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Dynamical analysis of the nonlinear complex fractional emerging telecommunication model with higher–order dispersive cubic–quintic

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Solitary waves

Abstract In this paper, a nonlinear fractional emerging telecommunication model with higher–order dispersive cubic–quintic is studied by using two recent computational schemes. This kind of model is arising in many applications such as machine learning and deep learning, cloud computing, data science, dense sensor network, artificial intelligence convergence, integration of Internet of Things, self–service IT for business users, self-powered data centers, and dense sensor networks (DSNs) that is used in the turbine blades monitoring and health monitoring. Two practical algorithms (modified Khater method and sech–tanh functions method) are applied to higher–order dispersive cubic–quintic nonlinear complex fractional Schrödinger ($\mathcal{NLCFS}$) equation. Many novel traveling wave solutions are constructed that do not exist earlier. These solutions are considered as the icon key in the emerging telecommunication field, were they able to explain the physical nature of the waves spread, especially in the dispersive medium. For more illustration, some attractive sketches are also depicted for the interpretation physically of the achieved solutions.

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1. Introduction

The contributions of the emerging technologies such as integration of Internet of things (IoT), cloud computing, self–powered data centers, data science, artificial intelligence convergence, dense sensor network, self–service IT for business users, and machine learning and deep learning are undeniable in daily life. Hundreds or thousands of cheap and small sensors are using the low–power multiloop wireless. These sensors have allowed engineers to develop dense sensor networks (DSNs). The low–power multihop wireless networks have customarily supported both of sensor networks and ultra–dense sense networks. In the former case, for example, these networks need to achieve efficient data forwarding among the sensors of the network, being reliable even in adversary conditions such as under distributed denial of service attacks. These networks may need to manage the services in these dense networks establishing priorities for assuring that urgent requests are attended in real–time. DSNs need to apply big data analysis for processing all the information generated by these sensors, as one can observe in smart health equipment such as intelligent beds for tracking sleeping poses in very critical patients. Several algorithms have developed for DSNs. For example, an algorithm was developed for detecting damage of wind turbine blades utilizing a DSN [4,10,13,36,33].

Recently, many integral equations have been derived and have been used in DSNs. For example, some authors proposed Nash equilibrium as the solution for managing integral equations in the context of decentralized activation in DSNs [14,30,38]. Also, Jindal and Psounis proposed a mathematical procedure for finding a correlation of data from DSN with their spatial information [3,24]. Their numerical process was generic and could be applied to different domains such as forest temperature and water contamination. Their procedure used several integral equations.

Partial differential equations (PDEs) have been playing an essential role in the emerging technologies where many nonlinear evolution equations have been derived to describe the dynamical behavior of several phenomena in several fields, for example, nonlinear optics, fluid dynamics, Bose–Einstein condensates, quantum mechanics and several other areas. However, the inadequate of the PDEs with an–integer order have been clarified because of the nonlocal property where this kind of equation does not explain that kind of features. Therefore, several natural phenomena have been formulated with nonlinear PDEs with fractional order. Thus, Partial differential equations (PDEs) have been playing an important role in the emerging technologies where many nonlinear evolution equations have been derived to describe the dynamical behaviour of several phenomenon in several fields, for example, nonlinear optics, fluid dynamics, Bose–Einstein condensates, quantum mechanics and several other areas. However, the inadequate of the PDEs with an–integer order have been clarified because of the nonlocal property where this kind of equation do not explain that kind of properties. Therefore, several nature phenomena have been formulated with nonlinear PDEs with fractional order [2,6,7,11,12,16–23,26,29].. Thus, many fractional operators have been derived such as conformable fractional derivative, fractional Riemann–Liouville derivatives, Caputo, Caputo–Fabrizio definition, and so on [1,5,9,15,27].

These definitions have been being employed to convert the fractional nonlinear partial differential equations to a nonlinear integer–order ordinary differential equation. Then the computational and numerical schemes can be applied to get various types of solutions for these models and the examples of these schemes.

The $\mathscr{ABR}$ fractional operator is considered as one of the most general recent fractional operators that is derived from avoiding the deficiencies and defects of some other fractional operator. This operator is defined as follows.

Definition 1.1. It is given by [28,32,39,35]

$$ {}^{\mathscr{ABR}}D^{\alpha}_{a+}f(t)=\frac{B(\alpha)}{1-\alpha}\frac{d}{dt}\int_a^t f(x)\,g_\alpha\left(\frac{-\alpha\,(t-\alpha)^{\alpha}}{1-\alpha}\right)dx, \tag{1} $$

where g_α stands for the Mittag–Leffler function, giving by [25,31]

$$ g_\alpha\left(\frac{-\alpha\,(t-\alpha)^{\alpha}}{1-\alpha}\right)=\sum_{n=0}^{\infty}\frac{\left(\frac{-\alpha}{1-\alpha}\right)^n(t-x)^{\alpha n}}{\Gamma(\alpha n+1)} $$

and $B(\alpha)$ being a normalization function. Thus

$$ {}^{\mathscr{ABR}}D^{\alpha}_{a+}f(x)=\frac{B(\alpha)}{1-\alpha}\sum_{n=0}^{\infty}\left(\frac{-\alpha}{1-\alpha}\right)^{n}\,{}^{RL}\mathscr{I}^{\alpha n}_{a}f(x), \tag{2} $$

This paper studies the analytical solutions of the $\mathscr{NLCFS}$ with higher–order dispersive cubic–quintic arising in the emerging telecommunication. This fractional model describes the wave function or state function of a quantum–

