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ORIGINAL ARTICLE

An explicit plethora of solution for the fractional nonlinear model of the low–pass electrical transmission lines via Atangana–Baleanu derivative operator

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Stability property;
Cubic & Septic B–spline schemes.

Abstract Novel explicit wave solutions are constructed for the fractional nonlinear model of the low–pass electrical transmission lines. A new fractional definition (Atangana–Baleanu derivative operator) is employed through the modified Khater method to get new wave solutions in distinct types of this model. The stability property of the obtained solutions is tested to show the ability of our obtained solutions in using through the physical experiments. Moreover, the obtained analytical solutions are used to evaluate the initial and boundary conditions that allows applying the cubic & septic B–spline schemes to investigate the numerical solutions of this model. The novelty and advantage of the proposed method are illustrated by applying to this model. Some sketches are plotted to show more about the dynamical behavior of this model.

1. Introduction

Fractional nonlinear evolution equation is one of the noticeable branches of science, particularly in recent years. Fractional calculus has a great profound physical background where it able to formulate many various phenomena in distinct fields such as physics, mechanical engineering, economics, chemistry, signal processing, food supplement, applied mathematics, quasi–chaotic dynamical systems, hydrodynamics,

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system identification, statistics, finance, fluid mechanics, solid–state biology, dynamical systems with chaotic dynamical behavior, optical fibers, electric control theory, economics and diffusion problems. The mathematical modeling of these phenomena will contain fractional derivative which provides a great explanation of the nonlocal property of these models since it depends on both of historical and current states of the problem in the contract of the classical calculus which depends on the current state only. Based on this importance of this kind of calculus, many definitions have been being derived such as conformable fractional derivative, fractional Riemann–Liouville derivatives, Caputo, Caputo–Fabrizio definition, and so on [11,12,19,20,37,39,51]. These definitions have been being employed to convert the fractional nonlinear partial differential equations to nonlinear integer–order ordinary differential equation and then the computational and numerical schemes can be applied to get various types of solutions for these models and the examples of these schemes [1,2,6,7,14,17,18,21,22,32–34,36,38,40,42–48,56,58].

Recently, The mK method is formulated and applied to distinct physical models such as the complex Ginzburg–Landau model, the (2 + 1)–dimensional KD equation and KdV equation, and the fractional (N + 1) Sinh–Gordon, biological population, equal width, modified equal width, Duffing equations and so on [4,5,10,25,26,29–31,35,50].

This method depends on a new auxiliary equation, which is equal to the auxiliary equation of the exp

$$-\Phi(\eth)$$

–expansion function method [3]. The auxiliary equation of the mK method is given by

$$\mathcal{E}'(\eth) = \frac{1}{ln(\mathcal{Q})}\left[\delta\,\mathcal{Q}^{\mathcal{E}(\eth)} + \varrho\,\mathcal{Q}^{-\mathcal{E}(\eth)} + \chi\right], \tag{1}$$

where δ, ϱ, χ, $\mathcal{Q}$ are arbitrary constants. While the auxiliary equation of the exp

$$-\Phi(\eth)$$

–expansion function method is given by

$$\Phi'(\eth) = e^{\Phi(\eth)} + \mu\,e^{-\Phi(\eth)} + \lambda, \tag{2}$$

$$\mathcal{S}(\eth) = \sum_{i=1}^{N} a_i\,\mathcal{Q}^{i\,\mathcal{E}(\eth)} + \sum_{i=1}^{N} b_i\,\mathcal{Q}^{-i\,\mathcal{E}(\eth)} + a_0 = a_1\,\mathcal{Q}^{\mathcal{E}(\eth)} + a_0 + b_1\,\mathcal{Q}^{-\mathcal{E}(\eth)}, \tag{9}$$

where μ, λ are arbitrary constants. So Eq. (1) and (2) are equal when $[\mathcal{E}(\wp) = \Phi(\wp),\ \mathcal{Q} = e,\ \varrho = \mu,\ \delta = 1,\ \chi = \lambda]$. Using this technique leads to the equal of the mK auxiliary equation with many analytical other methods. However, the mK method can obtain more solutions than almost of them. This equivalence shows a superiority, power, and productive of the mK method.

In this context, the mK method is employed to construct new formulas of solutions for the fractional nonlinear model of the low–pass electrical transmission lines which is given by [3,15,23,53–55,57]

$$D_{tt}^{2\vartheta}\mathcal{S} - \alpha D_{tt}^{2\vartheta}\mathcal{S}^2 - \sigma D_{tt}^{2\vartheta}\mathcal{S}^3 - \lambda^2 D_{xx}^{2\vartheta}\mathcal{S} - \frac{\lambda^4}{12}D_{xxxx}^{4\vartheta}\mathcal{S} = 0, \tag{3}$$

where $[\mathcal{S} = \mathcal{S}(x,t)]$ is the function that is used to describe the dynamical behavior of the nonlinear wave processes low–pass electrical transmission lines. Additionally, $[\alpha, \sigma, \lambda]$ are arbitrary constants while $[\vartheta \in]0,1[]$. Applying the next definition of $\mathcal{ABR}$ fractional operator [8,9,27,28] to Eq. (3)

