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Holographic Mesons in D4/D6 Model Revisited

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We revisit holographic mesons in the D4/D6 model to study holographic light vector mesons and the properties of heavy quarkonium in a confining phase. To treat light mesons and the heavy quarkonium on the same footing, we use the same compactification scale, M_{KK}, in both systems. We observe that like scalar and pseudo-scalar mesons, the vector meson mass is linearly proportional to the square root of the quark mass when the quark mass is large. With M_{KK} fixed by the light meson masses, we calculate the mass of the heavy quarkonium in a confining phase. In order to describe the heavy quarkonium in the D4/D6 model, we consider the classical open string configuration.

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I. INTRODUCTION

The properties of the heavy quarkonium both at zero and at finite temperature have been intensively studied (see Ref. 1 for a review). At zero temperature, the charmonium spectrum reflects detailed information about confinement and quark-antiquark potentials in QuantumChromoDynamics(QCD) [2]. At finite temperature, melting of heavy quarkonia could be a signal for the formation of a Quark-Gluon Plasma(QGP) in a relativistic heavy ion collision [3]. Moreover, lattice calculations suggest that the charmonium states will survive at finite temperature up to about 1.6 to 2 times the critical temperature T_c [4, 5]. This suggests that analyzing the charmonium data from heavy ion collisions inevitably requires detailed information about the properties of charmonium states in a QGP. Therefore, a very important theoretical challenge is to develop a consistent non-perturbative QCD picture for the heavy quark system both below and above the QCD phase transition temperature. In this respect, a promising attempt would be holographic QCD via Anti De Sitter/Conformal Field Theory (AdS/CFT) [6].

In a stringy D4/D6 model, scalar and pseudo-scalar bound states have been studied at zero and finite temperature in Refs. 7 and 8. In a more phenomenological approach, the bottom-up model, the mass spectrum of a charmonium and its dissociation temperature have been investigated [9–11]. However, we note that in the bottom-up model, different infrared scales are introduced to describe light mesons and heavy quarkonia. For example, in the hard wall model, the location of the infrared cutoff z_m varies from light mesons to heavy quarkonia: $1/z_m \simeq 320$ MeV [12, 13] for the light meson and $1/z_m \simeq 1315$ MeV [9] for the charmonium.

In this paper we study the spectrum of the light vector meson and heavy quarkonium by using the D4/D6 model [7] in a confining phase. To treat the light mesons and heavy quarkonium on the same footing, we use the same compactification scale M_{KK} in both systems. For the light vector meson, the spectrum was discussed in the D4/D8/$\bar{D}$8 model [14] where the chiral symmetry and its breaking are realized geometrically. However, in the Sakai-Sugimoto model [14], it is quite difficult to include the quark mass. Therefore, it is still of worth to study the spectrum of vector mesons in the D4/D6 model and to study the effect of the quark mass. We observe that light scalar and pseudo-scalar mesons [7], the vector meson

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Table 1. The brane configuration, the background D4 and the probe D6.

	Boundary				S^1	r(S^2)	S^2		$D6_\perp$	
	0	1	2	3	τ	z	ψ_1	ψ_2	r	ϕ
D4	•	•	•	•	•					
D6	•	•	•	•		•	•	•		

mass is proportional to the square root of the quark mass $M_v^2 \sim m_q$ for large quark masses. Then, we consider a spinning string in a confined phase [20] and calculate the energy of a long string attached on D6 branes in a D4 background to calculate the mass spectrum of the heavy quarkonium.

II. D4/D6 MODEL

We briefly summarize a pioneering holographic QCD model, the D4/D6 system [7]. The model contains N_c D4 branes and N_f flavor D6 branes, its configuration is given in Table 1.

In the probe limit, the N_c D4 branes are replaced by their supergravity background, and the N_f D6 branes are treated as probes. In this model, mesons of a QCD-like gauge theory are described by the fluctuations of the D6 brane in the D4 background. The geometry of confining D4 brane reads

$$\begin{aligned} ds^2 &= \left(\frac{U}{L}\right)^{3/2}(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2) \\ &\quad + \left(\frac{L}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \\ e^\phi &= g_s\left(\frac{U}{L}\right)^{3/4}, \\ F_4 &= dC_3 = \frac{2\pi N_c}{V_4}\epsilon_4, \\ f(U) &= 1 - \frac{U_{KK}^3}{U^3}, \end{aligned} \tag{1}$$

where the string coupling constant g_s and the period of τ are given by

$$g_s = \frac{g_{YM}^2}{2\pi M_{KK} l_s}, \quad \delta\tau \equiv \frac{4\pi}{3}\frac{L^{3/2}}{U_{KK}^{1/2}}. \tag{2}$$

The parameter L is given by the string coupling constant g_s and the string length l_s, $L^3 = \pi g_s N_c l_s^3$, the compactification scale M_{KK} reads

$$M_{KK} = \frac{2\pi}{\delta\tau} = \frac{3}{2}\frac{U_{KK}^{1/2}}{L^{3/2}}, \tag{3}$$

and the gauge theory parameters are converted by using the string parameters

$$\begin{aligned} L^3 &= \frac{1}{2}\frac{g_{YM}^2 N_c l_s^2}{M_{KK}}, \\ g_s &= \frac{1}{2\pi}\frac{g_{YM}^2}{M_{KK} l_s}, \\ U_{KK} &= \frac{2}{9} g_{YM}^2 N_c M_{KK} l_s^2. \end{aligned} \tag{4}$$

