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ORIGINAL ARTICLE

On new computational and numerical solutions of the modified Zakharov–Kuznetsov equation arising in electrical engineering

Choonkil Park [a], Mostafa M.A. Khater [b,*], Abdel-Haleem Abdel-Aty [d,e], Raghda A.M. Attia [b,c], Dianchen Lu [b]

[a] *Research Institute for Natural Sciences, Hanyang University, Seoul 04763, Republic of Korea*
[b] *Department of Mathematics, Faculty of Science, Jiangsu University, China*
[c] *Department of Basic Science, Higher Technological Institute 10th of Ramadan City, Egypt*
[d] *Department of Physics, College of Sciences, University of Bisha, Bisha 61922, P.O. Box 344, Saudi Arabia*
[e] *Physics Department, Faculty of Science, Al-Azhar University, 71524 Assiut, Egypt*

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Abstract In this research, analytical and numerical solutions are studied of a two–dimensional discrete electrical lattice, which is mathematically represented by the modified Zakharov–Kuznetsov equation. Moreover, the stability property of the obtained analytical solutions is investigated based on the Hamiltonian system, and then it is used to evaluate the initial and boundary conditions that are used in the numerical investigation. Many kinds of analytical solutions are obtained, such as complex, exponential, hyperbolic, and trigonometric function solutions. The functioning of both schemes of both techniques is tested and investigated.

1. Introduction

Partial differential equations have considered as a fundamental in many applications. This kind of equations has used to formulate many of natural, engineering, mechanical, and physical phenomena. That happens because it contains beforehand unknown multi-variable functions and its derivatives. During the last decade, many phenomena have been formulated in partial differential equations. Studying and investigation the solitary wave of these models are considered as one of the basic interesting of many researchers. Solitary wave is that kind of waves which propagates without any chronological evolution in shape or size. These properties and abilities of the nonlinear

* Corresponding author.
E-mail address: mostafa.khater2024@yahoo.com (M.M.A. Khater).

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partial differential equations are used to describe the natural phenomena. According to these properties, many mathematicians developed some methods and still trying to find new general methods to get exact and solitary traveling wave solutions of these models. For more details about these methods, you see [25–27,2,3,41,21,31,42,19,17,18,46,39,32,7,6,28,29, 36–38,16,50].

In this paper, a discrete nonlinear transmission line equation [14,20,30,48,40] is investigated by the modified Khater (mK) method [45,5,34,23,24,1], the Hamiltonian system [9,8,33,11,43], and B–spline scheme [12,51,22,10,4]. This equation is also known by the modified Zakharov–Kuznetsov (mZK) equation that helps in understanding the mechanism of various phenomena [52,15,35,13,44]. For example; the electrical transmission lines which is considered as a good example of systems for the investigation of the nonlinear excitations behave inside nonlinear media as shown in Fig. 1.

The nonlinear electrical transmission line is constructed based on periodically loading with var–actors or, alternatively, by arranging inductors and var–actors in a one–dimensional lattice. The nonlinear network with some couple nonlinear LC with dispersive transmission line is consisted in this model. There are many identical dispersive lines which are coupled by means of capacitance C_s at each node, as represented in Fig. 1 where a conductor L and a nonlinear capacitor of capacitance $C(v_{n,m})$ are consist in each line in the shunt branch. The mathematical model which describes the discrete nonlinear transmission is given by mZK equation that is formulated by Duan when he applied the Kirchhoff law on the model, is given by

$$\frac{\partial^2 \mathcal{Q}_{n,m}}{\partial \mathcal{T}} = \frac{1}{\mathcal{L}}(\mathcal{V}_{n+1,m} - 2\mathcal{V}_{n,m} + \mathcal{V}_{n-1,m}) + \mathcal{C}_s \frac{\partial^2}{\partial \mathcal{T}^2}(\mathcal{V}_{n,m+1} - 2\mathcal{V}_{n,m} + \mathcal{V}_{n,m-1}), \tag{1}$$

where $\mathcal{V}_{n,m} = \mathcal{V}_{n,m}(\mathcal{T})$ is the voltage so that the nonlinear charge is derived in the following form

$$\mathcal{Q}_{n,m} = \mathcal{C}_0\left(\mathcal{V}_{n,m} + \frac{\beta_1}{2}\mathcal{V}_{n,m}^2 + \frac{\beta_2}{3}\mathcal{V}_{n,m}^3\right), \tag{2}$$

where β_1, β_2 are arbitrary constants. Substituting Eq. (2) into Eq. (1), yields

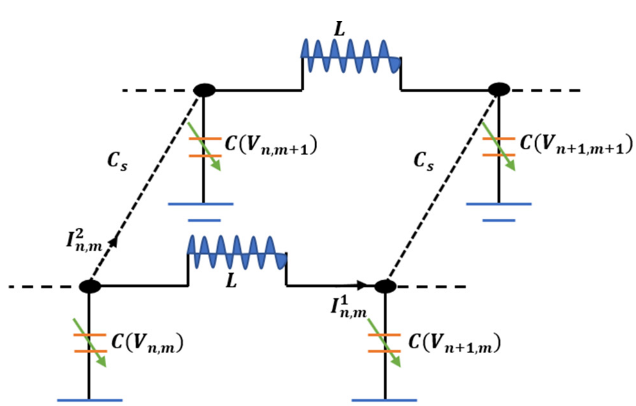

Fig. 1 Linear representation of the nonlinear electrical transmission line.

$$\mathcal{C}_0 \frac{\partial^2}{\partial \mathcal{T}^2}\left(\mathcal{V}_{n,m} + \frac{\beta_1}{2}\mathcal{V}_{n,m}^2 + \frac{\beta_2}{3}\mathcal{V}_{n,m}^3\right) = \frac{1}{\mathcal{L}}(\mathcal{V}_{n+1,m} - 2\mathcal{V}_{n,m} + \mathcal{V}_{n-1,m}) + \mathcal{C}_s \frac{\partial^2}{\partial \mathcal{T}^2}(\mathcal{V}_{n,m+1} - 2\mathcal{V}_{n,m} + \mathcal{V}_{n,m-1}). \tag{3}$$