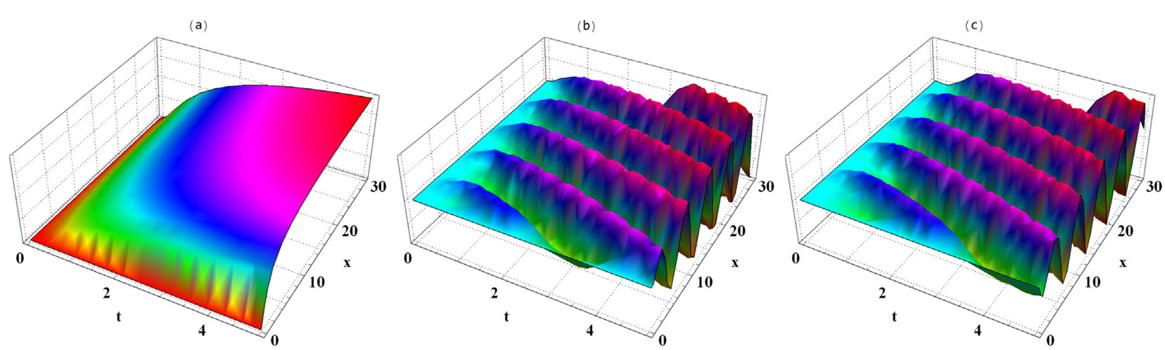

Fig. 1 Numerical simulation of Eq. (11) in three–dimensional sketches.

mechanical system. Moreover, it is also used in the optical fiber where it occurs in the Manakov system. The NLCFS equation is given by [8,34,37,40]

$$\imath D_x^\alpha \mathscr{S} - \frac{p_1}{2}\mathscr{S}_{tt} + q_1 \mathscr{S}|\mathscr{S}|^2 - \imath \frac{p_2}{6} D_{ttt}^{3\alpha}\mathscr{S} - \frac{p_3}{24} D_{tttt}^{4\alpha}\mathscr{S} + q_2 \mathscr{S}|\mathscr{S}|^4 = 0, \tag{3}$$

where $(0 < \alpha < 1)$, $\mathscr{S}$ describes the propagation of the wave through a nonlinear medium. Additionally, the p_1, p_2 and p_3 are dispersions of order 2^{nd} , 3^{nd} and 4^{nd} respectively, while the q_1 and q_2 are the coefficients of two nonlinearities of the medium, the function q is the gradually varying envelope of the electromagnetic material, the variables t and x are the retarded time and the distance along the direction of propagation respectively. The second and third terms in the above equations are revealed from the velocity dispersion and the Kerr effects. Using the next wave transformation $\left[\mathscr{S} = \mathscr{S}(x,t) = \mathscr{S}(\Upsilon)e^{\imath\Theta},\ \Upsilon = \frac{(1-\alpha)(l_1 x^{-n\alpha} + l_2 t^{-n\alpha})}{B(\alpha)\sum_{n=0}^{\infty}\left(-\frac{\alpha}{1-\alpha}\right)^n \Gamma(1-\alpha n)},\ \Theta = \frac{(1-\alpha)(d_1 x^{-n\alpha} + d_2 t^{-n\alpha} + d_3)}{B(\alpha)\sum_{n=0}^{\infty}\left(-\frac{\alpha}{1-\alpha}\right)^n \Gamma(1-\alpha n)}\right]$ to Eq. (3), yields

$$\begin{cases} l_2^2 p_3 \mathscr{S}^{(4)} + \left(12 p_1 l_2^2 - 6 p_3 d_2^2 l_2^2 - 12 p_2 d_2 l_2^2\right)\mathscr{S}'' + \left(p_3 d_2^4 + 4 p_2 d_1^3 - 12 p_1 d_2 + 24 d_1\right)\mathscr{S} \\ \quad -24 q_2 \mathscr{S}^5 - 24 q_1 \mathscr{S}^3 = 0, \\ \left(-4 p_3 d_2^3 l_2 - 12 p_2 d_2^2 l_2 + 24 p_1 d_2 l_2 - 24 k_1\right)\mathscr{S}' + 4 l_2^3 (p_3 d_2 + p_2)\mathscr{S}''' = 0, \end{cases} \tag{4}$$

where $[d_i, l_j, (i = 1,2,3), (j = 1,2)]$ are arbitrary constants. Differentiating second equation of the system (4) and then substitute the result into the first equation of the same equation, gives

$$k_1 \mathscr{S}'' + k_2 \mathscr{S}^5 + k_3 \mathscr{S}^3 + k_4 \mathscr{S} = 0, \tag{5}$$

where $\left[k_1 = l_2^2\left(12 p_1 - 6 p_3 d_2^2 - 12 p_2 d_2 \frac{4 p_3 d_2^3 l_2 + 12 p_3 d_2^2 l_2 - 24 p_1 d_2 l_2 + 24 k_1}{4 l_2^3 (p_3 d_2 + p_2)}\right),\right.$ $\left.k_2 = -24 q_2, k_3 = -24 q_1, k_4 = \left(p_3 d_2^4 + 4 p_2 d_2^3 - 12 p_1 d_2^2 + 24 d_1\right)\right]$. Applying the homogeneous balance principle to Eq. (5), leads to $\left(m = \frac{1}{2}\right)$. Thus, we use the next transformation $\left[\mathscr{S} = \mathscr{H}^{\frac{1}{2}}\right]$ to the Eq. (5), yields

$$\frac{-k_1}{4}\mathscr{H}'^2 + \frac{k_1}{2}\mathscr{H}\mathscr{H}'' + k_2 \mathscr{H}^4 + k_3 \mathscr{H}^3 + k_4 \mathscr{H}^2 = 0. \tag{6}$$

Applying the homogeneous balance principle to Eq. (6), obtains $(m = 1)$.

The rest of research paper is organized as follows: Section (2), applies the modified Khater method and sech–tanh functions expansion method to the suggested model to get novel solitary wave solutions of it [5,9]. Section (5), explains the conclusion of all the steps of our paper is detailed.

2. Application

Here in this section, the modified Khater method and sech–tanh functions expansion method are applied to the $\mathscr{NLCFS}$ equation to explain the restricted electromagnetic wave which stretches in media of nonlinear dispersive. Due to stability among nonlinearity and dispersion effects, the intensity of optical solitons are unchanged, and such categories of solitary waves are more significant because of their suppleness in optical of long distance.