By introducing K(ρ), the metric is simplified to

$$\begin{aligned} ds^2 &= \left(\frac{U}{L}\right)^{3/2}\eta_{\mu\nu}dx^\mu dx^\nu \\ &\quad + K(\rho)[dz^2 + z^2 d\Omega_2^2 + dr^2 + r^2 d\phi^2], \end{aligned} \tag{5}$$

where

$$\begin{aligned} K(\rho) &\equiv L^{3/2}\frac{U^{1/2}}{\rho^2}, \quad U(\rho) = \rho\left(1 + \frac{U_{KK}^3}{4\rho^3}\right)^{2/3}, \\ \text{with } \rho^2 &= z^2 + r^2. \end{aligned} \tag{6}$$

The position of the D6 brane is described by r(z) with ϕ = 0 and τ = constant. Then, the induced metric on D6 is

$$ds_{D6}^2 = \left(\frac{U}{L}\right)^{3/2}\eta_{\mu\nu}dx^\mu dx^\nu + K(\rho)\Big[(1+\dot{r}^2)dz^2 + z^2 d\Omega_2^2\Big]. \tag{7}$$

Now the action for D6 brane becomes

$$\begin{aligned} S_{D6} &= -T_6\int d^7\sigma e^{-\phi}\sqrt{-\det(g + 2\pi\alpha' F)}, \\ \text{where} \quad T_{D6} &= \frac{1}{(2\pi)^6 l_s^7}. \end{aligned} \tag{8}$$

By using the well-known identity

$$\det\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det A \cdot \det(D - CA^{-1}B) \tag{9}$$

up to quadratic order in fields, we obtain

$$\begin{aligned} \mathcal{L}_0 &= -\frac{T_6}{g_s}\sqrt{h}\left(1 + \frac{U_{KK}^3}{4\rho^3}\right)^2 z^2\sqrt{1+\dot{r}^2} \\ &\quad \times \left[1 + \frac{1}{4}(F_{\mu\nu}F^{\mu\nu} + 2F_{z\mu}F^{z\mu})\right], \end{aligned} \tag{10}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

III. MESON SPECTROSCOPY

In this section, we compute the vector meson masses by considering gauge field fluctuations on the D6 brane. Although we have two different scales of meson masses, light mesons and heavy quarkonia, the origin of the difference lies in the quark mass rather than the interaction.

There, we need to introduce only a single interaction scale M_{KK}. This scale is to be matched with the gauge theory scale ; Λ_{QCD}, hence, it is encoded in the background geometry while the quark masses are encoded in the geometry of the probe branes.

1. Embedding

We first find a D6 embedding geometry by solving the equation of motion for r(z). From the DBI action in Eq. (10), we obtain the equation of motion for r(z) to be

$$\partial_z \left[\left(1+\frac{U_{KK}^3}{4\rho^3}\right)^2 z^2 \frac{\dot r}{\sqrt{1+\dot r^2}} \right] = -\frac{3}{2}\frac{U_{KK}^3}{\rho^5}\left(1+\frac{U_{KK}^3}{4\rho^3}\right) z^2 r\sqrt{1+\dot r^2}. \quad (11)$$

With the following dimensionless variables

$$z \to U_{KK} z, \quad r \to U_{KK} r, \quad \rho \to U_{KK}\rho\,, \quad (12)$$

we rewrite Eq. (11) as

$$\partial_z \left[\left(1+\frac{1}{4\rho^3}\right)^2 z^2 \frac{\dot r}{\sqrt{1+\dot r^2}} \right] = -\frac{3}{2}\frac{1}{\rho^5}\left(1+\frac{1}{4\rho^3}\right) z^2 r\sqrt{1+\dot r^2}. \quad (13)$$

For large z, we can solve the equation of motion for $r(z)$ to obtain the asymptotic solution as

$$r(z) \sim r_\infty + \frac{c}{z} \quad (14)$$

where r_∞ and c are related to the quark mass and the chiral condensate respectively (see Ref. 7 for details).

2. Scalar and Pseudo-scalar Fluctuations

We start with scalar and pseudo-scalar fluctuations. Though these were extensively studied in Ref. [7], we include them for completeness, not to improve the results in Ref. 7. The transverse fluctuation of the D6 brane is given by

$$r(x^\mu, z) = r_v(z) + \delta r(x^\mu),\ \ \phi(x^\mu, z) = \delta\phi(x^\mu, z)\,, \quad (15)$$

where $r_v(z)$ is the solution of the embedding equation. Inserting Eq. (15) into the induced metric in Eq. (7) and the DBI action in Eq. (10), we obtain the induced metric

$$\begin{aligned} ds^2 \;=\; & \left(\frac{U}{L}\right)^{3/2} \eta_{\mu\nu} dx^\mu dx^\nu \\ & + K[(1+\dot r_v^2)dz^2 + z^2 d\Omega_2^2] + 2K\dot r_v \partial_a \delta r dz dx^a \\ & + K[\partial_a\delta r\partial_b\delta r + r_v^2\partial_a\delta\phi\partial_b\delta\phi]dx^a dx^b\,, \end{aligned} \quad (16)$$