Replacing $\mathcal{V}_{n,m}(T) = \mathcal{V}(n, m, \mathcal{T})$, leads to

$$\mathcal{C}_0 \frac{\partial^2}{\partial \mathcal{T}^2}\left(\mathcal{V} + \frac{\beta_1}{2}\mathcal{V}^2 + \frac{\beta_2}{3}\mathcal{V}^3\right) = \frac{1}{\mathcal{L}}\frac{\partial^2}{\partial n^2}\left(\mathcal{V} + \frac{1}{12}\frac{\partial^2 \mathcal{V}}{\partial n^2}\right) + \mathcal{C}_s \frac{\partial^4}{\partial \mathcal{T}^2 \partial m^2}\left(\mathcal{V} + \frac{1}{12}\frac{\partial^2 \mathcal{V}}{\partial m^2}\right). \tag{4}$$

Based on the reductive perturbation technique Eq. (4), is reduced to the following mZK equation

$$\mathcal{K}_t + l\mathcal{K}\mathcal{K}_x + m\mathcal{K}^2\mathcal{K}_x + d\mathcal{K}_{xxx} + q\mathcal{K}_{xyy} = 0, \tag{5}$$

where $\left[y = \sqrt{\chi}m,\ x = \sqrt{\chi}(n - v_s\mathcal{T}),\ t = \sqrt{\chi}\mathcal{T},\ \mathcal{V}(n,m,\mathcal{T}) = \chi\mathcal{K}(x,y,t),\ v_s^2 = \frac{1}{\mathcal{L}\mathcal{C}_0},\ l = -\beta_1 v_s,\ m = -\beta_2 v_s,\ d = \frac{1}{24\beta\beta_1\mathcal{L}v_s},\ q = \frac{\beta_1}{288\mathcal{L}^2 v_s \mathcal{C}_0^2}\right]$ since x, y, t are independent transformation variables. Applying the following wave transformation $\mathcal{K} = \mathcal{K}(x,y,t) = \mathcal{K}(\hbar),\ \hbar = h_1 x + h_2 y + h_3 t$ and integrate the obtained ODE once with zero constant of integration, give

$$6h_3\mathcal{K} + 3lh_1\mathcal{K}^2 + 2mh_1\mathcal{K}^3 + 6h_1\left(dh_1^2 + qh_2^2\right)\mathcal{K}'' = 0. \tag{6}$$

Balancing the highest order derivative term and nonlinear terms, yields $n = 1$.

The remaining of this paper is organized as follows: In Section 2, the mK method is used to obtain computational solutions of the mZK equation then the stability property of these solutions is tested. Moreover, the stable solution is used to find the initial and boundary conditions that allow applying the B–spline schemes to the same model to investigate the accuracy of obtained solutions. In Section 3, the comparison between our obtained solutions and that obtained in different research papers is represented. In Section 4, the conclusion is given.

2. Application

Here we apply the mK method to the mZK equation to establish the explicit wave solutions of both of models then test the stability property of these solutions. Moreover, we use these solutions in the investigation of numerical solution of the same models.

2.1. Explicit solutions of the mZK equation

According to the homogeneous balance value and the suggested general solution in the modified Khater method, the general solutions of Eq. (6) is in the following formula:

$$\mathcal{K}(\hbar) = \sum_{i=1}^{N} a_i \mathcal{M}^{i\Psi(\hbar)} + \sum_{i=1}^{N} b_i \mathcal{M}^{-i\Psi(\hbar)} + a_0 = a_1\mathcal{M}^{\Psi(\hbar)} + a_0 + b_1\mathcal{M}^{-\Psi(\hbar)}, \tag{7}$$

where a_0, a_1, b_1, $\mathcal{M}$ are arbitrary constants. Additionally $\Psi(\hbar)$ is the solution function of the next auxiliary equation

$$f'(\hbar) = \frac{1}{ln(\mathcal{M})}\left(\chi + \vartheta\mathcal{M}^{-\Psi(\hbar)} + \rho\mathcal{M}^{\Psi(\hbar)}\right), \tag{8}$$

where ϑ, χ, ρ are arbitrary constants to be determine later. Replacing Eq. (7) along (8) into Eq. (6) and gathering all terms with the same power of $\left[\mathcal{M}^{i\Psi(\hbar)}, i = -3, -2, \ldots, 2, 3\right]$, lead to system of algebraic equations. Solving this system, yields

Family I

$$\left[a_0 \to -\frac{l}{m}, a_1 \to -\frac{l\rho}{m\chi}, b_1 \to -\frac{l\vartheta}{m\chi}, h_2 \to -\frac{\sqrt{-6dh_1^2m\chi^2 - l^2}}{\sqrt{6}\sqrt{m}\sqrt{q}\chi}, h_3 \to \frac{h_1l^2\chi^2 - 4h_1l^2\rho\vartheta}{6m\chi^2}\right.$$

where $\left(l \neq 0,\ \rho \neq 0,\ \vartheta \neq 0,\ q > 0, m > 0, 6dh_1^2m\chi^2 < l^2\right)]$

