Results in Physics 18 (2020) 103317

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

On the numerical investigation of the interaction in plasma between (high & low) frequency of (Langmuir & ion-acoustic) waves

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ARTICLE INFO

Keywords:
The Zakharov equation in the dimensionless form
Langmuir waves
Ion-acoustic waves
Numerical schemes
B-spline schemes

ABSTRACT

In this paper, the Zakharov (Z.) equation in the dimensionless form is numerically investigated via (Cubic & Quantic & Septic) B-spline schemes to demonstrate the fidelity of the calculated computational solutions. The Z equation depicts the interaction in plasma between (high & low) frequency of (Langmuir & ion-acoustic) waves. This interaction is expounded in the prompts of the coastal engineering, electromagnetic field, signal handling in the optical fibres, plasma physics, and fluid dynamics. Three different computational schemes were applied to the Z equation for constructing many novel analytical solutions. In our paper, we try to check the accuracy of these solutions via the above-mentioned numerical schemes. Moreover, some separate sketches are given to indicate more physical features of this interaction. The originality of the obtained solutions is investigated by showing the similarities and differences between our obtained solutions and that was purchased in previously published papers.

Introduction

Recently, the nonlinear evaluation equations have been being exerted to characterize the dynamical and physical behavior of some natural phenomena at small scales [1,2]. The mathematical modeling of these phenomena in PDE formulas becomes manifest in figuring out the mechanical and electrical features of original materials, as well as in the characterized of rheological attributes of rocks [3–6]. Moreover, PDEs have been being played an essential role in other areas such as solid-state physics [7–9], plasma physics [10–12], chemical kinematics [13–15], optical fibers [16,17], control theory [18,19], condensed matter physics [20,21], signal processing [22,23], electrical circuits [24,25], bio-genetics [26,27], systems identification [28], and fluid flow [29]. These contributions of PDEs in discovering and analyzing many phenomena in previous areas have been attracted many researchers to in-depth investigation in this field. So, many researchers have been being paying deep attention to examine the closed-form of solutions for the mathematical modeling of these phenomena [30–32].

Studying the analytical and numerical solutions of these phenomena has been being gained considerable popularity and importance due to their realistic applications. Consequently, many systematic schemes which are considered as valuable tools for investigating the closed-form of solutions for many physical phenomena, have been proposed such as the modified simplest equation, the generalized tanh-expansion, the generalized Kudryashov, Khater, the modified Khater, the Adomian decomposition, the homotopy perturbation, the variational iteration, the generalized $\left(\frac{\Psi'}{\Psi}\right)$-expansion, the improved $\tanh\left(\frac{\varphi}{2}\right)$-expansion methods and others [33–50].

In this circumstance, we study the Z equation the dimensionless form that is given by [51–57]

$$\begin{cases} \imath\,\mathfrak{P}_t + \mathfrak{P}_{xx} + w_1\,\Psi(|\mathfrak{P}|^2)\,\mathfrak{P} = \mathfrak{P}\,\varphi, \\ \qquad \varphi_{tt} - \varphi_{xx} = (|\mathfrak{P}|^{2\gamma})_{xx}, \end{cases} \tag{1}$$

where $[\mathfrak{P} = \mathfrak{P}(x, t),\ \varphi = \varphi(x, t),\ w_1]$ respectively, elucidate the occurrence of the electric field in a high-frequency, the plasma density, and autocratic fixed.

For $[\Psi(|\mathfrak{P}|^2) = |\mathfrak{P}|^2,\ \gamma = 1]$, Eq. (1) takes the following formula

$$\begin{cases} \imath\,\mathfrak{P}_t + \mathfrak{P}_{xx} + w_1\,|\mathfrak{P}|^2\,\mathfrak{P} = \mathfrak{P}\,\varphi, \\ \qquad \varphi_{tt} - \varphi_{xx} = (|\mathfrak{P}|^2)_{xx}, \end{cases} \tag{2}$$

Using the following transformation $[\mathfrak{P} = \Upsilon e^{\imath\,\vartheta},\ \Upsilon = \Upsilon(\mathfrak{F}),\ \mathfrak{F} = ,x - r_1 t,$

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https://doi.org/10.1016/j.rinp.2020.103317
Received 7 July 2020; Received in revised form 24 July 2020; Accepted 6 August 2020
Available online 13 August 2020

