DE GRUYTER Open Math. 2019; 17:423–438

Open Mathematics

Research Article

Choonkil Park, Nasir Shah*, Noor Rehman, Abbas Ali, Muhammad Irfan Ali, and Muhammad Shabir

Soft covering based rough graphs and corresponding decision making

https://doi.org/10.1515/math-2019-0033
Received December 7, 2017; accepted June 27, 2018

Abstract: Soft set theory and rough set theory are two new tools to discuss uncertainty. Graph theory is a nice way to depict certain information. Particularly soft graphs serve the purpose beautifully. In order to discuss uncertainty in soft graphs, some new types of graphs called soft covering based rough graphs are introduced. Several basic properties of these newly defined graphs are explored. Applications of soft covering based rough graphs in decision making can be very fruitful. In this regard an algorithm has been proposed.

Keywords: soft graphs; rough sets; soft covering based rough graphs; approximation space; decision making

MSC: 05C72, 05C76, 05C85

1 Introduction

Many real world situations appearing in various spheres of life, such as physical sciences, chemistry, communications, computer sciences and several other areas, involve graphs. Graphs and operations on graphs are extensively studied by computer scientists. The main reason behind this is because graphs can be used to represent many real world problems in computer science that are otherwise abstract. The swiss mathematician Leonhard Euler [1] known as the father of graph theory is universally credited with having produced the first paper in 1736, when he settled a famous unsolved problem known as Königsburg Bridge problem by constructing the Eulerian graph. The subject of graph theory which is motivated by recreational mathematics and study of games may be considered as a part of combinatorial mathematics. The theory has greatly contributed to our understanding of programming, civil engineering, communication theory, switching circuits, operational research, economics and psychology. Many applications of graphs can be seen in [2, 3] (and the references in that respect).

Choonkil Park: Department of Mathematics, Research Institute of Natural Sciences, Hanyang University, Seoul 04763, Republic of Korea, E-mail: baak@hanyang.ac.kr
***Corresponding Author: Nasir Shah:** Department of Mathematics and Statistics, Riphah International University, I-14, Islamabad, Pakistan, E-mail: 8052@students.riu.edu.pk
Noor Rehman: Department of Mathematics and Statistics, Bacha Khan University Charsadda, KPK, Pakistan, E-mail: noorrehman82@yahoo.com
Abbas Ali: Department of Mathematics and Statistics, Riphah International University, I-14, Islamabad, Pakistan, E-mail: abbasali5068@gmail.com
Muhammad Irfan Ali: Department of Mathematics, Islamabad Model College for Girls , F-6/2, Islamabad, Pakistan, E-mail: mirfanali13@yahoo.com
Muhammad Shabir: Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan, E-mail: mshabirbhatti@yahoo.co.uk

In real situations, complexity and complications usually originate from uncertainty in the form of ambiguity. There are several real life problems involving uncertainty and vagueness where the classical mathematics is not successful and not absolutely prosperous. Most of our traditional and conventional mechanism of modeling, reasoning and computing are crisp, precise in character and deterministic. The dilemma and situations connecting with uncertainty are being handled by ancient and effective tools of probability. The drawback of probability theory is that it is applicable only when the occurrence of events is strictly determined by chance. In conjunction with probability theory, many other theories like fuzzy set theory, intuitionistic fuzzy set theory, rough set theory, neutrosophic set theory, soft set theory and blend of some of these theories to handle uncertainty which arises due to vagueness, have been introduced (for further details see [4, 5]).

