigma(c_n)\}$ instead of $\{T_\beta S_\sigma(c_n)\}$.

Theorem 3. *Let $(\mathfrak{I}, w)$ be a complete m.l.m. space. Assume that w is regular and satisfies the $\triangle_M$-condition. Let $r > 0$, $c_0 \in \overline{B_{w_1}(c_0, r)} \subseteq \mathfrak{I}$, $\alpha\colon \mathfrak{I} \times \mathfrak{I} \longrightarrow [0, \infty)$, and $\{S_\sigma\colon \sigma \in \Omega\}$, $\{T_\beta\colon \beta \in \Phi\}$ be two families of α-dominated mappings from $\mathfrak{I}$ to $\mathfrak{I}$. Then, there exist $\tau > 0$ and $U \in \digamma$ such that*

$$\tau + U\left(H_{w_1}\left(S_\sigma t, T_\beta g\right)\right) \leq U\left(\max\left\{w_1(t, g), w_1(t, S_\sigma t), \frac{w_2\left(t, T_\beta g\right)}{2}, \frac{w_1(t, S_\sigma t) \cdot w_1\left(g, T_\beta g\right)}{1 + w_1(t, g)}\right\}\right), \tag{47}$$

whenever $t, g \in \mathfrak{I} \cap \{T_\beta S_\sigma(c_n)\}$, $\alpha(t, g) \geq 1$, $\sigma \in \Omega$, $\beta \in \Phi$, and $w_1(S_\sigma t, T_\beta g) > 0$.

Then, $\{T_\beta S_\sigma(c_n)\}$ is a sequence in $\mathfrak{I}$, $\alpha(c_n, c_{n+1}) \geq 1$ for each $n \in \mathbb{N} \cup \{0\}$ and $\{T_\beta S_\sigma(c_n)\} \longrightarrow h \in \mathfrak{I}$. Also, if u satisfies (47) and either $\alpha(c_n, h) \geq 1$ or $\alpha(h, c_n) \geq 1$ for all $n \in \mathbb{N} \cup \{0\}$, then S_σ and T_β have a common fixed point h in $\mathfrak{I}$ for every $\sigma \in \Omega$ and $\beta \in \Phi$.

Proof. The proof is similar to the proof of Theorem 1.

If we take $\{S_\sigma\colon \sigma \in \Omega\} = \{T_\beta\colon \beta \in \Phi\}$ in Theorem 3, then we can get the following result. □

Corollary 2. *Let $(\mathfrak{I}, w)$ be a complete m.l.m. space. Assume that w is regular and satisfies the Δ_M-condition. Let $c_0 \in \mathfrak{I}$, $\alpha\colon \mathfrak{I} \times \mathfrak{I} \longrightarrow [0, \infty)$, and $\{S_\sigma\colon \sigma \in \Omega\}$ be a family of α-dominated mapping from $\mathfrak{I}$ to $\mathfrak{I}$. Then, there exist $\tau > 0$ and $U \in \digamma$ such that*

$$\tau + U\left(H_{w_1}\left(S_\sigma t, S_\beta g\right)\right) \leq U\left(\max\left\{w_1(t, g), w_1(t, S_\sigma t), \frac{w_2\left(t, T_\beta g\right)}{2}, \frac{w_1(t, S_\sigma t) \cdot w_1\left(g, S_\beta g\right)}{1 + w_1(t, g)}\right\}\right), \tag{48}$$

whenever $t, g \in \mathfrak{I} \cap \{\mathfrak{I}S_\sigma(c_n)\}$, $\alpha(t, g) \geq 1$, $\sigma, \beta \in \Omega$, and $w_1(S_\sigma t, S_\sigma g) > 0$. Then, $\{\mathfrak{I}S_\sigma(c_n)\}$ is a sequence in $\mathfrak{I}$, $\alpha(c_n, c_{n+1}) \geq 1$ for each $n \in \mathbb{N} \cup \{0\}$, and $\{\mathfrak{I}S_\sigma(c_n)\} \longrightarrow h \in \mathfrak{I}$. Also, if h satisfies (48) and either $\alpha(c_n, h) \geq 1$ or $\alpha(h, c_n) \geq 1$ for each $n \in \mathbb{N} \cup \{0\}$, then h is the fixed point of S_σ in $\mathfrak{I}$ for every $\sigma \in \Omega$.

