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Stochastic Lie bracket (derivation, derivation) in MB-algebras

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Abstract

By a stochastic controller, we make stable the pseudo stochastic Lie bracket (derivation, derivation) in complex MB-algebras. Next, we get an approximation by a stochastic Lie bracket (derivation, derivation) and calculate the maximum error of the estimate.

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1 Introduction

Let $(\Omega, \mathfrak{T}, \mu)$ be a probability measure space. Assume that $(T, \mathfrak{B}_T)$ is a Borel measureable space, in which T is an MB-space and $G, H : \Omega \times T \to T$ are random derivations. In MB-spaces, first we solve the (additive, additive)–(ω, ν) random operator inequality

$$\begin{aligned} &\xi_\tau^{G(\gamma,t+s)-G(\gamma,t)-G(\gamma,s)} * \xi_\tau^{H(\gamma,t+s)+H(\gamma,t-s)-2H(\gamma,t)} \\ &\quad \geq \xi_\tau^{\omega(2G(\gamma,\frac{t+s}{2})-G(\gamma,t)-G(\gamma,s))} * \xi_\tau^{\nu(2H(\gamma,\frac{t+s}{2})+2H(\gamma,\frac{t-s}{2})-2H(\gamma,t))}, \end{aligned} \tag{1.1}$$

where ω, ν are fixed nonzero complex numbers. By a stochastic controller we make stable the pseudo stochastic Lie bracket (derivation, derivation) in complex MB-algebras, associated to the above (additive, additive)–(ω, ν) random operator inequality and the following random operator inequality:

$$\xi_\tau^{[G,H](\gamma,ts)-[G,H](\gamma,t)s-t[G,H](\gamma,s)} * \xi_\tau^{H(\gamma,ts)-H(\gamma,t)s-tH(\gamma,s)} \geq \varphi_\tau^{t,s}. \tag{1.2}$$

The mentioned process is said to show Hyers–Ulam stability for the (additive, additive)–(ω, ν) random operator inequality (1.1).

2 Preliminaries

Let Ξ^+ be the set of distribution mappings, i.e., the set of all mappings $\rho : \mathbb{R} \cup \{-\infty, \infty\} \to [0, 1]$, writing ρ_τ for $\rho(\tau)$, such that ρ is left continuous and increasing on $\mathbb{R}$. $O^+ \subseteq \Xi^+$

includes all mappings $\rho \in \Xi^+$ for which $\ell^-\rho_{+\infty}$ is one and $\ell^-\rho_\tau$ is the left limit of the mapping ρ at the point τ, i.e., $\ell^-\rho_\tau = \lim_{\sigma\to\tau^-}\rho_\sigma$.

In Ξ^+, we define "$\leq$" as follows:

$$\rho \leq \varrho \quad \text{if and only if} \quad \rho_\tau \leq \varrho_\tau$$

for each τ in $\mathbb{R}$ (partially ordered). Note that the function ϑ^u defined by

$$\vartheta^u_s = \begin{cases} 0, & \text{if } s \leq u, \\ 1, & \text{if } s > u, \end{cases}$$

is an element of Ξ^+ and ϑ^0 is the maximal element in this space (for details, see [1–3]).

Definition 2.1 ([1, 4]) Denote by I the interval $[0,1]$. A *continuous triangular norm* (shortly, a *ct-norm*) is a continuous binary operation $*$ from I^2 to I such that

(a) $\varsigma * \tau = \tau * \varsigma$ and $\varsigma * (\tau * \upsilon) = (\varsigma * \tau) * \upsilon$ for all $\varsigma, \tau, \upsilon \in [0,1]$;
(b) $\varsigma * 1 = \varsigma$ for all $\varsigma \in I$;
(c) $\varsigma * \tau \leq \upsilon * \iota$ whenever $\varsigma \leq \upsilon$ and $\tau \leq \iota$ for all $\varsigma, \tau, \upsilon, \iota \in I$.

Some examples of *ct*-norms are as follows:

(1) $\varsigma *_P \tau = \varsigma\tau$;
(2) $\varsigma *_M \tau = \min\{\varsigma, \tau\}$;
(3) $\varsigma *_L \tau = \max\{\varsigma + \tau - 1, 0\}$ (the Lukasiewicz t-norm).

Definition 2.2 ([2]) Suppose that $*$ is a *ct*-norm, V is a linear space and ξ is a function from V to O^+. The ordered tuple $(V, \xi, *)$ is called a *Menger normed space* (in short, *MN-space*) if the following conditions are satisfied:

(MN1) $\xi^\nu_t = \vartheta^0_t$ for all $t > 0$ if and only if $\nu = 0$;
(MN2) $\xi^{\alpha\nu}_t = \xi^\nu_{\frac{t}{|\alpha|}}$ for all $\nu \in V$ and $\alpha \in \mathbb{C}$ with $\alpha \neq 0$;
(MN3) $\xi^{u+\nu}_{t+s} \geq \xi^u_t * \xi^\nu_s$ for all $u, \nu \in V$ and $t, s \geq 0$.

A complete MN-space is called Menger Banach space, in short, MB-space. Let $(V, \|\cdot\|)$ be a normed space. Then

$$\xi^\nu_s = \begin{cases} 0, & \text{if } s \leq 0, \\ \exp(-\frac{\|\nu\|}{s}), & \text{if } s > 0, \end{cases}$$

defines a Menger norm and the ordered tuple $(V, \xi, *_M)$ is an MN-space. Also,

$$\xi^\nu_s = \begin{cases} 0, & \text{if } s \leq 0, \\ \frac{s}{s+\|\nu\|}, & \text{if } s > 0, \end{cases}$$

defines a Menger norm and the ordered tuple $(V, \xi, *_M)$ is an MN-space.

Definition 2.3 ([5, 6]) A *Menger normed algebra* (in short, MN-algebra) $(V, \xi, *, \star)$ is an MN-space $(V, \xi, *)$ with algebraic structure such that

(FN-5) $\xi_{ts}^{uv} \geq \xi_t^u \star \xi_s^v$ for all $u, v \in V$ and all $t, s > 0$. in which $\star$ is a ct-norm.