It is believed that an important epoch in the evolution of modern theory of uncertainty arising due to vagueness was the publication of the pioneering paper by Zadeh [7] in 1965. He has defined fuzzy sets with an aspect to study, describe and develop mathematically those situations which are imprecise and defined vaguely. Pawlak [6] introduced the concept of rough sets which is an excellent mathematical tool to handle with the given information and an access to ambiguity and equivocalness. The main significance of rough set theory is that it does not involve any additional information about the data, like membership in fuzzy sets. The rough sets theory is based on equivalence relations, which are now extended to the notion of covering based rough sets [8, 9]. Probability theory, fuzzy set theory and rough set theory are different accessions to handle uncertainty, vagueness and imprecision. Each of these theories have their own restrictions and limitation. Many applications of these theories in data mining, pattern recognition, knowledge discovery and machine learning can be seen in [10–17]. While dealing with such theories, a question arises how to handle multi-attributes? Molodtsov [5] introduced the notion of soft sets to overcome the problem of dealing with attributes. This concept not only changed the role of above said theories as the sole representative of multi-attributes but also rectified in some disciplines to tackle many problems of uncertainty [18–20]. A number of applications, utilizations and practices have made with respect to multi-attributes modeling and decision making problems [24–28].

A useful and drastic theory has been established in [29–31] by connecting the covering soft sets to rough sets. Huge number of applications have been presented by many researchers in multi-attributes decision making problems, attributes reduction problems, data labeling problems, data mining problems and knowledge based systems [32–46].

The concept of soft graphs and their different operations can be seen in [47]. These concepts were required to tackle multi-attributes problems related to the theory of graphs. A number of generalizations of soft graphs are available in the literature [48–53]. To strengthen and enhance the applicability of soft graphs, an innovative approach by combining rough set with soft graphs, called soft covering based rough graphs are introduced. In the present paper we initiate the study of new types of graphs called soft covering based rough graphs. Several properties of these graphs are explored. As an application of soft covering based rough graphs in decision making problems an algorithm is proposed.

The rest of the paper is organized as follows: In Section 2, some basic concepts are revised. Section 3 is about basic definitions and characterization of soft covering based rough graph, lower/upper $\mathcal{S}$-soft vertex covering approximations, lower/upper $\mathcal{Q}$-soft edge covering approximations, $\mathcal{S}$-soft rough vertex covering graph, $\mathcal{Q}$-soft rough edge covering graph, soft covering based rough graphs, and basic theory is discussed with examples. Section 4 is devoted to present an application of soft covering based rough graphs in real life. To compute the effectiveness of some diseases amongst colleagues working in same factory, an algorithm is developed in a realistic way, using simple digraph with vertices as 20 colleagues and the edges as the interaction of these colleagues. Marginal fuzzy sets are defined with the help of lower and upper soft rough approximations of the given graph and using marginal fuzzy sets as weights, persons at high risk of having given diseases are found. Conclusion of the paper is presented in Section 5.

2 Preliminaries

In this section the basic ideas regarding the graphs, soft sets, soft graphs and soft rough sets are given which will help in the rest of sections.

Definition 1. *[2] A graph $G^{\star}$ is a pair (V, E) of sets, where V is a finite non-empty set whose members are called vertices (also called points or nodes) and E is a set of unordered pairs of distinct vertices called edges (also called lines or arcs). A graph is usually denoted as $G^{\star} = (V, E)$. Let $G^{\star}$ be a graph and $\{u, v\}$ be an edge of $G^{\star}$. It is often more convenient to represent this edge by uv or vu. The vertex set is usually denoted by $V(G^{\star})$ and the edge set by $E(G^{\star})$. An edge of a graph that joins a node to itself is called loop or self loop. In a multigraph no loops are allowed but more than one edge can join two vertices and these edges are called multiple edges or parallel edges. A graph $G^{\star}$ is called simple if it has no loops or multiple edges.*

Definition 2. *[2] A directed graph or digraph $G^{\star}$ containing a vertex set $V(G^{\star})$, and an edge set $E(G^{\star})$ whose elements are ordered pairs of elements of $V(G^{\star})$ called the directed edges. The first element of the ordered pair is called the tail of the edge and the second is called the head, together, they are the endpoints.*

Definition 3. *[20] Let T be the set of parameters. A pair (k, T) is called a soft set over the set U of universe, where $k : T \to P(U)$ is a set valued mapping and $P(U)$ is the power set of U.*

