

Soft b-separation axioms in neutrosophic soft topological structures

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ABSTRACT. The idea of neutrosophic set was floated by Smarandache by supposing a truth membership, an indeterminacy membership and a falsehood or falsity membership functions. Neutrosophic soft sets bonded by Maji have been utilized successfully to model uncertainty in several areas of application such as control, reasoning, pattern recognition and computer vision. The first aim of this article bounces the idea of neutrosophic soft b-open set, neutrosophic soft b-closed sets and their properties. Also the idea of neutrosophic soft b-neighborhood and neutrosophic soft b-separation axioms in neutrosophic soft topological structures are also reflected here. Later on the important results are discussed related to these newly defined concepts with respect to soft points. The concept of neutrosophic soft b-separation axioms of neutrosophic soft topological spaces is diffused in different results with respect to soft points. Furthermore, properties of neutrosophic soft $b-T^i$ -space ($i = 0, 1, 2, 3, 4$) and some linkage between them are discussed.

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1. INTRODUCTION

The traditional fuzzy sets is characterised by the membership value or the grade of membership value. Some times it may be very difficult to assign the membership value for a fuzzy sets. Consequently the concept of interval valued fuzzy sets was proposed to capture and grip the uncertainty of grade of membership value. In some real life problems in expert system, belief system, information fusion and so on, we must consider the truth-membership as well as the falsity-membership for

proper description of an object in uncertain,ambiguous environment. Neither the fuzzy sets nor the interval valued fuzzy sets is here to shoulder up the responsibilities for such a situation. The importance of intuitionistic fuzzy sets is automatically come in play in such a dangerous situation. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth-membership or association(or simply membership)and non-membership values. It does not handle the indeterminate and inconsistent information which exists in belief system. Smarandache [17] introduced new concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. The words "neutrosophy" and "neutrosophic" were introduced by Smarandache. "Neutrosophy" (noun) means knowledge of neutral thought, while "neutrosophic" (adjective), means having the nature of or having the characteristic of neutrosophy. This theory is straightforward generalization of crisp sets, fuzzy set theory [18], intuitionistic fuzzy set theory [1] etc. Some work have been supposed on neutrosophic sets by some researchers in many area of mathematics [4, 15]. Many practical problems in economics, engineering, environment, social science, medical science, etc.can not be dealt with by classical techniques, because classical methods have heritable complexities. These complexities may be taking birth due to the insufficiency of the theories of parameterization tools. Each of these theories has its transmissible difficulties, as was naked by Molodtsov [14]. Molodtsov originated an absolutely modern access to cope with uncertainty and vagueness and applied it progressively in different directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and so on. Honestly,Theory of soft set is free from the parameterization meagerness syndrome of fuzzy set theory, rough set theory. probability theory for dealing with uncertainty Shabir and Naz [16] first floated the notion of soft topological spaces, which are defined over an initial universe of discourse with a fixed set of parameters, and showed that a soft topological space gives a parameterized family of topological structures. Theoretical studies of soft topological spaces were also done by some authors in [2, 3, 6, 8]. The combination of Neutrosophic set with soft sets was first introduced by Maji [13]. This combination makes entirely a new mathematical model "Neutrosophic soft set" and later on this notion was improved by Deli and Broumi [7]. Work was progressively continue,later on mathematician came in action and defined a new mathematical structure known as neutrosophic soft topological spaces. Neutrosophic soft topological spaces were presented by Bera in [5]. Guzide.S [9] attempted to bring together the areas of spheres, soft real numbers and soft points. Relating spheres to soft real numbers and soft points provides a natural and intrinsic construction of soft spheres. Guzide [10] discussed the theory of soft topological space generated by L-soft sets is introduced. As a continuation of the study of operations on L-soft sets, the aim of this paper is to introduce new soft topologies using restricted and extended intersections on L-soft sets and to study the differences of these soft topologies. Guzide [11] discussed soft point sets and soft matrix form which were not described before is defined for each set of parameters. The matrix representation of soft points is useful for storing all soft points that can be obtained in all different parameters. The proposed soft matrix provides every soft point that changes with each parameter that takes place in a soft set is proved

and showed that it enables detailed examination in application of soft set theory. Guzide [12] discussed comparative research on the definition of Soft Point.

The first aim of this article bounces the idea of neutrosophic soft b-open set, neutrosophic soft b-neighborhood and neutrosophic soft b-separation axioms in neutrosophic soft topology which is defined on neutrosophic soft sets. Later on the important results are discussed related to these newly defined concepts with respect to soft points. Finally, the concept of b-separation axioms of neutrosophic soft topological spaces is diffused in different results with respect to soft points. Furthermore, properties of neutrosophic soft $b-T^i$ -space ($i = 0, 1, 2, 3, 4$) and some linkage between them are discussed. We hope that these results will best fit for future study on neutrosophic soft topology to carry out a general framework for practical applications.

2. PRELIMINARIES

In this section we now state certain useful definitions, theorems, and several existing results for neutrosophic soft sets that we require in the next sections.

Definition 2.1 ([17]). A neutrosophic set A on the universe set X is defined as:

$$A = \{ \langle x, T^A(x), I^A(x), F^A(x) \rangle : x \in X \},$$

where $T, I, F : X \rightarrow]-0, 1^+[$ and $-0 \leq T^A(x) + I^A(x) + F^A(x) \leq 3^+$.

Definition 2.2. [14] Let X be an initial universe, E be a set of all parameters, and $P(x)$ denote the power set of X . Then a pair (F, E) is called a soft set over X , where F is a mapping given by $F : E \rightarrow P(X)$.

In other words, the soft set is a parameterized family of subsets of the set X . For $\lambda \in E$, $F(\lambda)$ may be considered as the set of λ -elements of the soft set (F, E) , or as the set of λ -approximate element of the set, i.e.,

$$(F, E) = \{ \langle \lambda, F(\lambda) \rangle : \lambda \in E, F : E \rightarrow P(X) \}.$$

After the neutrosophic soft set was defined by Maji [13], this concept was modified by Deli and Broumi [7] as given below.