2.1. The modified Khater method

Applying the modified Khater method to Eq. (6), leads to formulate the general solution of this model in the following formula

$$\mathscr{H}(\Upsilon) = \sum_{i=1}^{m} a_i \mathscr{K}^{i\mathscr{F}(\Upsilon)} + \sum_{i=1}^{m} b_i \mathscr{K}^{-i\mathscr{F}(\Upsilon)} + a_0 = a_1 \mathscr{K}^{\mathscr{F}(\Upsilon)} + a_0 + b_1 \mathscr{K}^{-\mathscr{F}(\Upsilon)}, \tag{7}$$

where $[a_0, a_1, b_1]$ are arbitrary constants to be determined later. Additionally, $\mathscr{F}(\Upsilon)$ is the solution function of the following ordinary differential equation

$$\mathscr{F}'(\Upsilon) = \frac{1}{\ln(\mathscr{K})}\left[\delta \mathscr{K}^{\mathscr{F}(\Upsilon)} + \varrho \mathscr{K}^{-\mathscr{F}(\Upsilon)} + \chi\right], \tag{8}$$

where $[\delta, \varrho, \chi]$ are arbitrary constant. Substituting Eq. (7) along (8) into Eq. (6) and collecting all terms with the same power of $\left[\mathscr{K}^{i\mathscr{F}(\Upsilon)}, i = -5, -4, \ldots, 4, 5\right]$, give a system of algebraic equation. Using the Mathematica 12 program for solving this system, yields

Family I

$$\left[a_0 \to -\frac{3k_3}{4k_2}, a_1 \to -\frac{3\delta k_3}{4k_2\chi}, b_1 \to -\frac{3k_3\varrho}{4k_2\chi}, k_1 \to -\frac{3k_3^2}{4k_2\chi^2}, k_4 \to -\frac{3k_3^2(4\delta\varrho - \chi^2)}{16k_2\chi^2}\right]$$

Thus, the explicit wave solutions of Eq. (3) are formulated in the following formulas

For $[\chi^2 - 4\delta\varrho < 0 \,\&\, \delta \neq 0]$

$$\mathscr{S}_1(x,t) = \frac{1}{2}\sqrt{3}e^{\imath\Theta} \times \sqrt{-\frac{k_3(\chi^2 - 4\delta\varrho)}{k_2\chi\left(-\sqrt{4\delta\varrho - \chi^2}\sin\left(\Upsilon\sqrt{4\delta\varrho - \chi^2}\right) + \chi\cos\left(\Upsilon\sqrt{4\delta\varrho - \chi^2}\right) + \chi\right)}}, \tag{9}$$

$$\mathscr{S}_2(x,t) = \frac{1}{2}\sqrt{3}e^{\imath\Theta} \times \sqrt{\frac{k_3(\chi^2 - 4\delta\varrho)}{k_2\chi\left(\sqrt{4\delta\varrho - \chi^2}\sin\left(\Upsilon\sqrt{4\delta\varrho - \chi^2}\right) + \chi\cos\left(\Upsilon\sqrt{4\delta\varrho - \chi^2}\right) - \chi\right)}}. \tag{10}$$

For $[\chi^2 - 4\delta\varrho > 0 \,\&\, \delta \neq 0]$

$$\mathscr{S}_3(x,t) = \frac{1}{2}\sqrt{3}e^{\imath\Theta} \times \sqrt{-\frac{k_3(\chi^2 - 4\delta\varrho)}{k_2\chi\left(\sqrt{\chi^2 - 4\delta\varrho}\sinh\left(\Upsilon\sqrt{\chi^2 - 4\delta\varrho}\right) + \chi\cosh\left(\Upsilon\sqrt{\chi^2 - 4\delta\varrho}\right) + \chi\right)}}, \tag{11}$$

$$\mathscr{S}_4(x,t) = \frac{1}{2}\sqrt{3}e^{\imath\Theta} \times \sqrt{\frac{k_3(\chi^2 - 4\delta\varrho)}{k_2\chi\left(\sqrt{\chi^2 - 4\delta\varrho}\sinh\left(\Upsilon\sqrt{\chi^2 - 4\delta\varrho}\right) + \chi\cosh\left(\Upsilon\sqrt{\chi^2 - 4\delta\varrho}\right) - \chi\right)}}. \tag{12}$$

For $\left[\chi = \frac{\varrho}{2} = \kappa \,\&\, \delta = 0\right]$

$$\mathscr{S}_5(x,t) = \frac{1}{2}\sqrt{3}e^{\imath\Theta}\sqrt{-\frac{k_3 e^{\kappa\Upsilon}}{k_2(e^{\kappa\Upsilon} - 2)}}. \tag{13}$$

For $[\chi = \delta = \kappa \,\&\, \varrho = 0]$

$$\mathscr{S}_6(x,t)=\frac{1}{2}\sqrt{3}e^{i\Theta}\sqrt{\frac{k_3}{k_2(e^{\kappa\Upsilon}-1)}}. \tag{14}$$

For $[\varrho=0\,\&\,\chi\neq 0\,\&\,\delta\neq 0]$

$$\mathscr{S}_7(x,t)=\sqrt{\frac{3}{2}}e^{i\Theta}\sqrt{\frac{k_3}{k_2(\delta e^{\Upsilon\chi}-2)}}. \tag{15}$$

For $[\delta=0\,\&\,\chi\neq 0\,\&\,\varrho\neq 0]$

$$\mathscr{S}_8(x,t)=\frac{1}{2}\sqrt{3}e^{i\Theta}\sqrt{\frac{k_3\chi e^{\Upsilon\chi}}{k_2(\varrho-\chi e^{\Upsilon\chi})}}. \tag{16}$$

For $[\chi^2-4\delta\varrho=0]$

$$\mathscr{S}_9(x,t)=\frac{1}{2}\sqrt{\frac{3}{2}}e^{i\Theta}\times\sqrt{\frac{k_3\left(4\delta\varrho(\Upsilon\chi+2)^2-\Upsilon\chi^3(\Upsilon\chi+4)\right)}{k_2\Upsilon\chi^3(\Upsilon\chi+2)}}. \tag{17}$$

