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Inflation on a Pair of a D3-brane and a $\overline{\mathrm{D3}}$-brane in the Klebanov-Strassler Background

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We explain how to obtain the Klebanov-Strassler solution in the low-energy limit of type IIB superstring theory and describe slow-roll inflation on a system of parallelly-separated D3-brane and $\overline{\mathrm{D3}}$-brane in the Klebanov-Strassler background.

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I. INTRODUCTION

The systematic derivation of a viable cosmological model including a natural inflationary era in the context of superstring theory has been a challenging problem. Particularly, in string cosmology, construction of a bridge between a string-inspired brane-world scenario based on a warped geometry and obtaining a supergravity solution including this warped geometry has been an attractive subject. If such a supergravity solution supports nonvanishing fluxes and is consistent with moduli stabilization, it is even more intriguing.

In this note, we explain slow-roll inflation in the system of a D3-brane and an anti-D3-brane ($\overline{\mathrm{D3}}$) in the Klebanov-Strassler (KS) background [1]. In Sec. II, we briefly introduce the massless bosonic fields in the low-energy limit of type IIB superstring theory and then summarize the KS solution involving a warped geometry, a deformed conifold, a constant axion-dilaton, and various NS-NS and R-R form fields with nonvanishing fluxes. In Sec. III, the effective field theoretic description of the system of a D3-brane and a $\overline{\mathrm{D3}}$-brane is given, whose action is the sum of a Dirac-Born-Infeld(DBI)-type term and a Wess-Zumino(WZ)-type R-R coupling. In the KS background, the homogeneous time evolution of the separated D3$\overline{\mathrm{D3}}$ shows a slow-roll inflation for a wide range of parameter space. We conclude this paper with a discussion.

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II. KLEBANOV-STRASSLER SOLUTION

In this section, we briefly summarize the background geometry and fluxes on which the system of a separated D3-brane and a $\overline{\mathrm{D3}}$-brane lives. The specific form of the (1+9)-dimensional spacetime under consideration is from the KS solution with various fluxes and warp factors, which is obtained by solving the supergravity equations given in the low-energy limit of type IIB superstring theory.

1. IIB Superstring Theory and Low Energy Limit

In $(1+9)$ dimensions, five superstring theories are known: type IIB, type IIA, heterotic $\mathrm{E}_8 \times \mathrm{E}_8$, heterotic SO(32), and type I. We are interested in the type IIB superstring theory involving only closed oriented strings, whose characteristic mass scale is given by the string tension $1/\sqrt{\alpha'}$ (the corresponding string length scale is $l_{\mathrm{s}} = \sqrt{2\pi\alpha'}$) and whose mutual interaction is proportional to the square of string coupling g_{s}^2.

When the string tension approaches infinity ($\alpha' \to \infty$), all the massive modes of higher nodes decouple, and we obtain type IIB supergravity in ten dimensions as a low energy effective theory, which involves only massless fields (zero modes) and quadratic derivatives. The bosonic sector of the closed strings is composed of six fields completing a $\mathcal{N} = 2$ supergraviton multiplet, as summarized in Table 1. In a subsequent subsection we

Table 1. Massless bosonic fields in type IIB supergravity

Sector	Massless bosonic fields: NS-NS			R-R		
Names	Graviton	Dilaton	Two-form field	Axion	Two-form field	(self-dual) Four-form field
Fields	$G_{\mu\nu}$	Φ	$B_{\mu\nu}$ (B_2)	C	$C_{\mu\nu}$ (C_2)	$C_{\mu\nu\rho\sigma}$ (C_4)
Degrees	35	1	28	1	28	35
Field Strengths			$H_{\mu\nu\rho}$ (H_3)		$F_{\mu\nu\rho}$ (F_3)	$F_{\mu\nu\rho\sigma\tau}$ (F_5)
Role	Metric (Geometry)	Matter				