where a and b run from 0 to z and the DBI action, up to quadratic order, is

$$\begin{aligned} \mathcal{L} \;=\; & \mathcal{L}_0 - \frac{1}{2}T_{D6}U_{KK}^3 z^2\sqrt{h}\sqrt{1+\dot r_v^2}\Bigg[\frac{U^3}{\rho_v^3(1+\dot r_v^2)}\left(\frac{(\partial_z\delta r)^2}{1+\dot r_v^2} + r_v^2(\partial_z\delta\phi)^2\right) + \frac{L^3U^2}{U_{KK}\rho_v^5}\left(\frac{\partial_\mu\delta r\partial^\mu\delta r}{1+\dot r_v^2} + r_v^2\partial_\mu\phi\partial^\mu\phi\right) \\ & -\frac{3}{2\rho_v^7}\left(\left(1+\frac{1}{4\rho_v^3}\right)(z^2-4r_v^2) - \frac{3r_v^2}{4\rho_v^3}\right)(\delta r)^2 - \frac{3r_v\dot r_v}{2\rho_v^5}\left(1+\frac{1}{4\rho_v^3}\right)\frac{\partial_z(\delta r^2)}{1+\dot r_v^2}\Bigg]. \end{aligned} \quad (17)$$

Now we arrive at the linearized equation of motion for pseudo-scalar;

$$\begin{aligned} 0 \;=\; & \partial_z\left(\frac{z^2 r_v^2}{\sqrt{1+\dot r_v^2}}\left(1+\frac{1}{4\rho^3}\right)^2\partial_z\phi\right) \\ & + \frac{r_v^2 z^2\sqrt{1+\dot r_v^2}}{\rho_v^5}U^2\frac{9M_\phi^2}{4M_{KK}^2}\phi(z), \end{aligned} \quad (18)$$

and for the scalar,

$$\begin{aligned} 0 \;=\; & \partial_z\left[\frac{z^2}{(1+\dot r_v^2)^{3/2}}\left(1+\frac{1}{4\rho^3}\right)^2\partial_z\delta r\right] \\ & + \frac{z^2U^2}{\rho_v^5\sqrt{1+\dot r_v^2}}\frac{9M_{\delta r}^2}{4M_{KK}^2}\delta r \\ & + \frac{3z^2\sqrt{1+\dot r_v^2}}{2\rho_v^7}\left(\left(1+\frac{1}{4\rho_v^3}\right)(z^2-4r_v^2) - \frac{3r_v^2}{4\rho_v^3}\right)\delta r \\ & - \partial_z\left(\frac{3z^2}{2\rho_v^5}\frac{r_v\dot r_v}{\sqrt{1+\dot r_v^2}}\left(1+\frac{1}{4\rho_v^3}\right)\right)\delta r. \end{aligned} \quad (19)$$

These equations are numerically solved to get meson masses with proper boundary conditions [7].

3. Gauge Field Fluctuations

Now, we move on to the gauge field fluctuation. The relevant part of the Lagrangian density for the gauge field is given by

$$\begin{aligned}\mathcal{L} &\sim -\frac{1}{4}\left(1+\frac{U_{KK}^3}{4\rho^3}\right)^2 z^2\sqrt{1+\dot{r}_v^2}\left(\frac{L}{U}\right)^{3/2} \\ &\times \left[\left(\frac{L}{U}\right)^{3/2}\eta^{\mu\nu}\eta^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma}+2\frac{\eta^{\mu\nu}F_{z\mu}F_{\nu z}}{K(\rho)(1+\dot{r}_v^2)}\right], \quad (20)\end{aligned}$$

where r_v is the embedding solution. We decompose the gauge fields in terms of the orthonormal basis ψ_n, ϕ_n as

$$\begin{aligned}A_\mu(x^\mu, z) &= \sum_n B_\mu^{(n)}(x^\mu)\psi_n(z), \\ A_z(x^\mu, z) &= \sum_n \varphi^{(n)}(x^\mu)\phi_n(z). \quad (21)\end{aligned}$$

Then, the field strength takes the following form:

$$\begin{aligned}F_{\mu\nu}(x^\mu, z) &= \sum_n F_{\mu\nu}^{(n)}(x^\mu)\psi_n(z), \\ F_{\mu\nu}^{(n)}(x^\mu) &= \partial_\mu B_\nu^{(n)}(x^\mu) - \partial_\nu B_\mu^{(n)}(x^\mu), \\ F_{\mu z}(x^\mu, z) &= \sum_n \Big(\partial_\mu\varphi^{(n)}(x)\phi_n(z) \\ &\qquad - \partial_z\psi_n(z)B_\mu^{(n)}\Big). \quad (22)\end{aligned}$$

With the decomposition, the quadratic part of the Lagrangian for the B_μ field reads

$$\begin{aligned}\mathcal{L}_B &\sim -\frac{\sqrt{g_0}}{4}\sum_{m,n}\left[\left(\frac{R}{U}\right)^{3/2}F_{\mu\nu}^{(n)}F^{(n)\mu\nu}\psi_m\psi_n \right.\\ &\qquad \left. + \frac{2}{K(\rho)(1+\dot{r}_v^2)}\dot{\psi}_m\dot{\psi}_n B_\mu B^\mu\right], \quad (23)\end{aligned}$$

where $\sqrt{g_0} = \left(1+\frac{U_{KK}^3}{4\rho^3}\right)^2 z^2\sqrt{1+\dot{r}_v^2}\left(\frac{L}{U}\right)^{3/2}$. To recover the canonical kinetic term of the gauge field in 4D, we impose the normalization condition for the wave function $\psi(z)$ as

$$(2\pi\alpha')^2\tilde{T}_6\int dz\sqrt{g_0}\left(\frac{R}{U}\right)^{3/2}\psi_m\psi_n = \delta_{mn}, \quad (24)$$

where $\tilde{T}_6 = -T_6V_2/g_s$ and $V_2 = \int d\Omega_2\sqrt{g_{66}g_{77}}$. We will impose a similar condition for $\phi(z)$.