Definition 2.3 ([7]). Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X . Then a neutrosophic soft set (\tilde{F}, E) over X is a set defined by a set valued function \tilde{F} representing a mapping $\tilde{F} : E \rightarrow P(X)$, where \tilde{F} is called the approximate function of the neutrosophic soft set (\tilde{F}, E) . In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs:

$$(\tilde{F}, E) = \{ \langle \lambda, \langle x, T^{\tilde{F}(\lambda)}(x), I^{\tilde{F}(\lambda)}(x), F^{\tilde{F}(\lambda)}(x) \rangle : x \in X \rangle : \lambda \in E \},$$

where $T^{\tilde{F}(\lambda)}(x), I^{\tilde{F}(\lambda)}(x), F^{\tilde{F}(\lambda)}(x) \in [0, 1]$ are respectively called the truth-membership, indeterminacy-membership, and falsity-membership function of $\tilde{F}(\lambda)$. Since the supremum of each T, I, F is 1, the inequality

$$0 \leq T^{\tilde{F}(\lambda)}(x) + I^{\tilde{F}(\lambda)}(x) + F^{\tilde{F}(\lambda)}(x) \leq 3$$

is obvious.

Definition 2.4 ([5]). Let (\tilde{F}, E) be a neutrosophic soft set over the universe set X . The complement of (\tilde{F}, E) is denoted by $(\tilde{F}, E)^c$ and is defined by:

$$(\tilde{F}, E)^c = \{(\lambda, \langle x, F^{\tilde{F}(\lambda)}(x), 1 - I^{\tilde{F}(\lambda)}(x), T^{\tilde{F}(\lambda)}(x) \rangle : x \in X) : \lambda \in E\}.$$

It is obvious that $((\tilde{F}, E)^c)^c = (\tilde{F}, E)$.

Definition 2.5 ([13]). Let (\tilde{F}, E) and (\tilde{G}, E) be two neutrosophic soft sets over the universe set X . (\tilde{F}, E) is said to be a neutrosophic soft subset of (\tilde{G}, E) , if

$$T^{\tilde{F}(\lambda)}(x) \leq T^{\tilde{G}(\lambda)}(x), I^{\tilde{F}(\lambda)}(x) \leq I^{\tilde{G}(\lambda)}(x), F^{\tilde{F}(\lambda)}(x) \geq F^{\tilde{G}(\lambda)}(x), \forall \lambda \in E, \forall x \in X.$$

It is denoted by $(\tilde{F}, E) \subseteq (\tilde{G}, E)$. (\tilde{F}, E) is said to be neutrosophic soft equal to (\tilde{G}, E) if (\tilde{F}, E) is a neutrosophic soft subset of (\tilde{G}, E) and (\tilde{G}, E) is a neutrosophic soft subset of (\tilde{F}, E) . It is denoted by $(\tilde{F}, E) = (\tilde{G}, E)$.

3. NEUTROSOPHIC SOFT POINT AND RELATED CHARACTERISTICS

Definition 3.1. Let (\tilde{F}^1, E) and (\tilde{F}^2, E) be two neutrosophic soft sets over universe set X . Then their union, denoted by $(\tilde{F}^1, E) \sqcup (\tilde{F}^2, E) = (\tilde{F}^3, E)$, is defined by:

$$(\tilde{F}^3, E) = \{(\lambda, \langle x, T^{\tilde{F}^3(\lambda)}(x), I^{\tilde{F}^3(\lambda)}(x), F^{\tilde{F}^3(\lambda)}(x) \rangle : x \in X) : \lambda \in E\},$$

$$\begin{aligned} T^{\tilde{F}^3(\lambda)}(x) &= \max \{T^{\tilde{F}^1(\lambda)}(x), T^{\tilde{F}^2(\lambda)}(x)\}, \\ I^{\tilde{F}^3(\lambda)}(x) &= \min \{I^{\tilde{F}^1(\lambda)}(x), I^{\tilde{F}^2(\lambda)}(x)\}, \\ F^{\tilde{F}^3(\lambda)}(x) &= \min \{F^{\tilde{F}^1(\lambda)}(x), F^{\tilde{F}^2(\lambda)}(x)\}. \end{aligned}$$

Definition 3.2. Let (\tilde{F}^1, E) and (\tilde{F}^2, E) be two neutrosophic soft sets over the universe set X . Then their intersection, denoted by $(\tilde{F}^1, E) \cap (\tilde{F}^2, E) = (\tilde{F}^3, E)$, is defined by:

$$(\tilde{F}^3, E) = \{(\lambda, \langle x, T^{\tilde{F}^3(\lambda)}(x), I^{\tilde{F}^3(\lambda)}(x), F^{\tilde{F}^3(\lambda)}(x) \rangle : x \in X) : \lambda \in E\},$$

$$\begin{aligned} T^{\tilde{F}^3(\lambda)}(x) &= \min \{T^{\tilde{F}^1(\lambda)}(x), T^{\tilde{F}^2(\lambda)}(x)\}, \\ I^{\tilde{F}^3(\lambda)}(x) &= \max \{I^{\tilde{F}^1(\lambda)}(x), I^{\tilde{F}^2(\lambda)}(x)\}, \\ F^{\tilde{F}^3(\lambda)}(x) &= \max \{F^{\tilde{F}^1(\lambda)}(x), F^{\tilde{F}^2(\lambda)}(x)\}. \end{aligned}$$

Definition 3.3. A neutrosophic soft set (\tilde{F}, E) over the universe set X is said to be a null neutrosophic soft set, if for each $\lambda \in E$ and each $x \in X$,

$$T^{\tilde{F}(\lambda)}(x) = 0, I^{\tilde{F}(\lambda)}(x) = 0, F^{\tilde{F}(\lambda)}(x) = 1.$$

It is denoted by $0^{(X,E)}$.

Definition 3.4. A neutrosophic soft set (\tilde{F}, E) over the universe set X is said to be an absolute neutrosophic soft set, if for each $\lambda \in E$ and each $x \in X$,

$$T^{\tilde{F}(\lambda)}(x) = 1, I^{\tilde{F}(\lambda)}(x) = 1, F^{\tilde{F}(\lambda)}(x) = 0.$$

It is denoted by $1^{(X,E)}$.

Clearly, $0^c(X, E) = 1^{(X,E)}$ and $1^c(X, E) = 0^{(X,E)}$.

Definition 3.5. Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X and $\mathfrak{S} \sqsubset NSS(X, E)$. Then \mathfrak{S} is said to be a neutrosophic soft topology on X , if it satisfies the following axioms:

- (i) $0^{(X,E)}$ and $1^{(X,E)}$ belong to \mathfrak{S} ,
- (ii) the union of any number of neutrosophic soft sets in \mathfrak{S} belongs to \mathfrak{S} ,
- (iii) the intersection of a finite number of neutrosophic soft sets in \mathfrak{S} belongs to \mathfrak{S} .

Then (X, \mathfrak{S}, E) is said to be a neutrosophic soft topological space over X . Each member of \mathfrak{S} is said to be a neutrosophic soft open set.

Definition 3.6. Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X and (\tilde{F}, E) be a subset of neutrosophic soft topological space over X . Then (\tilde{F}, E) is said to be a neutrosophic soft closed set iff its complement is a neutrosophic soft open set.