Thus, the explicit wave solutions of Eq. (5) are given by When $[\chi^2 - 4\rho\vartheta < 0\,\&\,\rho \neq 0]$

$$\mathcal{K}_1(x,y,t) = -\frac{l(\chi^2-4\rho\vartheta)\sec^2\left(\frac{\sqrt{4\rho\vartheta-\chi^2}\left(h_1\left(l^2t\left(\chi^2-4\rho\vartheta\right)+6m\chi^2x\right)-\frac{\sqrt{6}\sqrt{m}\chi y\sqrt{-6dh_1^2m\chi^2-l^2}}{\sqrt{q}}\right)}{12m\chi^2}\right)}{2m\chi\left(\chi-\sqrt{4\rho\vartheta-\chi^2}\tan\left(\frac{\sqrt{4\rho\vartheta-\chi^2}\left(h_1\left(l^2t(\chi^2-4\rho\vartheta)+6m\chi^2x\right)-\frac{\sqrt{6}\sqrt{m}\chi y\sqrt{-6dh_1^2m\chi^2-l^2}}{\sqrt{q}}\right)}{12m\chi^2}\right)\right)}, \tag{9}$$

$$\mathcal{K}_2(x,y,t) = -\frac{l(\chi^2-4\rho\vartheta)\csc^2\left(\frac{\sqrt{4\rho\vartheta-\chi^2}\left(h_1\left(l^2t\left(\chi^2-4\rho\vartheta\right)+6m\chi^2x\right)-\frac{\sqrt{6}\sqrt{m}\chi y\sqrt{-6dh_1^2m\chi^2-l^2}}{\sqrt{q}}\right)}{12m\chi^2}\right)}{2m\chi\left(\chi-\sqrt{4\rho\vartheta-\chi^2}\cot\left(\frac{\sqrt{4\rho\vartheta-\chi^2}\left(h_1\left(l^2t(\chi^2-4\rho\vartheta)+6m\chi^2x\right)-\frac{\sqrt{6}\sqrt{m}\chi y\sqrt{-6dh_1^2m\chi^2-l^2}}{\sqrt{q}}\right)}{12m\chi^2}\right)\right)}. \tag{10}$$

When $[\chi^2 - 4\rho\vartheta > 0\,\&\,\rho \neq 0]$

$$\mathcal{K}_3(x,y,t) = -\frac{l(\chi^2-4\rho\vartheta)\mathrm{sech}^2\left(\frac{\sqrt{\chi^2-4\rho\vartheta}\left(h_1\left(l^2t\left(\chi^2-4\rho\vartheta\right)+6m\chi^2x\right)-\frac{\sqrt{6}\sqrt{m}\chi y\sqrt{-6dh_1^2m\chi^2-l^2}}{\sqrt{q}}\right)}{12m\chi^2}\right)}{2m\chi\left(\sqrt{\chi^2-4\rho\vartheta}\tanh\left(\frac{\sqrt{\chi^2-4\rho\vartheta}\left(h_1\left(l^2t(\chi^2-4\rho\vartheta)+6m\chi^2x\right)-\frac{\sqrt{6}\sqrt{m}\chi y\sqrt{-6dh_1^2m\chi^2-l^2}}{\sqrt{q}}\right)}{12m\chi^2}\right)+\chi\right)}, \tag{11}$$

$$\mathcal{K}_4(x,y,t) = \frac{l(\chi^2-4\rho\vartheta)\mathrm{csch}^2\left(\frac{\sqrt{\chi^2-4\rho\vartheta}\left(h_1\left(l^2t\left(\chi^2-4\rho\vartheta\right)+6m\chi^2x\right)-\frac{\sqrt{6}\sqrt{m}\chi y\sqrt{-6dh_1^2m\chi^2-l^2}}{\sqrt{q}}\right)}{12m\chi^2}\right)}{2m\chi\left(\sqrt{\chi^2-4\rho\vartheta}\coth\left(\frac{\sqrt{\chi^2-4\rho\vartheta}\left(h_1\left(l^2t(\chi^2-4\rho\vartheta)+6m\chi^2x\right)-\frac{\sqrt{6}\sqrt{m}\chi y\sqrt{-6dh_1^2m\chi^2-l^2}}{\sqrt{q}}\right)}{12m\chi^2}\right)+\chi\right)}. \tag{12}$$

When $\left[\chi = \frac{\vartheta}{2} = \kappa\,\&\,\rho = 0\right]$

$$\mathcal{K}_5(x,y,t) = \frac{l}{m}\left(-\frac{2}{\exp\left(h_1\left(\frac{\kappa l^2t}{6m}+\kappa x\right)-\frac{y\sqrt{-6dh_1^2\kappa^2m-l^2}}{\sqrt{6}\sqrt{m}\sqrt{q}}\right)-2}-1\right). \tag{13}$$

When $[\chi = \rho = \kappa\,\&\,\vartheta = 0]$

$$\mathcal{K}_6(x,y,t) = \frac{l}{m\left(\exp\left(h_1\left(\frac{\kappa l^2t}{6m}+\kappa x\right)-\frac{y\sqrt{-6dh_1^2\kappa^2m-l^2}}{\sqrt{6}\sqrt{m}\sqrt{q}}\right)-1\right)}. \tag{14}$$

When $[\vartheta = 0\,\&\,\chi \neq 0\,\&\,\rho \neq 0]$

$$\mathcal{K}_7(x,y,t) = -\frac{2l}{m\left(2-\rho\exp\left(h_1\left(\frac{l^2t\chi}{6m}+\chi x\right)-\frac{y\sqrt{-6dh_1^2m\chi^2-l^2}}{\sqrt{6}\sqrt{m}\sqrt{q}}\right)\right)}. \tag{15}$$

When $[\rho = 0\,\&\,\chi \neq 0\,\&\,\vartheta \neq 0]$

$$\mathcal{K}_8(x,y,t) = \frac{l}{m}\left(\frac{\vartheta}{\vartheta-\chi\exp\left(h_1\left(\frac{l^2t\chi}{6m}+\chi x\right)-\frac{y\sqrt{-6dh_1^2m\chi^2-l^2}}{\sqrt{6}\sqrt{m}\sqrt{q}}\right)}-1\right). \tag{16}$$