$\vartheta = -r_2 x + r_3 t + r_4,$ where $(r_i,\ (i = 1, 2, 3, 4))$ aretyrannicalconstants.] on Eq. (2), leads to

$$\begin{cases}(\imath r_1 \Upsilon' - r_3 \Upsilon + \Upsilon'' - 2\imath r_2 \Upsilon' - r_2^2 \Upsilon + w_1 \Upsilon^3 - \Upsilon \varphi) e^{\imath \vartheta} = 0,\\ (r_1^2 - 1)\varphi'' - (\Upsilon^2)'' = 0.\end{cases} \tag{3}$$

For $(r_1 + 2 r_2 = 0)$ and twice integration of the second equation in Eq. (3) with zero constant of integration then exchanging the result into first equation of the same system, yields

$$\Upsilon'' - q_1 \Upsilon + q_2 \Upsilon^3 = 0, \tag{4}$$

where $\left[q_1 = r_3 + r_2^2,\ q_2 = \left(w_1 - \frac{1}{r_1^2 - 1}\right)\right]$. Khater, Mostafa MA, et al. [58] have applied three different analytical schemes (the extended exp (ϕ)-expansion, generalized Kudryashov and the modified Khater methods) to examine the analytical solutions of this model. These solutions have been elicited according to these schemes as following:

1. *The extended exp($\phi(\Upsilon)$)-expansion method*:

$$\mathfrak{P}(x, t) = \frac{a_0 e^{\imath(r_3 t + r_2(-x) + r_4)}\left(\sqrt{s_1^2 - 4 s_2}\, s_1 \tanh\left(\frac{1}{2}\sqrt{s_1^2 - 4 s_2}\,(\eta - r_1 t + x)\right) + s_1^2 - 4 s_2\right)}{b_0 s_1\left(\sqrt{s_1^2 - 4 s_2}\tanh\left(\frac{1}{2}\sqrt{s_1^2 - 4 s_2}\,(\eta - r_1 t + x)\right) + s_1\right)}. \tag{5}$$

2. *The generalized Kudryashov method*:

$$\mathfrak{P}(x, t) = \frac{a_1 e^{i(r_3 t + r_2(-x) + r_4)}(e^{r_1 t} - A e^x)}{(2 b_0 - b_1)(A e^x + e^{r_1 t})}. \tag{6}$$

3. *The modified Khater method:*

$$\mathfrak{P}(x, t) = \frac{-1}{v_2}\left(a_0 \sqrt{v_2^2 - 4 v_1 v_3}\, e^{\imath(r_3 t + r_2(-x) + r_4)} \tanh\left(\frac{1}{2}\sqrt{v_2^2 - 4 v_1 v_3}\,(x - r_1 t)\right)\right). \tag{7}$$

Using The following conditions respectively $[(a_0 = 5,\ b_0 = 6,\ \eta = 4,\ s_1 = 3,\ s_2 = 2,\ s_3 = 1)\ \&\ (a_1 = 4,\ A = 5,\ b_0 = 2,\ b_1 = 3)\ \&\ (a_0 = 1,\ v_2 = 5,\ v_1 = 3,\ v_3 = 2)]$ on the above solutions, allow obtaining the exact solutions of Eq. (4) in a simple formulas. Here, we test these solutions via the B-spline schemes [59–62] to show which method of them obtained more accurate solutions than others.

The goal of our research paper is investigating the numerical solutions of the Z equation in the dimensionless form via the (Cubic & Quantic & Septic) B-spline. These numerical investigation aims to show the accurate of the obtained analytical solutions in [58].

The remainder partitions of this paper have been written in the next configuration: (Cubic & Quantic & Septic) B-spline techniques are examined to calculate the evaluate exact, numerical, and absolute error for the Z equation in Section "Numerical investigation of the Z equation". The figure interpretation is investigated in Section "Tables & Figures interpretation". The novelty of our paper is shown in Section "Results and discussion". The epilogue of the entire study is explained in Section "Conclusion"

Numerical investigation of the Z equation

Here, we try to find an approximate solutions of Eq. (4) via applying (Cubic & Quantic & Septic) B-spline schemes [63–66].

Cubic B-Spline

Employing the cubic spline technique to Eq. (4) with the above conditions, yields elicit its numerical solutions as following

$$\mathfrak{P}(\mathfrak{F}) = \sum_{\mathfrak{T}=-1}^{\mathfrak{M}+1} \mathfrak{C}_{\mathfrak{T}} \mathscr{E}_{\mathfrak{T}}, \tag{8}$$

where $\mathfrak{C}_{\mathfrak{T}},\ \mathscr{E}_{\mathfrak{T}}$ follow the next conditions, respectively:

$\mathfrak{L}\mathfrak{B}(\mathfrak{F}) = \mathscr{F}(\mathfrak{F}_{\mathfrak{T}}, \mathfrak{B}(\mathfrak{F}_{\mathfrak{T}}))$ where $(\mathfrak{T} = 0, 1, \ldots, n)$

and

$$\mathscr{E}_{\mathfrak{T}}(\mathfrak{F}) = \frac{1}{6\mathfrak{H}^3}\begin{cases}(\mathfrak{F} - \mathfrak{F}_{\mathfrak{T}-2})^3, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{T}-2}, \mathfrak{F}_{\mathfrak{T}-1}],\\ -3(\mathfrak{F} - \mathfrak{F}_{\mathfrak{T}-1})^3 + 3\mathfrak{H}(\mathfrak{F} - \mathfrak{F}_{\mathfrak{T}-1})^2 + 3\mathfrak{H}^2(\mathfrak{F} - \mathfrak{F}_{\mathfrak{T}-1}) + \mathfrak{H}^3, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{T}-1}, \mathfrak{F}_i],\\ -3(\mathfrak{F}_{\mathfrak{T}+1} - \mathfrak{F})^3 + 3\mathfrak{H}(\mathfrak{F}_{\mathfrak{T}+1} - \mathfrak{F})^2 + 3\mathfrak{H}^2(\mathfrak{F}_{\mathfrak{T}+1} - \mathfrak{F}) + \mathfrak{H}^3, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{T}}, \mathfrak{F}_{\mathfrak{T}+1}],\\ (\mathfrak{F}_{\mathfrak{T}+2} - \mathfrak{F})^3, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{T}+1}, \mathfrak{F}_{\mathfrak{T}+2}],\\ 0, & \text{otherwise.}\end{cases} \tag{9}$$

For $\mathfrak{T} \in [-2, \mathfrak{M} + 2]$, we obtain

$$\mathfrak{B}_{\mathfrak{T}}(\mathfrak{F}) = \mathfrak{C}_{\mathfrak{T}-1} + 4\mathfrak{C}_{\mathfrak{T}} + \mathfrak{C}_{\mathfrak{T}+1}. \tag{10}$$

Substituting Eq. (10) into Eq. (4), yields $(\mathfrak{M} + 3)$ of equations. Resolving this system gives

Quantic B-spline

Employing the cubic spline technique to Eq. (4) with the above conditions gives its numerical solutions in the next formula

$$\mathfrak{B}(\mathfrak{F}) = \sum_{\mathfrak{M}=-1}^{\mathfrak{M}+1} \mathfrak{C}_{\mathfrak{T}} \mathscr{E}_{\mathfrak{T}}, \tag{11}$$

where $\mathfrak{C}_{\mathfrak{T}},\ \mathscr{E}_{\mathfrak{T}}$ follow the next conditions, respectively:

$\mathfrak{L}\mathfrak{B}(\mathfrak{F}) = \mathscr{F}(\mathfrak{F}_{\mathfrak{T}}, \mathfrak{B}(\mathfrak{F}_{\mathfrak{T}}))$ where $(\mathfrak{T} = 0, 1, \ldots, n)$

and

$$\mathscr{E}_{\mathfrak{T}}(\mathfrak{F}) = \frac{1}{\mathfrak{H}^5}\begin{cases}(\mathfrak{F} - \mathfrak{F}_{\mathfrak{T}-3})^5, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{T}-3}, \mathfrak{F}_{\mathfrak{T}-2}],\\ (\mathfrak{F} - \mathfrak{F}_{\mathfrak{T}-3})^5 - 6(\mathfrak{F} - \mathfrak{F}_{\mathfrak{T}-2})^5, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{T}-2}, \mathfrak{F}_{\mathfrak{T}-1}],\\ (\mathfrak{F} - \mathfrak{F}_{\mathfrak{T}-3})^5 - 6(\mathfrak{F} - \mathfrak{F}_{\mathfrak{T}-2})^5 + 15(\mathfrak{F} - \mathfrak{F}_{\mathfrak{T}-1})^5, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{T}-1}, \mathfrak{F}_{\mathfrak{T}}],\\ (\mathfrak{F}_{\mathfrak{T}+3} - \mathfrak{F})^5 - 6(\mathfrak{F}_{\mathfrak{T}+2} - \mathfrak{F})^5 + 15(\mathfrak{F}_{\mathfrak{T}+1} - \mathfrak{F})^5, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{T}}, \mathfrak{F}_{\mathfrak{T}+1}],\\ (\mathfrak{F}_{\mathfrak{T}+3} - \mathfrak{F})^5 - 6(\mathfrak{F}_{\mathfrak{T}+2} - \mathfrak{F})^5, & \mathfrak{F} \in [\mathfrak{F}_{\mathfrak{T}+1}, \mathfrak{F}_{\mathfrak{T}+2}],\\ (\mathfrak{F}_{\mathfrak{T}+3} - \mathfrak{F})^5, & x \in [\mathfrak{F}_{\mathfrak{T}+2}, \mathfrak{F}_{\mathfrak{T}+3}],\\ 0, & \text{otherwise.}\end{cases} \tag{12}$$