Definition 3.7. Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X and (\tilde{F}, E) be a subset of neutrosophic soft topological space over X . Then (\tilde{F}, E) is said to be a neutrosophic soft b-open (NSBO) set, if

$$(\tilde{F}, E) \subseteq NSint(NScl((\tilde{F}, E))) \bigcup NScl(NSint((\tilde{F}, E))).$$

Definition 3.8. Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X and (\tilde{F}, E) be a subset of neutrosophic soft topological space over X . Then (\tilde{F}, E) is said to be a neutrosophic soft b-open (NSBC) set, if

$$(\tilde{F}, E) \supseteq NSint(NScl((\tilde{F}, E))) \bigcap NScl(NSint((\tilde{F}, E))).$$

Definition 3.9. Let NS be the family of all neutrosophic sets over the universe set X , let $x \in X$ and let $0 < \alpha, \beta, \gamma \leq 1$. Then the neutrosophic set $x^{(\alpha, \beta, \gamma)}$ is called a neutrosophic point and is defined as follow: for each $y \in X$,

$$(3.1) \quad x^{(\alpha, \beta, \gamma)}(y) = \begin{cases} (\alpha, \beta, \gamma), & \text{if } y = x \\ (0, 0, 1), & \text{if } y \neq x. \end{cases}$$

It is clear that every neutrosophic set is the union of its neutrosophic points.

Definition 3.10. Suppose that $X = \{x^1, x^2\}$. Then neutrosophic set

$$A = \{\langle x^1, 0.1, 0.3, 0.5 \rangle, \langle x^2, 0.5, 0.4, 0.7 \rangle\}$$

is the union of neutrosophic points $x^{1(0.1, 0.3, 0.5)}$ and $x^{2(0.1, 0.3, 0.5)}$.

Now we define the concept of neutrosophic soft points for neutrosophic soft sets.

Definition 3.11. Let $NSS(X, E)$ be the family of all neutrosophic soft sets over the universe set X and let $x \in X$, $0 < \alpha, \beta, \gamma \leq 1$, $\lambda \in E$. Then the neutrosophic soft set $x^{\lambda(\alpha, \beta, \gamma)}$ is called a neutrosophic soft point and is defined as follows: for each $y \in X$,

$$(3.2) \quad x^{\lambda(\alpha, \beta, \gamma)}(\lambda')(y) = \begin{cases} (\alpha, \beta, \gamma) & \text{if } \lambda' = \lambda \text{ and } y = x \\ (0, 0, 1), & \text{if } \lambda' \neq \lambda \text{ or } y \neq x. \end{cases}$$

Definition 3.12. Suppose that the universe set X is given by $X = \{x^1, x^2\}$ and the set of parameters by $E = \{\lambda^1, \lambda^2\}$. Let us consider neutrosophic soft sets (\tilde{F}, E) over the universe X as follows:

$$(3.3) \quad (\tilde{F}, E) = \left\{ \begin{array}{l} \lambda^1 = \{\langle x^1, 0.3, 0.7, 0.6 \rangle, \langle x^2, 0.4, 0.3, 0.8 \rangle\} \\ \lambda^2 = \{\langle x^1, 0.4, 0.6, 0.8 \rangle, \langle x^2, 0.3, 0.7, 0.2 \rangle\} \end{array} \right\}.$$

It is clear that (\tilde{F}, E) is union of its neutrosophic soft points $x^1\lambda^{1(0.3,0.7,0.6)}$, $x^1\lambda^{2(0.4,0.6,0.8)}$, $x^2\lambda^{1(0.4,0.3,0.8)}$, and $x^2\lambda^{2(0.3,0.7,0.2)}$. Here

$$(3.4) \quad x^1\lambda^{1(0.3,0.7,0.6)} = \left\{ \begin{array}{l} \lambda^1 = \{\langle x^1, 0.3, 0.7, 0.6 \rangle, \langle x^2, 0, 0, 1 \rangle\} \\ \lambda^2 = \{\langle x^1, 0, 0, 1 \rangle, \langle x^2, 0, 0, 1 \rangle\} \end{array} \right\},$$

$$(3.5) \quad x^1\lambda^{2(0.4,0.6,0.8)} = \left\{ \begin{array}{l} \lambda^1 = \{\langle x^1, 0.3, 0.7, 0.6 \rangle, \langle x^2, 0, 0, 1 \rangle\} \\ \lambda^2 = \{\langle x^1, 0.4, 0.6, 0.8 \rangle, \langle x^2, 0, 0, 1 \rangle\} \end{array} \right\},$$

$$(3.6) \quad x^2\lambda^{1(0.4,0.3,0.8)} = \left\{ \begin{array}{l} \lambda^1 = \{\langle x^1, 0, 0, 1 \rangle, \langle x^2, 0.4, 0.3, 0.8 \rangle\} \\ \lambda^2 = \{\langle x^1, 0, 0, 1 \rangle, \langle x^2, 0, 0, 1 \rangle\} \end{array} \right\},$$

$$(3.7) \quad x^2\lambda^{2(0.3,0.7,0.2)} = \left\{ \begin{array}{l} \lambda^1 = \{\langle x^1, 0, 0, 1 \rangle, \langle x^2, 0, 0, 1 \rangle\} \\ \lambda^2 = \{\langle x^1, 0, 0, 1 \rangle, \langle x^1, 0.3, 0.7, 0.2 \rangle\} \end{array} \right\}.$$

Definition 3.13. Let (\tilde{F}, E) be a neutrosophic soft set over the universe set X . We say that $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\tilde{F}, E)$ read as belonging to the neutrosophic soft set (\tilde{F}, E) , whenever $\alpha \leq T\tilde{F}^{(\lambda)}(x), \beta \leq I\tilde{F}^{(\lambda)}(x)$ and $\gamma \geq F\tilde{F}^{(\lambda)}(x)$.

Definition 3.14. Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X . A neutrosophic soft set (\tilde{F}, E) in (X, \mathfrak{S}, E) is called a neutrosophic soft b-neighborhood of the neutrosophic soft point $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\tilde{F}, E)$, if there exists a neutrosophic soft b-open set (\tilde{G}, E) such that $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\tilde{G}, E) \sqsubset (\tilde{F}, E)$.

Theorem 3.15. Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space and (\tilde{F}, E) be a neutrosophic soft set over X . Then (\tilde{F}, E) is a neutrosophic soft b-open set if and only if (\tilde{F}, E) is a neutrosophic soft b-neighborhood of its neutrosophic soft points.