Definition 1.1. It is given by [16]

$$^{\mathcal{ABR}}\mathcal{D}_{a+}^{\vartheta}\mathcal{F}(t) = \frac{\mathcal{B}(\vartheta)}{1-\vartheta}\frac{d}{dt}\int_a^t \mathcal{F}(x)\,\mathcal{G}_\vartheta\left(\frac{-\vartheta\,(t-\vartheta)^{\vartheta}}{1-\vartheta}\right)dx, \tag{4}$$

where $\mathcal{G}_\vartheta$ is the Mittag–Leffler function, and define by the following formula

$$\mathcal{G}_\vartheta\left(\frac{-\vartheta\,(t-\vartheta)^{\vartheta}}{1-\vartheta}\right) = \sum_{n=0}^{\infty}\frac{\left(\frac{-\vartheta}{1-\vartheta}\right)^n (t-x)^{\vartheta n}}{\Gamma(\vartheta n+1)} \tag{5}$$

and $\mathcal{B}(\vartheta)$ being a normalisation function. Thus

$$^{\mathcal{ABR}}\mathcal{D}_{a+}^{\vartheta}\mathcal{F}(x) = \frac{\mathcal{B}(\vartheta)}{1-\vartheta}\sum_{n=0}^{\infty}\left(\frac{-\vartheta}{1-\vartheta}\right)^n {}^{RL}\mathcal{I}_a^{\vartheta n}\mathcal{F}(x), \tag{6}$$

leads to

$$\mathcal{S}(x,t) = \mathcal{S}(\eth),\ \eth = x + \frac{k\,(1-\vartheta)\,t^{-\vartheta n}}{B(\vartheta)\sum_{n=0}^{\infty}\left(-\frac{\vartheta}{1-\vartheta}\right)^n\Gamma(1-\vartheta n)}, \tag{7}$$

where k are arbitrary constants.

This wave transformation converts Eq. (3) to ODE. Twice integration of the obtained ODEs with zero constant of the integration, gives

$$(k^2-\lambda)\,\mathcal{S} - \alpha k^2\,\mathcal{S}^2 + \sigma k^2\,\mathcal{S}^3 - \frac{\lambda^4}{12}\mathcal{S}'' = 0, \tag{8}$$

Calculating the homogeneous balance value in Eqs. (8) yields $N = 1$. Thus, the general formula of solution according to the mK method, is given by

where a_0, a_1, a_2, b_1, b_2 are arbitrary constants.

The order for the rest of this article is shown in the following order; Section 2 applies the mK method and two schemes of B–spline scheme [13,24,41,49,52] to the fractional nonlinear model of the low–pass electrical transmission lines. Moreover, some sketches are given to show more physical properties of both models. Section 3 discusses the stability property of the obtained solutions. Section 4 gives the conclusion of the whole research.

2. Computational and numerical solutions of the nonlinear fractional transmission lines mathematical model

2.1. Analytical wave solutions

Applying the mK method with its auxiliary equation and the suggested general solutions for the nonlinear fractional transmission lines equation, lead to a system of algebraic equations. Using the Mathematica 11.2 to find the values of the parameters in this system, leads to.

Family I

$$\left[a_0 \to \frac{6\alpha\sigma-\sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6-2\delta\lambda^3\varrho\right)+9\delta\lambda^3\sigma\varrho\right)}}{18\sigma^2}, a_1 \to -\frac{i\delta\lambda^{3/2}\sqrt{2\alpha^2-9\sigma}}{3\sqrt{6}\sigma}, b_1 \to 0, k \to -\frac{3\sqrt{\lambda}\sqrt{\sigma}}{\sqrt{9\sigma-2\alpha^2}},\right.$$
$$\left.\chi \to -\frac{2i\sqrt{\sigma^2\left(\alpha^2\left(6-2\delta\lambda^3\varrho\right)+9\delta\lambda^3\sigma\varrho\right)}}{\lambda^{3/2}\sigma\sqrt{2\alpha^2-9\sigma}}, \text{ where } (2\alpha^2-9\sigma<0,\ \sigma>0,\ \lambda>0)\right].$$

Consequently, the closed forms of solutions for the fractional transmission lines model are given by:

When $[\chi^2-4\delta\varrho<0\,\&\,\delta\neq 0]$

$$\mathcal{S}_1 = \frac{1}{18\sigma^2}\left[i\lambda^{3/2}\sigma\sqrt{3\alpha^2-\frac{27\sigma}{2}}\left(\chi-\sqrt{4\delta\varrho-\chi^2}\tan\left(\frac{1}{2}\Game\sqrt{4\delta\varrho-\chi^2}\right)\right)\right.$$
$$\left.-\sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6-2\delta\lambda^3\varrho\right)+9\delta\lambda^3\sigma\varrho\right)}+6\alpha\sigma\right], \tag{10}$$

$$\mathcal{S}_2 = \frac{1}{18\sigma^2}\left[i\lambda^{3/2}\sigma\sqrt{3\alpha^2-\frac{27\sigma}{2}}\left(\chi-\sqrt{4\delta\varrho-\chi^2}\cot\left(\frac{1}{2}\Game\sqrt{4\delta\varrho-\chi^2}\right)\right)\right.$$
$$\left.-\sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6-2\delta\lambda^3\varrho\right)+9\delta\lambda^3\sigma\varrho\right)}+6\alpha\sigma\right]. \tag{11}$$