For $\mathfrak{T} \in [-2, \mathfrak{M} + 2]$, we get

$$\mathfrak{B}_{\mathfrak{T}}(\mathfrak{F}) = \mathfrak{C}_{\mathfrak{T}-2} + 26\mathfrak{C}_{\mathfrak{T}-1} + 66\mathfrak{C}_{\mathfrak{T}} + 26\mathfrak{C}_{\mathfrak{T}+1} + \mathfrak{C}_{\mathfrak{T}+2}. \tag{13}$$

Substituting Eq. (13) into Eq. (4) gives $(\mathfrak{M} + 5)$ of equations. Resolving this system, leads to

Septic B-Spline

Employing the septic spline technique to Eq. (4) with the above conditions gives its numerical solutions in the next form

$$\mathfrak{B}(\mathfrak{F}) = \sum_{\mathfrak{T}=-1}^{\mathfrak{M}+1} \mathfrak{C}_{\mathfrak{M}} \mathscr{E}_{\mathfrak{M}}, \tag{14}$$

where $c_{\mathfrak{M}},\ \mathscr{E}_{\mathfrak{M}}$ follow the next conditions, respectively:

$\mathfrak{L}\mathfrak{B}(\mathfrak{F}) = \mathscr{F}(\mathfrak{F}_{\mathfrak{M}}, \mathfrak{B}(\mathfrak{F}_{\mathfrak{M}}))$ where $(\mathfrak{M} = 0, 1, \ldots, n)$

and

$$\mathscr{E}_{\mathfrak{T}}(\mathfrak{F}) = \frac{1}{\mathfrak{H}^5}\begin{cases} (\mathfrak{F}-\mathfrak{F}_{\mathfrak{T}-4})^7, & \mathfrak{F}\in[\mathfrak{F}_{\mathfrak{T}-4},\mathfrak{F}_{\mathfrak{T}-3}],\\ (\mathfrak{F}-\mathfrak{F}_{\mathfrak{T}-4})^7-8(\mathfrak{F}-\mathfrak{F}_{\mathfrak{T}-3})^7, & \mathfrak{F}\in[\mathfrak{F}_{\mathfrak{T}-3},\mathfrak{F}_{\mathfrak{T}-2}],\\ (\mathfrak{F}-\mathfrak{F}_{\mathfrak{T}-4})^7-8(\mathfrak{F}-\mathfrak{F}_{\mathfrak{T}-3})^7+28 & \mathfrak{F}\in[\mathfrak{F}_{\mathfrak{T}-2},\mathfrak{F}_{\mathfrak{T}-1}],\\ (\mathfrak{F}-\mathfrak{F}_{\mathfrak{T}-2})^7, & \\ (\mathfrak{F}-\mathfrak{F}_{\mathfrak{T}-4})^7-8(\mathfrak{F}-\mathfrak{F}_{\mathfrak{T}-3})^7+28 & \mathfrak{F}\in[\mathfrak{F}_{\mathfrak{T}-1},\mathfrak{F}_{\mathfrak{T}}],\\ (\mathfrak{F}-\mathfrak{F}_{\mathfrak{T}-2})^7+56(\mathfrak{F}-\mathfrak{F}_{\mathfrak{T}-1})^7, & \\ (\mathfrak{F}_{\mathfrak{T}+4}-\mathfrak{F})^7-8(\mathfrak{F}_{\mathfrak{T}+3}-\mathfrak{F})^7+28 & \mathfrak{F}\in[\mathfrak{F}_{\mathfrak{T}},\mathfrak{F}_{\mathfrak{T}+1}],\\ (\mathfrak{F}_{\mathfrak{T}+2}-\mathfrak{F})^7+56(\mathfrak{F}_{\mathfrak{T}+1}-\mathfrak{F})^7, & \\ (\mathfrak{F}_{\mathfrak{T}+4}-\mathfrak{F})^7-8(\mathfrak{F}_{\mathfrak{T}+3}-\mathfrak{F})^7+28 & \mathfrak{F}\in[\mathfrak{F}_{\mathfrak{T}+1},\mathfrak{F}_{\mathfrak{T}+2}],\\ (\mathfrak{F}_{\mathfrak{T}+2}-\mathfrak{F})^7, & \\ (\mathfrak{F}_{\mathfrak{T}+4}-\mathfrak{F})^7-8(\mathfrak{F}_{\mathfrak{T}+3}-\mathfrak{F})^7, & \mathfrak{F}\in[\mathfrak{F}_{\mathfrak{T}+2},\mathfrak{F}_{\mathfrak{T}+3}],\\ (\mathfrak{F}_{\mathfrak{T}+4}-\mathfrak{F})^7, & \mathfrak{F}\in[\mathfrak{F}_{\mathfrak{T}+3},\mathfrak{F}_{\mathfrak{T}+4}],\\ 0, & \text{otherwise}. \end{cases} \tag{15}$$