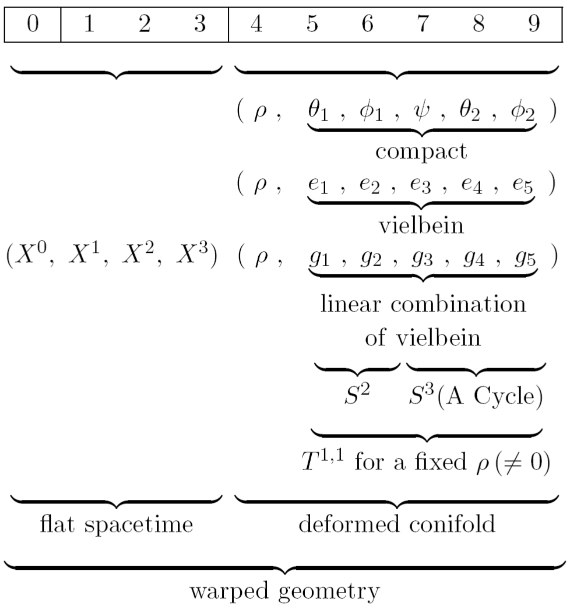

Fig. 1. Ten-dimensional spacetime given by the Klebanov-Strassler solution.

deal with the classical equations of motion for type IIB supergravity and obtain the KS solution.

2. Deformed Conifold and Klebanov-Strassler Background

Superstring theories are given in (1+9) dimensions, but the present Universe in which we live is (1+3) dimensions. To be consistent with the observed Universe, six spatial dimensions in a superstring theory should be unobservable and a usual method is to assume that six spatial dimensions are compactified. The ten-dimensional coordinates that we use are given in Fig. 1.

In the IIB superstring theory under consideration, the deformed conifold is utilized for the construction of a tip

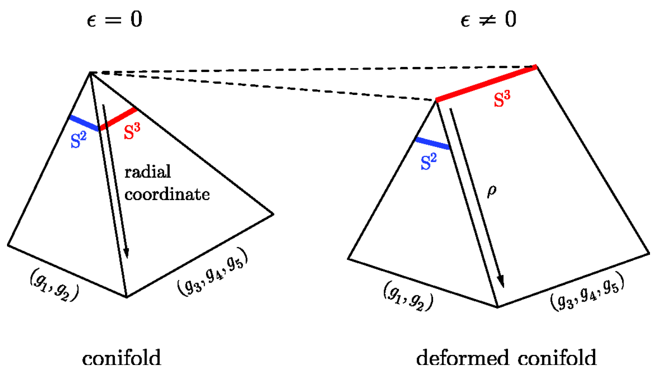

Fig. 2. (Color online) From conifold with $\epsilon = 0$ to deformed conifold with $\epsilon > 0$.

of six compact dimensions, for which the metric is

$$ds^2_{\text{def}} = \frac{1}{2}\epsilon^{\frac{4}{3}} K\left[\frac{1}{3K^3}(d\rho^2 + g_5^2) + \sinh^2\left(\frac{\rho}{2}\right)(g_1^2 + g_2^2) + \cosh^2\left(\frac{\rho}{2}\right)(g_3^2 + g_4^2)\right], \tag{1}$$

where the function K is a decreasing function of the radial coordinate ρ,

$$K(\rho) = \frac{(\sinh 2\rho - 2\rho)^{\frac{1}{3}}}{2^{\frac{1}{3}}\sinh\rho} \approx \begin{cases} \left(\frac{2}{3}\right)^{\frac{1}{3}}\left(1 - \frac{\rho^2}{10}\right) + \cdots & \text{as } \rho \to 0 \\ 2^{\frac{1}{3}} e^{-\frac{\rho}{3}} + \cdots & \text{as } \rho \to \infty, \end{cases} \tag{2}$$

and g_i's $(i = 1, ..., 5)$ stand for the fundamental one-forms (vielbeins) of the five angular coordinates. In the metric, ϵ is the deformation parameter that smooths the singular S^3 of (g_3, g_4, g_5) at the tip of the conifold, as schematically shown in Fig. 2.

Our (1+3)-dimensional spacetime is described by X^a's $(a, b = 0, 1, 2, 3)$, and the metric G_{ab} is assumed to be flat for the KS solution, $G_{ab} = \eta_{ab}$. In synthesis, the ansatz

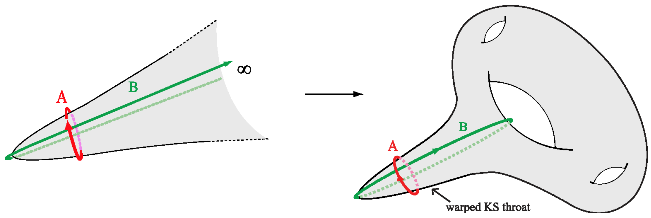

Fig. 3. (Color online) From noncompact deformed conifold to compact Calabi-Yau orientifold

of the (1+9)-dimensional metric is

$$ds^2 = \mathcal{H}^{-\frac{1}{2}} G_{ab} dX^a dX^b + \mathcal{H}^{\frac{1}{2}} ds^2_{\rm def}, \tag{3}$$

where $\mathcal{H}$ is a warp factor:

$$\mathcal{H}(\rho) = 2^{\frac{2}{3}} \epsilon^{-\frac{8}{3}} (g_s \mathcal{M} \alpha')^2 I(\rho). \tag{4}$$