Every normed algebra $(V, \|\cdot\|)$ defines an MN-algebra $(V, \xi, *_M, *_P)$, where

$$\xi_s^v = \begin{cases} 0, & \text{if } s \leq 0, \\ \exp(-\frac{\|v\|}{s}), & \text{if } s > 0, \end{cases}$$

if and only if

$$\|uv\| \leq \|u\|\|v\| + s\|v\| + t\|u\| \quad (u, v \in V; t, s > 0).$$

This space is called the induced MN-algebra. A complete MN-algebra is called Menger Banach algebra, in short, MB-algebra. Let (Γ, Σ, ξ) be a probability measure space. Assume that $(T, \mathfrak{B}_T)$ and $(S, \mathfrak{B}_S)$ are Borel measurable spaces, in which T and S are complete MN-spaces. A mapping $F : \Gamma \times T \to S$ is said to be a random operator if $\{\gamma : F(\gamma, t) \in B\} \in \Sigma$ for all t in T and $B \in \mathfrak{B}_S$. Also, F is a random operator if $F(\gamma, t) = s(\gamma)$ is an S-valued random variable for all t in T. A random operator $F : \Gamma \times T \to S$ is called *linear* if $F(\gamma, \alpha t_1 + \beta t_2) = \alpha F(\gamma, t_1) + \beta F(\gamma, t_2)$ almost everywhere for $t_1, t_2 \in T$ and α, β scalars, and *bounded* if there is a nonnegative random variable $M(\gamma)$ such that

$$\xi_{M(\gamma)\tau}^{F(\gamma,t)-F(\gamma,s)} \geq \xi_\tau^{t-s}$$

almost everywhere for each $t, s \in T$ and $\tau > 0$.

Let T be an MB-algebra. A linear random operator $\pi : \Gamma \times T \to T$ that satisfies

$$\pi(\gamma, ts) = \pi(\gamma, t)s + t\pi(\gamma, s)$$

for all $t, s \in T$ and $\gamma \in \Gamma$, is called stochastic derivation.

We denote by $\Pi(\Gamma, T)$ the set of $\mathbb{C}$-linear bounded stochastic derivations on $\Gamma \times T$. For $\pi_1, \pi_2 \in \Pi(\Gamma, T)$,

$$\pi_1 o \pi_2(\gamma, ts) = \pi_1 o \pi_2(\gamma, t)s + \pi_2(\gamma, t)\pi_1(\gamma, s) + \pi_1(\gamma, t)\pi_2(\gamma, s) + t\pi_1 o \pi_2(\gamma, s),$$
$$\pi_2 o \pi_1(\gamma, ts) = \pi_2 o \pi_1(\gamma, t)s + \pi_1(\gamma, t)\pi_2(\gamma, s) + \pi_2(\gamma, t)\pi_1(\gamma, t) + t\pi_2 o \pi_1(\gamma, s),$$

for all $t, s \in T$ and $\gamma \in \Gamma$. Assume that $[\pi_1, \pi_2] = \pi_1 o \pi_2 - \pi_2 o \pi_1$. Then

$$[\pi_1, \pi_2](\gamma, ts) = [\pi_1, \pi_2](\gamma, t)s + t[\pi_1, \pi_2](\gamma, s)$$

for all $t, s \in T$ and $\gamma \in \Gamma$. The $\mathbb{C}$-linearity of $[\pi_1, \pi_2]$ implies that $[\pi_1, \pi_2] \in \Pi(\Gamma, T)$ for all $\pi_1, \pi_2 \in \Pi(\Gamma, T)$. Then $\Pi(\Gamma, T)$ is a stochastic Lie algebra with stochastic Lie bracket $[\pi_1, \pi_2]$, $\pi_1 + \pi_2$ and $\beta\pi_1$ are $\mathbb{C}$-linear stochastic derivations in which $\beta \in \mathbb{C}$.

Definition 2.4 Consider an MB-algebra T and linear random operators $\Theta, \Phi : \Gamma \times T \to T$. Set $[\Theta, \Phi](\gamma, t) = \Theta(\gamma, \Phi(\gamma, t)) - \Phi(\gamma, \Theta(\gamma, t))$ for every $t \in T$ and $\gamma \in \Gamma$. The linear operator $[\Theta, \Phi] : \Gamma \times T \to T$ is said a stochastic Lie bracket (derivation, derivation) when

$$[\Theta,\Phi](\gamma,ts) = [\Theta,\Phi](\gamma,t)s + t[\Theta,\Phi](\gamma,s),$$
$$\Phi(\gamma,ts) = \Phi(\gamma,t)s + t\Phi(\gamma,s),$$

for all $t, s \in T$ and $\gamma \in \Gamma$.

Recently, some authors have published some papers on approximation of functional equations in various spaces by the direct technique and the fixed point technique, for example, fuzzy Menger normed algebras [5], fuzzy metric spaces [7], fuzzy normed spaces [8], non-Archimedian random Lie C^*-algebras [9], random multi-normed space [10], non-Archimedean random normed spaces [6]; see also [11–30].

Note that a $[0,\infty]$-valued metric is called a generalized metric.

Theorem 2.5 ([31–33]) *Consider a complete generalized metric space (T,δ) and a strictly contractive function $\Lambda : T \to T$ with Lipschitz constant $\beta < 1$. Then, for every given element $t \in T$, either*

$$\delta\bigl(\Lambda^n t, \Lambda^{n+1} t\bigr) = \infty$$

for each $n \in \mathbb{N}$ or there is an $n_0 \in \mathbb{N}$ such that

(1) *$\delta(\Lambda^n t, \Lambda^{n+1} t) < \infty$, for all $n \geq n_0$;*
(2) *the sequence $\{\Lambda^n t\}$ converges to a fixed point s^* of Λ;*
(3) *s^* is the unique fixed point of Λ in the set $V = \{s \in T \mid \delta(\Lambda^{n_0} t, s) < \infty\}$;*
(4) *$(1-\beta)\delta(s,s^*) \leq \delta(s, \Lambda s)$ for every $s \in V$.*

3 Stability of (additive, additive) (ω, ν)-random operator inequality: direct technique

Hereinafter we suppose that $* = *_M$.

Lemma 3.1 *Assume that random operators $G, H : \Gamma \times T \to T$ satisfy $G(\gamma,0) = H(\gamma,0) = 0$ and*

$$\xi_\tau^{G(\gamma,t+s)-G(\gamma,t)-G(\gamma,s)} * \xi_\tau^{H(\gamma,t+s)+H(\gamma,t-s)-2H(\gamma,t)} \geq \xi_\tau^{\omega(2G(\gamma,\frac{t+s}{2})-G(\gamma,t)-G(\gamma,s))} * \xi_\tau^{\nu(2H(\gamma,\frac{t+s}{2})+2H(\gamma,\frac{t-s}{2})-2H(\gamma,t))} \tag{3.1}$$

for all $t, s \in T$, $\gamma \in \Gamma$ and $\tau > 0$ in which $|\nu| < 1$ and $|\omega| < 1$. Then the random operators $G, H : \Gamma \times T \to T$ are additive.