Definition 4. *[47] A quadruple $\mathcal{G} = (G^{\star}, \lambda, \mu, T)$ is called a soft graph, where*
(1) *$G^{\star} = (V, E)$ is a simple graph,*
(2) *(λ, T) is a soft set over V,*
(3) *(μ, T) is a soft set over E,*
(4) *$(\lambda(a), \mu(a))$ is a subgraph of $G^{\star}$ for all $a \in T$.*

Definition 5. *[25] Let $S = (k, T)$ be a soft set over U. Then the pair $\mathcal{P} = (U, S)$ is called soft approximation space. Based on the soft approximation space $\mathcal{P}$, we define*

$$\underline{appr}_{\mathcal{P}}(X) = \{u \in U : \exists\, a \in T, [u \in k(a) \subseteq X]\},$$

$$\overline{appr}_{\mathcal{P}}(X) = \{u \in U : \exists\, a \in T, [x \in k(a), k(a) \cap X \neq \emptyset]\}$$

assigning to any set $X \subseteq U$, the sets $\underline{appr}_{\mathcal{P}}(X)$ and $\overline{appr}_{\mathcal{P}}(X)$ and are called soft $\mathcal{P}$-lower approximation and soft $\mathcal{P}$-upper approximation of X, respectively.
The sets

$$Pos(X) = \underline{appr}_{\mathcal{P}}(X),$$

$$Neg(X) = -\underline{appr}_{\mathcal{P}}(X),$$

$$\text{and } Bnd(X) = \overline{appr}_{\mathcal{P}}(X) - \underline{appr}_{\mathcal{P}}(X)$$

are called the soft $\mathcal{P}$-positive region, the soft $\mathcal{P}$-negative region, and the soft $\mathcal{P}$-boundary region of X, respectively. If $\overline{appr}_{\mathcal{P}}(X) = \underline{appr}_{\mathcal{P}}(X)$ then X is said to be soft $\mathcal{P}$-definable, otherwise X is called a soft $\mathcal{P}$-rough set.

3 Soft Covering Based Rough Graphs

Based upon the properties and usefulness of both rough sets and soft sets, a hybrid soft covering based rough graphs are defined in this section. Basic properties and results related to soft covering based rough graphs are discussed.

Definition 6. *Let $\mathcal{G} = (G^{\star}, \lambda, \mu, T)$ be a soft graph over the simple graph $G^{\star} = (V, E)$. Then $\mathcal{G}$ is called* *(i) full soft vertex graph if*

$$\bigcup_{\alpha \in T} \lambda(\alpha) = V.$$

(ii) full soft edge graph if

$$\bigcup_{\alpha \in T} \mu(\alpha) = E.$$

(iii) full soft graph if

$$G^* = \left(\bigcup_{\alpha \in T} \lambda(\alpha), \bigcup_{\alpha \in T} \mu(\alpha) \right).$$

(iv) covering soft vertex graph if $\lambda(\alpha) \neq \emptyset$ *for all* $\alpha \in T$. *In this case* (λ, T) *is called covering soft set over* V, *denoted by* $\mathcal{C}_{\mathcal{V}}$.

Denote by $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$ *and call it soft vertex covering approximation space.*

Definition 7. *Let* $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$ *be a soft vertex covering approximation space and* $v \in V$. *Then the set*

$$Mdes_{\mathcal{S}}(v) = \left\{ \begin{array}{c} \lambda(\alpha) : \alpha \in T \wedge v \in \lambda(\alpha) \wedge \\ (\text{for all } \beta \in T \wedge v \in \lambda(\beta) \subseteq \lambda(\alpha) \text{ implies } \lambda(\alpha) = \lambda(\beta)) \end{array} \right\}$$

is called soft minimal vertex description of $v \in V$.