Family II

$$\left[a_0\to\frac{k_1\chi^2}{k_3},a_1\to\frac{\delta k_1\chi}{k_3},b_1\to\frac{k_1\chi\varrho}{k_3},k_4\to\frac{1}{4}k_1\left(4\delta\varrho-\chi^2\right),k_2\to-\frac{3k_3^2}{4k_1\chi^2}\right]$$

Thus, the explicit wave solutions of Eq. (3) are formulated in the following formulas

For $[\chi^2-4\delta\varrho<0\,\&\,\delta\neq 0]$

$$\mathscr{S}_{10}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{k_1\chi(\chi^2-4\delta\varrho)\sec^2\left(\frac{1}{2}\Upsilon\sqrt{4\delta\varrho-\chi^2}\right)}{k_3\left(\chi-\sqrt{4\delta\varrho-\chi^2}\tan\left(\frac{1}{2}\Upsilon\sqrt{4\delta\varrho-\chi^2}\right)\right)}}}{\sqrt{2}}, \tag{18}$$

$$\mathscr{S}_{11}(x,t)=e^{i\Theta}\times\sqrt{-\frac{k_1\chi(\chi^2-4\delta\varrho)}{k_3\left(\sqrt{4\delta\varrho-\chi^2}\sin\left(\Upsilon\sqrt{4\delta\varrho-\chi^2}\right)+\chi\cos\left(\Upsilon\sqrt{4\delta\varrho-\chi^2}\right)-\chi\right)}}. \tag{19}$$

For $[\chi^2-4\delta\varrho>0\,\&\,\delta\neq 0]$

$$\mathscr{S}_{12}(x,t)=e^{i\Theta}\times\sqrt{\frac{k_1\chi(\chi^2-4\delta\varrho)}{k_3\left(\sqrt{\chi^2-4\delta\varrho}\sinh\left(\Upsilon\sqrt{\chi^2-4\delta\varrho}\right)+\chi\cosh\left(\Upsilon\sqrt{\chi^2-4\delta\varrho}\right)+\chi\right)}}, \tag{20}$$

$$\mathscr{S}_{13}(x,t)=e^{i\Theta}\times\sqrt{-\frac{k_1\chi(\chi^2-4\delta\varrho)}{k_3\left(\sqrt{\chi^2-4\delta\varrho}\sinh\left(\Upsilon\sqrt{\chi^2-4\delta\varrho}\right)+\chi\cosh\left(\Upsilon\sqrt{\chi^2-4\delta\varrho}\right)-\chi\right)}}. \tag{21}$$

For $\left[\chi=\frac{\varrho}{2}=\kappa\,\&\,\delta=0\right]$

$$\mathscr{S}_{14}(x,t)=e^{i\Theta}\sqrt{\frac{\kappa^2k_1e^{\kappa\Upsilon}}{k_3(e^{\kappa\Upsilon}-2)}}. \tag{22}$$

For $[\chi=\delta=\kappa\,\&\,\varrho=0]$

$$\mathscr{S}_{15}(x,t)=e^{i\Theta}\sqrt{\frac{\kappa^2k_1}{k_3-k_3e^{\kappa\Upsilon}}}. \tag{23}$$

For $[\varrho=0\,\&\,\chi\neq 0\,\&\,\delta\neq 0]$

$$\mathscr{S}_{16}(x,t)=\sqrt{2}e^{i\Theta}\sqrt{-\frac{k_1\chi^2}{k_3(\delta e^{\Upsilon\chi}-2)}}. \tag{24}$$

For $[\delta=0\,\&\,\chi\neq 0\,\&\,\varrho\neq 0]$

$$\mathscr{S}_{17}(x,t)=e^{i\Theta}\sqrt{\frac{k_1\chi^3e^{\Upsilon\chi}}{k_3(\chi e^{\Upsilon\chi}-\varrho)}}. \tag{25}$$

For $[\chi^2-4\delta\varrho=0]$

$$\mathscr{S}_{18}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{k_1\left(\Upsilon\chi^3(\Upsilon\chi+4)-4\delta\varrho(\Upsilon\chi+2)^2\right)}{k_3\Upsilon\chi(\Upsilon\chi+2)}}}{\sqrt{2}}. \tag{26}$$

Family III

$$\left[a_0\to\frac{k_1k_3(\chi^2-4\delta\varrho)-\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}}{2k_3^2},a_1\to-\frac{\delta\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}}{k_3^2\chi},b_1\to 0,k_4\to\frac{1}{4}k_1\left(4\delta\varrho-\chi^2\right),\right.$$
$$\left.k_2\to\frac{3k_3^2}{16\delta k_1\varrho-4k_1\chi^2},\text{ where }(k_1\neq 0,\ k_2\neq\neq 0,\ k_3\neq 0)\right]$$

Thus, the explicit wave solutions of Eq. (3) are formulated in the following formulas

For $[\chi^2-4\delta\varrho<0\,\&\,\delta\neq 0]$

$$\mathscr{S}_{19}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{k_1k_3\chi(\chi^2-4\delta\varrho)-\sqrt{4\delta\varrho-\chi^2}\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}\tan\left(\frac{1}{2}\Upsilon\sqrt{4\delta\varrho-\chi^2}\right)}{k_3^2\chi}}}{\sqrt{2}}, \tag{27}$$

$$\mathscr{S}_{20}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{k_1k_3\chi(\chi^2-4\delta\varrho)-\sqrt{4\delta\varrho-\chi^2}\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}\cot\left(\frac{1}{2}\Upsilon\sqrt{4\delta\varrho-\chi^2}\right)}{k_3^2\chi}}}{\sqrt{2}}. \tag{28}$$