Proof. Let (\tilde{F}, E) be a neutrosophic soft b-open set and $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\tilde{F}, E)$. Then $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\tilde{F}, E) \sqsubset (\tilde{F}, E)$. Thus (\tilde{F}, E) is a neutrosophic soft b-neighborhood of $x^{\lambda^{(\alpha,\beta,\gamma)}}$.

Conversely, let (\tilde{F}, E) be a neutrosophic soft b-neighborhood of its neutrosophic soft points. Let $x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\tilde{F}, E)$. Since (\tilde{F}, E) is a neutrosophic soft b-neighborhood of the neutrosophic soft point $x^{\lambda^{(\alpha,\beta,\gamma)}}$, there exists $(\tilde{G}, E) \in \mathfrak{S}$ such that

$$x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\tilde{G}, E) \sqsubset (\tilde{F}, E).$$

Since $(\tilde{F}, E) = \sqcup \{x^{\lambda^{(\alpha,\beta,\gamma)}} : x^{\lambda^{(\alpha,\beta,\gamma)}} \in (\tilde{F}, E)\}$, it follows that (\tilde{F}, E) is a union of neutrosophic soft b-open sets. Then (\tilde{F}, E) is a neutrosophic soft b-open set.

The b-neighborhood system of a neutrosophic soft point $x^{\lambda^{(\alpha,\beta,\gamma)}}$, denoted by $U(x^{\lambda^{(\alpha,\beta,\gamma)}}, E)$, is the family of all its b-neighborhoods. \square

Theorem 3.16. *The neighborhood system $U(x^{\lambda(\alpha,\beta,\gamma)}, E)$ at $x^{\lambda(\alpha,\beta,\gamma)}$ in a neutrosophic soft topological space (X, \mathfrak{S}, E) has the following properties.*

- (1) *If $(\tilde{F}, E) \in U(x^{\lambda(\alpha,\beta,\gamma)}, E)$, then $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{F}, E)$.*
- (2) *If $(\tilde{F}, E) \in U(x^{\lambda(\alpha,\beta,\gamma)}, E)$ and $(\tilde{F}, E) \sqsubset (\tilde{H}, E)$, then $(\tilde{H}, E) \in U(x^{\lambda(\alpha,\beta,\gamma)}, E)$.*
- (3) *If $(\tilde{F}, E), (\tilde{G}, E) \in U(x^{\lambda(\alpha,\beta,\gamma)}, E)$, then $(\tilde{F}, E) \sqcap (\tilde{G}, E) \in U(x^{\lambda(\alpha,\beta,\gamma)}, E)$.*
- (4) *If $(\tilde{F}, E) \in U(x^{\lambda(\alpha,\beta,\gamma)}, E)$, then there exists a $(\tilde{G}, E) \in U(x^{\lambda(\alpha,\beta,\gamma)}, E)$ such that $(\tilde{G}, E) \in U(y^{\lambda'(\alpha',\beta',\gamma')}, E)$, for each $y^{\lambda'(\alpha',\beta',\gamma')} \in (\tilde{G}, E)$.*

Proof. The proofs of (1), (2) and (3) is obvious from Definition 3.12.

(4) Suppose $(\tilde{F}, E) \in U(x^{\lambda(\alpha,\beta,\gamma)}, E)$. Then there exists a neutrosophic soft b-open set (\tilde{G}, E) such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{G}, E) \sqsubset (\tilde{F}, E)$. Thus by Proposition 3.1, $(\tilde{G}, E) \in U(x^{\lambda(\alpha,\beta,\gamma)}, E)$. So for each $y^{\lambda'(\alpha',\beta',\gamma')} \in (\tilde{G}, E)$, $(\tilde{G}, E) \in U(y^{\lambda'(\alpha',\beta',\gamma')}, E)$. \square

Definition 3.17. Let $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ be two neutrosophic soft points. For the neutrosophic soft points $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ over a common universe X, we say that neutrosophic soft points are distinct points, if

$$x^{\lambda(\alpha,\beta,\gamma)} \sqcap y^{\lambda'(\alpha',\beta',\gamma')} = 0^{(X,E)}.$$

It is clear that $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ are distinct neutrosophic soft points if and only if $x \neq y$ or $\lambda' \neq \lambda$.

4. NEUTROSOPHIC SOFT B-SEPARATION STRUCTURES

In this section, we consider neutrosophic soft b-separation axioms and neutrosophic soft topological subspace consisting of distinct neutrosophic soft points of neutrosophic soft topological space over X.

Definition 4.1. (i) Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X, and $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ are distinct neutrosophic soft points. If there exist neutrosophic soft b-open sets (\tilde{F}, E) and (\tilde{G}, E) such that

$$\begin{aligned} x^{\lambda(\alpha,\beta,\gamma)} &\in (\tilde{F}, E) \text{ and } x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\tilde{G}, E) = 0^{(X,E)} \text{ or} \\ y^{\lambda'(\alpha',\beta',\gamma')} &\in (\tilde{G}, E) \text{ and } y^{\lambda'(\alpha',\beta',\gamma')} \sqcap (\tilde{F}, E) = 0^{(X,E)}, \end{aligned}$$

then (X, \mathfrak{S}, E) is called a neutrosophic soft b- T^0 -space.

(ii) Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X and $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ are distinct neutrosophic soft points. If there exist neutrosophic soft b-open sets (\tilde{F}, E) and (\tilde{G}, E) such that

$$\begin{aligned} x^{\lambda(\alpha,\beta,\gamma)} &\in (\tilde{F}, E), \quad x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\tilde{G}, E) = 0^{(X,E)} \text{ or} \\ y^{\lambda'(\alpha',\beta',\gamma')} &\in (\tilde{G}, E), \quad y^{\lambda'(\alpha',\beta',\gamma')} \sqcap (\tilde{F}, E) = 0^{(X,E)}, \end{aligned}$$

then (X, \mathfrak{S}, E) is called a neutrosophic soft b- T^1 -space.

(iii) Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X , and $x^{\lambda(\alpha, \beta, \gamma)}$ and $y^{\lambda'(\alpha', \beta', \gamma')}$ are distinct neutrosophic soft points. If there exist neutrosophic soft b-open sets (\tilde{F}, E) and (\tilde{G}, E) such that

$$x^{\lambda(\alpha, \beta, \gamma)} \in (\tilde{F}, E), y^{\lambda'(\alpha', \beta', \gamma')} \in (\tilde{G}, E) \text{ and } (\tilde{F}, E) \cap (\tilde{G}, E) = 0^{(X, E)},$$

then (X, \mathfrak{S}, E) is called a neutrosophic soft b- T^2 -space.

