## Search for the decay $\boldsymbol{B}^{\boldsymbol{0}} \rightarrow \boldsymbol{D} K^{* \boldsymbol{0}}$ followed by $\boldsymbol{D} \rightarrow \boldsymbol{K}^{-} \boldsymbol{\pi}^{+}$

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We report a study of the decay $B^{0} \rightarrow D K^{+} \pi^{-}$followed by $D \rightarrow K^{-} \pi^{+}$, where $D$ indicates $D^{0}$ or $\bar{D}^{0}$. We reconstruct the $D K^{+} \pi^{-}$state in a phase space corresponding to $D K^{*}(892)^{0}$. The $C P$-violating angle $\phi_{3}$ affects its decay rate via the interference between $b \rightarrow u$ and $b \rightarrow c$ transitions. The result is obtained from a $711 \mathrm{fb}^{-1}$ data sample that contains $772 \times 10^{6} B \bar{B}$ pairs collected at the $\mathrm{Y}(4 S)$ resonance with the Belle detector at the KEKB asymmetric-energy $e^{+} e^{-}$collider. We measure the ratio $\mathcal{R}_{D K^{* 0}} \equiv \Gamma\left(B^{0} \rightarrow\right.$ $\left.\left[K^{-} \pi^{+}\right]_{D} K^{+} \pi^{-}\right) / \Gamma\left(B^{0} \rightarrow\left[K^{+} \pi^{-}\right]_{D} K^{+} \pi^{-}\right)$to be $\left(4.5_{-5.0-1.8}^{+5.6+2.8}\right) \times 10^{-2}$, and set an upper limit of $\mathcal{R}_{D K^{* 0}}<0.16$ at the $95 \%$ credible interval.

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Determination of the parameters of the standard model is important as a consistency check and as a way to search for new physics. In the standard model, the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] gives a successful description of current experimental measurements of $C P$ violation. The three $C P$-violating phases $\phi_{1}, \phi_{2}$ and $\phi_{3}$ are defined as the angles of one particular CKM unitarity triangle with the latter defined as $\phi_{3} \equiv$ $\arg \left(-V_{u d} V_{u b}{ }^{*} / V_{c d} V_{c b}{ }^{*}\right)$. This phase is less accurately determined than the other two [2]. In the usual quark-phase convention where large complex phases appear only in $V_{u b}$ and $V_{t d}$ [3], the measurement of $\phi_{3}$ is equivalent to the extraction of the phase of $V_{u b}$ relative to the phases of other CKM matrix elements. To date, the $\phi_{3}$ measurement has been advanced mainly by exploiting charged $B$ meson decays into $D^{(*)} K^{ \pm}$final states [4-13] wherein the $C P$ sensitivity is due to the interference between the two amplitudes of $\bar{D}^{(*) 0}$ and $D^{(*) 0}$ decays into a common final state.

In this paper, we consider the neutral meson decay $B^{0} \rightarrow D K^{* 0}$ as an alternative process for measuring the angle $\phi_{3}$. As shown by the Feynman diagrams in Fig. 1, a weak decay of the $B$ meson is tagged by the $K^{* 0}$ decaying into $K^{+} \pi^{-}[14]$. We measure the ratio $\mathcal{R}_{D K^{* 0}}[15,16]$ defined as

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$$
\begin{align*}
& \mathcal{R}_{D K^{* 0}} \\
& \qquad \begin{array}{l}
\Gamma\left(B^{0} \rightarrow\left[K^{-} \pi^{+}\right]_{D} K^{+} \pi^{-}\right)+\Gamma\left(\bar{B}^{0} \rightarrow\left[K^{+} \pi^{-}\right]_{D} K^{-} \pi^{+}\right) \\
\\
\quad=r_{S}^{2}+r_{D}^{2}+2 k r_{S} r_{D} \cos \left(\delta_{S}+\delta_{D}\right) \cos \phi_{3},
\end{array}
\end{align*}
$$