The wave function $\psi(z)$ satisfies the following mode equation derived from the quadratic action

$$\partial_z\left(\sqrt{g_0}\frac{\partial_z\psi_n}{K(\rho)(1+\dot{r}_v^2)}\right) = -\sqrt{g_0}\left(\frac{L}{U}\right)^{3/2}m_n^2\psi_n, \quad (25)$$

where $m_n^2 = -q^2$. Then, we obtain

$$(2\pi\alpha')^2\tilde{T}_6\int dz\sqrt{g_0}\frac{1}{K(\rho)(1+\dot{r}_v^2)}\dot{\psi}_m\dot{\psi}_n = m_n^2\delta_{mn}, \quad (26)$$

where m_n is the eigenvalue. From Eqs. (24) and (26), we now have

$$S_{D6} = -N\int d^4x\sum_{n=1}^{\infty}\left(\frac{1}{4}F_{\mu\nu}^{(m)}F^{(n)\mu\nu}+\frac{1}{2}m_n^2B_\mu^{(n)}B^{(n)\mu}\right). \quad (27)$$

We rescale the coordinate by U_{KK} to obtain

$$\partial_z\left(\sqrt{g_0}\frac{\partial_z\psi_n}{K(\rho)(1+\dot{r}_v^2)}\right) = -\frac{\sqrt{g_0}}{U^{3/2}}\frac{9}{4}\frac{m_n^2}{M_{KK}^2}\psi_n. \quad (28)$$

To solve this equation, we impose two boundary conditions: at the IR, either $\psi_n(0)$ or $\dot{\psi}_n(0) = 0$, and at the UV, $\psi_n \sim z^\alpha$ with $\alpha \le 1/2$ from the normalizability condition in Eq. (24).

Now we consider ϕ_n. The normalization of ϕ_n is similar to Eq. (26) :

$$(2\pi\alpha')^2\tilde{T}_6\int dz\sqrt{g_0}\frac{1}{K(\rho)(1+\dot{r}_v^2)}\phi_m\phi_n = \delta_{mn}. \quad (29)$$

As explained below and also in Ref. 14, once we set the field as $\phi_n = \dot{\psi}_n/m_n$ for n ≥ 1, it can be gauged away as a part of the B_μ field. However, the zero mode, which is orthogonal to the other modes, is exceptional :

$$\begin{aligned}(\phi_0, \phi_n) &= \frac{(2\pi\alpha')^2}{m_n}\tilde{T}_6\int dz\sqrt{g_0}\frac{1}{K(\rho)(1+\dot{r}_v^2)}\phi_0\dot{\psi}_n \\ &= 0 \quad (\text{for n} \ge 1). \quad (30)\end{aligned}$$

If we take $\phi_0 = CK(\rho)(1+\dot{r}_v^2)/\sqrt{g_0}$,

$$\begin{aligned}(\phi_0, \phi_n) &= \int_0^\infty dz\ \dot{\psi}_n = \psi_n(\infty) - \psi_n(0) \\ &= 0 \quad (\text{for n} \ge 1). \quad (31)\end{aligned}$$

Then, the constant C is given by

$$\begin{aligned}1 &= (\phi_0, \phi_0) \ \rightarrow\ C \\ &= \left((2\pi\alpha')^2\tilde{T}_6\int dz\frac{K(\rho)(1+\dot{r}_v^2)}{\sqrt{g_0}}\right)^{-1/2}. \quad (32)\end{aligned}$$

The field strength is written as

$$\begin{aligned}F_{\mu z}(x^\mu, z) &= \partial_\mu\varphi^0(x)\phi_0(z) \\ &\quad + \sum_n\left(m_n^{-1}\partial_\mu\varphi^{(n)}(x) - B_\mu^{(n)}\right)\dot{\psi}_n(z). \quad (33)\end{aligned}$$

By a gauge transformation, B_μ absorbs $\partial_\mu\varphi^{(n)}$:

$$B_\mu^{(n)} \rightarrow B_\mu^{(n)} + m_n^{-1}\partial_\mu\varphi^{(n)}(x), \quad (34)$$

therefore, the action in Eq. (23) becomes

$$S_{D6} = \int d^4x \left[\frac{1}{2}\partial_\mu\varphi^0\partial^\mu\varphi^0 + \sum_{n=1}^{\infty}\left(\frac{1}{4}F_{\mu\nu}^{(m)}F^{(n)\mu\nu} + \frac{1}{2}m_n^2 B_\mu^{(n)}B^{(n)\mu}\right)\right]. \quad (35)$$

Note that for the heavy quarkonium system,

$$\begin{aligned} C &= \left((2\pi\alpha')^2\tilde{T}_6\int_0^\infty dz\frac{\sqrt{1+\dot{r}_v^2}}{z^2}\left(1+\frac{1}{4\rho^3}\right)^{-2/3}\right)^{-1/2} \\ &= \left((2\pi\alpha')^2\tilde{T}_6\int_0^\infty dz\frac{1}{z^2}\right)^{-1/2} \\ &= \left((2\pi\alpha')^2\tilde{T}_6\frac{1}{z}\Big|_\infty^0\right)^{-1/2} = 0\,, \end{aligned} \quad (36)$$

so $\varphi_0 = 0$ due to Eq. (32). To impose the UV boundary condition for ψ_n more precisely, we consider the mode equation, Eq. (28), at large z ;

$$\partial_z(z^2\partial_z\psi_n) = -\frac{m_n^2}{z}\psi_n\,. \quad (37)$$

With $\psi_n \sim z^\alpha$, we obtain $\alpha(\alpha-3) = 0$. Since the normalizability condition dictates $\alpha \le 1/2$, we should choose $\alpha = 0$.