5. Application in Integral Equations

In this section, we discuss the application of our work in integral equations. First of all, we present our main result without a closed ball for self-mappings and without α_*-dominated functions and then apply it to attain an application in integral equations. We also discuss the uniqueness.

Theorem 4. *Let $(\mathfrak{I}, w)$ be a complete m.l.m. space. Assume that w is regular and satisfies the Δ_M-condition. Let $c_0 \in \mathfrak{I}$ and $\{S_\sigma\colon \sigma \in \Omega\}$ and $\{T_\beta\colon \beta \in \Phi\}$ be the families of self-mappings. Then, there exist $\tau > 0$ and a strictly increasing function $U\colon \mathbb{R}_+ \longrightarrow \mathbb{R}$ such that*

$$\tau + U\left(H_{w_1}\left(S_\sigma t, T_\beta g\right)\right) \leq U\left(\max\left\{w_1(t, g), w_1(t, S_\sigma t), \frac{w_2\left(t, T_\beta g\right)}{2}, \frac{w_1(t, S_\sigma t) \cdot w_1\left(g, T_\beta g\right)}{1 + w_1(t, g)}\right\}\right), \tag{49}$$

whenever $t, g \in \{T_\beta S_\sigma(c_n)\}$, $\sigma \in \Omega$, $\beta \in \Phi$, and $w_1(S_\sigma t, T_\beta g) > 0$. Then, $\{T_\beta S_\sigma(c_n)\} \longrightarrow h \in \mathfrak{I}$. Also, if inequality (49) holds for $t, g \in \{h\}$, then S_σ and T_β have a unique common fixed point h in $\mathfrak{I}$ for all $\sigma \in \Omega$ and $\beta \in \Phi$.

Proof. The proof is similar to the proof of Theorem 1. We prove only uniqueness. Let S_σ and T_β have another common fixed point v. Suppose $w_1(S_\sigma u, T_\beta v) > 0$. Then,

$$\tau + U\left(w_1\left(S_\sigma u, T_\beta v\right)\right) \leq U\left(\max\left\{w_1(u, v), w_1(u, S_\sigma u), \frac{w_2\left(u, T_\beta v\right)}{2}, \frac{w_1(u, S_\sigma u) \cdot w_1\left(v, T_\beta v\right)}{1 + w_1(u, v)}\right\}\right). \tag{50}$$

This implies that

$$w_1(u,v) < \mu_1 w_1(u,v) < w_1(u,v), \tag{51}$$

which is not true. So, $w_1(S_\sigma u, T_\beta v) = 0$. Hence, $u = v$.

Let $W = C([0,1], \mathbb{R}_+)$ be the set of all continuous functions on $[0,1]$. Consider the families of integral equations

$$u(k) = \int_0^k H_\sigma(k,h,u(h))\mathrm{d}h + \epsilon, \tag{52}$$

$$c(k) = \int_0^k G_\beta(k,h,c(h))\mathrm{d}h + \epsilon, \tag{53}$$

for all $k \in [0,1]$, $\sigma \in \Omega$, $\beta \in \Phi$, and H_σ, G_β are the functions from $[0,1] \times [0,1] \times W$ to $\mathbb{R}$. For $c \in C([0,1], \mathbb{R}_+)$, define the supremum norm as $\|c\|_\tau = \sup_{k\in[0,1]}\{|c(k)|e^{-\eta k}\}$, where $\eta > 0$ is arbitrarily taken. Define

$$w_1(c,p) = \frac{1}{2}\sup_{k\in[0,1]}\left\{|c(k)+p(k)|e^{-\tau k}\right\} = \frac{1}{2}\|c+p\|_\tau, \tag{54}$$

for all $c, p \in C([0,1], \mathbb{R}_+)$; with these settings, $(C([0,1], \mathbb{R}_+), d_\tau)$ becomes a complete *m.l.m.* space.

Now, we prove the following theorem to ensure the uniqueness and existence of a solution of families of non-linear integral equations (52) and (53). □

Theorem 5. *Assume the following constraints are satisfied:*

(i) $\{H_\sigma, \sigma \in \Omega\}$ and $\{G_\beta, \beta \in \Phi\}$ are two families of mappings from $[0,1] \times [0,1] \times C([0,1], \mathbb{R}_+)$ to $\mathbb{R}$.