Example 4.2. Let $X = \{x^1, x^2\}$ be a universe set, $E = \{\lambda^1, \lambda^2\}$ be a parameters set, and $(x^1)\lambda^1(0.1, 0.4, 0.7)$, $(x^1)\lambda^2(0.2, 0.5, 0.6)$, $(x^2)\lambda^1(0.3, 0.3, 0.5)$, and $(x^2)\lambda^2(0.4, 0.4, 0.4)$ be neutrosophic soft points. Then the family $\mathfrak{S} = \{0^{(X, E)}, 1^{(X, E)}, (\tilde{F}^1, E), (\tilde{F}^2, E), (\tilde{F}^3, E), (\tilde{F}^4, E), (\tilde{F}^5, E), (\tilde{F}^6, E), (\tilde{F}^7, E), (\tilde{F}^8, E)\}$, where

$$\begin{aligned} (\tilde{F}^1, E) &= \{x^1\lambda^1(0.1, 0.4, 0.7)\}, \\ (\tilde{F}^2, E) &= \{(x^1)\lambda^2(0.2, 0.5, 0.6)\}, \\ (\tilde{F}^3, E) &= \{(x^2)\lambda^1(0.3, 0.3, 0.5)\}, \\ (\tilde{F}^4, E) &= (\tilde{F}^1, E) \sqcup (\tilde{F}^2, E), \\ (\tilde{F}^5, E) &= (\tilde{F}^1, E) \sqcup (\tilde{F}^3, E), \\ (\tilde{F}^6, E) &= (\tilde{F}^2, E) \sqcup (\tilde{F}^3, E), \\ (\tilde{F}^7, E) &= (\tilde{F}^1, E) \sqcup (\tilde{F}^2, E) \sqcup (\tilde{F}^3, E), \\ (\tilde{F}^8, E) &= \{(x^1)\lambda^1(0.1, 0.4, 0.7), (x^1)\lambda^2(0.2, 0.5, 0.6), (x^2)\lambda^1(0.3, 0.3, 0.5), (x^2)\lambda^2(0.4, 0.4, 0.4)\} \end{aligned}$$

is a neutrosophic soft topology over X . Thus (X, \mathfrak{S}, E) is a neutrosophic soft topological space over X . Also, (X, \mathfrak{S}, E) is a neutrosophic soft b- T^0 -space but not a neutrosophic soft b- T^1 -space because for neutrosophic soft points $(x^1)\lambda^1(0.1, 0.4, 0.7)$ and $(x^2)\lambda^2(0.4, 0.4, 0.4)$, (X, \mathfrak{S}, E) is not a neutrosophic soft b- T^1 -space.

Example 4.3. Let $X = \mathbb{N}$ be a natural numbers set and $E = \{\lambda\}$ be a parameters set. Here $n^{\lambda(\alpha n, \beta n, \gamma n)}$ are neutrosophic soft points. Here we can give $(\alpha n, \beta n, \gamma n)$ appropriate values and the neutrosophic soft points $n^{\lambda(\alpha n, \beta n, \gamma n)}, m^{\lambda(\alpha n, \beta n, \gamma n)}$ are distinct neutrosophic soft points if and only if $n \neq m$. It is clear that there is one-to-one compatibility between the set of natural numbers and the set of neutrosophic soft points $N^\lambda = \{n^{\lambda(\alpha n, \beta n, \gamma n)}\}$.

Then we give cofinite topology on this set. Then neutrosophic soft set (\tilde{F}, E) is a neutrosophic soft b-open set if and only if the finite neutrosophic soft point is discarded from N^λ . Hence, (X, \mathfrak{S}, E) is a neutrosophic soft b- T^1 -space but not a neutrosophic soft b- T^2 -space.

Example 4.4. Let $X = \{x^1, x^2\}$ be a universe set, $E = \{\lambda^1, \lambda^2\}$ be a parameters set, and $x^1\lambda^1(0.1, 0.4, 0.7)$, $x^1\lambda^2(0.2, 0.5, 0.6)$, $x^2\lambda^1(0.3, 0.3, 0.5)$, and $x^2\lambda^2(0.4, 0.4, 0.4)$ be neutrosophic soft points. Then the family $\mathfrak{S} = \{0^{(X, E)}, 1^{(X, E)}, (\tilde{F}^1, E), (\tilde{F}^2, E), \dots, (\tilde{F}^{15}, E)\}$, where

$$\begin{aligned} (\tilde{F}^1, E) &= \{x^1\lambda^1(0.1, 0.4, 0.7)\}, \\ (\tilde{F}^2, E) &= \{(x^1)\lambda^2(0.2, 0.5, 0.6)\}, \\ (\tilde{F}^3, E) &= \{(x^2)\lambda^1(0.3, 0.3, 0.5)\}, \\ (\tilde{F}^4, E) &= \{(x^2)\lambda^2(0.4, 0.4, 0.4)\}, \\ (\tilde{F}^5, E) &= (\tilde{F}^1, E) \sqcup (\tilde{F}^2, E), \end{aligned}$$

$$\begin{aligned}
 (\tilde{F}^6, E) &= (\tilde{F}^1, E) \sqcup (\tilde{F}^3, E), \\
 (\tilde{F}^7, E) &= (\tilde{F}^1, E) \sqcup (\tilde{F}^4, E), \\
 (\tilde{F}^8, E) &= (\tilde{F}^2, E) \sqcup (\tilde{F}^3, E), \\
 (\tilde{F}^9, E) &= (\tilde{F}^2, E) \sqcup (\tilde{F}^4, E), \\
 (\tilde{F}^{10}, E) &= (\tilde{F}^3, E) \sqcup (\tilde{F}^4, E), \\
 (\tilde{F}^{11}, E) &= (\tilde{F}^1, E) \sqcup (\tilde{F}^2, E) \sqcup (\tilde{F}^3, E), \\
 (\tilde{F}^{12}, E) &= (\tilde{F}^1, E) \sqcup (\tilde{F}^2, E) \sqcup (\tilde{F}^4, E), \\
 (\tilde{F}^{13}, E) &= (\tilde{F}^2, E) \sqcup (\tilde{F}^3, E) \sqcup (\tilde{F}^4, E), \\
 (\tilde{F}^{14}, E) &= (\tilde{F}^1, E) \sqcup (\tilde{F}^3, E) \sqcup (\tilde{F}^4, E), \\
 (\tilde{F}^{15}, E) &= \{(x^1)\lambda^{1(0.1,0.4,0.7)}, (x^1)\lambda^{2(0.2,0.5,0.6)}, (x^2)\lambda^{2(0.3,0.3,0.5)}, (x^2)\lambda^{2(0.4,0.4,0.4)}\}
 \end{aligned}$$

is a neutrosophic soft topology over X. Thus (X, \mathfrak{S}, E) is a neutrosophic soft topological space over X. Also, (X, \mathfrak{S}, E) is a neutrosophic soft b- T^2 -space.