When $[\chi^2-4\delta\varrho>0\,\&\,\delta\neq 0]$

$$\mathcal{S}_3 = \frac{1}{36\sigma^2}\left[i\sqrt{6}\lambda^{3/2}\sigma\sqrt{2\alpha^2-9\sigma}\sqrt{\chi^2-4\delta\varrho}\tanh\left(\frac{1}{2}\Game\sqrt{\chi^2-4\delta\varrho}\right)\right.$$
$$\left.+\sqrt{6}\left(-2\sqrt{\sigma^2\left(\alpha^2\left(6-2\delta\lambda^3\varrho\right)+9\delta\lambda^3\sigma\varrho\right)}+i\lambda^{3/2}\sigma\chi\sqrt{2\alpha^2-9\sigma}\right)+12\alpha\sigma\right], \tag{12}$$

$$\mathcal{S}_4 = \frac{1}{36\sigma^2}\left[i\sqrt{6}\lambda^{3/2}\sigma\sqrt{2\alpha^2-9\sigma}\sqrt{\chi^2-4\delta\varrho}\coth\left(\frac{1}{2}\Game\sqrt{\chi^2-4\delta\varrho}\right)\right.$$
$$\left.+\sqrt{6}\left(-2\sqrt{\sigma^2\left(\alpha^2\left(6-2\delta\lambda^3\varrho\right)+9\delta\lambda^3\sigma\varrho\right)}+i\lambda^{3/2}\sigma\chi\sqrt{2\alpha^2-9\sigma}\right)+12\alpha\sigma\right]. \tag{13}$$

When $[\delta\varrho>0\,\&\,\varrho\neq 0\,\&\,\delta\neq 0\,\&\,\chi=0]$

$$\mathcal{S}_5 = \frac{-1}{18\sigma^2}\left[i\sqrt{6}\lambda^{3/2}\sigma\sqrt{2\alpha^2-9\sigma}\sqrt{\delta\varrho}\tan\left(\Game\sqrt{\delta\varrho}\right)+\sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6-2\delta\lambda^3\varrho\right)+9\delta\lambda^3\sigma\varrho\right)}-6\alpha\sigma\right], \tag{14}$$

$$\mathcal{S}_6 = \frac{1}{18\sigma^2}\left[i\sqrt{6}\lambda^{3/2}\sigma\sqrt{2\alpha^2-9\sigma}\sqrt{\delta\varrho}\cot\left(\Game\sqrt{\delta\varrho}\right)-\sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6-2\delta\lambda^3\varrho\right)+9\delta\lambda^3\sigma\varrho\right)}+6\alpha\sigma\right]. \tag{15}$$

When $[\delta\varrho<0\,\&\,\varrho\neq 0\,\&\,\delta\neq 0\,\&\,\chi=0]$

$$\mathcal{S}_7 = \frac{1}{18\sigma^2}\left[i\sqrt{6}\lambda^{3/2}\sigma\sqrt{2\alpha^2-9\sigma}\sqrt{-\delta\varrho}\tanh\left(\Game\sqrt{-\delta\varrho}\right)-\sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6-2\delta\lambda^3\varrho\right)+9\delta\lambda^3\sigma\varrho\right)}+6\alpha\sigma\right], \tag{16}$$

$$\mathcal{S}_8 = \frac{1}{18\sigma^2}\left[i\sqrt{6}\lambda^{3/2}\sigma\sqrt{2\alpha^2-9\sigma}\sqrt{-\delta\varrho}\coth\left(\Game\sqrt{-\delta\varrho}\right)-\sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6-2\delta\lambda^3\varrho\right)+9\delta\lambda^3\sigma\varrho\right)}+6\alpha\sigma\right]. \tag{17}$$

When $[\chi=0\,\&\,\varrho=-\delta]$

$$\mathcal{S}_9 = \frac{-1}{18\sigma^2}\left[-i\sqrt{6}\lambda^{3/2}\sigma\varrho\sqrt{2\alpha^2-9\sigma}\coth(\Game\varrho)+\sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(2\lambda^3\varrho^2+6\right)-9\lambda^3\sigma\varrho^2\right)}-6\alpha\sigma\right]. \tag{18}$$

When $[\chi = \delta = \kappa \,\&\, \varrho = 0]$

$$\mathcal{S}_{10} = \frac{1}{18\sigma^2}\left[6\alpha\sigma + \sqrt{6}\left(-\sqrt{6}\sqrt{\alpha^2\sigma^2} + \frac{i\kappa\lambda^{3/2}\sigma\sqrt{2\alpha^2 - 9\sigma}\,e^{\kappa\eth}}{e^{\kappa\eth} - 1}\right)\right]. \tag{19}$$

When $[\varrho = 0 \,\&\, \chi \neq 0 \,\&\, \delta \neq 0]$

$$\mathcal{S}_{11} = \frac{1}{18\sigma^2}\left[\frac{i\sqrt{6}\delta\lambda^{3/2}\sigma\chi\sqrt{2\alpha^2 - 9\sigma}\,e^{\eth\chi}}{\delta e^{\eth\chi} - 2} - 6\sqrt{\alpha^2\sigma^2} + \alpha\sigma\right]. \tag{20}$$

When $[\chi = \varrho = 0 \,\&\, \delta \neq 0]$

$$\mathcal{S}_{12} = \frac{1}{18\eth\sigma^2}\left[6\alpha\eth\sigma + \sqrt{6}\left(-\sqrt{6}\eth\sqrt{\alpha^2\sigma^2} + i\lambda^{3/2}\sigma\sqrt{2\alpha^2 - 9\sigma}\right)\right]. \tag{21}$$

When $[\chi = 0 \,\&\, \varrho = \delta]$

$$\mathcal{S}_{13} = \frac{-1}{18\sigma^2}\left[\sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6 - 2\lambda^3\varrho^2\right) + 9\lambda^3\sigma\varrho^2\right)} - 6\alpha\sigma + i\sqrt{6}\lambda^{3/2}\sigma\varrho\sqrt{2\alpha^2 - 9\sigma}\tan(C + \eth\varrho)\right]. \tag{22}$$