where $r_{D} \equiv\left|A\left(D^{0} \rightarrow K^{+} \pi^{-}\right) / A\left(D^{0} \rightarrow K^{-} \pi^{+}\right)\right|$is the ratio for $D$ decay amplitudes and $\delta_{D}$ is the strong phase difference of the two $D$ decays appearing in this ratio. Both $r_{D}$ and $\delta_{D}$ have been obtained experimentally [17]. The parameters $r_{S}, \delta_{S}$ and $k$ are defined as

$$
\begin{gather*}
r_{S}^{2} \equiv \frac{\Gamma\left(B^{0} \rightarrow D^{0} K^{+} \pi^{-}\right)}{\Gamma\left(B^{0} \rightarrow \bar{D}^{0} K^{+} \pi^{-}\right)}=\frac{\int d p A_{b \rightarrow u}^{2}(p)}{\int d p A_{b \rightarrow c}^{2}(p)},  \tag{2}\\
k \mathrm{e}^{i \delta_{S}} \equiv \frac{\int d p A_{b \rightarrow c}(p) A_{b \rightarrow u}(p) \mathrm{e}^{i \delta(p)}}{\sqrt{\int d p A_{b \rightarrow c}^{2}(p) \int d p A_{b \rightarrow u}^{2}(p)}}, \tag{3}
\end{gather*}
$$

where $A_{b \rightarrow c}(p)$ and $A_{b \rightarrow u}(p)$ are the magnitudes of the amplitudes for the $b \rightarrow c$ and $b \rightarrow u$ transitions, respectively, and $\delta(p)$ is the relative strong phase. The variable $p$ indicates the position in the $D K^{+} \pi^{-}$Dalitz plot. In this analysis, we calculate the integrals over a phase space of the state $D K^{*}(892)^{0}$. In the case of a two-body $B$ decay, $r_{S}$ becomes the ratio of the amplitudes for $b \rightarrow u$ and $b \rightarrow c$ and $k$ becomes 1 . The value of $r_{S}$ is expected to be around

SEARCH FOR THE DECAY $B^{0} \rightarrow D K *^{0} \ldots$


FIG. 1. Diagrams for the $B^{0} \rightarrow D^{0} K^{* 0}$ and $B^{0} \rightarrow \bar{D}^{0} K^{* 0}$ decays. The $\phi_{3}$ dependence in the $b \rightarrow u$ transition is extracted from the interference of the two decay paths, which occurs when the $\bar{D}^{0}$ and $D^{0}$ mesons decay to the same final state.
0.4 , which is obtained from $\left|V_{u b} V_{c s}^{*}\right| /\left|V_{c b} V_{u s}^{*}\right|$, and depends on strong interaction effects. According to a simulation study using a Dalitz model based on recent measurements [18], the value of $k$ is around 0.95 in the phase space of interest here. One observable $\mathcal{R}_{D K^{* 0}}$ is not enough to extract the four unknowns $\phi_{3}, r_{S}, k$, and $\delta_{S}$. However, the measurements for other $D$ decays such as $D \rightarrow K^{+} K^{-}$ and $K_{S} \pi^{0}$ provide additional information needed to extract $\phi_{3}$, where the observable $\mathcal{R}_{D K^{* 0}}$ should be defined in the same phase space of the $B^{0}$ decay between different $D$ decays so that the same parameters $r_{S}, k$, and $\delta_{S}$ can be used. The decay in the numerator of Eq. (1) is the signal mode, referred to as the "suppressed mode," while the decay in the denominator is the calibration mode referred to as the "favored mode."

This result is based on a data sample that contains $772 \times$ $10^{6} B \bar{B}$ pairs, collected with the Belle detector at the KEKB asymmetric-energy $e^{+} e^{-}$(3.5 on 8 GeV ) collider [19] operating at the $\mathrm{Y}(4 S)$ resonance. The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector, a 50 -layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of $\mathrm{CsI}(\mathrm{Tl})$ crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented to detect $K_{L}^{0}$ mesons and to identify muons. The detector is described in detail elsewhere [20].