Theorem 4.5. *Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X. Then (X, \mathfrak{S}, E) is a neutrosophic soft b- T^1 -space if and only if each neutrosophic soft point is a neutrosophic soft b-closed set.*

Proof. Let (X, \mathfrak{S}, E) be a neutrosophic soft b- T^1 -space and $x^{\lambda(\alpha, \beta, \gamma)}$ be an arbitrary neutrosophic soft point. We show that $(x^{\lambda(\alpha, \beta, \gamma)})^\lambda$ is a neutrosophic soft b-open set. Let $y^{\lambda'(\alpha', \beta', \gamma')} \in (x^{\lambda(\alpha, \beta, \gamma)})^\lambda$. Then $x^{\lambda(\alpha, \beta, \gamma)}$ and $y^{\lambda'(\alpha', \beta', \gamma')}$ are distinct neutrosophic soft points. Thus $x \neq y$ or $\lambda' \neq \lambda$.

Since (X, \mathfrak{S}, E) is a neutrosophic soft b- T^1 -space, there exists a neutrosophic soft b-open set (\tilde{G}, E) such that

$$y^{\lambda'(\alpha', \beta', \gamma')} \in (\tilde{G}, E) \text{ and } x^{\lambda(\alpha, \beta, \gamma)} \cap (\tilde{G}, E) = 0^{(X, E)}.$$

Since $x^{\lambda(\alpha, \beta, \gamma)} \cap (\tilde{G}, E) = 0^{(X, E)}$, we have $y^{\lambda'(\alpha', \beta', \gamma')} \in (\tilde{G}, E) \sqsubset (x^{\lambda(\alpha, \beta, \gamma)})^\lambda$. Thus $(x^{\lambda(\alpha, \beta, \gamma)})^\lambda$ is a neutrosophic soft b-open set, i.e., $x^{\lambda(\alpha, \beta, \gamma)}$ is a neutrosophic soft b-closed set.

Suppose that each neutrosophic soft point $x^{\lambda(\alpha, \beta, \gamma)}$ is a neutrosophic soft b-closed set. Then $(x^{\lambda(\alpha, \beta, \gamma)})^\lambda$ is a neutrosophic soft open set. Let $x^{\lambda(\alpha, \beta, \gamma)} \cap y^{\lambda'(\alpha', \beta', \gamma')} = 0^{(X, E)}$. Thus $y^{\lambda'(\alpha', \beta', \gamma')} \in (x^{\lambda(\alpha, \beta, \gamma)})^\lambda$ and $x^{\lambda(\alpha, \beta, \gamma)} \cap (x^{\lambda(\alpha, \beta, \gamma)})^\lambda = 0^{(X, E)}$. So (X, \mathfrak{S}, E) is a neutrosophic soft b- T^1 -space over X. \square

Theorem 4.6. *Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X. Then (X, \mathfrak{S}, E) is a neutrosophic soft b- T^2 -space iff for distinct neutrosophic soft points $x^{\lambda(\alpha, \beta, \gamma)}$ and $y^{\lambda'(\alpha', \beta', \gamma')}$, there exists a neutrosophic soft b-open set (\tilde{F}, E) containing $x^{\lambda(\alpha, \beta, \gamma)}$ but not $y^{\lambda'(\alpha', \beta', \gamma')}$ such that $y^{\lambda'(\alpha', \beta', \gamma')}$ does not belong to $(\overline{\tilde{F}}, E)$.*

Proof. Let $x^{\lambda(\alpha, \beta, \gamma)}$ and $y^{\lambda'(\alpha', \beta', \gamma')}$ be two neutrosophic soft points in neutrosophic soft b- T^2 -space (X, \mathfrak{S}, E) . Then there exist disjoint neutrosophic soft b-open set (\tilde{F}, E) , (\tilde{G}, E) such that $x^{\lambda(\alpha, \beta, \gamma)} \in (\tilde{F}, E)$, $y^{\lambda'(\alpha', \beta', \gamma')} \in (\tilde{G}, E)$. Since $x^{\lambda(\alpha, \beta, \gamma)}$

\square $y^{\lambda'(\alpha',\beta',\gamma')}$ $= 0^{(X,E)}$ and $(\tilde{F}, E) \cap (\tilde{G}, E) = 0^{(X,E)}$, $y^{\lambda'(\alpha',\beta',\gamma')}$ does not belong to (\tilde{F}, E) . It implies that $y^{\lambda'(\alpha',\beta',\gamma')}$ does not belong to $(\overline{\tilde{F}, E})$.

Next suppose that, for distinct neutrosophic soft points $x^{\lambda(\alpha,\beta,\gamma)}$, $y^{\lambda'(\alpha',\beta',\gamma')}$, there exists a neutrosophic soft b-open set (\tilde{F}, E) containing $x^{\lambda(\alpha,\beta,\gamma)}$ but not $y^{\lambda'(\alpha',\beta',\gamma')}$ such that $y^{\lambda'(\alpha',\beta',\gamma')}$ does not belong to $(\overline{\tilde{F}, E})$. Then $y^{\lambda'(\alpha',\beta',\gamma')} \in ((\tilde{F}, E))^c$, i.e., (\tilde{F}, E) and $((\tilde{F}, E))^c$ are disjoint neutrosophic soft b-open sets containing $x^{\lambda(\alpha,\beta,\gamma)}$, $y^{\lambda'(\alpha',\beta',\gamma')}$, respectively. \square

Theorem 4.7. Let (X, \mathfrak{S}, E) be a neutrosophic soft $b-T^1$ -space for every neutrosophic soft point $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{F}, E) \in \mathfrak{S}$. If there exists a neutrosophic soft b-open set (\tilde{G}, E) such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{G}, E) \subset (\overline{\tilde{G}, E}) \subset (\tilde{F}, E)$, then (X, \mathfrak{S}, E) is a neutrosophic soft $b-T^2$ -space.