In the warp factor $\mathcal{H}$, the constant $\mathcal{M}$ is the R-R three-form flux and $I(\rho)$ decreases exponentially for large ρ:

$$I(\rho) = \int_\rho^\infty dx\, \frac{x \coth x - 1}{\sinh^2 x} (\sinh 2x - 2x)^{\frac{1}{3}} \tag{5}$$

$$\approx \begin{cases} I(0) - 2\left(\frac{1}{6}\right)^{\frac{4}{3}} \rho^2 + \cdots & \text{as } \rho \to 0, \\ \quad I(0) \approx 0.71805 & \\ 3 \cdot 2^{-\frac{1}{3}} \rho e^{-\frac{4}{3}\rho} + \cdots & \text{as } \rho \to \infty \end{cases}. \tag{6}$$

The KS solution involves various NS-NS and the R-R form fluxes and their field configurations are given in Table 2.

Since the deformed conifold is not compact along the ρ-coordinate, the NS-NS 3-form flux from the NS-NS 2-form field B_2 is not explicitly given. In the phenomenological viewpoint of superstring theory, this ρ-direction should also be compactified, which means that the large-ρ region is chopped and is replaced by a compact geometry. An appropriate known candidate is the compact Calabi-Yau (CY) orientifold whose schematic shape is shown in Fig. 3. The R-R 3-form flux lives along compact A cycle (red line) and the NS-NS 3-form does along a compactified B cycle (green line) in the compact CY orientifold. This surgery is not important for describing the cosmological evolution of the early Universe which will be discussed in a subsequent section.

III. A PAIR OF D-BRANE AND $\overline{\rm D}$-BRANE IN KS BACKGROUND

In addition to the perturbative degrees whose bosonic fields are summarized in Table 1, type IIB superstring theory also involves various branes as nonperturbative degrees. They are p-dimensional Dirichlet branes (Dp-branes), a 5-dimensional Neveu-Schwarz brane (NS5-brane), and a fundamental string (F1) summarized in the following: D(-1), D1, D3, D5, D7, NS5, and F1.

Though each D-brane is stable and supersymmetric, a pair of a D-brane and an anti-D-brane ($\overline{\rm D}$-brane) does not possess supersymmetry and becomes unstable [2]. Between the Dp-brane and the $\overline{\rm D}p$-brane, open string degrees live, whose low energy modes are a complex tachyon field $T = \tau e^{i\chi}$ ($\bar{T} = \tau e^{-i\chi}$) depicting instability, two gauge fields $A^a_{(n)}$ living on each brane, and two sets of transverse coordinates $X^i_{(n)}$ representing the positions of the Dp-brane and the $\overline{\rm D}p$-brane with distance $\ell^i = X^i_{(1)} - X^i_{(2)}$. The dynamics of the system of a Dp-brane and a $\overline{\rm D}p$-brane is described by an effective action that consists of DBI-type term [3] and WZ-type R-R coupling [4], respectively:

$$S_{\rm D\bar{D}} = -\mathcal{T}_p \int d^{p+1}\xi \Big[V_{(1)}(\tau,\ell) e^{-\Phi(X_{(1)})} \sqrt{-\det A_{(1)}} + V_{(2)}(\tau,\ell) e^{-\Phi(X_{(2)})} \sqrt{-\det A_{(2)}} \Big], \tag{7}$$

$$S_{\rm WZ} = \mathcal{T}_p \int V(\tau)\, C \wedge {\rm Str}\, e^{B_2 + 2\pi\alpha' \tilde{\mathcal{F}}}. \tag{8}$$