Charged kaon and pion candidates are identified using ionization loss in the CDC and information from the ACC and the TOF. The efficiency is $85-95 \%$ and the probability of misidentification is $10-20 \%$. We reconstruct $D$ mesons from pairs of oppositely-charged kaon and pion candidates. We require that the invariant mass is within $\pm 15 \mathrm{MeV} / c^{2}$ ( $\pm 3 \sigma$ ) of the nominal $D^{0}$ mass. $K^{* 0}$ candidates are reconstructed from $K^{+} \pi^{-}$pairs. We require that the invariant mass is within $\pm 50 \mathrm{MeV} / c^{2}$ of the nominal $K^{* 0}$ mass. We combine $D$ and $K^{* 0}$ candidates to form $B^{0}$ mesons. Candidate events are identified by the energy difference
$\Delta E \equiv \sum_{i} E_{i}-E_{\mathrm{b}}$ and the beam-constrained mass $M_{\mathrm{bc}} \equiv$ $\sqrt{E_{\mathrm{b}}^{2}-\left|\sum_{i} \vec{p}_{i}\right|^{2}}$, where $E_{\mathrm{b}}$ is the beam energy and $\vec{p}_{i}$ and $E_{i}$ are the momenta and energies, respectively, of the $B^{0}$ meson decay products in the $e^{+} e^{-}$center-of-mass (CM)

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frame. We select events with $5.271 \mathrm{GeV} / c^{2}<M_{\mathrm{bc}}<$ $5.287 \mathrm{GeV} / c^{2}$ and $-0.1 \mathrm{GeV}<\Delta E<0.3 \mathrm{GeV}$. In the rare case where there are multiple candidates in an event, the candidate with $M_{\mathrm{bc}}$ closest to its nominal value is chosen.

Among other $B$ decays, the most serious background for the suppressed mode comes from $\bar{B}^{0} \rightarrow\left[\bar{K}^{* 0} K^{+}\right]_{D^{+}} \pi^{-}$. This decay produces the same final state as the $B^{0} \rightarrow$ $D K^{* 0}$ signal, and the product branching fraction is about 10 times higher than that expected for the signal. To suppress this background, we exclude candidates for which the invariant mass of the $K^{-} \pi^{+} K^{+}$system is within $\pm 18 \mathrm{MeV} / c^{2}( \pm 3 \sigma)$ of the nominal $D^{+}$mass. The relative loss in the signal efficiency is $0.5 \%$.

Large combinatorial background of true $D^{0}$ and random $K^{+}$and $\pi^{-}$combinations from the $e^{+} e^{-} \rightarrow c \bar{c}$ process and other $B \bar{B}$ decays is reduced if the $D^{0}$ is a decay product of $D^{*+} \rightarrow D^{0} \pi^{+}$by using the mass difference $\Delta M$ between the $\left[K^{-} \pi^{+}\right]_{D} \pi^{+}$and $\left[K^{-} \pi^{+}\right]_{D}$ systems, where a $\pi^{+}$candidate is added to the latter to form the former. If $\Delta M>0.15 \mathrm{GeV} / c^{2}$ for any additional $\pi^{+}$candidate not used in the $B$ candidate reconstruction, the event is retained. This requirement removes $24 \%$ of $c \bar{c}$ background and $14 \%$ of $B \bar{B}$ background according to Monte Carlo (MC) simulation. The relative loss in signal efficiency is $5.0 \%$.