4. Numerical Results

We solve the mode equations for scalar, pseudo-scalar, and gauge field fluctuations numerically. We first compute the mass of light mesons to fix the model parameters r_∞^l and M_{KK}. Because the D4/D6 model has no non-Abelian chiral symmetry except for U$(1)_A$, the pseudo-scalar meson in this model corresponds to η' in QCD [7]. In QCD, however, U$(1)_A$ symmetry is explicitly broken by the axial anomaly, and the observed mass of η', $m_{\eta'} = 958$ MeV, is much larger than the pion or kaon mass. Note that some portion of the η' mass comes from the anomaly effect, which scales as N_f/N_c. Because we are working in the large N_c limit, we may use the mass of η' with the anomaly contribution turned-off, so we use a non-anomalous η' mass to obtain a rough number for the model input. To this end, we use the mass relation for the Goldstone boson obtained in chiral perturbation theory at large N_c [16]:

$$m_\pi^2 = \frac{2m_q\Sigma}{f_\pi^2}, \quad m_{\eta'}^2 = \frac{2\Sigma(2m_q+m_s)}{3f_\pi^2} + \frac{6\tau}{f_\pi^2}\,, \quad (38)$$

where $m_u = m_d \equiv m_q$. The term with τ is from the axial anomaly. Now we take $\tau = 0$ to estimate the η' mass from the non-anomalous contribution. With $m_q = 7$ MeV, $m_s = 150$ MeV, $f_\pi = 93$ MeV, and $\Sigma = (230\text{ MeV})^3$, we obtain $m_\pi \sim 140$ MeV and $m'_\eta \sim 390$ MeV. Note that the mass of the $q\bar{q}$ bound state, such as the ρ meson mass, stays almost constant as we increase N_c: for instance, the light meson mass at large N_c is extensively studied in a unitarized chiral perturbation theory [17] and in lattice QCD [18].

Table 2. Light meson masses to fix the free parameters, M_{KK} and r_∞, in the model.

Mode	Input (MeV)	M/M$_{KK}$	M (MeV)
Ps (η')	390	0.375	390
Rs (σ)		0.918	954
V (ρ)	770	0.741	770

We use the ρ-meson mass and η' mass in the large N_c limit as inputs to fix $r_\infty = 0.191$ and $M_{KK} = 1.039$ GeV. Our fitting results are summarized in Table 2. As observed in Ref. 7, for large r_∞, the meson mass becomes degenerate and increases monotonically with increasing r_∞. This is simply because the equations of motion for scalar, pseudo-scalar and vector for a heavy quark system, $r_\infty \gg U_{KK}$, are degenerated. To see these more explicitly, we expand the equations of motion of fluctuations as r_∞ goes to infinity. Then, we rescale the coordinate $z = y\, r_\infty$, which means that both r_∞ and z are very large, but their ratio is finite :

$$\frac{1}{y^2}\partial_y(y^2\partial_y\Psi) + \frac{1}{r_\infty}\left(\frac{3}{2M_{KK}}\right)^2\frac{M^2}{(1+y^2)^{3/2}}\Psi = 0, \quad (39)$$

where Ψ can be a real scalar δr, a pseudo-scalar ϕ, or a vector ψ, and y is a rescaled coordinate $y = z/r_\infty$. In Eq. (39), the meson mass scales as

$$\begin{aligned} &M^2 \sim r_\infty M_{KK}^2 \sim \frac{m_q M_{KK}}{\lambda}, \\ \text{where}\quad &m_q = \frac{U_{KK}r_\infty}{2\pi l_s^2} \\ &\quad= \frac{r_\infty}{9\pi}g_{YM}^2 N_c M_{KK} = \frac{r_\infty}{9\pi}M_{KK}\lambda\,, \end{aligned} \quad (40)$$

with $\lambda = g_{YM}^2 N_c$ being the t' Hooft coupling. This means that for the heavy quark system, the fluctuating field has mass proportional to $\sqrt{\frac{m_q M_{KK}}{\lambda}}$. This is a bit at odds with heavy quarkonia in QCD because we would expect the mass of heavy quarkonium in QCD to be proportional to the heavy quark mass not its square root. Because of this aspect, we will study the heavy quarkonium system based on the spinning string picture in the next section. As discussed in Ref. 7, the reason for the degeneracy is supersymmetry restoration. All of the fluctuating fields are in the same supermultiplet and for small m_q limit, supersymmetry is broken so their masses split. However, for the large quark mass or large separation between D4 and D6, the embedding is nearly flat, so the D6 brane restores supersymmetry.

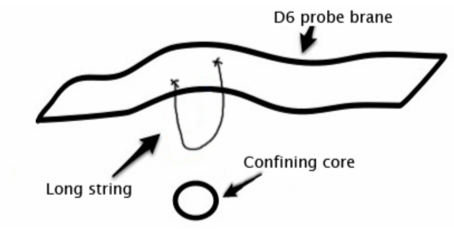

Fig. 1. A classical string configuration ending on a D6 brane.