Family II

$$\left[a_0 \to -\frac{\frac{l\chi}{\sqrt{\chi^2-4\rho\vartheta}}+l}{2m}, a_1 \to -\frac{l\rho}{m\sqrt{\chi^2-4\rho\vartheta}}, b_1 \to 0, h_2 \to \frac{\sqrt{6dh_1^2m(\chi^2-4\rho\vartheta)+l^2}}{\sqrt{6}\sqrt{mq(4\rho\vartheta-\chi^2)}}, h_3 \to \frac{h_1l^2}{6m}\right.$$

where $\left(mq < 0,\ \chi^2 - 4\rho\vartheta > 0,\ 6dh_1^2m\left(\chi^2 - 4\rho\vartheta\right) + l^2 > 0,\ l \neq 0,\ \rho \neq 0,\ m \neq 0,\ h_1 \neq 0\right)]$

Thus, the solitary wave solutions of Eq. (5) are given by When $[\chi^2 - 4\rho\vartheta > 0\,\&\,\rho \neq 0]$

$$\mathcal{K}_9(x,y,t) = \frac{l}{2m}\left(\tanh\left(\frac{1}{2}\sqrt{\chi^2-4\rho\vartheta}\left(\frac{y\sqrt{6dh_1^2m(\chi^2-4\rho\vartheta)+l^2}}{\sqrt{6}\sqrt{mq(4\rho\vartheta-\chi^2)}}+h_1\left(\frac{l^2t}{6m}+x\right)\right)\right)+\frac{2\chi}{\sqrt{\chi^2-4\rho\vartheta}}+1\right), \tag{17}$$

$$\mathcal{K}_{10}(x,y,t) = \frac{l}{2m}\left(\coth\left(\frac{1}{2}\sqrt{\chi^2-4\rho\vartheta}\left(\frac{y\sqrt{6dh_1^2m(\chi^2-4\rho\vartheta)+l^2}}{\sqrt{6}\sqrt{mq(4\rho\vartheta-\chi^2)}}+h_1\left(\frac{l^2t}{6m}+x\right)\right)\right)+\frac{2\chi}{\sqrt{\chi^2-4\rho\vartheta}}+1\right). \tag{18}$$

When $[\rho\vartheta < 0\,\&\,\vartheta \neq 0\,\&\,\rho \neq 0\,\&\,\chi = 0]$

$$\mathcal{K}_{11}(x,y,t) = \frac{l}{2m}\left(\tanh\left(\sqrt{\rho(-\vartheta)}\left(\frac{y\sqrt{l^2-24dh_1^2m\rho\vartheta}}{2\sqrt{6}\sqrt{mq\rho\vartheta}}+h_1\left(\frac{l^2t}{6m}+x\right)\right)\right)+1\right), \tag{19}$$

$$\mathcal{K}_{12}(x,y,t) = \frac{l}{2m}\left(\coth\left(\sqrt{\rho(-\vartheta)}\left(\frac{y\sqrt{l^2-24dh_1^2m\rho\vartheta}}{2\sqrt{6}\sqrt{mq\rho\vartheta}}+h_1\left(\frac{l^2t}{6m}+x\right)\right)\right)+1\right). \tag{20}$$

When $[\chi = 0\&\vartheta = -\rho]$

$$\mathcal{K}_{13}(x,y,t) = \frac{1}{2m}\left(\frac{l\vartheta\coth\left(\frac{1}{6}\vartheta\left(\frac{\sqrt{\frac{3}{2}}y\sqrt{24dh_1^2m\vartheta^2+l^2}}{\sqrt{-mq\vartheta^2}}+h_1\left(\frac{l^2t}{m}+6x\right)\right)\right)}{\sqrt{\vartheta^2}}+l\right). \tag{21}$$

When $[\chi = \rho = \kappa\,\&\,\vartheta = 0]$

$$\mathcal{K}_{14}(x,y,t) = \frac{l}{2\sqrt{\kappa^2}m}\left(\kappa\left(\coth\left(\frac{1}{12}\kappa\left(\frac{\sqrt{6}y\sqrt{6dh_1^2\kappa^2m+l^2}}{\sqrt{\kappa^2(-m)q}}+h_1\left(\frac{l^2t}{m}+6x\right)\right)\right)+2\right)+\sqrt{\kappa^2}\right). \tag{22}$$

2.2. Stability property

This section studies the stability property of the obtained solutions by using the properties of the Hamiltonian system. The momentum Υ in the Hamiltonian system is given by

$$\Upsilon = \frac{1}{2}\int_{-\varsigma}^{\varsigma} \mathcal{K}^2(\hbar)\, d\hbar, \tag{23}$$

where $\mathcal{K}(\hbar)$ is the solution of the model and the necessary condition for stability is formulated in the next form

$$\frac{\partial \mathcal{K}}{\partial h_3} > 0, \tag{24}$$

where h_3 is the wave velocity. Thus, the studying of the stability property of the mZK equation by using Eq. (19) when $[d = -1, h_1 = 2, l = 6, m = 3, \rho = 1, q = 4, \chi = 0, y = 0, \vartheta = -4]$:

$$\Upsilon = \frac{\log\left(e^{40-10h_3} + e^{10h_3}\right) - \log\left(e^{-10h_3} + e^{10(h_3+4)}\right)}{8h_3} + 100$$

and thus

$$\frac{\partial \Upsilon}{\partial \epsilon}|_{h_3=4} = 0.312500000 > 0.$$

Consequently, this solution is stable.

2.3. Numerical Simulations

This section studies the numerical solution of the mZK equation according to B–spline schemes and based on the obtained stable analytical solution (19). Applying the B–spline schemes to Eq. (6), allows obtaining the next values of absolute value of error.

Table 1 Absolute value of error by using B–spline schemes.