To discriminate the large combinatorial background dominated by the two-jet-like $e^{+} e^{-} \rightarrow q \bar{q}$ continuum process, where $q$ indicates $u, d$, $s$ or $c$, a multivariate analysis is performed using the following nine variables. (1) A variable obtained from the Fisher discriminants based on modified Fox-Wolfram moments [21] where the coefficients of the Fisher discriminants are optimized using the signal and $q \bar{q}$ MC samples. This variable exploits the event topology, which is spherical and jet-like for $B \bar{B}$ and $q \bar{q}$ events, respectively. (2) The angle in the CM frame between the thrust axes of the $B$ decay and the detected remainders. For the latter, we assign the pion mass to all the charged particles and use photons with energy above 0.1 GeV . (3) The signed difference of the vertices between the $B$ candidate and the remaining charged tracks. For the signal event, the absolute value tends to be larger because of the longer lifetime of the $B$ meson. (4) The angle between the $K$ candidate from the $D$ decay and the $B$ candidate in the rest frame of the $D$ candidate. Its distribution is flat for signal events but peaked near the extreme values for $q \bar{q}$ background. (5) The expected flavor dilution factor described in Ref. [22]. It ranges from zero for no flavor information to unity for unambiguous flavor assignment. $B$ candidates tend to have a larger flavor dilution factor than $q \bar{q}$ background. (6) The angle $\theta$ between the $B$ meson momentum direction and the beam axis in the CM frame. The $B$ decays follow a $1-\cos ^{2} \theta$ distribution, while the $q \bar{q}$ background is nearly flat in $\cos \theta$. (7) The distance of closest approach between the trajectories of the $K^{*}$ and $D$ candidates. The value is close to zero for the signal but
tends to be larger for the $c \bar{c}$ background. (8) The difference between the sum of the particle charges in the $D$ hemisphere and the sum in the opposite hemisphere, excluding those used in the reconstruction of the $B$ meson. The average charge difference is 0 for the signal events but $\pm 4 / 3$ for the $c \bar{c}$ events, depending on the flavor of the $B$ candidate. (9) The angle between the $D$ and $\Upsilon(4 S)$ directions in the rest frame of the $B$ candidate. The cosine distribution is about flat for signal events but peaks toward +1 for $c \bar{c}$ events.

To effectively combine these nine variables, we employ the NeuroBayes neural network package [23]. The NeuroBayes output is denoted as $C_{\mathrm{NB}}$ with a range of $[-1,1]$. For example, events at $C_{\mathrm{NB}} \sim 1$ are signal-like and events at $C_{\mathrm{NB}} \sim-1$ are $q \bar{q}$-like. The training for the neural network optimization is performed by using the signal and the $q \bar{q}$ MC samples, each of which contains 100,000 events after the event-selection requirements. For the latter sample, we loosen the requirement on $M_{\mathrm{bc}}$ to $5.23 \mathrm{GeV} / c^{2}<M_{\mathrm{bc}}<5.27 \mathrm{GeV} / c^{2}$ to obtain a larger number of events, since all the input parameters have little correlation with $M_{\mathrm{bc}}$. Consistencies of the distributions of the inputs and the output between data and MC samples are checked using a control mode $B^{-} \rightarrow\left[K^{-} \pi^{+}\right]_{D} \pi^{-}$and a $M_{\mathrm{bc}}$ sideband for the signal and the background, respectively.

The $C_{\mathrm{NB}}$ distribution peaks at $\left|C_{\mathrm{NB}}\right| \sim 1$ and is therefore difficult to represent with a simple analytic function. However, the transformed variable

$$
\begin{equation*}
C_{\mathrm{NB}}^{\prime}=\ln \frac{C_{\mathrm{NB}}-C_{\mathrm{NB}, \text { low }}}{C_{\mathrm{NB}, \text { high }}-C_{\mathrm{NB}}}, \tag{4}
\end{equation*}
$$

where $C_{\mathrm{NB}, \text { low }}=-0.6$ and $C_{\mathrm{NB}, \text { high }}=1.0$, has a distribution that can be modelled by a Gaussian. The events with $C_{\text {NB }}<-0.6$ are rejected. The background rejection rate is $70.5 \%$, while the signal loss is $3.9 \%$.