IV. ROTATING STRING AS A MESON

We believe that almost all of the heavy quarkonium mass comes from the string connecting two quarks. Here, we calculate the energy of a long string attached on D6 branes in the D4 background to see another aspect of the holographic meson mass [20] and to calculate the mass spectrum of the heavy quarkonium.

A confining flux tube or a confining string in QCD can be realized holographically as a long string in a certain background. The image of a long string, which is identified with the confining flux tube of gauge theory, in the bulk is projected on a holographic screen, R^4, where our gauge theory is living.

By solving the classical equation of motion for the string derived from the Nambu-Goto action, we obtain a classical string configuration, (Fig. 1), and using the solution, we compute the energy and the angular momentum of an open string. In this section, we first review the work of Ref. 20 on spinning strings in the confining D4/D6 model. Then we use their result to study the mass of the heavy quarkonium.

To describe a meson as a bound state of a quark and an anti-quark placed at the endpoints of a rotating open string, we begin with the Nambu Goto action with a proper ansatz:

$$\begin{aligned} S_{NG} &= \frac{1}{2\pi l_s^2}\int d\sigma d\tau \sqrt{X'^2\dot{X}^2-(X'\cdot\dot{X})^2}, \\ \text{where } X^{\mu\prime} &= \partial_\sigma X^\mu, \quad \dot{X}^\mu = \partial_\tau X^\mu \\ X^0 &= e\tau, \quad \theta = ew\tau, \quad R = R(\sigma), \\ r &= r(\sigma), \quad z = z(\sigma)\,, \end{aligned} \tag{41}$$

with X^μ being the string coordinate in four dimensional target space-time and $X'\cdot\dot{X} = X'_\mu\dot{X}^\mu$. Here, we choose the string coordinate R as one of the D4 brane's world-volume coordinates :

$$dX^\mu dX_\mu = -(dX^0)^2 + dR^2 + R^2 d\theta^2 + (dX^3)^2. \tag{42}$$

The endpoint of this open string is attached on D6, so the boundary conditions are given by

$$\begin{aligned} 0 &= \Big(\partial_\sigma X^0 = \partial_\sigma\theta = \partial_\sigma R = \partial_\sigma z\Big)|_{\sigma=0,\pi}, \\ r(\sigma) &= r(z(\sigma))|_{\sigma=-\pi/2,\pi/2}\,. \end{aligned} \tag{43}$$

Then, the Nambu-Goto action for the rotating string becomes

$$S = -T_s\int d\sigma d\tau\sqrt{\left(\frac{U}{L}\right)^3\Big((\partial_\tau X^0)^2 - R^2(\partial_\tau\theta)^2\Big)\left((\partial_\sigma R)^2 + K\left(\frac{U}{L}\right)^{-3/2}(\partial_\sigma\rho)^2\right)}\,, \tag{44}$$

where $\rho^2 = r^2 + z^2$. When $z = 0$, the range of ρ is from $\rho_0 = U_{KK}/4^{1/3}$, at which the confining core is located, to $\rho_f = r_\infty$. We will take $z = 0$ to see the long string configuration clearly, which is also energetically favored. After re-scaling of variables,

$$\begin{aligned} \rho &\to U_{KK}\rho, \quad U \to U_{KK}U, \\ R &\to \sqrt{\frac{L^3}{U_{KK}}}R = \frac{3}{2}\frac{R}{M_{KK}}, \\ w &\to \frac{2}{3}M_{KK}w, \quad \sigma \to U_{KK}\sigma\,, \end{aligned} \tag{45}$$

we obtain

$$S = -T_s U_{KK}\int d\sigma d\tau\sqrt{U^3 e^2\Big(1-w^2R^2\Big)\left((\partial_\sigma R)^2 + \frac{(\partial_\sigma\rho)^2}{U\rho^2}\right)}. \tag{46}$$

The equation of motion derived from this action is

$$\frac{d}{dR}\left[\frac{U\mathcal{E}}{\sqrt{U\rho^2+\dot{\rho}^2}}\frac{\dot{\rho}}{\rho}\right] = \frac{\mathcal{E}}{\rho^2\sqrt{U\rho^2+\dot{\rho}^2}}\left[\frac{\partial U}{\partial\rho}\left(\frac{3}{2}U\rho^2+\dot{\rho}^2\right)\rho - U\dot{\rho}^2\right]\,, \tag{47}$$

where "dot" denotes R-derivative. Note that we take the world sheet coordinate σ as R and $\mathcal{E} = e\sqrt{1-w^2R^2}$.

Since there are no explicit X^0 and θ dependences, we have two constants of motion :

$$E = T_s U_{KK} \int dR \frac{U\sqrt{U+\dot{\rho}^2/\rho^2}}{\sqrt{1-w^2R^2}},$$
$$J = T_s U_{KK}\sqrt{\frac{L^3}{U_{KK}}} \int dR \frac{wR^2 U\sqrt{U+\dot{\rho}^2/\rho^2}}{\sqrt{1-w^2R^2}}. \quad (48)$$

We identify E (J) as the energy (angular momentum) of the spinning string. Notice that the solution of Eq. (47) has two separable regions that we will consider separately below: $\dot{\rho} \to \infty$ or 0.