For $\mathfrak{T}\in[-3,\mathfrak{M}+3]$, we get

$$\mathfrak{B}_{\mathfrak{T}}(\mathfrak{F}) = \mathfrak{C}_{\mathfrak{T}-3}+120\mathfrak{C}_{\mathfrak{T}-2}+1191\mathfrak{C}_{\mathfrak{T}-1}+2416\mathfrak{C}_{\mathfrak{T}}+1191\mathfrak{C}_{\mathfrak{T}+1}+120\,\mathfrak{C}_{\mathfrak{T}+2}+\mathfrak{C}_{\mathfrak{T}+3}. \tag{16}$$

Substituting Eq. (16) into Eq. (4) gives $(\mathfrak{M}+7)$ of equations. Resolving this system leads to

Tables & Figures interpretation

This section gives the clarification and interpretation of the above tables [1, 2 3] & figures [1 2, 3] as following:

- Table 1, exhibits values of the guaranteed computational solutions and cubic spline numerical solutions of Eq. (4) with above-mentioned stipulations with the various values of $\Upsilon\in[0,1]$.
- Fig. 1 illustrates a two-dimensional plot for the gained numerical solutions through the cubic spline technique and the acquired analytical solutions through the extended exp $(\phi(\Upsilon))$-expansion method (**left**), the gained analytical solutions through the generalized Kudryashov method (**middle**), and the obtained analytical solutions through the modified Khater method (**right**).
- Table 2, shows values of the ensured computational solutions and quantic spline numerical solutions of Eq. (4) with above-aforementioned conditions with the various values of $\Upsilon\in[0,1]$.
- Fig. 2 clarifies a two-dimensional plot for the gained numerical solutions through the quantic spline technique and the obtained analytical solutions through the extended exp $(\phi(\Upsilon))$-expansion method (**left**), the acquired analytical solutions through the generalized Kudryashov method (**middle**), and the obtained analytical solutions through the modified Khater method (**right**).
- Table 3, represents values of the warranted computational solutions and septic spline numerical solutions of Eq. (4) with above-aforesaid provisions with the diverse values of $\Upsilon\in[0,1]$. Fig. 3 illustrates a two-dimensional plot for the obtained numerical solutions through the septic spline technique and the gained analytical solutions through the extended exp $(\phi(\Upsilon))$-expansion method (**left**), the acquired analytical solutions through the generalized Kudryashov method (**middle**), and the obtained analytical solutions through the modified Khater method (**right**).

Results and discussion

In this section, we try to explain the novelty and originality of our discussion as following:

- We have investigated the numerical solutions of the dimensionless form of the Z equation via the B-spline (cubic & quantic & septic) schemes. Still, we did not select just one exact solution of this equation. Nevertheless, we have chosen three distinct exact solutions that were obtained via three recent computational schemes [58]. The main goal of this study is investigating which one of these methods able to get a more accurate solution than the others.
- For achieving our goal of this research, we have calculated the absolute values of error for each method of the B-spline schemes as shown in 4, 5
- These Tables 4, 5 the accuracy of the modified Khater method in (cubic & quantic & septic) B-spline scheme over the extended exp ϕ-expansion method, and the generalized Kudryashov method where its absolute error between its exact and numerical is smaller than the others.
- Fig. 4 illustrates a two-dimensional plot for the gained absolute values of error between analytical and numerical solutions through the above-mentioned analytical schemes and (cubic (**left**) & quantic (**middle**) & septic (**right**)) spline.

Conclusion

In this research paper, the B-spline schemes have been utilized to estimate the numerical solutions of the Z equation in the dimensionless form. The precision of the solution which was gained by Khater, Mostafa MA, et al. [58] have been demonstrated. Furthermore, the preponderance of the modified Khater method over the other two used computational schemes is also elucidated. Many computational and numerical solutions have been obtained with respect to different value of Υ to explain the matching between both kind of solutions. Additionally, some distinct sketches were used to prove the superiority and accuracy of the modified Khater method over the other two used analytical schemes.