The number of signal events is obtained by a twodimensional unbinned extended maximum likelihood fit to $\Delta E$ and $C_{\mathrm{NB}}^{\prime}$. The fits are applied separately for favored and suppressed modes. For both modes, we categorize five common contributions. These are the $D K^{* 0}$ signal, the $\bar{D}^{0} \rho^{0}$ background, the combinatorial $B \bar{B}$ background, the $q \bar{q}$ background, and the backgrounds that have peaks in the signal region of $\Delta E$ and $C_{\mathrm{NB}}^{\prime}$ ("peaking background"). In the favored mode, we include two more components: $\bar{D}^{0} K^{+}$and $\bar{D}^{0} \pi^{+}$. The $B^{0} \rightarrow \bar{D}^{0} \rho^{0}$ decay satisfies the selection criteria when a pion from the $\rho^{0}$ decay is misidentified as a kaon. This component also includes other decays that satisfy the selection criteria when a pion in the final state is misidentified as a kaon. The peaking background for the suppressed mode consists of $B^{0} \rightarrow$ $\left[K^{-} \pi^{+} \pi^{-}\right]_{D^{-}} K^{+}$and $B^{0} \rightarrow\left[K^{+} K^{-}\right]_{D^{0}} \pi^{+} \pi^{-}$while the peaking background for the favored mode consists of $B^{0} \rightarrow$ $\left[K^{+} \pi^{-} \pi^{-}\right]_{D^{-}} K^{+}$. For the $B^{+} \rightarrow \bar{D}^{0} K^{+}$and $\bar{D}^{0} \pi^{+}$backgrounds, a pion candidate is added to reconstruct $K^{* 0}$, where the latter satisfies the selection when the $\pi^{+}$is
misidentified as $K^{+}$. We prepare two-dimensional probability density functions (PDFs) for each component as a product of one-dimensional PDFs on $\Delta E$ and $C_{\mathrm{NB}}^{\prime}$, since the correlation between $\Delta E$ and $C_{\mathrm{NB}}^{\prime}$ is found to be small.

The $\Delta E$ PDFs for a favored mode are parameterized by a double Gaussian for signal, a double Gaussian for $\bar{D}^{0} \rho^{0}$, an exponential function for $B \bar{B}$ background, a linear function for $q \bar{q}$ background, a Crystal Ball function for $\bar{D}^{0} K^{+}$, and a double bifurcated Gaussian for $\bar{D}^{0} \pi^{+}$. The means, the widths and the fractions of yields of the double-Gaussian PDFs for the signal and $\bar{D}^{0} \rho^{0}$ components are fixed from MC samples respectively. The mean of the $\Delta E$ distribution for $\bar{D}^{0} \rho^{0}$ is higher than that for the signal by about 70 MeV due to misidentification of a pion as a kaon. The parameters of the exponential and linear PDFs are allowed to float. The $\Delta E$ PDF for the peaking background is defined to be that of the signal, and the yield is fixed by the world-average value of the branching fraction [24]. The mean values of $\Delta E$ for $\bar{D}^{0} K^{+}$and $\bar{D}^{0} \pi^{+}$are higher than those for the signal due to one additional pion and a misidentification for the latter mode. The shape parameters of the $\Delta E$ PDFs for these components are determined from MC and their yields are fixed by the world-average value of the branching fraction.