For $[\chi^2-4\delta\varrho>0\,\&\,\delta\neq 0]$

$$\mathscr{S}_{21}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{\sqrt{\chi^2-4\delta\varrho}\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}\tanh\left(\frac{1}{2}\Upsilon\sqrt{\chi^2-4\delta\varrho}\right)+k_1k_3\chi(\chi^2-4\delta\varrho)}{k_3^2\chi}}}{\sqrt{2}}, \tag{29}$$

$$\mathscr{S}_{22}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{\sqrt{\chi^2-4\delta\varrho}\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}\coth\left(\frac{1}{2}\Upsilon\sqrt{\chi^2-4\delta\varrho}\right)+k_1k_3\chi(\chi^2-4\delta\varrho)}{k_3^2\chi}}}{\sqrt{2}}. \tag{30}$$

For $[\chi=\delta=\kappa\,\&\,\varrho=0]$

$$\mathscr{S}_{23}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{\sqrt{\kappa^4k_1^2k_3^2}\coth\left(\frac{\kappa\Upsilon}{2}\right)+\kappa^2k_1k_3}{k_3^2}}}{\sqrt{2}}. \tag{31}$$

For $[\varrho=0\,\&\,\chi\neq 0\,\&\,\delta\neq 0]$

$$\mathscr{S}_{24}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{\frac{\sqrt{k_1^2k_3^2\chi^4}\left(\delta e^{\Upsilon\chi}+2\right)}{\delta e^{\Upsilon\chi}-2}+k_1k_3\chi^2}{k_3^2}}}{\sqrt{2}}. \tag{32}$$

Family IV

$$\Bigg[a_0 \to \frac{k_1k_3(\chi^2-4\delta\varrho)-\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}}{2k_3^2}, a_1 \to 0,$$

$$b_1 \to -\frac{\varrho\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}}{k_3^2\chi}, k_4 \to \frac{1}{4}k_1(4\delta\varrho-\chi^2),$$

$$k_2 \to \frac{3k_3^2}{16\delta k_1\varrho-4k_1\chi^2}, \text{ where } (k_1 \neq 0, k_2 \neq \neq 0, k_3 \neq 0)\Bigg]$$

Thus, the explicit wave solutions of Eq. (3) are formulated in the following formulas

For $[\chi^2-4\delta\varrho<0\,\&\,\delta\neq 0]$

$$\mathscr{S}_{25}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{\frac{4\delta\varrho\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}}{\chi\left(\chi-\sqrt{4\delta\varrho-\chi^2}\tan\left(\frac{1}{2}\Upsilon\sqrt{4\delta\varrho-\chi^2}\right)\right)}-\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}+k_1k_3(\chi^2-4\delta\varrho)}{k_3^2}}}{\sqrt{2}}, \tag{33}$$

$$\mathscr{S}_{26}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{\frac{4\delta\varrho\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}}{\chi\left(\chi-\sqrt{4\delta\varrho-\chi^2}\cot\left(\frac{1}{2}\Upsilon\sqrt{4\delta\varrho-\chi^2}\right)\right)}-\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}+k_1k_3(\chi^2-4\delta\varrho)}{k_3^2}}}{\sqrt{2}}. \tag{34}$$

For $[\chi^2-4\delta\varrho>0\,\&\,\delta\neq 0]$

$$\mathscr{S}_{27}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{\frac{4\delta\varrho\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}}{\chi\left(\sqrt{\chi^2-4\delta\varrho}\tanh\left(\frac{1}{2}\Upsilon\sqrt{\chi^2-4\delta\varrho}\right)+\chi\right)}-\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}+k_1k_3(\chi^2-4\delta\varrho)}{k_3^2}}}{\sqrt{2}}, \tag{35}$$

$$\mathscr{S}_{28}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{\frac{4\delta\varrho\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}}{\chi\left(\sqrt{\chi^2-4\delta\varrho}\coth\left(\frac{1}{2}\Upsilon\sqrt{\chi^2-4\delta\varrho}\right)+\chi\right)}-\sqrt{k_1^2k_3^2\chi^2(\chi^2-4\delta\varrho)}+k_1k_3(\chi^2-4\delta\varrho)}{k_3^2}}}{\sqrt{2}}. \tag{36}$$

For $\left[\chi=\frac{\varrho}{2}=\kappa\,\&\,\delta=0\right]$

$$\mathscr{S}_{29}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{\kappa^2k_1k_3-\frac{\left(e^{\kappa\Upsilon}+2\right)\sqrt{\kappa^4k_1^2k_3^2}}{e^{\kappa\Upsilon}-2}}{k_3^2}}}{\sqrt{2}}. \tag{37}$$

For $[\delta=0\,\&\,\chi\neq 0\,\&\,\varrho\neq 0]$

$$\mathscr{S}_{30}(x,t)=\frac{e^{i\Theta}\sqrt{\frac{\frac{\sqrt{k_1^2k_3^2\chi^4}\left(\chi e^{\Upsilon\chi}+\varrho\right)}{\varrho-\chi e^{\Upsilon\chi}}+k_1k_3\chi^2}{k_3^2}}}{\sqrt{2}}. \tag{38}$$

2.2. The sech–tanh functions expansion method

Applying the sech–tanh functions expansion method to Eq. (6), leads to formulate the general solution of this model in the following formula

$$\begin{aligned}\mathscr{H}(\Upsilon)&=\sum_{i=1}^{m}\mathrm{sech}^{i-1}(\Upsilon)(a_i\mathrm{sech}(\Upsilon)+b_i\tanh(\Upsilon))+a_0\\&=a_1\mathrm{sech}(\Upsilon)+a_0+b_1\tanh(\Upsilon),\end{aligned} \tag{39}$$

where $[a_0, a_1, b_1]$ are arbitrary constants to be determined later. Substituting Eq. (39) into Eq. (6) and collecting all terms with the same power of $[\mathrm{sech}(\Upsilon), \tanh(\Upsilon)]$, give a system of algebraic equation. Using the Mathematica 12 program for solving this system, yields