Proof. Suppose that $x^{\lambda(\alpha,\beta,\gamma)} \cap y^{\lambda'(\alpha',\beta',\gamma')} = 0^{(X,E)}$. Since (X, \mathfrak{S}, E) is a neutrosophic soft T^1 -space, $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ are neutrosophic soft b-closed sets in \mathfrak{S} . Then $x^{\lambda(\alpha,\beta,\gamma)} \in (y^{\lambda'(\alpha',\beta',\gamma')})^c \in \mathfrak{S}$. Thus there exists a neutrosophic soft b-open set (\tilde{G}, E) in \mathfrak{S} such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{G}, E) \subset (\overline{\tilde{G}, E}) \subset (y^{\lambda'(\alpha',\beta',\gamma')})^c$. So we have $y^{\lambda'(\alpha',\beta',\gamma')} \in ((\overline{\tilde{G}, E}))^c$, $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{G}, E)$ and $(\tilde{G}, E) \cap ((\overline{\tilde{G}, E}))^c = 0^{(X,E)}$, i.e., (X, \mathfrak{S}, E) is a neutrosophic soft $b-T^2$ -space. \square

Remark 4.8. Let (X, \mathfrak{S}, E) be a neutrosophic soft $b-T^1$ -space for $i = 0, 1, 2$. For each $x \neq y$, neutrosophic points $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ have neighborhoods satisfying conditions of $b-T^i$ -space in neutrosophic topological space $(X, \mathfrak{S}^\lambda)$ for each $\lambda \in E$ because $x^{\lambda(\alpha,\beta,\gamma)}$ and $y^{\lambda'(\alpha',\beta',\gamma')}$ are distinct neutrosophic soft points.

Definition 4.9. Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X , (\tilde{F}, E) be a neutrosophic soft b-closed set and $x^{\lambda(\alpha,\beta,\gamma)} \cap (\tilde{F}, E) = 0^{(X,E)}$. If there exist neutrosophic soft b-open open sets (\tilde{G}^1, E) and (\tilde{G}^2, E) such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{G}^1, E)$, $(\tilde{F}, E) \subset (\tilde{G}^2, E)$, and $(\tilde{G}^1, E) \cap (\tilde{G}^2, E) = 0^{(X,E)}$, then (X, \mathfrak{S}, E) is called a neutrosophic soft b-regular space. (X, \mathfrak{S}, E) is said to be a neutrosophic soft $b-T^3$ -space if is both a neutrosophic soft b-regular and neutrosophic soft $b-T^1$ -space.

Theorem 4.10. Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X , (X, \mathfrak{S}, E) is a neutrosophic soft $b-T^3$ -space if and only if for every $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{F}, E) \in \mathfrak{S}$, there exists $(\tilde{G}, E) \in \mathfrak{S}$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{G}, E) \subset (\overline{\tilde{G}, E}) \subset (\tilde{F}, E)$.

Proof. Let (X, \mathfrak{S}, E) be a neutrosophic soft $b-T^3$ -space and $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{F}, E) \in \mathfrak{S}$. Since (X, \mathfrak{S}, E) is a neutrosophic soft $b-T^3$ -space for the neutrosophic soft point $x^{\lambda(\alpha,\beta,\gamma)}$ and neutrosophic soft b-closed set $(\tilde{F}, E)^c$, there exist (\tilde{G}^1, E) , $(\tilde{G}^2, E) \in \mathfrak{S}$ such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{G}^1, E)$, $(\tilde{F}, E)^c \subset (\tilde{G}^2, E)$, and $(\tilde{G}^1, E) \cap (\tilde{G}^2, E) = 0^{(X,E)}$. Then we have $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{G}^1, E) \subset (\overline{\tilde{G}^1, E}) \subset (\tilde{F}, E)$. Since $(\tilde{G}^2, E)^c$ is a neutrosophic soft b-closed set, $(\overline{\tilde{G}^1, E}) \subset (\overline{\tilde{G}^2, E})^c$.

Conversely, let $x^{\lambda(\alpha,\beta,\gamma)} \sqcap (\tilde{H}, E) = 0^{(X,E)}$ and (\tilde{H}, E) be a neutrosophic soft b-closed set. Then $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{H}, E)^c$ and from the condition of the theorem, we have $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{G}, E) \sqsubset (\overline{\tilde{G}, E}) \sqsubset (\tilde{H}, E)^c$.

Thus $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{G}, E)$, $(\tilde{H}, E) \sqsubset ((\overline{\tilde{G}, E}))^c$, and $(\tilde{G}, E) \sqcap (\overline{\tilde{G}, E})^c = 0^{(X,E)}$. So (X, \mathfrak{S}, E) is a neutrosophic soft b- T^3 -space. \square

Definition 4.11. A neutrosophic soft topological space (X, \mathfrak{S}, E) over X is called a neutrosophic soft b-normal space, if for every pair of disjoint neutrosophic soft b-closed set (\tilde{F}^1, E) , (\tilde{F}^2, E) , there exists disjoint neutrosophic soft b-open sets (\tilde{G}^1, E) , (\tilde{G}^2, E) such that $(\tilde{F}^1, E) \sqsubset (\tilde{G}^1, E)$ and $(\tilde{F}^2, E) \sqsubset (\tilde{G}^2, E)$.

(X, \mathfrak{S}, E) is said to be a neutrosophic soft b- T^4 -space, if it is both a neutrosophic soft b-normal and neutrosophic soft b- T^1 -space.

Theorem 4.12. Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X . Then (X, \mathfrak{S}, E) is a neutrosophic soft b- T^4 -space if and only if, for each neutrosophic soft b-closed set (\tilde{F}, E) and neutrosophic soft b-open set (\tilde{G}, E) with $(\tilde{F}, E) \sqsubset (\tilde{G}, E)$, there exists a neutrosophic soft b-open set (\tilde{D}, E) such that

$$(\tilde{F}, E) \sqsubset (\tilde{D}, E) \sqsubset (\overline{\tilde{D}, E}) \sqsubset (\tilde{G}, E).$$

Proof. Let (X, \mathfrak{S}, E) be a neutrosophic soft b- T^4 -space, let (\tilde{F}, E) be a neutrosophic soft b-closed set and let $(\tilde{F}, E) \sqsubset (\tilde{G}, E) \in \mathfrak{S}$. Then $(\tilde{G}, E)^c$ is a neutrosophic soft b-closed set and $(\tilde{F}, E) \sqcap (\tilde{G}, E)^c = 0^{(X,E)}$. Since (X, \mathfrak{S}, E) is a neutrosophic soft b- T^4 -space, there exist neutrosophic soft b-open sets (\tilde{D}^1, E) and (\tilde{D}^2, E) such that

$$(\tilde{F}, E) \sqsubset (\tilde{D}^1, E), (\tilde{G}, E)^c \sqsubset (\tilde{D}^2, E) \text{ and } (\tilde{D}^1, E) \sqcap (\tilde{D}^2, E) = 0^{(X,E)}.$$

Thus $(\tilde{F}, E) \sqsubset (\tilde{D}^1, E) \sqsubset (\tilde{D}^2, E)^c \sqsubset (\tilde{G}, E)$, $(\tilde{D}^2, E)^c$ is a neutrosophic soft b-closed set and $(\tilde{D}^1, E) \sqsubset (\tilde{D}^2, E)^c$. So $(\tilde{F}, E) \sqsubset (\tilde{D}^1, E) \sqsubset (\overline{\tilde{D}^1, E}) \sqsubset (\tilde{G}, E)$.