Value of $\hbar$	Cubic	Quintic	Septic
0	0	0	0
0.1	0.00122084	5.2426×10^{-6}	1.2709×10^{-7}
0.2	0.00201838	9.46474×10^{-6}	1.63978×10^{-7}
0.3	0.00222625	7.63285×10^{-6}	4.31598×10^{-8}
0.4	0.00196322	4.12028×10^{-6}	1.41841×10^{-8}
0.5	0.00147327	1.18457×10^{-6}	2.83815×10^{-8}
0.6	0.000965422	3.25168×10^{-7}	1.5585×10^{-8}
0.7	0.000552262	7.52359×10^{-7}	5.23207×10^{-9}
0.8	0.000264813	6.26445×10^{-7}	7.07674×10^{-10}
0.9	8.99409E−05	2.62173×10^{-7}	1.122×10^{-9}
1	0	2.22045×10^{-16}	2.22045×10^{-16}

3. Results and discussion

This section studies the novelty of our presented paper by making a comparison between our solutions and that obtained in previous research paper:

1. The analytical solutions:
   - In [49], E. Tala-Tebue, Z.I. Djoufacka, S.B. Yamgouéb, A. Kenfack–Jiotsac, T.C. Kofanéd applied the Jacobi elliptical function method to the mZK equation and by analysis our solutions and those solutions, we find Eq. (19) and Eq. (23) in [49] are equal when $\left[B = mA, \sqrt{-\rho\vartheta} = 1\right]$ and all our other solutions are different from that obtained in this paper.
   - In [47], A. Sardar, S. M. Husnine, S. T. R. Rizvi, M. Younis, and K. Ali used the $\left(\frac{G\prime}{G}\right)$–expansion method, Tanh method, and Sine–cosine method. Investigating their and our solution, we find Eq. (19) is equal to (18) when $\left[-2\rho\vartheta = \lambda^2 - 4\mu,\ A = \frac{-1}{3\lambda m} - \lambda, B = 24m^2 (r_1^2 M + r_2^2 N)\ (\lambda^2 - 4\mu)\right]$ and Eq. (19) is equal to Eq. (36) when $\left[B = \frac{6a_0}{12c_1 + r_1(1+3m)}, \rho\vartheta = -1\right]$.

2. Numerical solutions:
   - according to the shown Table 1 and Fig. 5, the septic B–spline scheme obtain the most accurate value of numerical solutions of the mZK equation.

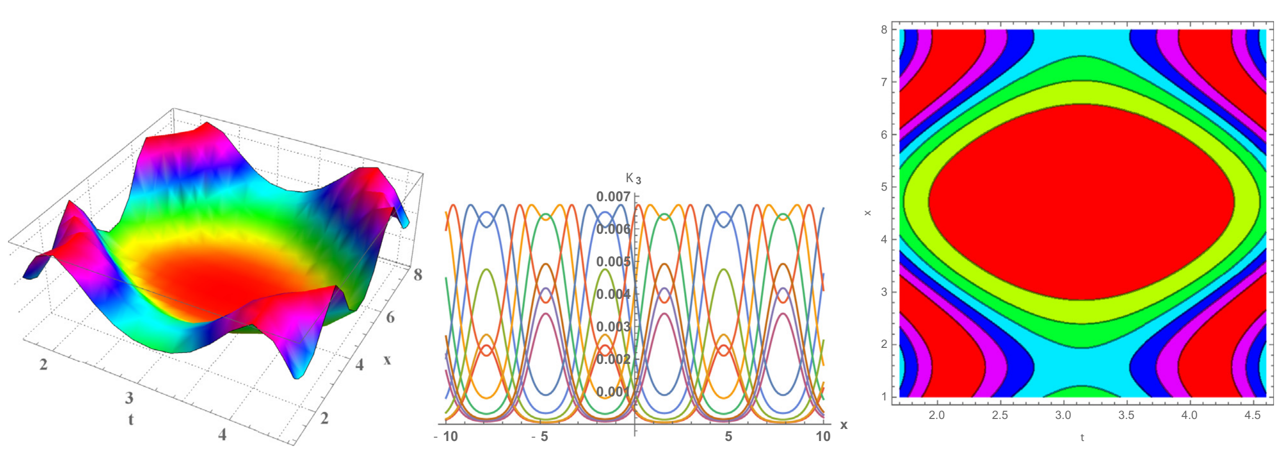

Fig. 2 Solitary wave of (11) in three, two-dimensional, and contour plots when $[h_1 = 2, l = -1, m = 3, \rho = 6, X = 5, y = 0, \vartheta = 1]$.

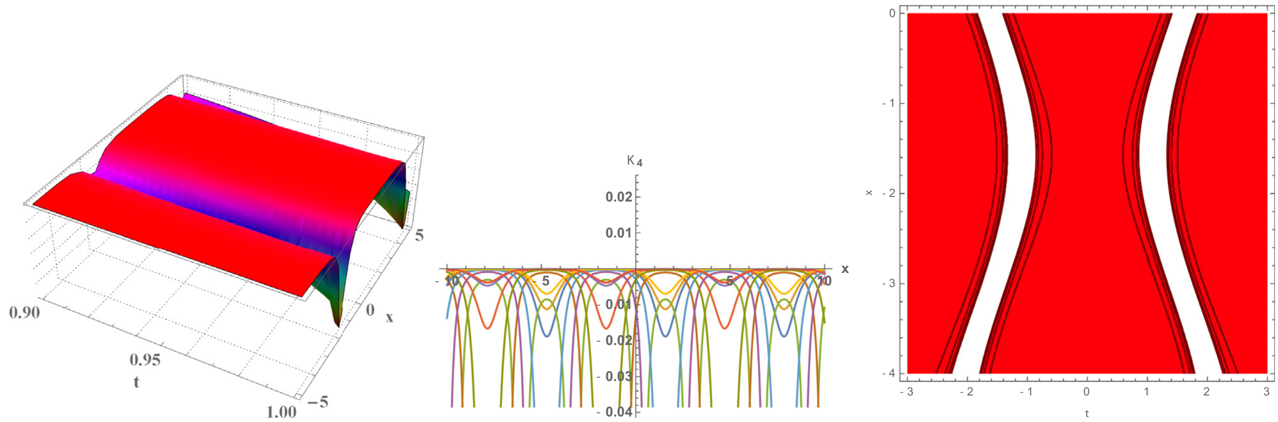

Fig. 3 Solitary wave of (12) in three, two-dimensional, and contour plots when $[h_1 = 2, l = -1, m = 3, \rho = 6, X = 5, y = 0, \vartheta = 1]$.