Vertical string

In the first region ($\dot{\rho} \to \infty$), a vertically extended string, the equation is simplified to

$$\frac{d}{dR}\left(\frac{U\mathcal{E}}{\rho}\right) = \left(\frac{dU}{d\rho} - \frac{U}{\rho}\right)\mathcal{E}\frac{\dot{\rho}}{\rho}. \quad (49)$$

This means $\dot{R} \to 0$, so $R = R_0$. The corresponding energy and angular momentum are given as

$$E_I = \frac{2m_q}{\sqrt{1-w^2R_0^2}},$$
$$J_I = \frac{2m_q w R_0^2}{\sqrt{1-w^2R_0^2}}\frac{3}{2M_{KK}},$$
$$\left(\frac{U_{KK}}{L}\right)^{3/2} = \frac{2}{3}M_{KK}U_{KK}. \quad (50)$$

The mass of the dynamical quark is

$$m_q = T_s U_{KK}\int_{\rho_f}^{\rho_Z} d\rho\sqrt{g_{00}g_{\rho\rho}} = T_s U_{KK}\int_{\rho_f}^{\rho_z} d\rho\frac{U}{\rho}$$
$$= T_s U_{KK}\left[x\left(1+\frac{1}{4x^3}\right)^{2/3}\left(\frac{4x^3\ {}_2F_1\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-4x^3\right]}{(1+4x^3)^{2/3}} - 1\right)\right]_{x=\rho_0}^{x=\rho_f} \quad (51)$$

Horizontal string

In the second region $\dot{\rho} \to 0$, ($\rho = \rho_0$, U_0 =1) , horizontally extended, Eq. (47) reads

$$\frac{d}{dR}\left(\frac{\sqrt{U}\mathcal{E}\dot{\rho}}{\rho^2}\right) = \frac{3}{2}\frac{\partial U}{\partial\rho}\frac{\sqrt{U}}{\rho}\mathcal{E}. \quad (52)$$

From these, we obatin

$$E_{II} = T_g U_{KK}\frac{2}{w}\arcsin(wR_0),$$
$$J_{II} = 2T_g\frac{3U_{KK}}{2M_{KK}}\frac{1}{2w^2}\left(\arcsin(wR_0) - wR_0\sqrt{1-w^2R_0^2}\right). \quad (53)$$

These two regions are combined by the sewing condition. After re-scaling,

$$1 - w^2R_0^2 = \frac{w^2R_0^2 m_q}{T_g R_0}\frac{2}{3}M_{KK} \quad (54)$$

Let us define T_g, an effective string tension, as

$$T_g = T_s\left(\frac{U_{KK}}{L}\right)^{3/2} = \frac{1}{2\pi l_s^2}\left(\frac{U_{KK}}{L}\right)^{3/2} = \frac{2\lambda}{27\pi}M_{KK}^2. \quad (55)$$

An effective string slope is given by $\tilde{\alpha}' = \frac{1}{2\pi T_g}$. Here we identify the quark mass from the D6 fluctuation with the one from open string dynamics. Now, the total energy and angular momentum are given by

$$E = E_I + E_{II} = \frac{2m_q}{\sqrt{1-w^2R_0^2}} + T_g U_{KK}\frac{2}{w}\arcsin(wR_0)$$
$$J = J_I + J_{II} = \frac{2m_q w R_0^2}{\sqrt{1-w^2R_0^2}}\frac{3}{2M_{KK}} + 2T_g\frac{3U_{KK}}{2M_{KK}}\frac{1}{2w^2}\left(\arcsin(wR_0) - wR_0\sqrt{1-w^2R_0^2}\right) \quad (56)$$

We rewrite the total energy and angular momentum as [20]

$$E = \frac{2T_g U_{KK}}{w}\left(\arcsin x + \frac{1}{x}\sqrt{1-x^2}\right),$$
$$J = \frac{3}{2M_{KK}}\frac{T_g U_{KK}}{w^2}\left(\arcsin x + x\sqrt{1-x^2}\right), \quad (57)$$

where $x = wR_0$ is the speed of the string's endpoint. Note that $T_s U_{KK} = \frac{\lambda M_{KK}}{9\pi}$. By using the constraint in

Eq. (54), we find

$$E = \frac{2m_q\sqrt{1+q}}{q}\frac{2}{3}U_{KK}M_{KK}\left(\arcsin\frac{1}{\sqrt{1+q}}+\sqrt{q}\right)$$
$$J = \frac{m_q^2(1+q)}{T_g q^2}\frac{2}{3}U_{KK}M_{KK} \times\left(\arcsin\frac{1}{\sqrt{1+q}}+\frac{3}{2}\sqrt{\frac{q}{1+q}}\right) \quad (58)$$

where $q = \frac{2M_{KK}}{3}\frac{m_q}{T_g R_0}$.