Proof Putting $s = t$ in (3.1), we get

$$\xi_\tau^{G(\gamma,2t)-2G(\gamma,t)} * \xi_\tau^{H(\gamma,2t)-2H(\gamma,t)} \geq \vartheta_\tau^0$$

for all $t \in T$ and $\gamma \in \Gamma$. Then $G(\gamma,2t) = 2G(\gamma,t)$ and $H(\gamma,2t) = 2H(\gamma,t)$ for all $t \in T$ and $\gamma \in \Gamma$. By (3.1) we have

$$\begin{aligned}
&\xi_\tau^{G(\gamma,t+s)-G(\gamma,t)-G(\gamma,s)} * \xi_\tau^{H(\gamma,t+s)+H(\gamma,t-s)-2H(\gamma,t)} \\
&\quad \geq \xi_\tau^{\omega(G(\gamma,t+s)-G(\gamma,t)-G(\gamma,s))} * \xi_\tau^{\nu(H(\gamma,t+s)+H(\gamma,t-s)-2H(\gamma,t))}
\end{aligned}$$

for all $t,s \in T$, $\gamma \in \Gamma$ and $\tau > 0$. So $|\nu| < 1$ and $|\omega| < 1$ imply that $G(\gamma,t+s) - G(\gamma,t) - G(\gamma,s) = 0$ and $H(\gamma,t+s) + H(\gamma,t-s) - 2H(\gamma,t) = 0$ for all $t \in T$ and $\gamma \in \Gamma$. Thus the random operators $G,H : \Gamma \times T \to T$ are additive. □

Lemma 3.2 ([34, Theorem 2.1]) *Assume that a random operator $F : \Gamma \times T \to T$ is additive and*

$$F(\gamma,dt) = dF(\gamma,t)$$

for all $d \in \mathbb{D}^1 := \{c \in \mathbb{C} : |c| = 1\}$ and each $t \in T$ and $\gamma \in \Gamma$. Then the random operator $F : \Gamma \times T \to T$ is $\mathbb{C}$-linear.

Theorem 3.3 *Let $(T,\xi,*,*)$ be an MB-algebra. Let $\varphi : T^2 \to O^+$ be a distribution function such that there exists a $\beta \in (0,1)$ with*

$$\varphi_{\frac{\beta}{2}\tau}^{\frac{t}{2},\frac{s}{2}} \geq \varphi_{\frac{\beta}{4}\tau}^{\frac{t}{2},\frac{s}{2}} \geq \varphi_\tau^{t,s} \tag{3.2}$$

for all $t,s \in T$ and $\tau > 0$. Suppose that random operators $G,H : \Gamma \times T \to T$ satisfy $G(\gamma,0) = H(\gamma,0) = 0$ and

$$\begin{aligned}
&\xi_\tau^{G(\gamma,d(t+s))-dG(\gamma,t)-dG(\gamma,s)} * \xi_\tau^{H(\gamma,d(t+s))+H(\gamma,d(t-s))-2dH(\gamma,t)} \\
&\quad \geq \xi_\tau^{\omega(2G(\gamma,d\frac{t+s}{2})-dG(\gamma,t)-dG(\gamma,s))} \\
&\qquad * \xi_\tau^{\nu(2H(\gamma,d\frac{t+s}{2})+2H(\gamma,d\frac{t-s}{2})-2dH(\gamma,t))} * \varphi_\tau^{t,s}
\end{aligned} \tag{3.3}$$

for all $d \in \mathbb{D}^1$, $t,s \in T$, $\gamma \in \Gamma$ and $\tau > 0$. Assume that the random operators $G,H : \Gamma \times T \to T$ satisfy

$$\xi_\tau^{[G,H](\gamma,ts)-[G,H](\gamma,t)s-t[G,H](\gamma,s)} * \xi_\tau^{H(\gamma,ts)-H(\gamma,t)s-tH(\gamma,s)} \geq \varphi_\tau^{t,s} \tag{3.4}$$

for all $t,s \in T$, $\gamma \in \Gamma$ and $\tau > 0$. Then there are a unique $\mathbb{C}$-linear random operator $\Theta : \Gamma \times T \to T$ and a unique stochastic derivation $\pi : \Gamma \times T \to T$ such that $[\Theta,\pi] : \Gamma \times T \to T$ is a stochastic derivation and

$$\xi_\tau^{G(\gamma,t)-\Theta(\gamma,t)} * \xi_\tau^{H(\gamma,t)-\pi(\gamma,t)} \geq \varphi_{\frac{2(1-\beta)}{\beta}\tau}^{t,t} \tag{3.5}$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$.

Proof In (3.3), putting $d = 1$ and $s = t$, one obtains

$$\xi_\tau^{G(\gamma,2t)-2G(\gamma,t)} * \xi_\tau^{H(\gamma,2t)-2H(\gamma,t)} \geq \varphi_\tau^{t,t} \tag{3.6}$$

and so

$$\begin{aligned} \xi_\tau^{G(\gamma,t)-2G(\gamma,\frac{t}{2})} * \xi_\tau^{H(\gamma,t)-2H(\gamma,\frac{t}{2})} &\geq \varphi_\tau^{\frac{t}{2},\frac{t}{2}} \\ &\geq \varphi_{\frac{2}{\beta}\tau}^{t,t} \end{aligned} \tag{3.7}$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$. Replacing t by $\frac{t}{2^n}$ in (3.7), we get

$$\begin{aligned} \xi_\tau^{2^n G(\gamma,\frac{t}{2^n})-2^{n+1}G(\gamma,\frac{t}{2^{n+1}})} * \xi_\tau^{2^n H(\gamma,\frac{t}{2^n})-2^{n+1}H(\gamma,\frac{t}{2^{n+1}})} &\geq \varphi_{\frac{2}{\beta}\tau}^{\frac{t}{2^{n+1}},\frac{t}{2^{n+1}}} \\ &\geq \varphi_{\frac{2}{\beta^{n+1}}\tau}^{t,t} \end{aligned} \tag{3.8}$$

for all $t \in T$, $\gamma \in \Gamma$, $\tau > 0$ and $n \in \mathbb{N}$. Since

$$2^n G\left(\gamma, \frac{t}{2^n}\right) - G(\gamma,t) = \sum_{k=1}^{n} 2^k G\left(\gamma, \frac{t}{2^k}\right) - 2^{k-1} G\left(\gamma, \frac{t}{2^{k-1}}\right),$$