Table 1
Cubic B-Spline scheme.

	The extended exp(ϕ)-expansion method		The generalized Kudryashov method		The modified Khater method	
Value of Υ	Exact	Numerical	Exact	Numerical	Exact	Numerical
0	0.0925926	0.0925926	−3.45173	−2.66667	0	−3.38813E−21
0.001	0.092716	0.092716	−2.66778	−2.66778	−0.0001	−0.0001
0.002	0.0928394	0.0928394	−2.66889	−2.66889	−0.0002	−0.0002
0.003	0.0929628	0.0929628	−2.67	−2.67	−0.0003	−0.0003
0.004	0.0930861	0.0930861	−2.67111	−2.67111	−0.000399999	−0.000399999
0.005	0.0932094	0.0932094	−2.67221	−2.67221	−0.000499999	−0.000499999
0.006	0.0933326	0.0933326	−2.67332	−2.67332	−0.000599998	−0.000599998
0.007	0.0934558	0.0934558	−2.67443	−2.67443	−0.000699997	−0.000699997
0.008	0.0935789	0.0935789	−2.67553	−2.67553	−0.000799996	−0.000799996
0.009	0.093702	0.093702	−2.67664	−2.67664	−0.000899994	−0.000899994
0.01	0.0938251	0.0938251	−2.67774	−2.67774	−0.000999992	−0.000999992

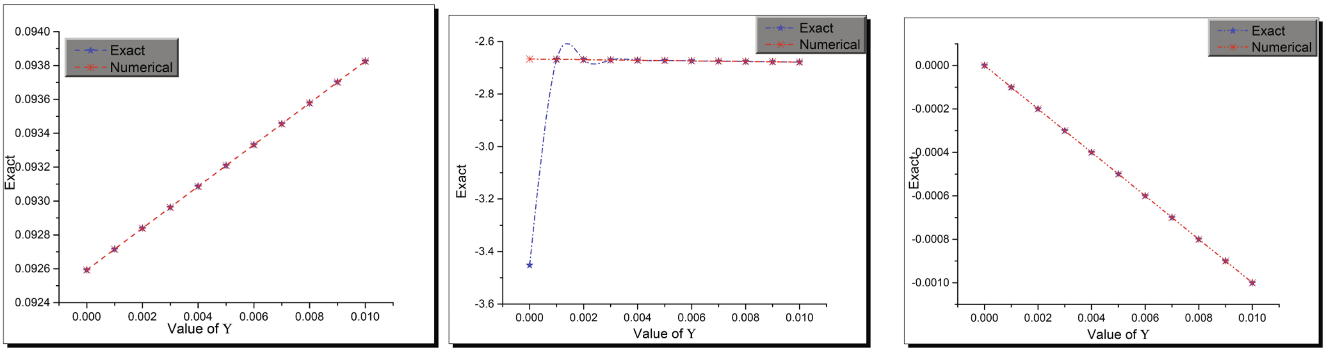

Fig. 1. Exact and numerical values by using cubic B-spline scheme for the three analytical schemes.

Table 2
Quantic B-Spline scheme.

Value of ϒ	The extended exp(ϕ)-expansion method		The generalized Kudryashov method		The modified Khater method	
	Exact	Numerical	Exact	Numerical	Exact	Numerical
0	0.0925926	0.0925926	−2.66667	−2.66667	0	0
0.001	0.092716	0.0999562	−2.66778	−2.72517	−0.0001	−0.0068788
0.002	0.0928394	0.111121	−2.66889	−2.81381	−0.0002	−0.0173165
0.003	0.0929628	0.121267	−2.67	−2.89438	−0.0003	−0.0268008
0.004	0.0930861	0.131685	−2.67111	−2.9771	−0.000399999	−0.0365394
0.005	0.0932094	0.142035	−2.67221	−3.05928	−0.000499999	−0.0462144
0.006	0.0933326	0.152385	−2.67332	−3.14145	−0.000599998	−0.0558894
0.007	0.0934558	0.162803	−2.67443	−3.22417	−0.000699997	−0.065628
0.008	0.0935789	0.172949	−2.67553	−3.30472	−0.000799996	−0.0751118
0.009	0.093702	0.184117	−2.67664	−3.39341	−0.000899994	−0.085551
0.01	0.0938251	0.191465	−2.67774	−3.45173	−0.000999992	−0.0924234