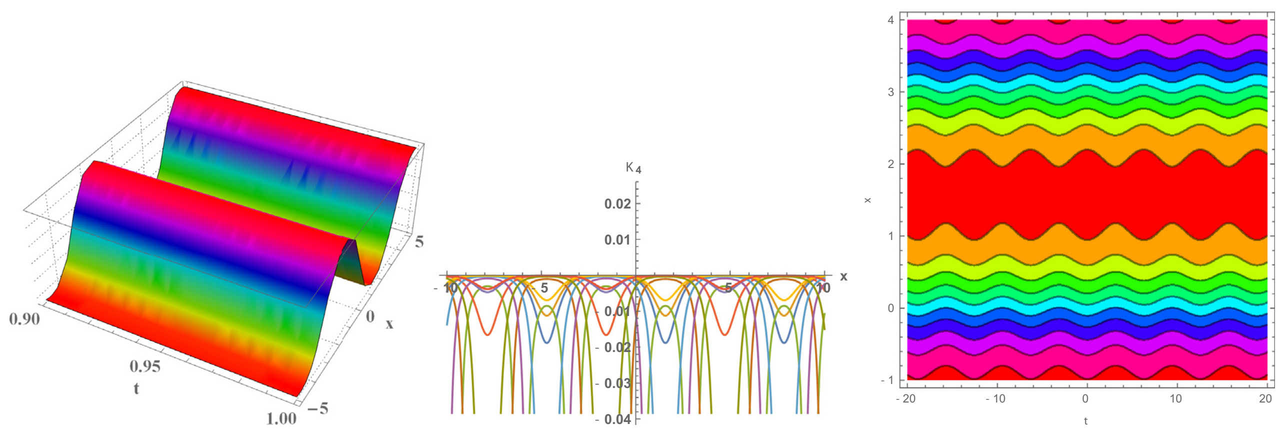

Fig. 4 Periodic solitary wave of (17) in three, two-dimensional, and contour plots when $[h_1 = 2, l = -1, m = 3, \rho = 6, X = 5, y = 0, \vartheta = 1]$.

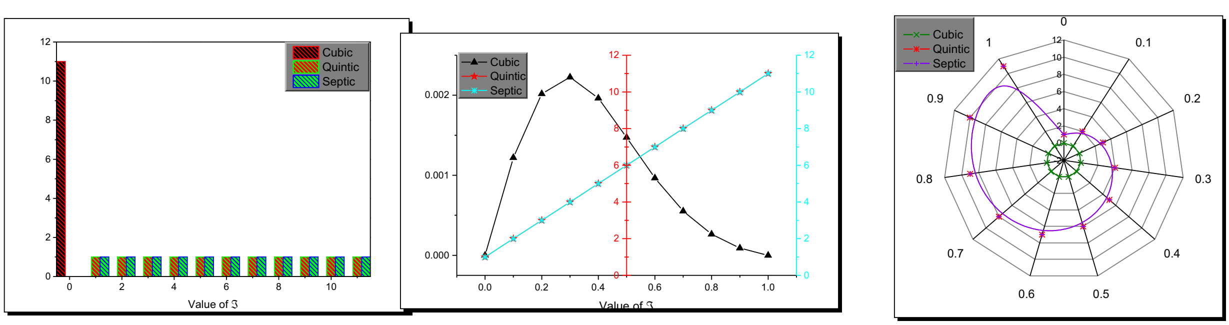

Fig. 5 Absolute value of error between exact and numerical solutions that obtained respectively by B–spline schemes (cubic & quintic & septic) to explain that the B–spline septic scheme is the most accurate method for this model (6).

4. Conclusion

This paper succeeded in the implementation of the mK method on the mZK equation to show more physical properties of the transportation of the energy in nonlinear electrical transmission lines. Moreover, the stability property of the obtained solutions was discussed and explained by using the momentum Υ in the Hamiltonian system. Some sketches were plotted to illustrate the more physical properties of these models (Figs. 2–4). The performance of the used method shows the effective and power of this method and its ability to apply other nonlinear evolutions equations.

Declarations of Competing Interest

None.

References

[1] Lump soliton wave solutions for the (2+ 1)-dimensional Konopelchenko-Dubrovsky equation and V equation, author = Khater, Mostafa M.A., Lu, Dianchen and Attia, Raghda AM, journal = Modern Physics Letters B, pages = 1950199, year = 2019, publisher = World Scientific.

[2] Mahmoud A.E. Abdelrahman, Mostafa M.A. Khater, Exact traveling wave solutions for Fitzhugh-Nagumo (FN) equation and modified Liouville Equation, Int. J. Comput. Appl. 113 (2015) 1–7.

[3] Mahmoud A.E. Abdelrahman, Mostafa M.A. Khater, Traveling wave solutions for the couple Boiti–Leon–Pempinelli system by using extended Jacobian elliptic function expansion method, J.: J. Adv. Phys. 11 (3) (2015).

[4] S.H. Alfalqi, J.F. Alzaidi, Dianchen Lu, Mostafa Khater, On exact and approximate solutions of (2+ 1)-dimensional Konopelchenko-Dubrovsky equation via modified simplest equation and cubic B-spline schemes, Therm. Sci. (2019) 349.

[5] Ahmad T. Ali, Mostafa M.A. Khater, Raghda A.M. Attia, Abdel-Haleem Abdel-Aty, Dianchen Lu. Abundant numerical and analytical solutions of the generalized formula of Hirota-Satsuma coupled KdV system. Chaos, Solitons Fract. 2019, p. 109473.

[6] Aliyu Isa Aliyu, Mustafa Inc, Abdullahi Yusuf, Dumitru Baleanu, A fractional model of vertical transmission and cure of vector-borne diseases pertaining to the Atangana-Baleanu fractional derivatives, Chaos, Solitons Fract. 116 (2018) 268–277.