Conversely, let (\tilde{F}^1, E) , (\tilde{F}^2, E) be two disjoint neutrosophic soft b-closed sets. Then $(\tilde{F}^1, E) \sqsubset (\tilde{F}^2, E)^c$. From the condition of theorem, there exists a neutrosophic soft b-open set (\tilde{D}, E) such that $(\tilde{F}^1, E) \sqsubset (\tilde{D}, E) \sqsubset (\overline{\tilde{D}^1, E}) \sqsubset (\tilde{F}^2, E)^c$. Thus (\tilde{D}, E) , $((\overline{\tilde{D}, E}))^c$ are neutrosophic soft b-open sets and $(\tilde{F}^1, E) \sqsubset (\tilde{D}, E)$, $(\tilde{F}^2, E) \sqsubset ((\overline{\tilde{D}, E}))^c$ and $(\tilde{D}, E) \sqcap ((\overline{\tilde{D}, E}))^c = 0^{(X,E)}$. So (X, \mathfrak{S}, E) is a neutrosophic soft b- T^4 -space. \square

Definition 4.13. Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X and (\tilde{F}, E) be an arbitrary neutrosophic soft set. Then $\mathfrak{S}^{(\tilde{F}, E)} = \{(\tilde{F}, E) \sqcap (\tilde{H}, E) : (\tilde{H}, E) \in \mathfrak{S}\}$ is said to be neutrosophic soft topology on (\tilde{F}, E) and $((\tilde{F}, E), \mathfrak{S}^{(\tilde{F}, E)}, E)$ is called a neutrosophic soft topological subspace of (X, \mathfrak{S}, E) .

Theorem 4.14. Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X . If (X, \mathfrak{S}, E) is a neutrosophic soft b- T^i -space, then the neutrosophic soft topological subspace $((\tilde{F}, E), \mathfrak{S}^{(\tilde{F}, E)}, E)$ is a neutrosophic soft b- T^i -space for $i = 0, 1, 2, 3$.

Proof. Let $x^{\lambda(\alpha,\beta,\gamma)}, y^{\lambda'(\alpha',\beta',\gamma')} \in ((\tilde{F}, E), \mathfrak{S}^{(\tilde{F}, E)}, E)$ such that $x^{\lambda(\alpha,\beta,\gamma)} \sqcap y^{\lambda'(\alpha',\beta',\gamma')} = 0^{(X,E)}$. Then there exist neutrosophic soft b-open set (\tilde{F}^1, E) and (\tilde{F}^2, E) satisfying the conditions of neutrosophic soft b- T^i -space such that $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{F}^1, E)$,

$y^{\lambda'(\alpha',\beta',\gamma')} \in (\tilde{F}^2, E)$. Thus $x^{\lambda(\alpha,\beta,\gamma)} \in (\tilde{F}^1, E) \cap (\tilde{F}, E)$ and $y^{\lambda'(\alpha',\beta',\gamma')} \in (\tilde{F}^2, E) \cap (\tilde{F}, E)$. Also, the neutrosophic soft b-open set $(\tilde{F}^1, E) \cap (\tilde{F}, E)$, $(\tilde{F}^2, E) \cap (\tilde{F}, E)$ in $\mathfrak{S}(\tilde{F}, E)$ satisfy the conditions of neutrosophic soft b- T^i -space for $i = 0, 1, 2, 3$. \square

Theorem 4.15. *Let (X, \mathfrak{S}, E) be a neutrosophic soft topological space over X . If (X, \mathfrak{S}, E) is a neutrosophic soft b- T^4 -space and (\tilde{F}, E) is a neutrosophic soft b-closed set in (X, \mathfrak{S}, E) , then $((\tilde{F}, E), \mathfrak{S}(\tilde{F}, E), E)$ is a neutrosophic soft b- T^4 -space.*

Proof. Let (X, \mathfrak{S}, E) be a neutrosophic soft b- T^4 -space and (\tilde{F}, E) be a neutrosophic soft b-closed set in (X, \mathfrak{S}, E) . Let (\tilde{F}^1, E) and (\tilde{F}^2, E) be two neutrosophic soft b-closed sets in $((\tilde{F}, E), \mathfrak{S}(\tilde{F}, E), E)$ such that $(\tilde{F}^1, E) \cap (\tilde{F}^2, E) = 0^{(X, E)}$. When (\tilde{F}, E) is a neutrosophic soft b-closed set in (X, \mathfrak{S}, E) , (\tilde{F}^1, E) and (\tilde{F}^2, E) are neutrosophic soft b-closed sets in (X, \mathfrak{S}, E) . Since (X, \mathfrak{S}, E) is a neutrosophic soft b- T^4 -space, there exist neutrosophic soft b-open sets (\tilde{G}^1, E) and (\tilde{G}^2, E) such that $(\tilde{F}^1, E) \sqsubset (\tilde{G}^1, E)$, $(\tilde{F}^2, E) \sqsubset (\tilde{G}^2, E)$ and $(\tilde{G}^1, E) \cap (\tilde{G}^2, E) = 0^{(X, E)}$. Then $(\tilde{F}^1, E) = (\tilde{G}^1, E) \cap (\tilde{F}, E)$, $(\tilde{F}^2, E) = (\tilde{G}^2, E) \cap (\tilde{F}, E)$ and $((\tilde{G}^1, E) \cap (\tilde{F}, E)) \cap ((\tilde{G}^2, E) \cap (\tilde{F}, E)) = 0^{(X, E)}$. This implies that $((\tilde{F}, E), \mathfrak{S}(\tilde{F}, E), E)$ is a neutrosophic soft b- T^4 -space. \square

5. CONCLUSION

Neutrosophic soft b-separation structures are the most imperative and fascinating notions in neutrosophic soft topology we have introduced neutrosophic soft b-separation axioms in neutrosophic soft topological structures with respect to soft points, which are defined over an initial universe of discourse with a fixed set of variables. We further investigated and scrutinized some essential features of the initiated neutrosophic soft b-separation structures. It is supposed that these results will be very very useful for future studies on neutrosophic soft topology to carry out a general framework for practical applications. Applications of neutrosophic soft b-separation structures in neutrosophic soft topological spaces can be traced out in decision making problems.

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