From Eq. (57), we get the Regge trajectory

$$J = \tilde{\alpha}'_1 E^2,$$
$$\text{where} \quad \tilde{\alpha}'_1 = \frac{27}{4\lambda M_{KK}^2} = \frac{1}{2\pi T_g} = \tilde{\alpha}'$$
$$\text{when} \quad x \to 1 \quad (59)$$

The subscript 1 of α'_1 means α' for $x \to 1$, which eventually equals the effective string slope α'. From the experimental data, $1/\alpha'_{uu} = 1.2$ GeV^2, $1/\alpha'_{cc} = 3.2$ GeV^2 and $1/\alpha'_{bb} = 4.2$ GeV^2 [21,22]. In addition, the limit $x \to 1$, means, $m_q \to 0$. In this limit, we have

$$J = n = \frac{\lambda}{12w^2}, \quad E(J=n) = \frac{2\sqrt{3}}{9}\sqrt{\lambda}M_{KK}\sqrt{n}\,. \quad (60)$$

For the large quark mass, we expand Eq. (58) in large q to obtain

$$E \sim \frac{2}{3}U_{KK}M_{KK}\left(2m_q + \frac{9}{2M_{KK}}T_g R_0 + O(1/m_q)\right),$$
$$J \sim \frac{2}{3}U_{KK}M_{KK} \times\left(\frac{m_q}{M_{KK}}R_0 + \sqrt{\frac{m_q}{M_{KK}}}\sqrt{\frac{27}{8}\frac{T_g R_0^3}{M_{KK}^2}} + O(1)\right). \quad (61)$$

By varying w, we obtain the energy (mass) spectrum of mesons. Now we consider a charmonium J/ψ as an example for phenomenological applications of spinning strings. Requiring Bohr-Sommerfeld quantization, we obtain the quantization condition

$$J = n\hbar = n, \quad n \text{ is integer}. \quad (62)$$

Here we take natural units, c = $\hbar$ =1. We take the value of M_{KK} fixed by the η' mass, $M_{KK} = 1.039$ GeV. We summarize our results in Tables 3 and 4, where we take the mass of the heavy quarkonium from Ref. 21.

We choose λ =11.5 to fit the slope of the Regge trajectory. We also calculate the inter-quark distance in heavy quarkonia, R_0. The sewing condition gives the relation between the inter-quark distance R_0 and w,

$$R_0 = \frac{\sqrt{9T_g^2 + M_{KK}^2 m_q^2 w^2} - M_{KK}m_q w}{3T_g w} = \frac{9\pi}{2\lambda}\frac{1}{w}\left(\sqrt{\left(\frac{2\lambda}{9\pi}\right)^2 + \left(\frac{m_q w}{M_{KK}}\right)^2} - \frac{m_q w}{M_{KK}}\right). \quad (63)$$

Table 3. Charmonium mass and size.

Charmonium : $m_q^c = 1.2$ GeV			
J	E (GeV)	M_{exp}(GeV)	$2R_0$
J = 1	3.15	3.096	0.34 fm
J = 2	3.57	3.686	0.53 fm
J = 3	3.92	4.04	0.68 fm
J = 4	4.22	4.415	0.81 fm

Table 4. Bottomium mass and size.

Bottomium : m_q^b= 4.4 GeV			
J	E (GeV)	M_{exp}(GeV)	$2R_0$
J = 1	9.5	9.46	0.23 fm
J = 2	9.79	10.02	0.36 fm
J = 3	10.03	10.36	0.47 fm
J = 4	10.25	10.58	0.58 fm

V. REMARKS

As in Eq. (40) and Eq. (57), the energies from the fluctuation and the long (or spinning) string have different scalings of λ and M_{KK}. For the fluctuation, mass is proportional to $E_{fl} = (m_q M_{KK}/\lambda)^{1/2}$ or $\sqrt{r_\infty}\ M_{KK}$ while for the spinning string, it is given by $E_{long} = \lambda M_{KK}$. In AdS/CFT, the t' Hooft coupling λ is assumed to be very large, and in this limit, the long string and fluctuation spectra are different from each other by a factor of λ. These two totally different classes of energy spectra originate from the intrinsic difference of the two approaches; a long string is a classical object that can be viewed as solitonic state while fluctuating modes are local fields on a classical background. Note that fluctuating modes do not have correct Regge behavior while a long string does. This is because higher modes of fluctuating fields come from the Kaluza-Klein mode of the compactified scale M_{KK} while the spectrum of a long string comes from its classical configuration. Therefore, we may argue that these two different approaches describe two different physical systems: one is for the light meson, and the other is for the heavy quarkonium. For large λ, a long string is not relevant to describe the light mesons because its mass is too heavy in units of M_{KK}.

It is still a generic problem in holographic QCD models to describe both the light and the heavy quarkonium state simultaneously in a single model. In this work, we try to analyze them in a unified manner. We describe two different systems by using different string configurations with the same M_{KK} in a D4/D6 model: one is fluctuating field on the brane, and the other is the classical configuration of an open string.

VI. SUMMARY

We re-analyzed the D4/D6 model to study holographic light vector mesons and the properties of the heavy quarkonium in a confining phase. To treat the light mesons and heavy quarkonium on the same footing, we used the same compactification scale M_{KK} in both systems. In a confined phase, we observed that the meson spectroscopy of the light meson and heavy quarkonium could be described with a single M_{KK}. We found that like scalar and pseudo-scalar mesons, the vector meson mass is linearly proportional to the square root of the quark mass when the quark mass is large. With M_{KK} fixed by the light meson masses, we calculated the mass of the heavy quarkonium in a confining phase. In order to describe the heavy quarkonium in the D4/D6 model, we considered a classical open string configuration.

Certainly there are many things to be improved for our study to be closer to QCD. We list some of them here. Surely, chiral symmetry should be the first one. As is well known, non-Abelian chiral symmetry is essential to understand light mesons, and it is also important for a heavy-light system due to the light quark. The second thing is how to include the heavy-light meson in this picture with correct chiral symmetry and heavy quark spin symmetry.

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