Family I

$$\left[a_0 \to -\frac{3k_3}{8k_2}, a_1 \to \frac{3k_3}{8k_2}, b_1 \to 0, k_1 \to \frac{3k_3^2}{16k_2}, k_4 \to \frac{15k_3^2}{64k_2}\right]$$

Thus, the explicit wave solutions of Eq. (3) are formulated in the following formulas

$$\mathscr{S}_{31}(x,t)=\frac{1}{2}\sqrt{\frac{3}{2}}e^{i\Theta}\sqrt{\frac{k_3(\mathrm{sech}(\Upsilon)-1)}{k_2}}. \tag{40}$$

Family II

$$\left[a_0 \to -\frac{3k_3}{8k_2}, a_1 \to 0, b_1 \to -\frac{3k_3}{8k_2}, k_1 \to -\frac{3k_3^2}{16k_2}, k_4 \to \frac{3k_3^2}{16k_2}\right]$$

Thus, the explicit wave solutions of Eq. (3) are formulated in the following formulas

$$\mathscr{S}_{32}(x,t)=\frac{1}{2}\sqrt{\frac{3}{2}}e^{i\Theta}\sqrt{-\frac{k_3(\tanh(\Upsilon)+1)}{k_2}}. \tag{41}$$

Family III

$$\left[a_0 \to -\frac{3k_3}{8k_2}, a_1 \to \frac{3\imath k_3}{8k_2}, b_1 \to -\frac{3k_3}{8k_2}, k_1 \to -\frac{3k_3^2}{4k_2}, k_4 \to \frac{3k_3^2}{16k_2}\right]$$

Thus, the explicit wave solutions of Eq. (3) are formulated in the following formulas

$$\mathscr{S}_{33}(x,t)=\frac{1}{2}\sqrt{\frac{3}{2}}e^{i\Theta}\sqrt{-\frac{k_3(\tanh(\Upsilon)-\imath\,\mathrm{sech}(\Upsilon)+1)}{k_2}}. \tag{42}$$

3. Figure interpretation

This section gives the physical interpretation of the shown figures in our paper. All our obtained solutions are considered as optical soliton wave solutions. This kind of solutions have a basic importance to illustrate the dynamical behavior of the particles in the optical waves where the optical soliton is restricted electromagnetic wave which stretches in media of nonlinear dispersive. Due to stability among nonlinearity and dispersion effects, the intensity of optical solitons are unchanged, and such categories of solitary waves are more significant because of their suppleness in optical of long distance. Our interpretation of the shown figures is given as following.

1. Figs. 1 and 2 show the W–shapes of the absolute, real, and imaginary solution (11) in the three–dimensional plot (a, b, c) to explain the perspective view of the solution and the absolute, real, and imaginary sketches in two– dimensional plot (d, e, f) to explain the wave propagation pattern of the wave along x axis when $[\alpha=0.5, d_1=1, d_2=2, d_3=-1, \delta=1, k_2=-2, k_3=-3, l_1=4, l_2=5, \chi=3, \varrho=2]$.
2. Figs. 3 and 4 show the W–shapes of the absolute, real, and imaginary solution (13) in the three–dimensional plot (a, b, c) to explain the perspective view of the solution and the

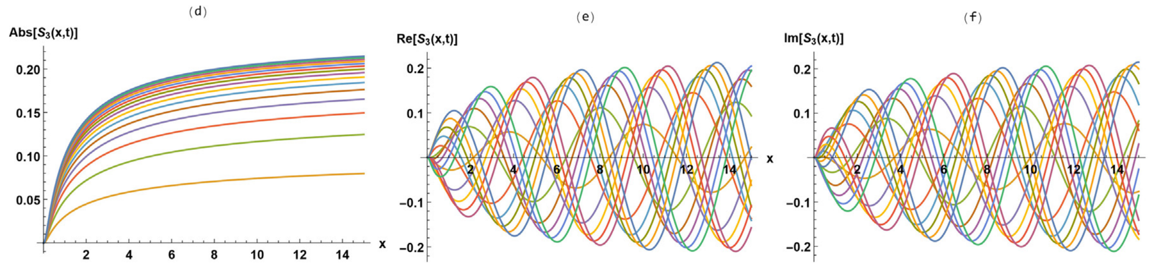

Fig. 2 Numerical simulation of Eq. (11) in two–dimensional sketches.

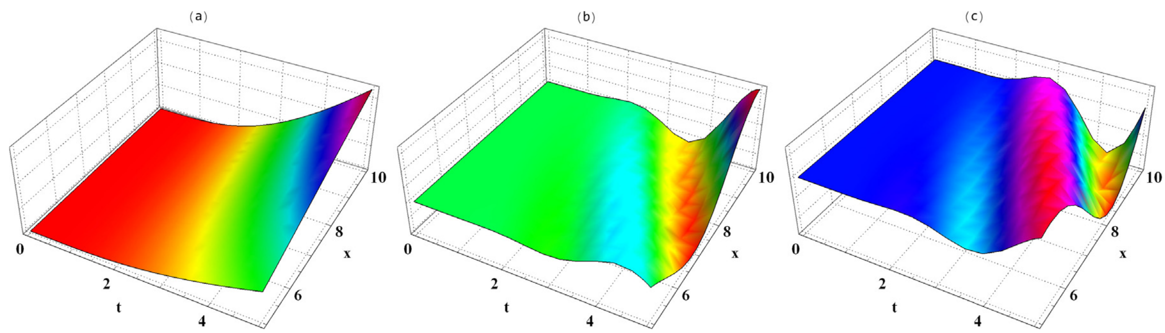

Fig. 3 Numerical simulation of Eq. (13) in three–dimensional sketches.