(ii) Define

$$\begin{aligned}(S_\sigma u)(k) &= \int_0^k H_\sigma(k,h,u(h))\mathrm{d}h + \epsilon,\\ (T_\beta c)(k) &= \int_0^k G_\beta(k,h,c(h))\mathrm{d}h + \epsilon.\end{aligned} \tag{55}$$

Suppose there exists $\tau > 0$ such that

$$\left|H_\sigma(k,h,u) + G_\beta(k,h,c)\right| \le \frac{\tau E_{(\sigma,\beta)}(u,c)}{\tau E_{(\sigma,\beta)}(u,c)+1}, \tag{56}$$

for all $k, h \in [0,1]$ and $u, c \in C([0,1], \mathbb{R}^+)$, where

$$E_{(\sigma,\beta)}(u,c) = \max\left(\frac{1}{2}\left\{\begin{array}{c}\|u+c\|_\tau, \left\|u+S_\sigma u\right\|_\tau,\\ \dfrac{\left\|u+S_\sigma u\right\|_\tau + \left\|c+T_\beta c\right\|_\tau}{2},\\ \dfrac{\left\|u+S_\sigma u\right\|_\tau \cdot \left\|c+T_\beta c\right\|_\tau}{1+\|u+c\|_\tau}\end{array}\right\}\right). \tag{57}$$

Then, integral equations (52) and (53) have a unique solution.

Proof. By assumption (ii),

$$\begin{aligned}\left|S_\sigma u + T_\beta c\right| &= \int_0^k \left|H_\sigma(k,h,u) + G_\beta(k,h,c)\right|\mathrm{d}h \le \int_0^k \frac{\tau E_{(\sigma,\beta)}(u,c)}{\tau E_{(\sigma,\beta)}(u,c)+1} e^{\tau h}\mathrm{d}h\\ &\le \frac{\tau E_{(\sigma,\beta)}(u,c)}{\tau E_{(\sigma,\beta)}(u,c)+1}\int_0^k e^{\tau h}\mathrm{d}h \le \frac{E_{(\sigma,\beta)}(u,c)}{\tau E_{(\sigma,\beta)}(u,c)+1} e^{\tau k}.\end{aligned} \tag{58}$$

This implies

$$\begin{aligned}&\left|S_\sigma u + T_\beta c\right| e^{-\tau k} \le \frac{E_{(\sigma,\beta)}(u,c)}{\tau E_{(\sigma,\beta)}(u,c)+1}\left\|S_\sigma u + T_\beta c\right\|_\tau \le \frac{E_{(\sigma,\beta)}(u,c)}{\tau E_{(\sigma,\beta)}(u,c)+1}\\ &\frac{\tau E_{(\sigma,\beta)}(u,c)+1}{E_{(\sigma,\beta)}(u,c)} \le \frac{1}{\left\|S_\sigma u + T_\beta c\right\|_\tau}\tau + \frac{1}{E_{(\sigma,\beta)}(u,c)} \le \frac{1}{\left\|S_\sigma u + T_\beta c\right\|_\tau},\end{aligned} \tag{59}$$

which further implies

$$\tau - \frac{1}{\left\|S_\sigma u(k) + T_\beta c(k)\right\|_\tau} \le \frac{-1}{E_{(\sigma,\beta)}(u,c)}. \tag{60}$$

So, all the requirements of Theorem 5 are satisfied for $U(f) = -1/\sqrt{f}$; $f > 0$ and $w_1(f,c) = 1/2\|f+c\|_\tau$. Hence, two families of integral equations given in (52) and (53) have a unique common solution. □

6. Conclusion

In this article, we have achieved some new results for set-valued mappings belonging to two families which satisfy generalized rational-type Wardowski's contraction. Dominated mappings are applied to find out the fixed point results. Applications in the subject of integral equations and graph theory are presented. Moreover, we investigate our results in new generalized modular-like metric spaces. Many consequences of our results in dislocated metric spaces, metric spaces, and partial metric spaces (even with a partial order) can be established easily.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Each author contributed equally to this paper, read, and approved the final manuscript.

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