The D$\overline{\rm D}$ potential in Eqs. (7) and (8) is based on the tachyon potential of an unstable Dp-brane $V(\tau,\ell)$ as $V_{(n)}(\tau,\ell) = V(\tau)\sqrt{\det Q_{(n)}}$, and $A_{(n)}$ in the square roots are two $(1+p)\times(1+p)$ matrices:

$$\begin{aligned} A_{(n)ab} &= P_{(n)ab}\left[E_{\mu\nu}(X_{(n)}) - \frac{\tau^2}{2\pi\alpha' \det Q_{(n)}} E_{\mu i}(X_{(n)}) \ell^i \ell^j E_{j\nu}(X_{(n)}) \right] + 2\pi\alpha' F_{(n)ab} + \frac{1}{\det Q_{(n)}} \\ &\times \left\{ \frac{2\pi\alpha}{2} (\overline{D_a T} D_b T + \overline{D_b T} D_a T) + \frac{i}{2} \left[E_{ai}(X_{(n)}) + \partial_a X_{(n)j} E_{ji}(X_{(n)}) \right] \ell^i (T\overline{D_b T} - \bar{T} D_b T) \right. \\ &\left. + \frac{i}{2} (T\overline{D_a T} - \bar{T} D_a T) \ell^i \left[E_{ib}(X_{(n)}) - E_{ij}(X_{(n)}) \partial_b X_{(n)j} \right] \right\}. \end{aligned} \tag{9}$$

In the previous expression, $P^{ab}_{(n)}[\cdots]$ means pull-back of the closed string fields on the n-th brane, $E_{\mu\nu} = G_{\mu\nu} +$

Table 2. NS-NS and R-R form fields and corresponding fluxes.

Field or field strength	Solution
Dilaton	$\Phi = \Phi_0$
Axion	$C = 0$
R-R three-form field strength	$F_3 = \frac{\mathcal{M}\alpha'}{2}\left\{g_5 \wedge g_3 \wedge g_4 + d\left[F(g_1 \wedge g_3 + g_2 \wedge g_4)\right]\right\}$
	with $F(\rho) = \frac{\sinh\rho - \rho}{2\sinh\rho}$
NS-NS two-form field	$B_2 = \frac{g_s\mathcal{M}\alpha'}{2}(f g_1 \wedge g_2 + k g_3 \wedge g_4)$
	with $f(\rho) = \frac{\rho\coth\rho - 1}{2\sinh\rho}(\cosh\rho - 1)$
	with $k(\rho) = \frac{\rho\coth\rho - 1}{2\sinh\rho}(\cosh\rho + 1)$
Self-dual R-R five-form field strength	$\tilde{F}_5 = \mathcal{F}_5 + *\mathcal{F}_5, \quad \mathcal{F}_5 = B_2 \wedge F_3$
	with $l(\rho) = f(1-F) + kF$

$B_{\mu\nu}$ raises and lowers the indices in the action, and

$$\det Q_{(n)} = 1 + \frac{\tau^2}{2\pi\alpha}\ell^i\ell^j G_{ij}(X_{(n)}). \qquad (10)$$

The field strength tensor of a U(1) gauge field on the n-th brane is $F^{ab}_{(n)} = \partial^a A^b_{(n)} - \partial^b A^a_{(n)}$, and the covariant derivative of complex tachyon field is $D^aT = \partial^a T - i(A^a_{(1)} - A^a_{(2)})T$. In (8), Str denotes supertrace and

$$\tilde{\mathcal{F}} = \begin{pmatrix} F_{(1)} - \mathrm{i}_{d\Phi_1} & i^{3/2}\left[DT + iT(\mathrm{i}_{\Phi_1} - \mathrm{i}_{\Phi_2})\right] \\ i^{3/2}\left[\overline{DT} - i\bar{T}(\mathrm{i}_{\Phi_1} - \mathrm{i}_{\Phi_2})\right] & F_{(2)} - \mathrm{i}_{d\Phi_2} \end{pmatrix}, \qquad (11)$$

where i_{Φ_n} denotes the interior product by Φ_n regarded as a vector in the transverse space.

Now, we introduce a D3-brane and a $\overline{\rm D3}$-brane in the KS background. If we consider the toal action $S_{{\rm D}\overline{\rm D}} + S_{\rm WZ}$, the potential terms of the D3-brane which are inversely proportional to the warp factor, $\mathcal{H}^{-1}(\rho_{(1)})$, do not appear due to cancellation between the contribution from the DBI-type action in Eq. (7) (or the NS-NS coupling) and that from the WZ-type action in Eq. (8) (or the R-R coupling), while the contributions are added up for the $\bar{\rm D}$3-brane, which is proportional to $2\mathcal{H}^{-1}(\rho_{(2)})$. This means that the D3-brane experiences no net force from the background, but the $\overline{\rm D3}$-brane does experience the attractive force from the background. Resultantly, as a natural initial configuration, the D3-brane is located at the position of the warped throat while the $\overline{\rm D3}$-brane is located at the tip of the deformed conifold as shown in Fig. 4.