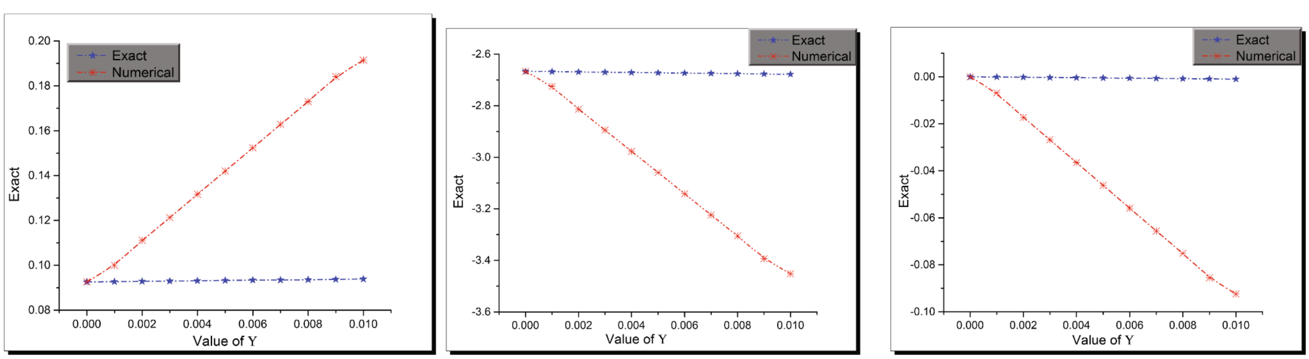

Fig. 2. Exact and numerical values by using quantic B-spline scheme for the three analytical schemes.

Table 3
Septic B-Spline scheme.

Value of ϒ	The extended exp(ϕ)-expansion method		The generalized Kudryashov method		The modified Khater method	
	Exact	Numerical	Exact	Numerical	Exact	Numerical
0	0.0925926	0.0925926	−2.66667	−2.66667	0	0
0.001	0.092716	0.092716	−2.66778	−2.66778	−0.0001	−0.0001
0.002	0.0928394	0.0928394	−2.66889	−2.66889	−0.0002	−0.0002
0.003	0.0929628	0.0929628	−2.67	−2.67	−0.0003	−0.0003
0.004	0.0930861	0.0930861	−2.67111	−2.67111	−0.000399999	−0.000399999
0.005	0.0932094	0.0932094	−2.67221	−2.67221	−0.000499999	−0.000499999
0.006	0.0933326	0.0933326	−2.67332	−2.67332	−0.000599998	−0.000599998
0.007	0.0934558	0.0934558	−2.67443	−2.67443	−0.000699997	−0.000699997
0.008	0.0935789	0.0935789	−2.67553	−2.67553	−0.000799996	−0.000799996
0.009	0.093702	0.093702	−2.67664	−2.67664	−0.000899994	−0.000899994
0.01	0.0938251	0.0938251	−2.67774	−2.67774	−0.000999992	−0.000999992

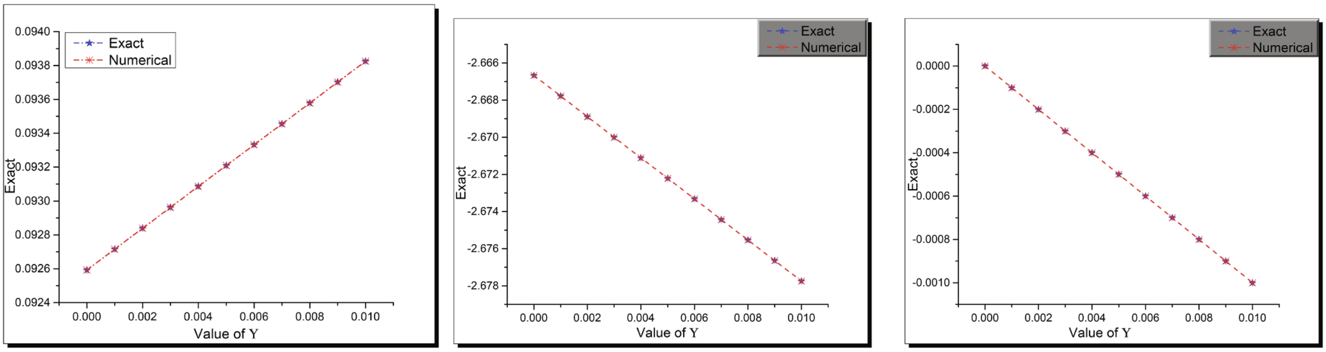

Fig. 3. Exact and numerical values by using septic B-spline scheme for the three analytical schemes.

Table 4
Absolute values of error via the B-Spline scheme.