The $C_{\mathrm{NB}}^{\prime} \mathrm{PDF}$ is a sum of two Gaussians for each component. The shapes for the signal and $B \bar{B}$ background are fixed from the MC samples of each decay model. The $C_{\mathrm{NB}}^{\prime} \mathrm{PDF}$ for $\bar{D}^{0} \rho^{0}$ is defined by the same function as that of $B \bar{B}$ background. The $C_{\mathrm{NB}}^{\prime}$ PDF for the peaking background is described as a weighted sum of MC-based PDFs for all the constituents. The shape for the $q \bar{q}$ background is fixed from the $M_{\mathrm{bc}}$ sideband data sample defined by $5.23 \mathrm{GeV} / c^{2}<M_{\mathrm{bc}}<5.27 \mathrm{GeV} / c^{2}$. The validity of this use of the $M_{\mathrm{bc}}$ sideband sample, which is reasonable since all inputs for $C_{\mathrm{NB}}^{\prime}$ have little correlation with $M_{\mathrm{bc}}$, is checked using MC samples. The $C_{\text {NB }}^{\prime}$ PDF for $\bar{D}^{0} K^{+}$and $\bar{D} \pi^{+}$is the same as that of the $B \bar{B}$ background.

The results of the fits for suppressed and favored modes are shown in Fig. 2 and presented in Table I. We obtain the ratio $\mathcal{R}_{D K^{* 0}}$ to be

$$
\mathcal{R}_{D K^{* 0}}=\frac{N_{\text {sup }} / \epsilon_{\text {sup }}}{N_{\text {fav }} / \epsilon_{\text {fav }}}=\left(4.5_{-5.0-1.8}^{+5.6+2.8}\right) \times 10^{-2},
$$

where $N_{\text {sup(fav) }}$ is the signal yield for the suppressed (favored) mode, $\boldsymbol{\epsilon}_{\text {sup(fav) }}$ is the detection efficiency obtained from a MC study for the suppressed (favored) mode.

We list the sources of systematic uncertainties in Table II. The uncertainties of the PDF shape parameters are estimated by varying the determined parameters of the PDFs independently by $\pm 1 \sigma$. The uncertainties due to the $C_{\mathrm{NB}}^{\prime}$ PDFs for $\bar{D}^{0} \rho^{0}$, combinatorial $B \bar{B}, \bar{D}^{0} K^{+}$, and $\bar{D}^{0} \pi^{+}$ are estimated by replacing their PDFs with the signal PDF. The uncertainty due to the PDF shape for $q \bar{q}$ is the largest systematic uncertainty. The uncertainty due to the yields of the peaking background is conservatively estimated by applying 0 and 2 times the nominal expected yields. The


FIG. 2 (color online). The projections of the fits to data for the suppressed mode (upper) and the favored mode (lower): the $\Delta E$ projection for $3<C_{\mathrm{NB}}^{\prime}<10$ (left) and the $C_{\mathrm{NB}}^{\prime}$ projection for $|\Delta E|<0.03 \mathrm{GeV}$ (right). The fitted data samples are shown by the dots with error bars and the total PDFs are shown by the solid blue curve. Individual components are shown by the dashed red ( $D K^{* 0}$ signal), the dash-dotted magenta ( $\bar{D}^{0} \rho^{0}$ ), the short dashed green (combinatorial $B \bar{B}$ background), the long dashed brown ( $q \bar{q}$ background), the very long dashed black (peaking backgrounds), the dash-dot-dotted gray ( $\bar{D}^{0} K^{+}$), and the dash-dot-dot-dotted aqua ( $\bar{D}^{0} \pi^{+}$).
systematic uncertainty associated with the peaking background is small because of its small expected yield. The uncertainty due to the yields of $\bar{D}^{0} K^{+}$and $\bar{D}^{0} \pi^{+}$is estimated by taking into account the uncertainty of the efficiencies and the branching fractions. We check the fit bias by generating 10,000 pseudo-experiments for each of the suppressed and favored modes. We obtain an almost standard Gaussian distribution for the pull, and take the product of the mean of the pull and the error of the nominal fit. The suppressed mode is biased, while the favored mode is not significantly biased. We correct the result for $\mathcal{R}_{D K^{* 0}}$ with the fit bias. The limited MC sample size and the uncertainties in the efficiencies of particle identification dominate the systematic uncertainty in detection efficiency. The uncertainties in the efficiencies of particle identifications are determined from the decay $D^{*+} \rightarrow D^{0} \pi^{+}$followed by $D^{0} \rightarrow K^{-} \pi^{+}$. The uncertainty due to the charmless