we have

$$\begin{aligned} &\xi_{\sum_{k=1}^{n}\frac{1}{2}\beta^k\tau}^{2^n G(\gamma,\frac{t}{2^n})-G(\gamma,t)} * \xi_{\sum_{k=1}^{n}\frac{1}{2}\beta^k\tau}^{2^n H(\gamma,\frac{t}{2^n})-H(\gamma,t)} \\ &\quad\geq \prod_{k=1}^{n}\left[\xi_{\frac{1}{2}\beta^k\tau}^{2^k G(\gamma,\frac{t}{2^k})-2^{k-1}G(\gamma,\frac{t}{2^{k-1}})} * \xi_{\frac{1}{2}\beta^k\tau}^{2^k H(\gamma,\frac{t}{2^k})-2^{k-1}H(\gamma,\frac{t}{2^{k-1}})}\right] \\ &\quad\geq \varphi_\tau^{t,t} \end{aligned} \tag{3.9}$$

and so

$$\xi_\tau^{2^n G(\gamma,\frac{t}{2^n})-G(\gamma,t)} * \xi_\tau^{2^n H(\gamma,\frac{t}{2^n})-H(\gamma,t)} \geq \varphi_{\frac{\tau}{\sum_{k=1}^{n}\frac{1}{2}\beta^k}}^{t,t} \tag{3.10}$$

for all $t \in T$, $\gamma \in \Gamma$, $\tau > 0$ and $n \in \mathbb{N}$.

Replacing t by $\frac{t}{2^m}$ in (3.10), we get

$$\begin{aligned} \xi_\tau^{2^{n+m} G(\gamma,\frac{t}{2^{n+m}})-2^m G(\gamma,\frac{t}{2^m})} * \xi_\tau^{2^{n+m} H(\gamma,\frac{t}{2^{n+m}})-2^m H(\gamma,\frac{t}{2^{n+m}})} &\geq \varphi_{\frac{2^m\tau}{\sum_{k=1}^{n}\frac{1}{2}\beta^k}}^{\frac{t}{2^m},\frac{t}{2^m}} \\ &\geq \varphi_{\frac{\tau}{\sum_{k=m+1}^{n+m}\frac{1}{2}\beta^k}}^{t,t}, \end{aligned} \tag{3.11}$$

for all $t \in T$, $\gamma \in \Gamma$, $\tau > 0$ and $n, m \in \mathbb{N}$.

Let $m, n \to \infty$ in (3.11), since $\beta \in (0,1)$, we conclude that $\varphi_{\frac{\tau}{\sum_{k=m+1}^{n+m}\frac{1}{2}\beta^k}}^{t,t}$ tends to 1 for all $\tau > 0$. Thus this shows that $\{2^n G(\gamma, \frac{t}{2^n})\}$ and $\{2^n H(\gamma, \frac{t}{2^n})\}$ are Cauchy sequences for each

$t \in T$, $\gamma \in \Gamma$. Since T is complete, the mentioned sequences converge. Now we define the random operators $\Theta, \pi : \Gamma \times T \to T$ by

$$\Theta(\gamma,t) := \lim_{n\to+\infty} 2^n G\left(\gamma, \frac{t}{2^n}\right), \qquad \pi(\gamma,t) := \lim_{n\to+\infty} 2^n H\left(\gamma, \frac{t}{2^n}\right) \tag{3.12}$$

for each $t \in T$, $\gamma \in \Gamma$. Putting $m = 0$ and $n \to +\infty$ in (3.11), we obtain (3.5).

Using (3.3), (3.12) and letting n tend to $+\infty$, we have

$$\begin{aligned}
&\xi_\tau^{\Theta(\gamma,d(t+s))-d\Theta(\gamma,t)-d\Theta(\gamma,s)} * \xi_\tau^{\pi(\gamma,d(t+s))+\pi(\gamma,d(t-s))-2d\pi(\gamma,s)} \\
&= \xi_{\frac{\tau}{2^n}}^{G(\gamma,d(\frac{t+s}{2^n}))-dG(\gamma,\frac{t}{2^n})-dG(\gamma,\frac{t}{2^n})} * \xi_{\frac{\tau}{2^n}}^{H(\gamma,d(\frac{t+s}{2^n}))+H(\gamma,d(\frac{t-s}{2^n}))-2dH(\gamma,\frac{s}{2^n})} \\
&\geq \xi_{\frac{\tau}{2^n}}^{\omega(2G(\gamma,d\frac{t+s}{2^{n+1}})-dG(\gamma,\frac{t}{2^n})-dG(\gamma,\frac{s}{2^n}))} * \xi_{\frac{\tau}{2^n}}^{\nu(2H(\gamma,d\frac{t+s}{2^{n+1}})+2H(\gamma,d\frac{t-s}{2^{n+1}})-2dH(\gamma,\frac{t}{2^n}))} * \varphi_{\frac{\tau}{2^n}}^{\frac{t}{2^n},\frac{s}{2^n}} \\
&\geq \xi_\tau^{\omega(2\Theta(\gamma,d\frac{t+s}{2})-d\Theta(\gamma,t)-d\Theta(\gamma,s))} * \xi_\tau^{\nu(2\pi(\gamma,d\frac{t+s}{2})+2\pi(\gamma,d\frac{t-s}{2})-2d\pi(\gamma,s))}
\end{aligned}$$

for all $d \in \mathbb{D}^1$, $t,s \in T$, $\gamma \in \Gamma$ and $\tau > 0$. Then

$$\begin{aligned}
&\xi_\tau^{\Theta(\gamma,d(t+s))-d\Theta(\gamma,t)-d\Theta(\gamma,s)} * \xi_\tau^{\pi(\gamma,d(t+s))+\pi(\gamma,d(t-s))-2d\pi(\gamma,s)} \\
&\geq \xi_\tau^{\omega(2\Theta(\gamma,d\frac{t+s}{2})-d\Theta(\gamma,t)-d\Theta(\gamma,s))} * \xi_\tau^{\nu(2\pi(\gamma,d\frac{t+s}{2})+2\pi(\gamma,d\frac{t-s}{2})-2d\pi(\gamma,s))}
\end{aligned} \tag{3.13}$$

for all $d \in \mathbb{D}^1$ and $t,s \in T$, $\gamma \in \Gamma$, $\tau > 0$. Putting $d = 1$ in (3.13) and using Lemma 3.1, we see that the random operators $\Theta, \pi : \Gamma \times T \to T$ are additive.