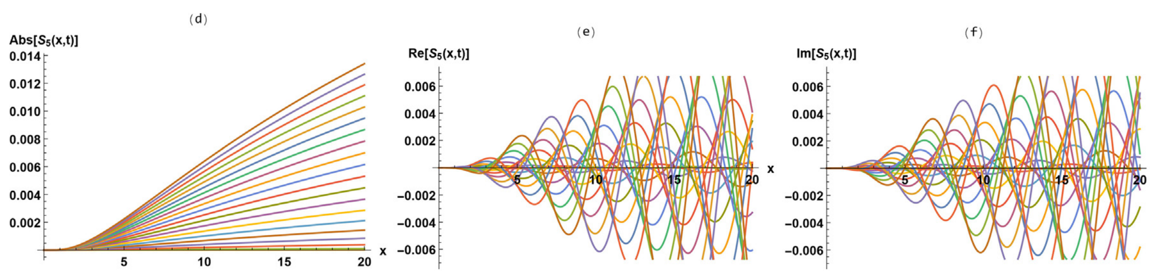

Fig. 4 Numerical simulation of Eq. (13) in two–dimensional sketches.

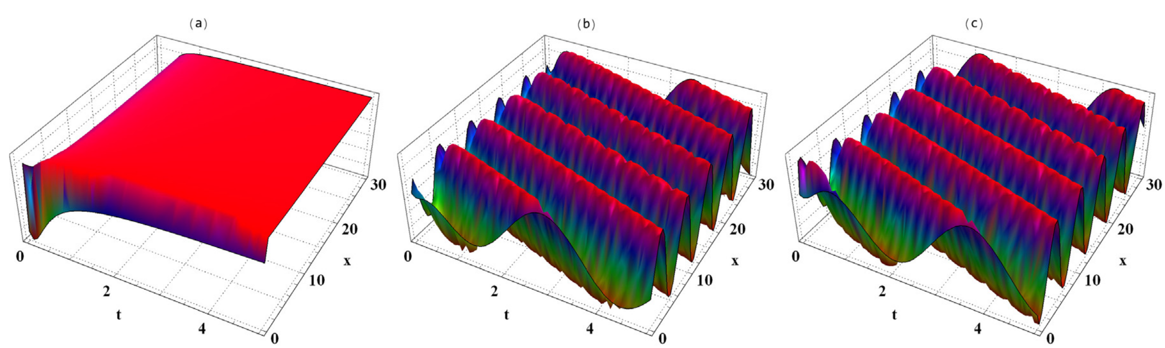

Fig. 5 Numerical simulation of Eq. (27) in three–dimensional sketches.

absolute, real, and imaginary sketches in two– dimensional plot (d, e, f) to explain the wave propagation pattern of the wave along x axis when [$\alpha = 0.5$, $d_1 = 1$, $d_2 = 2$, $d_3 = -1$, $\kappa = 6$, $k_2 = -2$, $k_3 = -3$, $l_1 = 4$, $l_2 = 5$].

3. Figs. 5 and 6 show the W–shapes of the absolute, real, and imaginary solution (27) in the three–dimensional plot (a, b, c) to explain the perspective view of the solution and the absolute, real, and imaginary sketches in two– dimensional

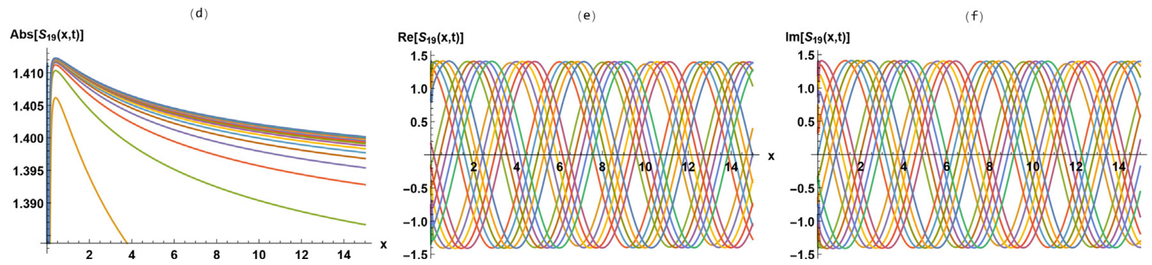

Fig. 6 Numerical simulation of Eq. (27) in two–dimensional sketches.

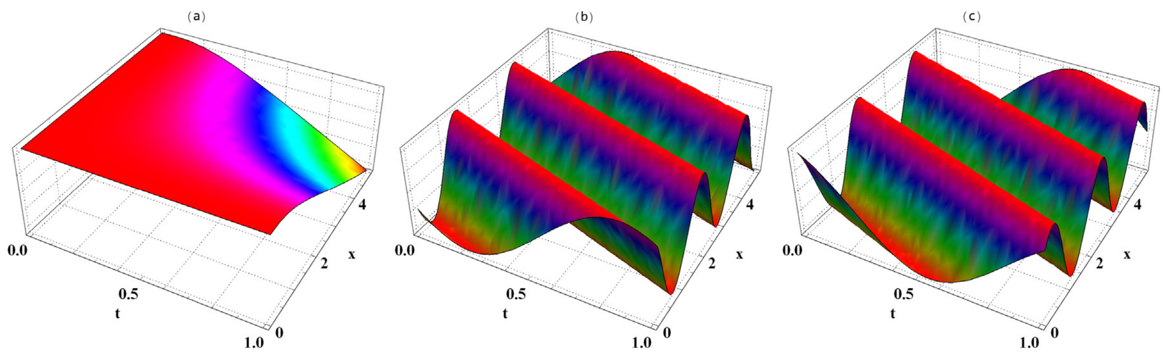

Fig. 7 Numerical simulation of Eq. (40) in three–dimensional sketches.