[7] Ebru Cavlak Aslan, Mustafa Inc, Optical soliton solutions of the NLSE with quadratic–cubic–Hamiltonian perturbations and modulation instability analysis, Optik 196 (2019) 162661.

[8] Bjorn Augner, Well-posedness and stability of infinite-dimensional linear port-Hamiltonian systems with nonlinear boundary feedback, SIAM J. Control Optim. 57 (3) (2019) 1818–1844.

[9] Boris S. Bardin, Evgeniya A. Chekina, On the constructive algorithm for stability analysis of an equilibrium point of a periodic hamiltonian system with two degrees of freedom in the case of combinational resonance, Regul. Chaotic Dynam. 24 (2) (2019) 127–144.

[10] ALİ BAŞHAN, Seydi Battal Gazi Karakoc, Turabi Geyikli, Approximation of the KdVB equation by the quintic B-spline differential quadrature method, Kuwait J. Sci., 42(2) (2019).

[11] Alexander Batkhin. Some properties of doubly symmetric periodic solutions to Hamiltonian system, in: International Conference Polynomial Computer Algebra, 2019, pp. 15–20.

[12] QingZhen Bi, Jie Huang, Lu. YaoAn, LiMin Zhu, Han Ding, A general, fast and robust B-spline fitting scheme for micro-line tool path under chord error constraint, Sci. China Technol. Sci. 62 (2) (2019) 321–332.

[13] Amiya Das, Explicit weierstrass traveling wave solutions and bifurcation analysis for dissipative Zakharov-Kuznetsov modified equal width equation, Comput. Appl. Math. 37 (3) (2018) 3208–3225.

[14] Guy Roger Deffo, Serge Bruno Yamgoue, Francois Beceau Pelap, Modulational instability and peak solitary wave in a discrete nonlinear electrical transmission line described by the modified extended nonlinear Schrödinger equation, Eur. Phys. J. B 91 (10) (2018) 242.

[15] Zhongzhou Dong, Ling Wang, Solitary wave solutions and rational solutions for modified Zakharov-Kuznetsov equation with initial value problem, Scient. Res. Publish. 6 (2018) 949–959.

[16] Du. Xue-Dong, Chong-Chen Wang, Jian-Guo Liu, Xu-Dong Zhao, Jun Zhong, Yu-Xuan Li, Jin Li, Peng Wang, Extensive and selective adsorption of ZIF-67 towards organic dyes: performance and mechanism, J. Colloid Interface Sci. 506 (2017) 437–441.

[17] S.O. Edeki, I. Adinya, G.O. Akinlabi, O.P. Ogundile, Conformable decomposition for analytical solutions of a time-fractional one-factor markovian model for bond pricing, Appl. Math. 13 (4) (2019) 539–544.

[18] Feng Gao, Xiao-Jun Yang, Ju. Yang, Exact traveling-wave solutions for one-dimensional modified Korteweg–De Vries equation defined on cantor sets, Fractals 27 (01) (2019) 1940010.

[19] Laura R González-Ramírez, Mark A Kramer, The effect of inhibition on the existence of traveling wave solutions for a neural field model of human seizure termination, J. Comput. Neurosci. 44 (3) (2018) 393–409.

[20] Tiague Takongmo Guy, Jean Roger Bogning, Construction of Breather soliton solutions of a modeled equation in a discrete nonlinear electrical line and the survey of modulationnal instability, J. Phys. Commun. 2 (11) (2018) 115007.

[21] G.M. Ismail, H.R. Abdl-Rahim, A. Abdel-Aty, R. Kharabsheh, W. Alharbi, M. Abdel-Aty, An analytical solution for fractional oscillator in a resisting medium, Chaos, Solitons Fract. 130 (2020) 109395.

[22] C. Kadapa, W.G. Dettmer, D. Perić, A stabilised immersed framework on hierarchical b-spline grids for fluid-flexible structure interaction with solid–solid contact, Comput. Methods Appl. Mech. Eng. 335 (2018) 472–489.

[23] Mostafa Khater, Raghda A.M. Attia, Lu Dianchen, Explicit lump solitary wave of certain interesting (3+ 1)-dimensional waves in physics via some recent traveling wave methods, Entropy 21 (4) (2019) 397.

[24] Mostafa M.A. Khater, Dianchen Lu, Raghda A.M. Attia, Dispersive long wave of nonlinear fractional Wu-Zhang system via a modified auxiliary equation method, AIP Adv. 9 (2) (2019) 025003.

[25] Mostafa M.A. Khater, Aly R. Seadawy, Lu Dianchen, Dispersive solitary wave solutions of new coupled Konno-Oono, Higgs field and Maccari equations and their applications, J. King Saud Univ.-Sci. 30 (3) (2018) 417–423.

[26] Mostafa M.A. Khater, Aly R. Seadawy, Dianchen Lu, New optical soliton solutions for nonlinear complex fractional Schrödinger equation via new auxiliary equation method and novel (G′/G)-expansion method, Pramana, 90(5) (2018) 59.

[27] Mostafa MA Khater, Aly R Seadawy, Lu. Dianchen, Optical soliton and bright–dark solitary wave solutions of nonlinear complex Kundu-Eckhaus dynamical equation of the ultra–short femtosecond pulses in an optical fiber, Opt. Quant. Electron. 50 (3) (2018) 155.

[28] Zeliha Korpinar, Mustafa Inc, Numerical simulations for fractional variation of (1+ 1)-dimensional Biswas-Milovic equation, Optik 166 (2018) 77–85.

[29] Zeliha Korpinar, Mustafa Inc, Mustafa Bayram, Theory and application for the system of fractional burger equations with mittag leffler kernel, Appl. Math. Comput. 367 (124781) (2020).