Here, we introduce the distance ℓ between the D3-brane and the anti-D3-brane as

$$\ell^i = \begin{cases} \rho_{(1)} - \rho_{(2)} \equiv \ell & \text{for } i = \rho \\ 0 & \text{otherwise.} \end{cases} \qquad (12)$$

In order to study the cosmological implication of the $\rm D\overline{D}$ system, whose main topic is the realization of an inflationary era, we choose the static gauge $\xi^a = X^a$, assume

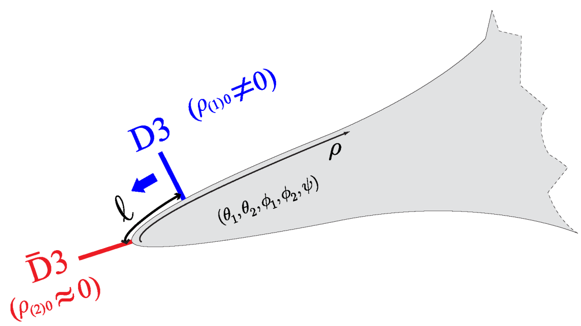

Fig. 4. (Color online) D3 at $\rho = \rho_{(1)0} \gtrsim l_s$ (blue color) and $\overline{\rm D3}$ at $\rho = \rho_{(2)0} \approx 0$ (red color) in a warped throat of the deformed conifold.

the absence of nontrivial gauge fields, $A^a_{(n)} = 0$, and consider homogeneous open string fields

$$\tau = \tau(t), \quad \chi = \chi(t), \quad \ell = \ell(t). \qquad (13)$$

In the world-volume of the $\rm D\overline{D}$ system we assume the flat Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - {\rm k}r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \quad ({\rm k} = 0), \qquad (14)$$

instead of the (1+3)-dimensional Minkowski spacetime in Eq. (3). In the 6-dimensional space of the deformed conifold, we also read and substitute the string coupling $g_s = e^{\Phi_0}$, the trivial axion field $C = 0$, the nonvanishing NS-NS 2-form field B_2, the R-R 2-form field C_2, and the R-R 4-form field C_4 from the self-dual R-R 5-form field $\tilde{F}_5$, as summarized in Table 2.

The dynamics of the closed string degrees in the weak coupling limit is of order $1/g_s^2$ but that of open string degrees is of order $\mathcal{T}_p = g_s^{-1}(2\pi)^{-p}(\alpha')^{-\frac{p+1}{2}}$. Therefore, the interaction between the Dp-brane and the $\overline{\text{D}}p$-brane from the closed string degrees, which is a $1/g_s$ correction, should be taken into account. When the transverse distance is large enough ($\ell > \ell_c$), the 1-loop correction [5] provides $\mathcal{O}(1/\ell^4)$ order corrections from gravitation and R-R coupling, whose magnitudes and signatures are exactly the same. As the distance between D and $\overline{\text{D}}$ reaches a critical distance, the 1-loop amplitude diverges. A natural assumption for the coincident D$\overline{\text{D}}$ ($\ell = 0$) is to introduce a finite binding energy per unit area $\mathcal{E}_b$. Interpolation of both limits suggests the following correction to the tachyon potential in the last square bracket:

$$V_{(1)}(\tau,\ell) = V_{(2)}(\tau,\ell) = V(\tau,\ell) = \frac{1}{\cosh(\sqrt{\pi}\,\tau)}\sqrt{1+\frac{\tau^2\ell^2}{2\pi\alpha'}}\left[1-\frac{\mathcal{E}_b/2\mathcal{T}_3}{1+(\ell/\ell_c)^4}\right], \quad (15)$$

reflecting the gravitational and the R-R attractions between the D-brane and the $\overline{\text{D}}$-brane. In Eq. (15), $\ell_c = \left(2\kappa_{10}^2\mathcal{T}_3^2/\mathcal{E}_b\right)^{1/4}$, where κ_{10}^2 is ten-dimensional gravitational constant.

In Fig. 4, the buoyant D3-brane starts to move to the sunken $\overline{\text{D}}$3-brane because of these attractive forces, but they are weak enough due to the nontrivial warp factor. We assume that $\rho_{(2)}$ is always sufficiently small. Furthermore, to perform the numerical analysis, we set the position of the anti D3-brane $\rho_{(2)}$ to be fixed at the tip of the warped throat in the KS background, which naturally gives $\dot{\rho}_{(2)} = 0$. Note that we consider only the motion along the radial coordinate ρ and omit the dynamics of the angular variables $(\theta_1, \theta_2, \phi_1, \phi_2, \psi)$ in the deformed conifold for simplicity, which do not lose generality. In addition, we do not consider the dynamics of the tachyon phase field $\chi(t) = 0$.