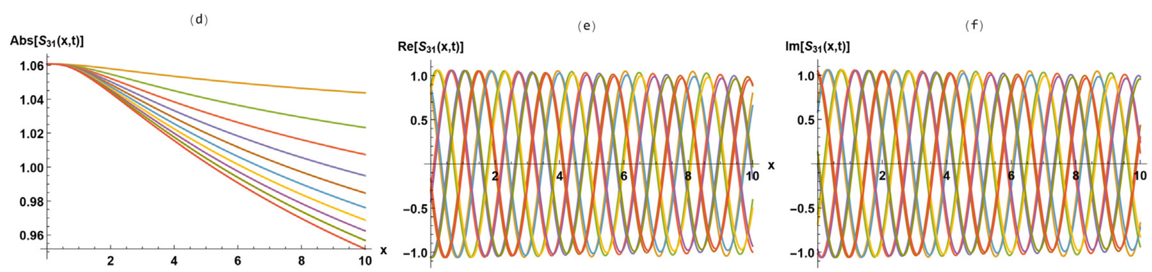

Fig. 8 Numerical simulation of Eq. (40) in two–dimensional sketches.

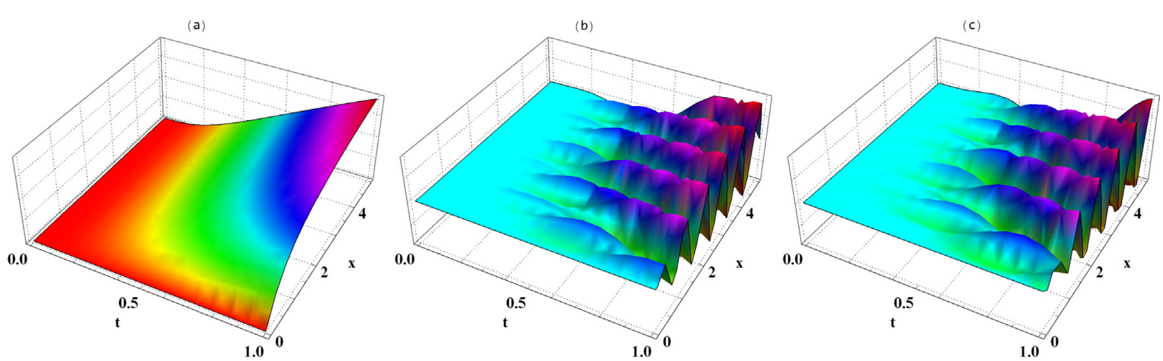

Fig. 9 Numerical simulation of Eq. (42) in three–dimensional sketches.

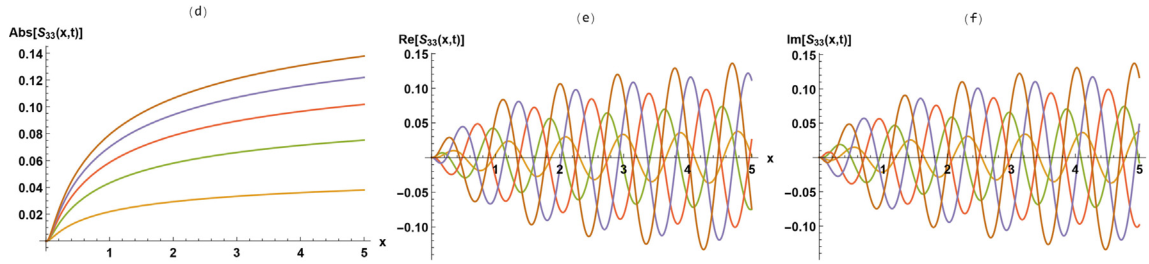

Fig. 10 Numerical simulation of Eq. (42) in two–dimensional sketches.

plot (d, e, f) to explain the wave propagation pattern of the wave along x axis when $[\alpha = 0.5, d_1 = 1, d_2 = 2, d_3 = -1, \delta = 1, k_2 = -2, k_3 = -3, k_1 = 6, l_1 = 4, l_2 = 5, \chi = 3, \varrho = 2]$.

4. Figs. 7 and 8 show the W–shapes of the absolute, real, and imaginary solution (40) in the three–dimensional plot (a, b, c) to explain the perspective view of the solution and the absolute, real, and imaginary sketches in two– dimensional plot (d, e, f) to explain the wave propagation pattern of the wave along x axis when $[\alpha = 0.5, d_1 = 1, d_2 = 2, d_3 = -1, \delta = 1, k_3 = -3, k_1 = 6, l_1 = 4, l_2 = 5, \chi = 3, \varrho = 2]$.
5. Figs. 9 and 10 show the breath W–shapes of the absolute, real, and imaginary solution (42) in the three–dimensional plot (a, b, c) to explain the perspective view of the solution and the absolute, real, and imaginary sketches in two–dimensional plot (d, e, f) to explain the wave propagation pattern of the wave along x axis when $[\alpha = 0.5, d_1 = 7, d_2 = 8, d_3 = 8, k_3 = 1, k_2 = 2, l_1 = 3, l_2 = 7]$.

4. Results and discussion

This section shows the novelty of this research paper by explain the comparison between our obtained solutions and that obtained in previous paper.

1. **Computational schemes:**This paper investigated the analytical solutions of the complex fractional Schrödinger equation by using two recent computational schemes (modified Khater method and sech–tanh functions expansion method). These methods are considered as recent analytical schemes in this field and they were not applied to this model yet.
2. **Obtained computational wave solutions:**
   - Eq. (29) equal Eq. (41) when $\left[k_3^2 = \frac{4k_2(-4\delta\varrho+\chi^2)}{-9}\right]$.
   - All our obtained solutions are different from that obtained in [8,34,37,40] where the authors of [8,34,37,40] used the integral from of the nonlinear complex fractional Schrödinger equation. On the other hand we investigated the fractional form of this model.

5. Conclusion

In this paper, we investigated the optical soliton wave solutions of the nonlinear complex fractional Schrödinger equation by using two recent analytical schemes. A new fractional operator is used to convert the fractional form of this model to a nonlinear partial differential equation with an–integer order. The modified Khater method and sech–tanh functions expansion method were applied to this model. Some new soliton optical wave solutions were obtained, and some of them were explained by plotting them in two, three–dimensional in absolute, real, and imaginary values of these solutions. The novelty of our paper was shown by making the comparison between our obtained solutions and that were purchased in previously published articles.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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