[30] Md. Lukasz Krzczanowicz, Asif Iqbal, Ian Phillips, Mingming Tan, Pavel Skvortcov, Paul Harper, Wladek Forysiak, Low transmission penalty dual-stage broadband discrete raman amplifier, Opt. Exp. 26 (6) (2018) 7091–7097.

[31] D. Kumar, J. Singh, A. Prakash, R. Swroop, Numerical simulation for system of time–fractional linear and nonlinear differential equations, Progr. Fract. Differ. Appl. 5 (1) (2019) 65–77.

[32] Sachin Kumar, Qin Zhou, Wenjun Liu, Invariant traveling wave solutions of parity–time–symmetric mixed linear–nonlinear

optical lattices with three types of nonlinearity, Laser Phys. 29 (4) (2019) 045401.
[33] Chenhao Li, Yusheng Xue, Effects of cascading failure intervals on synchronous stability, Int. J. Electr. Power Energy Syst. 106 (2019) 502–510.
[34] Jing Li, Yuyang Qiu, Dianchen Lu, Raghda AM Attia, Mostafa Khater, Study on the solitary wave solutions of the ionic currents on microtubules equation by using the modified Khater method, Therm. Sci., (00) (2019) 370–370.
[35] Felipe Linares, Gustavo Ponce, On special regularity properties of solutions of the Zakharov-Kuznetsov equation, Commun. Pure Appl. Anal. 17 (4) (2018) 1561–1572.
[36] Jian-Guo Liu, Jian-Qiang Du, Zhi-Fang Zeng, Guo-Ping Ai, Exact periodic cross–kink wave solutions for the new (2+ 1)-dimensional KdV equation in fluid flows and plasma physics, Chaos: An Interdiscip. J. Nonlinear Sci. 26 (10) (2016) 103114.
[37] Jian-Guo Liu, Du. Jian-Qiang, Zhi-Fang Zeng, Bin Nie, New three-wave solutions for the (3+ 1)-dimensional Boiti–Leon–Manna–Pempinelli equation, Nonlinear Dyn. 88 (1) (2017) 655–661.
[38] Jian-Guo Liu, Yan He, Abundant lump and lump–kink solutions for the new (3+ 1)-dimensional generalized Kadomtsev-Petviashvili equation, Nonlinear Dyn. 92 (3) (2018) 1103–1108.
[39] Jian-Guo Liu, Yu Tian, Jian-Guo Hu, New non–traveling wave solutions for the (3+ 1)-dimensional Boiti–Leon–Manna–Pempinelli equation, Appl. Math. Lett. 79 (2018) 162–168.
[40] A.B. Togueu Motcheyo, C. Tchawoua, J.D. Tchinang Tchameu, Supratransmission induced by waves collisions in a discrete electrical lattice, Phys. Rev. E 88 (4) (2013) 040901.
[41] Saud Owyed, M.A. Abdou, Abdel-Haleem Abdel-Aty, S. Saha Ray, New optical soliton solutions of nolinear evolution equation describing nonlinear dispersion, Commun. Theor. Phys. 71 (9) (2019) 1063.
[42] Okkes Ozturk, Resat Yilmazer, On applications of the fractional calculus for some singular differential equations, Progr. Fract. Differ. Appl 4 (2018) 27–33.
[43] Orlando Ragnisco, Massimo Scalia, Stability for generalized volterra's model in the Hamiltonian formalism. Preprint on Researchgate, 2018, pages 22–11.
[44] Nauman Raza, Muhammad Abdullah, Asma Rashid Butt, Isma Ghulam Murtaza, Sultan Sial, New exact periodic elliptic wave solutions for extended quantum Zakharov-Kuznetsov equation, Opt. Quant. Electron., 50(4) (2018) 177.
[45] Hadi Rezazadeh, Alper Korkmaz, Mostafa M.A. Khater, Mostafa Eslami, Dianchen Lu, Raghda A.M. Attia, New exact traveling wave solutions of biological population model via the extended rational sinh-cosh method and the modified khater method, Modern Phys. Lett. B, 33(28) (2019) 1950338,.
[46] Hadi Rezazadeh, Jalil Manafian, Farid Samsami Khodadad, Fakhroddin Nazari, Traveling wave solutions for density-dependent conformable fractional diffusion–reaction equation by the first integral method and the improved Tanh-expansion method, Opt. Quant. Electron., 50(3) (2018) 121.
[47] Ali Sardar, S.M. Husnine, Syed Tahir Raza Rizvi, M. Younis, K. Ali, Multiple travelling wave solutions for electrical transmission line model, Nonlinear Dyn. 82 (3) (2015) 1317–1324.
[48] S. Shige, K. Miyasaka, W. Shi, Y. Soga, M. Sato, A.J. Sievers, Experimentally observed evolution between dynamic patterns and intrinsic localized modes in a driven nonlinear electrical cyclic lattice, EPL (Europhys. Lett.) 121 (3) (2018) 30003.
[49] E. Tala-Tebue, Z.I. Djoufack, S.B. Yamgoué, A. Kenfack-Jiotsa, T.C. Kofané, New soliton solutions for a discrete electrical lattice using the Jacobi elliptical function method, Chin. J. Phys. 56 (3) (2018) 1010–1020.
[50] Yu-Hang Yin, Wen-Xiu Ma, Jian-Guo Liu, Xing Lü, Diversity of exact solutions to a (3+ 1)-dimensional nonlinear evolution equation and its reduction, Comput. Math. Appl. 76 (6) (2018) 1275–1283.
[51] Baoxing Zhang, Hongchan Zheng, Lulu Pan, A generalized cubic exponential b-spline scheme with shape control, Math. Probl. Eng. 2019 (2019).
[52] Xuan Zhou, Wenrui Shan, Zhilei Niu, Pengcheng Xiao, Ying Wang, Lie symmetry analysis and some exact solutions for modified Zakharov-Kuznetsov equation, Mod. Phys. Lett. B 32 (31) (2018) 1850383.