	Cubic			Quantic		
Value of ϒ	Extended exp.	Gen. Kudryashov	Mod. Khater	The extended exp.	Gen. Kudryashov	Mod. Khater
0	8.32667×10^{-17}	0	3.38813×10^{-21}	8.32667×10^{-17}	0	0
0.001	2.57711×10^{-14}	1.84297×10^{-13}	1.37545×10^{-16}	0.0072402	0.0573958	0.0067788
0.002	4.59771×10^{-14}	3.27294×10^{-13}	2.66768×10^{-16}	0.0182815	0.144924	0.0171165
0.003	6.01186×10^{-14}	4.2899×10^{-13}	3.79362×10^{-16}	0.0283046	0.224382	0.0265008
0.004	6.88616×10^{-14}	4.8983×10^{-13}	4.66857×10^{-16}	0.0385993	0.305991	0.0361394
0.005	7.17343×10^{-14}	5.0937×10^{-13}	5.21176×10^{-16}	0.048826	0.387063	0.0457144
0.006	6.88061×10^{-14}	4.88942×10^{-13}	5.33536×10^{-16}	0.0590527	0.468134	0.0552894
0.007	6.01602×10^{-14}	4.27214×10^{-13}	4.95914×10^{-16}	0.0693476	0.549747	0.064928
0.008	4.58661×10^{-14}	3.25517×10^{-13}	4.00071×10^{-16}	0.0793696	0.629193	0.0743118
0.009	2.57711×10^{-14}	1.82521×10^{-13}	2.37549×10^{-16}	0.0904148	0.716769	0.084651
0.01	1.38778×10^{-17}	0	0	0.0976402	0.773991	0.0914234

Table 5
Absolute error via the B-Spline scheme.

	Septic		
Value of ϒ	Extended exp. method	Gen. Kudryashov method	Mod. Khater method
0	8.32667×10^{-17}	0	0
0.001	0	0	1.35525×10^{-20}
0.002	1.66533×10^{-16}	0	2.71051×10^{-20}
0.003	1.38778×10^{-17}	0	1.0842×10^{-19}
0.004	1.38778×10^{-16}	4.44089×10^{-16}	5.42101×10^{-20}
0.005	1.38778×10^{-16}	8.88178×10^{-16}	2.1684×10^{-19}
0.006	8.32667×10^{-17}	4.44089×10^{-16}	1.0842×10^{-19}
0.007	1.38778×10^{-17}	8.88178×10^{-16}	1.0842×10^{-19}
0.008	2.77556×10^{-17}	4.44089×10^{-16}	0
0.009	2.77556×10^{-17}	4.44089×10^{-16}	0
0.01	0	0	2.1684×10^{-19}

CRediT authorship contribution statement

Mostafa M.A. Khater: Conceptualization, Methodology, Software, Investigation, Writing - review & editing, Visualization, Supervision. **Raghda A.M. Attia:** Conceptualization, Methodology, Software, Investigation, Writing - review & editing, Visualization, Supervision. **Choonkil Park:** Data curation, Writing - original draft, Software, Investigation. **Dianchen Lu:** Data curation, Writing - original draft, Software, Investigation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

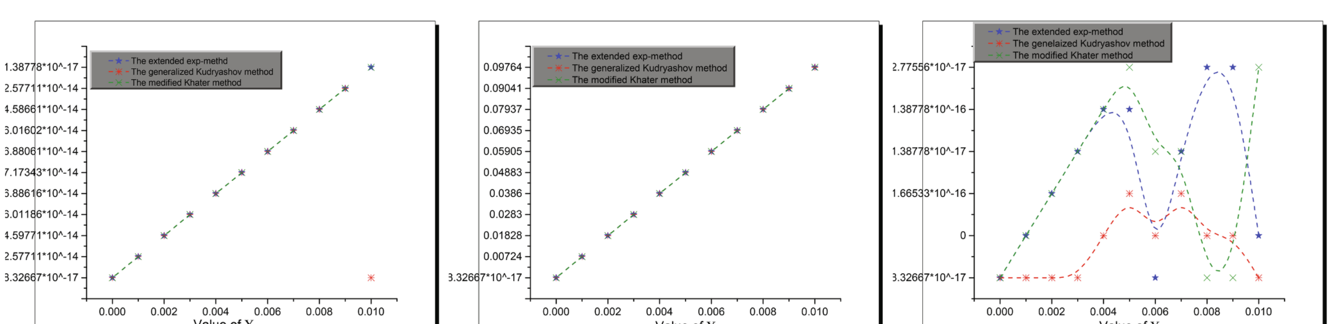

Fig. 4. Absolute values of error by using the B-spline schemes for the three analytical schemes.

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