When $[\chi^2 - 4\delta\varrho = 0]$

$$\mathcal{S}_{14} = \frac{1}{18\eth\sigma^2\chi^2}\left[\eth\chi^2\left(6\alpha\sigma - \sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6 - 2\delta\lambda^3\varrho\right) + 9\delta\lambda^3\sigma\varrho\right)}\right) + 2i\sqrt{6}\delta\lambda^{3/2}\sigma\varrho\sqrt{2\alpha^2 - 9\sigma}(\eth\chi + 2)\right]. \tag{23}$$

When $[\chi^2 - 4\delta\varrho < 0 \,\&\, \delta \neq 0]$

$$\mathcal{S}_{15} = \frac{1}{18\sigma^2}\left[6\alpha\sigma + \sqrt{6}\left(-\sqrt{\sigma^2\left(\alpha^2\left(6 - 2\delta\lambda^3\varrho\right) + 9\delta\lambda^3\sigma\varrho\right)} + \frac{2i\delta\lambda^{3/2}\sigma\varrho\sqrt{2\alpha^2 - 9\sigma}}{\chi - \sqrt{4\delta\varrho - \chi^2}\tan\left(\frac{1}{2}\eth\sqrt{4\delta\varrho - \chi^2}\right)}\right)\right], \tag{24}$$

$$\mathcal{S}_{16} = \frac{1}{18\sigma^2}\left[6\alpha\sigma + \sqrt{6}\left(-\sqrt{\sigma^2\left(\alpha^2\left(6 - 2\delta\lambda^3\varrho\right) + 9\delta\lambda^3\sigma\varrho\right)} + \frac{2i\delta\lambda^{3/2}\sigma\varrho\sqrt{2\alpha^2 - 9\sigma}}{\chi - \sqrt{4\delta\varrho - \chi^2}\cot\left(\frac{1}{2}\eth\sqrt{4\delta\varrho - \chi^2}\right)}\right)\right]. \tag{25}$$

When $[\chi^2 - 4\delta\varrho > 0 \,\&\, \delta \neq 0]$

$$\mathcal{S}_{17} = \frac{1}{18\sigma^2}\left[6\alpha\sigma + \sqrt{6}\left(-\sqrt{\sigma^2\left(\alpha^2\left(6 - 2\delta\lambda^3\varrho\right) + 9\delta\lambda^3\sigma\varrho\right)} + \frac{2i\delta\lambda^{3/2}\sigma\varrho\sqrt{2\alpha^2 - 9\sigma}}{\sqrt{\chi^2 - 4\delta\varrho}\tanh\left(\frac{1}{2}\eth\sqrt{\chi^2 - 4\delta\varrho}\right) + \chi}\right)\right], \tag{26}$$

$$\mathcal{S}_{18} = \frac{1}{18\sigma^2}\left[6\alpha\sigma + \sqrt{6}\left(-\sqrt{\sigma^2\left(\alpha^2\left(6 - 2\delta\lambda^3\varrho\right) + 9\delta\lambda^3\sigma\varrho\right)} + \frac{2i\delta\lambda^{3/2}\sigma\varrho\sqrt{2\alpha^2 - 9\sigma}}{\sqrt{\chi^2 - 4\delta\varrho}\coth\left(\frac{1}{2}\eth\sqrt{\chi^2 - 4\delta\varrho}\right) + \chi}\right)\right]. \tag{27}$$

When $[\delta\varrho > 0 \,\&\, \varrho \neq 0 \,\&\, \delta \neq 0 \,\&\, \chi = 0]$

$$\mathcal{S}_{19} = \frac{-1}{18\sigma^2}\left[i\sqrt{6}\lambda^{3/2}\sigma\sqrt{2\alpha^2 - 9\sigma}\sqrt{\delta\varrho}\cot\left(\eth\sqrt{\delta\varrho}\right) + \sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6 - 2\delta\lambda^3\varrho\right) + 9\delta\lambda^3\sigma\varrho\right)} - 6\alpha\sigma\right], \tag{28}$$

$$\mathcal{S}_{20} = \frac{1}{18\sigma^2}\left[i\sqrt{6}\lambda^{3/2}\sigma\sqrt{2\alpha^2 - 9\sigma}\sqrt{\delta\varrho}\tan\left(\eth\sqrt{\delta\varrho}\right) - \sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6 - 2\delta\lambda^3\varrho\right) + 9\delta\lambda^3\sigma\varrho\right)} + 6\alpha\sigma\right]. \tag{29}$$

Where

$$\left[\eth = \frac{3\sqrt{\lambda}\sqrt{\sigma}(\vartheta - 1)t^{-m\vartheta}}{\sqrt{9\sigma - 2\alpha^2}B(\vartheta)\sum_{m=0}^{\infty}\left(-\frac{\vartheta}{1-\vartheta}\right)^m\Gamma(1 - m\vartheta)} + x\right].$$