TABLE I. Summary of the results. The errors for $N$ and $\mathcal{R}_{D K^{* 0}}$ are statistical only.

| Mode | $\epsilon(\%)$ | $N$ | $\mathcal{R}_{D K^{* 0}}$ |
| :--- | :---: | :---: | :---: |
| $B^{0} \rightarrow\left[K^{+} \pi^{-}\right]_{D} K^{* 0}$ | $21.0 \pm 0.3$ | $190_{-21.2}^{+22.3}$ | $\left(4.5_{-5.0}^{+5.6}\right) \times 10^{-2}$ |
| $B^{0} \rightarrow\left[K^{-} \pi^{+}\right]_{D} K^{* 0}$ | $20.9 \pm 0.3$ | $7.7_{-9.5}^{+10.6}$ |  |

TABLE II. Summary of the systematic uncertainties for $\mathcal{R}_{\text {DK }}{ }^{* 0}$.

| Source | Uncertainty $\left[10^{-2}\right]$ |
| :--- | :---: |
| Signal PDFs | $+0.1-0.2$ |
| $\bar{D}^{0} \rho^{0}$ PDFs | $+0.0-0.1$ |
| Combinatorial $B \bar{B}$ PDFs | $+1.8-1.2$ |
| Peaking background PDFs | $+0.1-0.1$ |
| $q \bar{q}$ PDFs | $+2.2-1.4$ |
| $\bar{D}^{0} K^{+}$and $\bar{D}^{0} \pi^{+}$PDFs | $+0.0-0.1$ |
| Fit bias | $+0.1-0.1$ |
| Efficiency | $+0.1-0.1$ |
| Charmless decay | $+0.0-0.3$ |
| Total | $+2.8-1.8$ |

$B^{0} \rightarrow K^{* 0} K^{+} \pi^{-}$decay is obtained from the upper limit of its branching ratio [24] and the efficiency estimated by assuming a nonresonant distribution in phase space. The uncertainties due to the favored mode are estimated in a similar manner as for the suppressed mode and are found to be small. The total systematic error is taken as a quadratic sum of the above uncertainties.

The distribution of the likelihood $\mathcal{L}$ is obtained by convolving the likelihood in the $\left(\Delta E, C_{\mathrm{NB}}^{\prime}\right)$ twodimensional fit and an asymmetric Gaussian whose widths are the negative and positive systematic errors. The likelihood in the fit is obtained as a one-dimensional function of the signal yield of the suppressed mode divided by the central value for the favored mode. The statistical and systematic errors for the favored mode are included in the asymmetric Gaussian. We set a $95 \%$ credible upper limit for $\mathcal{R}_{D K^{* 0}}$ to be $\mathcal{R}_{D K^{* 0}}<0.16$ so that the integral of $\mathcal{L}$ up to this limit becomes $95 \%$ of the total integral, where the integrals are done in the physical region of positive $\mathcal{R}_{D K^{* *}}$. The uncertainties due to the signal yield of the favored mode are found to be negligible.

In summary, we report a result of the measurement of the ratio $\mathcal{R}_{D K^{* 0}}$, using a $711 \mathrm{fb}^{-1}$ data sample collected by the Belle detector. We obtain $\mathcal{R}_{D K^{* 0}}=\left(4.5_{-5.0-1.8}^{+5.6+2.8}\right) \times 10^{-2}$, which can be used to extract $\phi_{3}$ by combining with other observables related to the same dynamical parameters $r_{S}$, $\delta_{S}$ and $k$. Since the value of $\mathcal{R}_{D K^{* 0}}$ is not significant, we set a credible upper limit of $\mathcal{R}_{D K^{* 0}}<0.16$ ( $95 \%$ ); this is the most stringent limit to date.

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