To perform the numerical analysis for the slow-roll inflation in the D$\overline{\text{D}}$ system, we introduce the dimensionless quantities

$$\frac{t}{l_s},\quad a,\quad \tau,\quad \frac{\rho_{(1)}}{l_s},\quad \tilde{\mathcal{T}}_3 = \frac{\mathcal{T}_3 l_s^2}{M_P^2},\quad e_b = \frac{\mathcal{E}_b}{\mathcal{T}_3},$$
$$\frac{\ell_c}{l_s},\quad \mathcal{M},\quad \epsilon = \exp\left(-\frac{\pi\mathcal{K}}{g_s\mathcal{M}}\right), \quad (16)$$

where M_P is the Planck mass and the deformation parameter ϵ is given by the R-R 3-form flux $\mathcal{M}$ and the NS-NS 3-form flux $\mathcal{K}$. An appropriate set of initial conditions is

$$\tau(0) = \tau_0, \quad \ell(0) = \ell_0, \quad \dot{\tau}(0) = \dot{\tau}_0, \quad \dot{\ell}(0) = \dot{\ell}_0 \quad (17)$$

because $a(0)$ can always be fixed to be unity for k = 0 from the Einstein equations. An actual numerical analysis will be performed under a natural condition of no initial time derivatives, $\dot{\tau}_0 = \dot{\ell}_0 = 0$. In the synthesis, the expansion rate represented by the value of e-folding is studied in the space of four dimensionless parameters, $(\tilde{\mathcal{T}}_3, e_b, \tau_0, \ell_0)$.

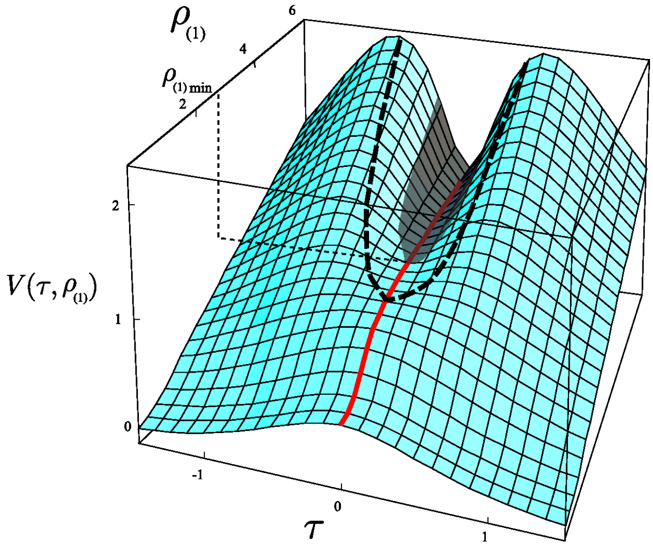

Fig. 5. (Color online) Tachyon potential with $\mathcal{E}_b = 0.5$, $\mathcal{T}_3 = 1$, $\ell_c = 1$, and $\rho_{(1)\min} = 3.18$.

As shown in Fig. 5, the area shaded by thick-grey color stands for the region of sufficient slow-roll inflation over 60 e-folding in the KS background. For comparison, the area bounded by the dashed line represents the region of the 60 e-folding condition in the flat background. As the tension $\mathcal{T}_3$ increases, the initial distance ℓ_0 decreases, and the initial tachyon amplitude τ_0 increases.

IV. DISCUSSION

The KS solution has constant dilaton and axion configuration, which is proven to be a dynamically-favorable configuration [6], and other moduli, including the volume moduli, can also be stabilized by adding D7-branes and $\overline{\text{D}}$3-branes [7]. Though we obtained a slow-roll inflation model based on D$\overline{\text{D}}$ system in the IIB superstring thoery [8], it suffered from huge supergravity corrections, so its present form does not generate a natural inflationary era [9].

When the fluxes are generated on the D3-brane and are left in the present Universe, it may jeopardize the cosmological model. As long as the dilaton moduli are fixed, they can sufficiently be diluted through the cosmological expansion [10]. Because this result is obtained without R-R form fluxes, an intriguing research direction may be the inclusion of fluxes both in the D-brane and the compactified directions.

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