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$$\left[a_0 \to \frac{6\alpha\sigma - \sqrt{6}\sqrt{-2\alpha^2\delta\lambda^3\sigma^2\varrho + 6\alpha^2\sigma^2 + 9\delta\lambda^3\sigma^3\varrho}}{18\sigma^2}, a_1 \to 0, b_1 \to -\frac{i\lambda^{3/2}\varrho\sqrt{2\alpha^2 - 9\sigma}}{3\sqrt{6}\sigma}, k \to -\frac{3\sqrt{\lambda}\sqrt{\sigma}}{\sqrt{9\sigma - 2\alpha^2}},\right.$$
$$\left.\chi \to -\frac{2i\sqrt{\sigma^2\left(\alpha^2\left(6 - 2\delta\lambda^3\varrho\right) + 9\delta\lambda^3\sigma\varrho\right)}}{\lambda^{3/2}\sigma\sqrt{2\alpha^2 - 9\sigma}}, \text{where}\,(2\alpha^2 - 9\sigma < 0, \sigma > 0, \lambda > 0)\right].$$

Consequently, the closed forms of solutions for the fractional transmission lines model are given by:

When $[\delta\varrho < 0 \,\&\, \varrho \neq 0 \,\&\, \delta \neq 0 \,\&\, \chi = 0]$

$$\mathcal{S}_{21} = \frac{-1}{18\sigma^2}\left[i\sqrt{6}\lambda^{3/2}\sigma\sqrt{2\alpha^2 - 9\sigma}\sqrt{-\delta\varrho}\coth\left(\backepsilon\sqrt{-\delta\varrho}\right) + \sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6 - 2\delta\lambda^3\varrho\right) + 9\delta\lambda^3\sigma\varrho\right)} - 6\alpha\sigma\right], \tag{30}$$

$$\mathcal{S}_{22} = \frac{-1}{18\sigma^2}\left[i\sqrt{6}\lambda^{3/2}\sigma\sqrt{2\alpha^2 - 9\sigma}\sqrt{-\delta\varrho}\tanh\left(\backepsilon\sqrt{-\delta\varrho}\right) + \sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6 - 2\delta\lambda^3\varrho\right) + 9\delta\lambda^3\sigma\varrho\right)} - 6\alpha\sigma\right]. \tag{31}$$

When $[\chi = 0 \,\&\, \varrho = -\delta]$

$$\mathcal{S}_{23} = \frac{-1}{18\sigma^2}\left[i\sqrt{6}\lambda^{3/2}\sigma\varrho\sqrt{2\alpha^2 - 9\sigma}\tanh(\backepsilon\varrho) + \sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(2\lambda^3\varrho^2 + 6\right) - 9\lambda^3\sigma\varrho^2\right)} - 6\alpha\sigma\right]. \tag{32}$$

When $[\chi = \kappa \,\&\, \varrho = 2\kappa \,\&\, \delta = 0]$

$$\mathcal{S}_{24} = \frac{-1}{18\sigma^2}\left[\frac{2i\sqrt{6}\kappa\lambda^{3/2}\sigma\sqrt{2\alpha^2 - 9\sigma}}{e^{\kappa\backepsilon} - 2} + 6\sqrt{\alpha^2\sigma^2} - 6\alpha\sigma\right]. \tag{33}$$

When $[\varrho \neq 0 \,\&\, \chi = 0 \,\&\, \delta = 0]$

$$\mathcal{S}_{25} = \frac{1}{18\backepsilon\sigma^2}\left[6\alpha\backepsilon\sigma + \sqrt{6}\left(-\sqrt{6}\backepsilon\sqrt{\alpha^2\sigma^2} - i\lambda^{3/2}\sigma\sqrt{2\alpha^2 - 9\sigma}\right)\right]. \tag{34}$$

When $[\chi = 0 \,\&\, \delta = \varrho]$

$$\mathcal{S}_{26} = \frac{-1}{18\sigma^2}\left[\sqrt{6}\sqrt{\sigma^2\left(\alpha^2\left(6 - 2\lambda^3\varrho^2\right) + 9\lambda^3\sigma\varrho^2\right)} - 6\alpha\sigma + i\sqrt{6}\lambda^{3/2}\sigma\varrho\sqrt{2\alpha^2 - 9\sigma}\cot(C + \backepsilon\varrho)\right]. \tag{35}$$

When $[\delta = 0]$

$$\mathcal{S}_{27} = \frac{1}{18\sigma^2}\left[\frac{i\sqrt{6}\lambda^{3/2}\sigma\chi\varrho\sqrt{2\alpha^2 - 9\sigma}}{\varrho - \chi e^{\backepsilon\chi}} - 6\sqrt{\alpha^2\sigma^2} + 6\alpha\sigma\right]. \tag{36}$$

When $[\chi^2 - 4\delta\varrho = 0]$

$$\mathcal{S}_{28} = \frac{1}{36\sigma^2}\left[\frac{i\sqrt{6}\lambda^{3/2}\backepsilon\sigma\chi^2\sqrt{2\alpha^2 - 9\sigma}}{\backepsilon\chi + 2} - 12\sqrt{\alpha^2\sigma^2} + 12\alpha\sigma\right]. \tag{37}$$

Where

$$\left[\backepsilon = \frac{3\sqrt{\lambda}\sqrt{\sigma}(\vartheta - 1)t^{-m\vartheta}}{\sqrt{9\sigma - 2\alpha^2}B(\vartheta)\sum_{m=0}^{\infty}\left(-\frac{\vartheta}{1-\vartheta}\right)^m\Gamma(1 - m\vartheta)} + x\right].$$

2.2. Numerical solutions

This section studies the numerical solutions of the fractional nonlinear model of the low–pass electrical transmission lines by applying the cubic & septic B–spline techniques that are considered as the most accurate numerical tools to get this type of solutions.