The additivity of Θ and π and (3.13) imply that

$$\begin{aligned}
&\xi_\tau^{\Theta(\gamma,d(t+s))-d\Theta(\gamma,t)-d\Theta(\gamma,s)} * \xi_\tau^{\pi(\gamma,d(t+s))+\pi(\gamma,d(t-s))-2d\pi(\gamma,s)} \\
&\geq \xi_\tau^{\omega(\Theta(\gamma,d(t+s))-d\Theta(\gamma,t)-d\Theta(\gamma,s))} * \xi_\tau^{\nu(\pi(\gamma,d(t+s))+\pi(\gamma,d(t-s))-2d\pi(\gamma,s))}
\end{aligned} \tag{3.14}$$

for all $d \in \mathbb{D}^1$ and $t,s \in T$, $\gamma \in \Gamma$, $\tau > 0$, which implies that

$$\begin{aligned}
&\Theta\big(\gamma, d(t+s)\big) - d\Theta(\gamma,t) - d\Theta(\gamma,s) = 0, \\
&\pi\big(\gamma, d(t+s)\big) + \pi\big(\gamma, d(t-s)\big) - 2d\pi(\gamma,s) = 0.
\end{aligned}$$

Then $\Theta(\gamma,dt) = d\Theta(\gamma,t)$ and $\pi(\gamma,dt) = d\pi(\gamma,t)$ for all $d \in \mathbb{D}^1$ and $t \in T$, $\gamma \in \Gamma$. Now, Lemma 3.2 implies that the additive mappings Θ and π are $\mathbb{C}$-linear.

The additivity of Θ and π and (3.4) imply that

$$\begin{aligned}
&\xi_\tau^{[\Theta,\phi](\gamma,ts)-[\Theta,\phi](\gamma,t)s-t[\Theta,\phi](\gamma,s)} * \xi_\tau^{\pi(\gamma,ts)-\pi(\gamma,t)s-t\pi(\gamma,s)} \\
&\geq \xi_{\frac{\tau}{4^n}}^{[G,H](\gamma,\frac{ts}{4^n})-[G,H](\gamma,\frac{t}{2^n})\frac{s}{2^n}-\frac{t}{2^n}[G,H](\gamma,\frac{s}{2^n})} * \xi_{\frac{\tau}{4^n}}^{H(\gamma,\frac{ts}{4^n})-H(\gamma,\frac{t}{2^n})\frac{s}{2^n}-\frac{t}{2^n}H(\gamma,\frac{s}{2^n})} \\
&\geq \varphi_{\frac{\tau}{4^n}}^{\frac{t}{2^n},\frac{s}{2^n}} \geq \varphi_{\frac{\tau}{\beta^n}}^{t,t},
\end{aligned} \tag{3.15}$$

which tends to 1 as $n \to +\infty$. Then

$$[\Theta,\phi](\gamma,ts) - [\Theta,\phi](\gamma,t)s - t[\Theta,\phi](\gamma,s) = 0,$$

$$\pi(\gamma, ts) - \pi(\gamma, t)s - t\pi(\gamma, s) = 0,$$

for all $t, s \in T$, $\gamma \in \Gamma$. Thus $[\Theta, \phi]$ and π are stochastic derivations. □

Corollary 3.4 *Let $(T, \xi, *, *)$ be an MB-algebra. Assume that $q > 0$ and $p > 1$. Suppose that random operators $G, H : \Gamma \times T \to T$ satisfy $G(\gamma, 0) = H(\gamma, 0) = 0$ and*

$$\begin{aligned} &\xi_\tau^{G(\gamma,d(t+s))-dG(\gamma,t)-dG(\gamma,s)} * \xi_\tau^{H(\gamma,d(t+s))+H(\gamma,d(t-s))-2dH(\gamma,t)} \\ &\quad \geq \xi_\tau^{\omega(2G(\gamma,d\frac{t+s}{2})-dG(\gamma,t)-dG(\gamma,s))} \\ &\qquad * \xi_\tau^{\nu(2H(\gamma,d\frac{t+s}{2})+2H(\gamma,d\frac{t-s}{2})-2dH(\gamma,t))} * \frac{\tau}{\tau + q(\|t\|^p + \|s\|^p)} \end{aligned} \tag{3.16}$$

for all $d \in \mathbb{D}^1$, $t, s \in T$, $\gamma \in \Gamma$ and $\tau > 0$. Let

$$\xi_\tau^{[G,H](\gamma,ts)-[G,H](\gamma,t)s-t[G,H](\gamma,s)} * \xi_\tau^{H(\gamma,ts)-H(\gamma,t)s-tH(\gamma,s)} \geq \frac{\tau}{\tau + q(\|t\|^p + \|s\|^p)} \tag{3.17}$$

for all $t, s \in T$, $\gamma \in \Gamma$ and $\tau > 0$. Then there are a unique $\mathbb{C}$-linear random operator $\Theta : \Gamma \times T \to T$ and a unique stochastic derivation $\pi : \Gamma \times T \to T$ such that $[\Theta, \pi] : \Gamma \times T \to T$ is a stochastic derivation and

$$\xi_\tau^{G(\gamma,t)-\Theta(\gamma,t)} * \xi_\tau^{H(\gamma,t)-\pi(\gamma,t)} \geq \frac{\tau}{\tau + q(\frac{2}{2^p-2}\|t\|^p)} \tag{3.18}$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$.

Proof In Theorem 3.3, putting

$$\varphi_\tau^{t,s} = \frac{\tau}{\tau + q(\|t\|^p + \|s\|^p)}$$

and letting $\beta = 2^{1-p}$, we get the desired result. □

Theorem 3.5 *Let $(T, \xi, *, *)$ be an MB-algebra. Let $\varphi : T^2 \to O^+$ be a distribution function such that there exists a $\beta \in (0, 1)$ with*

$$\varphi_{4\beta\tau}^{t,s} \geq \varphi_\tau^{\frac{t}{2},\frac{s}{2}} \tag{3.19}$$

for all $t, s \in T$ and $\tau > 0$. Suppose that the random operators $G, H : \Gamma \times T \to T$ satisfy $G(\gamma, 0) = H(\gamma, 0) = 0$, (3.3) and (3.4). Then there are a unique $\mathbb{C}$-linear random operator $\Theta : \Gamma \times T \to T$ and a unique stochastic derivation $\pi : \Gamma \times T \to T$ such that $[\Theta, \pi] : \Gamma \times T \to T$ is a stochastic derivation and

$$\xi_\tau^{G(\gamma,t)-\Theta(\gamma,t)} * \xi_\tau^{H(\gamma,t)-\pi(\gamma,t)} \geq \varphi_{2(1-\beta)\tau}^{t,t} \tag{3.20}$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$.