2.2.1. Cubic-spline

According to the cubic B–spline, the numerical solution of the fractional nonlinear model of the low–pass electrical transmission lines (8) is given by

$$\mathcal{S}(\backepsilon) = \sum_{i=-1}^{n+1}\mathcal{A}_i\,\mathcal{B}_i, \tag{38}$$

where $\boldsymbol{\mathcal{A}}_i$, $\boldsymbol{\mathcal{B}}_i$ fulfill the next conditions:

$$\mathcal{L}\,\mathcal{S}(\backepsilon) = \emptyset(\backepsilon_i, \mathcal{S}(\backepsilon_i)) \text{ where } (i = 0, 1, ..., n)$$

and

$$\mathcal{B}_i(\backepsilon) = \frac{1}{6\,h^3}\begin{cases} (\backepsilon - \backepsilon_{i-2})^3, & \backepsilon \in [\backepsilon_{i-2}, \backepsilon_{i-1}], \\ -3\,(\backepsilon - \backepsilon_{i-1})^3 + 3\,h\,(\backepsilon - \backepsilon_{i-1})^2 + 3\,h^2\,(\backepsilon - \backepsilon_{i-1}) + h^3, & \backepsilon \in [\backepsilon_{i-1}, \backepsilon_i], \\ -3\,(\backepsilon_{i+1} - \backepsilon)^3 + 3\,h\,(\backepsilon_{i+1} - \backepsilon)^2 + 3\,h^2\,(\backepsilon_{i+1} - \backepsilon) + h^3, & \backepsilon \in [\backepsilon_i, \backepsilon_{i+1}], \\ (\backepsilon_{i+2} - \backepsilon)^3, & \backepsilon \in [\backepsilon_{i+1}, \backepsilon_{i+2}], \\ 0, & \text{otherwise}, \end{cases} \tag{39}$$

Table 1 Computational, numerical, and absolute value of error that obtained by using cubic B–spline scheme

Value of $\eth$	Val. Com.	Val. Num.	Value of abs. error
0	0.122706	0.122706	0
0.001	0.122456	0.122455	$8.42148\,10^{-7}$
0.002	0.122206	0.122205	$1.4961\,10^{-6}$
0.003	0.121956	0.121954	$1.96225\,10^{-6}$
0.004	0.121706	0.121704	$2.24099\,10^{-6}$
0.005	0.121456	0.121454	$2.33272\,10^{-6}$
0.006	0.121206	0.121204	$2.23782\,10^{-6}$
0.007	0.120956	0.120954	$1.9567\,10^{-6}$
0.008	0.120706	0.120705	$1.48976\ \ 10^{-6}$
0.009	0.120456	0.120455	$8.37389\ \ 10^{-7}$
0.01	0.120206	0.120206	$1.38778\ \ 10^{-17}$

Table 2 Computational, numerical, and absolute value of error that obtained by using septic B–spline scheme

Value of $\eth$	Val. Com.	Val. Num.	Value of abs. error
0	0.122706	0.122706	0
0.001	0.122456	0.122455	$7.47116\,10^{-7}$
0.002	0.122206	0.122205	$1.5887\,10^{-6}$
0.003	0.121956	0.121954	$1.96668\,10^{-6}$
0.004	0.121706	0.121704	$2.28605\,10^{-6}$
0.005	0.121456	0.121454	$2.35458\,10^{-6}$
0.006	0.121206	0.121204	$2.28277\,10^{-6}$
0.007	0.120956	0.120954	$1.96121\,10^{-6}$
0.008	0.120706	0.120705	$1.58179\,10^{-6}$
0.009	0.120456	0.120456	$7.43057\,10^{-7}$
0.01	0.120206	0.120206	$2.77556\ \ 10^{-17}$

where $i \in [-2, n+2]$. So that, the numerical formula of the solution is given as

$$\mathcal{S}_i(\eth) = \mathcal{A}_{i-1} + 4\,\mathcal{A}_i + \mathcal{A}_{i+1}. \tag{40}$$

Substituting Eq. (40) into (8), leads to a system of equations. Solving this system of equations, gives the value of $\mathcal{A}_i$. Replacing the values of $\mathcal{A}_i$, $\mathcal{B}_i$ into Eq. (38), gives the following data that are shown in the next Table 1

2.2.2. Septic-spline

Based on the septic B-spline, the suggested solution of the ordinary differential form of the fractional nonlinear model of the low–pass electrical transmission lines (8) is given as follow

$$\mathcal{S}(\eth) = \sum_{i=-1}^{n+1} \mathcal{A}_i\,\mathcal{B}_i, \tag{41}$$

where $\mathcal{A}_i$, $\mathcal{B}_i$ satisfies the next conditions

$$\mathcal{L}\,\mathcal{S}(\eth) = \emptyset(\eth_i, \mathcal{S}(x_i)) \text{ where } (i = 0, 1, \ldots, n)$$

and

$$\mathcal{B}_i(\eth) = \frac{1}{h^5}\begin{cases} (\eth-\eth_{i-4})^7, & \eth \in [\eth_{i-4}, \eth_{i-3}],\\ (\eth-\eth_{i-4})^7 - 8(\eth-\eth_{i-3})^7, & \eth \in [\eth_{i-3}, \eth_{i-2}],\\ (\eth-\eth_{i-4})^7 - 8(\eth-\eth_{i-3})^7 + 28\eth(\eth-\eth_{i-2})^7, & \eth \in [\eth_{i-2}, \eth_{i-1}],\\ (\eth-\eth_{i-4})^7 - 8(\eth-\eth_{i-3})^7 + 28(\eth-\eth_{i-2})^7 + 56(\eth-\eth_{i-1})^7, & \eth \in [\eth_{i-1}, \eth_i],\\ (\eth_{i+4}-\eth)^7 - 8(\eth_{i+3}-\eth)^7 + 28(\eth_{i+2}-\eth)^7 + 56(\eth_{i+1}-\eth)^7, & \eth \in [\eth_i, \eth_{i+1}],\\ (\eth_{i+4}-\eth)^7 - 8(\eth_{i+3}-\eth)^7 + 28(\eth_{i+2}-\eth)^7, & \eth \in [\eth_{i+1}, \eth_{i+2}],\\ (\eth_{i+4}-\eth)^7 - 8(\eth_{i+3}-\eth)^7, & \eth \in [\eth_{i+2}, \eth_{i+3}],\\ (\eth_{i+4}-\eth)^7, & \eth \in [\eth_{i+3}, \eth_{i+4}],\\ 0, & \text{otherwise}, \end{cases} \tag{42}$$

where $i \in [-3, n+3]$. Thus, the approximate solution is given by

$$\mathcal{S}_i(\eth) = \mathcal{A}_{i-3} + 120\,\mathcal{A}_{i-2} + 1191\,\mathcal{A}_{i-1} + 2416\,\mathcal{A}_i + 1191\,\mathcal{A}_{i+1} + 120\,\mathcal{A}_{i+2} + \mathcal{A}_{i+3}. \tag{43}$$

Substituting Eq. (43) into Eq. (8), obtains a system of equations. Solving this system, gives the following data that are shown in the next Table 2

3. Stability

This section of our research paper investigates one of the basic properties of any model. It examines the stability property for the fractional nonlinear model of the low–pass electrical transmission lines by using a Hamiltonian system. The momentum in the Hamiltonian system given by the following formula:

$$\mathcal{M} = \frac{1}{2}\int_{-\backepsilon}^{\backepsilon} \mathcal{S}^2(\eth)\,d\eth, \tag{44}$$

where $\backepsilon$ is arbitrary constant. Thus, the condition for stability is given in the next condition:

$$\frac{\partial \mathcal{M}}{\partial k}\bigg|_{k=\flat} > 0. \tag{45}$$

where c, $\flat$ are arbitrary constants.

For an example of studying the stability of the solution of Eq. (3) by using (12) with the following values of the constants $\left[\alpha = \frac{3}{2},\ \delta = 1,\ \lambda = 1,\ \sigma = 2,\ \varrho = -1\right]$, yields:

$$\mathcal{M} = \frac{1}{432k}\Big[50\left(61 - 6\sqrt{21}\right)k + 27\log\left(e^{10-5k} + e^{5k}\right) - 27\log\left(e^{-5k} + e^{5(k+2)}\right)\Big] \tag{46}$$

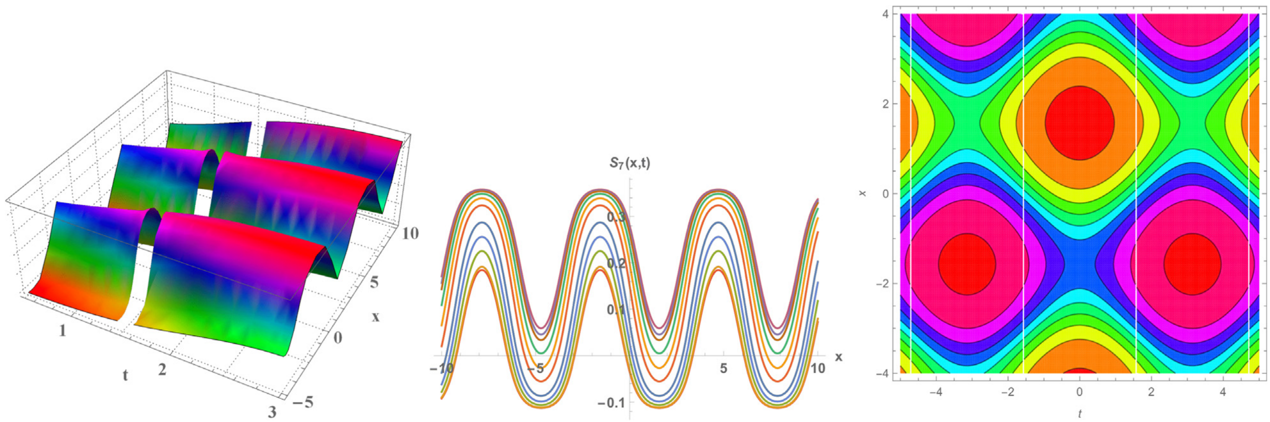

Fig. 1 Numerical simulations of Eq. (16) in three different types $\left[\alpha = \frac{3}{2}, \delta = 1, \lambda = 1, k = -\frac{2}{\sqrt{3}}, \sigma = 2, \varrho = -1\right]$.