Proof Using (3.6), we get

$$\xi_{\tau}^{G(\gamma,t)-\frac{1}{2}G(\gamma,2t)} * \xi_{\tau}^{H(\gamma,t)-\frac{1}{2}H(\gamma,2t)} \geq \varphi_{2\tau}^{2t,2t} \geq \varphi_{\frac{\tau}{2\beta}}^{t,t} \tag{3.21}$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$.

Replacing t by $2^n t$ in (3.21), we get

$$\begin{aligned}\xi_{\tau}^{\frac{1}{2^n}G(\gamma,2^n t)-\frac{1}{2^{n+1}}G(\gamma,2^{n+1}t)} * \xi_{\tau}^{\frac{1}{2^n}H(\gamma,2^n t)-\frac{1}{2^{n+1}}H(\gamma,2^{n+1}t)} &\geq \varphi_{2^{n+1}\tau}^{2^{n+1}t,2^{n+1}t} \\ &\geq \varphi_{\frac{2^{n+1}}{(4\beta)^n}\tau}^{t,t}\end{aligned} \tag{3.22}$$

for all $t \in T$, $\gamma \in \Gamma$, $\tau > 0$ and $n \in \mathbb{N}$. Since

$$\frac{1}{2^n}G(\gamma,2^n t) - G(\gamma,t) = \sum_{k=0}^{n-1} \frac{1}{2^{k+1}}G(\gamma,2^{k+1}t) - \frac{1}{2^k}G(\gamma,2^k t),$$

we have

$$\begin{aligned}&\xi_{\sum_{k=0}^{n-1}\frac{(4\beta)^k}{2^{k+1}}\tau}^{\frac{1}{2^n}G(\gamma,2^n t)-G(\gamma,t)} * \xi_{\sum_{k=0}^{n-1}\frac{(4\beta)^k}{2^{k+1}}\tau}^{\frac{1}{2^n}H(\gamma,2^n t)-H(\gamma,t)} \\ &\quad\geq \prod_{k=0}^{n-1}\bigl[\xi_{\frac{(4\beta)^k}{2^{k+1}}\tau}^{\frac{1}{2^{k+1}}G(\gamma,2^{k+1}t)-\frac{1}{2^k}G(\gamma,2^k t)} * \xi_{\frac{(4\beta)^k}{2^{k+1}}\tau}^{\frac{1}{2^{k+1}}H(\gamma,2^{k+1}t)-\frac{1}{2^k}H(\gamma,2^k t)}\bigr] \\ &\quad\geq \varphi_{\tau}^{t,t}\end{aligned} \tag{3.23}$$

and so

$$\xi_{\tau}^{\frac{1}{2^n}G(\gamma,2^n t)-G(\gamma,t)} * \xi_{\tau}^{\frac{1}{2^n}H(\gamma,2^n t)-H(\gamma,t)} \geq \varphi_{\frac{\tau}{\sum_{k=0}^{n-1}\frac{(4\beta)^k}{2^{k+1}}}}^{t,t} \tag{3.24}$$

for all $t \in T$, $\gamma \in \Gamma$, $\tau > 0$ and $n \in \mathbb{N}$.

Replacing t by $2^m t$ in (3.24), we get

$$\begin{aligned}\xi_{\tau}^{\frac{1}{2^{n+m}}G(\gamma,2^{n+m}t)-\frac{1}{2^m}G(\gamma,2^m t)} * \xi_{\tau}^{\frac{1}{2^{n+m}}H(\gamma,2^{n+m}t)-\frac{1}{2^m}H(\gamma,2^m t)} &\geq \varphi_{\frac{\frac{1}{2^m}\tau}{\sum_{k=0}^{n-1}\frac{(4\beta)^k}{2^{k+1}}}}^{2^m t,2^m t} \\ &\geq \varphi_{\frac{\tau}{\sum_{k=m}^{n+m}\frac{(4\beta)^k}{2^{k+1}}}}^{t,t}\end{aligned} \tag{3.25}$$

for all $t \in T$, $\gamma \in \Gamma$, $\tau > 0$ and $n, m \in \mathbb{N}$.

Letting $m, n \to +\infty$ in (3.25), since $\beta \in (0,1)$, we conclude that $\varphi_{\frac{\tau}{\sum_{k=m}^{n+m}\frac{(4\beta)^k}{2^{k+1}}}}^{t,t}$ tends to 1 for all $\tau > 0$. This shows that $\{\frac{1}{2^n}G(\gamma,2^n t)\}$ and $\{\frac{1}{2^n}H(\gamma,2^n t)\}$ are Cauchy sequences for each $t \in T$, $\gamma \in \Gamma$. Since T is complete, the mentioned sequences converge. Now we define the random operators $\Theta, \pi : \Gamma \times T \to T$ by

$$\Theta(\gamma,t) := \lim_{n\to+\infty}\frac{1}{2^n}G(\gamma,2^n t), \qquad \pi(\gamma,t) := \lim_{n\to+\infty}\frac{1}{2^n}G(\gamma,2^n t), \tag{3.26}$$

for each $t \in T$, $\gamma \in \Gamma$. Putting $m = 0$ and $n \to \infty$ in (3.25), we get (3.5). By the same method in the proof of Theorem 3.3, the random operators $\Theta, \pi : \Gamma \times T \to T$ are $\mathbb{C}$-linear.

The additivity of Θ and π and (3.4) imply that

$$\begin{aligned}
&\xi_{\tau}^{[\Theta,\phi](\gamma,ts)-[\Theta,\phi](\gamma,t)s-t[\Theta,\phi](\gamma,s)} * \xi_{\tau}^{\pi(\gamma,ts)-\pi(\gamma,t)s-t\pi(\gamma,s)} \\
&\quad \geq \xi_{4^n\tau}^{[G,H](\gamma,4^n ts)-[G,H](\gamma,2^n t)2^n s-2^n t[G,H](\gamma,2^n s)} * \xi_{4^n\tau}^{H(\gamma,4^n ts)-H(\gamma,2^n t)2^n s-2^n tH(\gamma,2^n s)} \\
&\quad \geq \varphi_{4^n\tau}^{2^n t,2^n s} \\
&\quad \geq \varphi_{\frac{\tau}{\beta^n}}^{t,t},
\end{aligned} \tag{3.27}$$

which tends to 1 as $n \to +\infty$. Then

$$[\Theta,\phi](\gamma,ts) - [\Theta,\phi](\gamma,t)s - t[\Theta,\phi](\gamma,s) = 0,$$

$$\pi(\gamma,ts) - \pi(\gamma,t)s - t\pi(\gamma,s) = 0$$

for all $t, s \in T$, $\gamma \in \Gamma$. Thus $[\Theta,\phi]$ and π are stochastic derivations. □

Corollary 3.6 *Let $(T, \xi, *, *)$ be an MB-algebra. Assume that $q > 0$ and $p < 1$. Suppose that random operators $G, H : \Gamma \times T \to T$ satisfy $G(\gamma,0) = H(\gamma,0) = 0$, (3.16) and (3.17). Then there are a unique $\mathbb{C}$-linear random operator $\Theta : \Gamma \times T \to T$ and a unique stochastic derivation $\pi : \Gamma \times T \to T$ such that $[\Theta,\pi] : \Gamma \times T \to T$ is a stochastic derivation and*

$$\xi_{\tau}^{G(\gamma,t)-\Theta(\gamma,t)} * \xi_{\tau}^{H(\gamma,t)-\pi(\gamma,t)} \geq \frac{\tau}{\tau + q(\frac{2}{2-2^p}\|t\|^p)} \tag{3.28}$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$.

Proof In Theorem 3.5, putting

$$\varphi_{\tau}^{t,s} = \frac{\tau}{\tau + q(\|t\|^p + \|s\|^p)},$$

and letting $\beta = 2^{p-1}$, we get the desired result. □

4 Stability of (additive, additive) (ω, ν)-random operator inequality (1.1) via fixed point technique

Theorem 4.1 *Let $(T, \xi, *, *)$ be an MB-algebra. Let $\varphi : T^2 \to O^+$ be a distribution function such that there exists a $\beta \in (0,1)$ with*

$$\varphi_{\frac{\beta}{2}\tau}^{\frac{t}{2},\frac{s}{2}} \geq \varphi_{\frac{\beta}{4}\tau}^{\frac{t}{2},\frac{s}{2}} \geq \varphi_{\tau}^{t,s} \tag{4.1}$$

for all $t, s \in T$ and $\tau > 0$. Suppose that random operators $G, H : \Gamma \times T \to T$ satisfy $G(\gamma,0) = H(\gamma,0) = 0$, (3.3) and (3.4). Then there are a unique $\mathbb{C}$-linear random operator $\Theta : \Gamma \times T \to T$ and a unique stochastic derivation $\pi : \Gamma \times T \to T$ such that $[\Theta,\pi] : \Gamma \times T \to T$

is a stochastic derivation and

$$\xi_\tau^{G(\gamma,t)-\Theta(\gamma,t)} * \xi_\tau^{H(\gamma,t)-\pi(\gamma,t)} \geq \varphi^{t,t}_{\frac{2(1-\beta)}{\beta}\tau} \tag{4.2}$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$.

Proof By Theorem 3.3, there exist a unique $\mathbb{C}$-linear random operator $\Theta : \Gamma \times T \to T$ and a unique stochastic derivation $\pi : \Gamma \times T \to T$ such that $[\Theta,\pi] : \Gamma \times T \to T$ is a stochastic a derivation.

In (3.3), putting $d = 1$ and $s = t$, we get

$$\xi_\tau^{G(\gamma,2t)-2G(\gamma,t)} * \xi_\tau^{H(\gamma,2t)-2H(\gamma,t)} \geq \varphi^{t,t}_{\tau} \tag{4.3}$$

and so

$$\begin{aligned}\xi_\tau^{G(\gamma,t)-2G(\gamma,\frac{t}{2})} * \xi_\tau^{H(\gamma,t)-2H(\gamma,\frac{t}{2})} &\geq \varphi^{\frac{t}{2},\frac{t}{2}}_{\tau} \\ &\geq \varphi^{t,t}_{\frac{2}{\beta}\tau}\end{aligned}$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$.

On the set

$$S := \bigl\{(G,H) \mid G,H : \Gamma \times T \to T, G(\gamma,0) = H(\gamma,0) = 0\bigr\},$$

we define the following generalized metric on S:

$$\begin{aligned}&\delta\bigl((G,H),(G_1,H_1)\bigr) \\ &\quad = \inf\bigl\{\mu \in \mathbb{R}_+ : \xi_\tau^{G(\gamma,t)-G_1(\gamma,t)} * \xi_\tau^{H(\gamma,t)-H_1(\gamma,t)} \geq \varphi^{t,t}_{\frac{\tau}{\mu}}, \forall t \in T, \gamma \in \Gamma, \tau > 0\bigr\}.\end{aligned}$$

In [35], Miheţ and Radu proved that (S,δ) is complete (see also [36]).

Now, we consider the linear mapping $\Lambda : S \to S$ such that

$$\Lambda(G,H)(\gamma,t) := \left(2G\left(\gamma,\frac{t}{2}\right), 2H\left(\gamma,\frac{t}{2}\right)\right)$$

for all $t \in T$, $\gamma \in \Gamma$.

Let $(G,H),(G_1,H_1) \in S$ be given such that $\delta((G,H),(G_1,H_1)) = \varepsilon$. Then

$$\xi_\tau^{G(\gamma,t)-G_1(\gamma,t)} * \xi_\tau^{H(\gamma,t)-H_1(\gamma,t)} \geq \varphi^{t,t}_{\frac{\tau}{\varepsilon}}$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$. So

$$\xi_\tau^{2G(\gamma,\frac{t}{2})-2G_1(\gamma,\frac{t}{2})} * \xi_\tau^{2H(\gamma,\frac{t}{2})-H_1(\gamma,\frac{t}{2})} \geq \varphi^{\frac{t}{2},\frac{t}{2}}_{\frac{\tau}{\varepsilon}} \geq \varphi^{t,t}_{\frac{\tau}{\beta\varepsilon}}$$

for all $t \in T$, $\gamma \in \Gamma$, $\tau > 0$ and $\delta(\Lambda(G,H),\Lambda(G_1,H_1)) \leq \beta\varepsilon$. This means that

$$\delta\bigl(\Lambda(G,H),\Lambda(G_1,H_1)\bigr) \leq \beta\delta\bigl((G,H),(G_1,H_1)\bigr)$$

for all $(G,H),(G_1,H_1) \in S$.