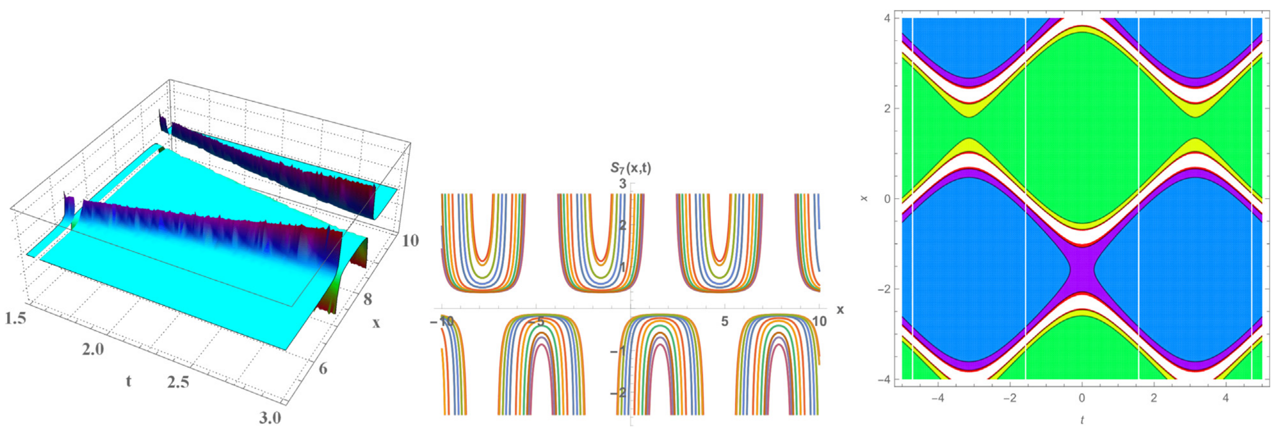

Fig. 2 Numerical simulations of Eq. (17) in three different types for $\left[\alpha = \frac{3}{2}, \delta = 1, \lambda = 1, k = -\frac{2}{\sqrt{3}}, \sigma = 2, \varrho = -1\right]$.

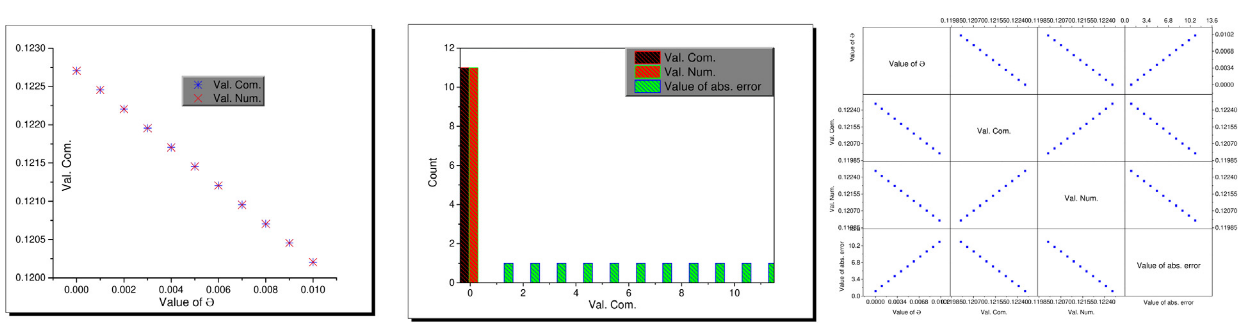

Fig. 3 Numerical simulations in three different types to show the relation between computational and numerical values according the values of 1.

Thus, we obtain

$$\left.\frac{\partial \mathcal{M}}{\partial k}\right|_{k=-\frac{2}{\sqrt{3}}} = -0.3647005019 < 0. \tag{47}$$

This means, this solution is unstable, and by applying the same steps to other obtained solutions, the stability property of each one of them can be determined.

4. Conclusion

This research is successfully applied the modified Khater method and B–spline schemes with a new fractional operator for the fractional nonlinear model of the low–pass electrical transmission lines. This new operator is used to avoid the disadvantage of the other fractional operator. Distinct, solitary wave solutions were obtained of this equation, and for more

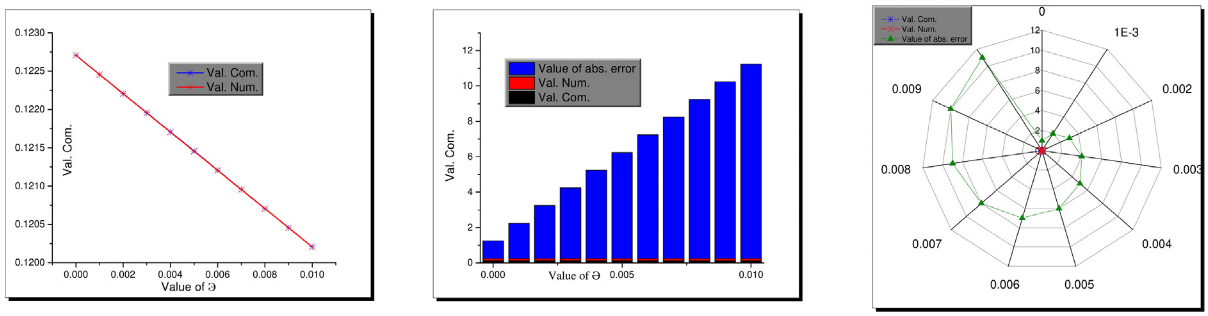

Fig. 4 Numerical simulations in three different types to show the relation between computational and numerical values according the values of 2.

illustration of the dynamical behavior of this kind of an electrical transmission, some solutions were sketched Figs. 1 and 2 in three different formula of each figure (two, three–dimensional, and contour plots). Moreover, the numerical solutions were also investigated to check the accurate values of the obtained analytical solutions. This accuracy was shown by Tables 1 and 2 and Figs. 3 and 4.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Mostafa Khater would like to dedicate this paper to my mother, son (Adam) and the soul of my father. their love, support, and constant care will never be forgotten.

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