It follows from (3.3) that

$$\xi_\tau^{G(\gamma,t)-2G_1(\gamma,\frac{t}{2})} * \xi_\tau^{H(\gamma,t)-H_1(\gamma,\frac{t}{2})} \geq \varphi_\tau^{\frac{t}{2},\frac{t}{2}} \geq \varphi_{\frac{2\tau}{\beta}}^{t,t}$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$. So $\delta((G,H),\Lambda(G,H)) \leq \frac{\beta}{2}$. By Theorem 2.5, there exist random operators $\Theta,\pi : \Gamma \times T \to T$ satisfying the following:

(1) There is a fixed point (Θ,π) for the function Λ such that

$$\Theta(\gamma,t) := 2\Theta\left(\gamma,\frac{t}{2}\right), \qquad \pi(\gamma,t) := 2\pi\left(\gamma,\frac{t}{2}\right) \tag{4.4}$$

for all $t \in T$, $\gamma \in \Gamma$. The random operator (Θ,π) is a unique fixed point of Λ in the set

$$M = \{(G,H) \in S : \delta((G,H),(G_1,H_1)) < \infty\}.$$

(2) $\delta(\Lambda^n(G,H),(\Theta,\pi)) \to 0$ as $n \to +\infty$. which implies

$$\Theta(\gamma,t) := \lim_{n\to+\infty} 2^n G\left(\gamma,\frac{t}{2^n}\right), \qquad \pi(\gamma,t) := \lim_{n\to+\infty} 2^n H\left(\gamma,\frac{t}{2^n}\right).$$

(3) $\delta((G,H),(\Theta,\pi)) \leq \frac{1}{1-\beta}\delta((G,H),\Lambda(G,H))$, which implies

$$\xi_\tau^{G(\gamma,t)-\Theta(\gamma,t)} * \xi_\tau^{H(\gamma,t)-\pi(\gamma,t)} \geq \varphi_{\frac{2(1-\beta)}{\beta}\tau}^{t,t}$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$. □

Corollary 4.2 *Let $(T,\xi,*,*)$ be an MB-algebra. Assume that $q > 0$ and $p > 1$. Suppose that random operators $G,H : \Gamma \times T \to T$ satisfy $G(\gamma,0) = H(\gamma,0) = 0$, (3.16) and (3.17). Then there are a unique $\mathbb{C}$-linear random operator $\Theta : \Gamma \times T \to T$ and a unique stochastic derivation $\pi : \Gamma \times T \to T$ such that $[\Theta,\pi] : \Gamma \times T \to T$ is a stochastic derivation and*

$$\xi_\tau^{G(\gamma,t)-\Theta(\gamma,t)} * \xi_\tau^{H(\gamma,t)-\pi(\gamma,t)} \geq \exp\left(-\frac{q(\frac{2}{2^p-2}\|t\|^p)}{\tau}\right)$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$.

Proof In Theorem 4.1, putting

$$\varphi_\tau^{t,s} = \exp\left(-\frac{q(\frac{2}{2^p-2}\|t\|^p)}{\tau}\right),$$

and letting $\beta = 2^{1-p}$, we get the desired result. □

Theorem 4.3 *Let $(T,\xi,*,*)$ be an MB-algebra. Let $\varphi : T^2 \to O+$ be a distribution function such that there exists a $\beta \in (0,1)$ with*

$$\varphi_{4\beta\tau}^{t,s} \geq \varphi_\tau^{\frac{t}{2},\frac{s}{2}} \tag{4.5}$$

for all $t, s \in T$ *and* $\tau > 0$. *Suppose that random operators* $G, H : \Gamma \times T \to T$ *satisfy* $G(\gamma, 0) = H(\gamma, 0) = 0$, (3.3) *and* (3.4). *Then there are a unique* $\mathbb{C}$*-linear random operator* $\Theta : \Gamma \times T \to T$ *and a unique stochastic derivation* $\pi : \Gamma \times T \to T$ *such that* $[\Theta, \pi] : \Gamma \times T \to T$ *is a stochastic derivation and*

$$\xi_\tau^{G(\gamma,t)-\Theta(\gamma,t)} * \xi_\tau^{H(\gamma,t)-\pi(\gamma,t)} \geq \varphi_{2(1-\beta)\tau}^{t,t} \tag{4.6}$$

for all $t \in T$, $\gamma \in \Gamma$ *and* $\tau > 0$.

Proof By Theorem 3.5, there exist a unique $\mathbb{C}$-linear random operator $\Theta : \Gamma \times T \to T$ and a unique stochastic derivation $\pi : \Gamma \times T \to T$ such that $[\Theta, \pi] : \Gamma \times T \to T$ is a stochastic a derivation.

Let (S, δ) be the generalized metric space defined in the proof of Theorem 4.1. Now, we consider the linear mapping $\Lambda : S \to S$ such that

$$\Lambda(G, H)(\gamma, t) := \left(\frac{1}{2}G(\gamma, 2t), \frac{1}{2}H(\gamma, 2t)\right)$$

for all $t \in T$, $\gamma \in \Gamma$. It follows from (4.3) that

$$\begin{aligned} \xi_\tau^{G(\gamma,t)-\frac{1}{2}G(\gamma,2t)} * \xi_\tau^{H(\gamma,t)-\frac{1}{2}H(\gamma,2t)} &\geq \varphi_{2\tau}^{2t,2t} \\ &\geq \varphi_{\frac{\tau}{2\beta}}^{t,t} \end{aligned}$$

for all $t \in T$, $\gamma \in \Gamma$ and $\tau > 0$. The proof will be finished by a similar method to the one used in the proofs of Theorems 3.3 and 4.1. □

Corollary 4.4 *Let* $(T, \xi, *, *)$ *be an MB-algebra. Assume that* $q > 0$ *and* $p < 1$. *Suppose that random operators* $G, H : \Gamma \times T \to T$ *satisfy* $G(\gamma, 0) = H(\gamma, 0) = 0$, (3.16) *and* (3.17). *Then there are a unique* $\mathbb{C}$*-linear random operator* $\Theta : \Gamma \times T \to T$ *and a unique stochastic derivation* $\pi : \Gamma \times T \to T$ *such that* $[\Theta, \pi] : \Gamma \times T \to T$ *is a stochastic derivation and*

$$\xi_\tau^{G(\gamma,t)-\Theta(\gamma,t)} * \xi_\tau^{H(\gamma,t)-\pi(\gamma,t)} \geq \exp\left(-\frac{q(\frac{2}{2-2^p}\|t\|^p)}{\tau}\right)$$

for all $t \in T$, $\gamma \in \Gamma$ *and* $\tau > 0$.

Proof In Theorem 4.3, putting

$$\varphi_\tau^{t,s} = \exp\left(-\frac{q(\frac{2}{2-2^p}\|t\|^p)}{\tau}\right),$$

and letting $\beta = 2^{p-1}$, we get the desired result. □

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Authors' contributions
All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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