Definition 8. *Let* $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$ *be a soft vertex covering approximation space. Based on* $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$, *the sets defined by*

$$\underline{appr}_{\mathcal{S}}(X) = \bigcup_{\alpha \in T} \{\lambda(\alpha) : \lambda(\alpha) \subseteq X\},$$

and

$$\overline{appr}_{\mathcal{S}}(X) = \bigcup \{Mdes_{\mathcal{S}}(v) : v \in X\}$$

for the subset $X \subseteq V$, *are called the lower* $\mathcal{S}$*-soft vertex covering and upper* $\mathcal{S}$*-soft vertex covering approximations of X, respectively.*
Furthermore

$$Post_{\mathcal{S}}(X) = \underline{appr}_{\mathcal{S}}(X),$$

$$Negt_{\mathcal{S}}(X) = V - \overline{appr}_{\mathcal{S}}(X)$$

$$\text{and } Bnd_{\mathcal{S}}(X) = \overline{appr}_{\mathcal{S}}(X) - \underline{appr}_{\mathcal{S}}(X).$$

are called $\mathcal{S}$*-soft positive vertex covering region,* $\mathcal{S}$*-soft negative vertex covering region and* $\mathcal{S}$*-soft boundary vertex covering region respectively. If* $\overline{appr}_{\mathcal{S}}(X) = \underline{appr}_{\mathcal{S}}(X)$, *for* $X \subseteq V$, *then X is called* $\mathcal{S}$*-soft vertex covering definable set and* $G_{\mathcal{S}} = (V, E)$ *is called* $\mathcal{S}$*-soft vertex covering definable graph. On the other hand if* $\overline{appr}_{\mathcal{S}}(X) \neq \underline{appr}_{\mathcal{S}}(X)$ *then X is called* $\mathcal{S}$*-soft vertex covering based rough set and* $G_{\mathcal{S}}$ *is called an* $\mathcal{S}$*-soft rough vertex covering graph. Define and denote the lower and upper* $\mathcal{S}$*-soft vertex covering approximations of* $G_{\mathcal{S}}$ *by*

$$\underline{G_{\mathcal{S}}} = \left(\underline{appr}_{\mathcal{S}}(X), E \right)$$

and

$$\overline{G_{\mathcal{S}}} = (\overline{appr}_{\mathcal{S}}(X), E)$$

for any $X \subseteq V$.

Definition 9. *Let* $G_{\mathcal{S}}$ *be an* $\mathcal{S}$*-soft rough vertex covering graph. Then the* $\mathcal{S}$*-roughness membership function of* $X \subseteq V$ *is given by*

$$\xi_{G_{\mathcal{S}}}(X) = 1 - \frac{1}{2} \left[1 + \frac{\left| \underline{appr}_{\mathcal{S}}(X) \right|}{\left| \overline{appr}_{\mathcal{S}}(X) \right|} \right].$$

Thus if $\overline{appr}_{\mathcal{S}}(X) = \underline{appr}_{\mathcal{S}}(X)$ *then* $\xi_{G_{\mathcal{S}}}(X) = 0$, *and the graph* $G_{\mathcal{S}}$ *is soft vertex covering definable, i.e., no roughness.*

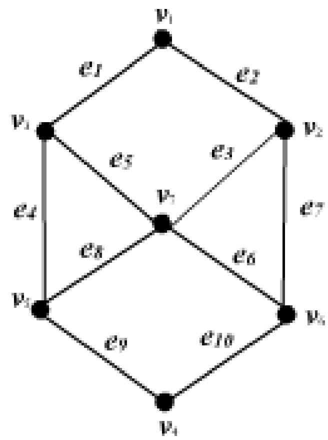

Figure 1

Table 1

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
α_1	0	0	1	1	1	1	0
α_2	1	1	0	0	0	0	0
α_3	0	0	1	0	1	1	0
α_4	0	0	1	1	0	0	0
α_5	1	1	1	0	0	0	1
α_6	1	1	0	0	1	1	0

Example 1. *Consider a simple graph $G^\star = (V, E)$, where $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ is set of vertices and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$ is the set of edges as shown in figure below. Let $T = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$ be the set of parameters and $\mathcal{G} = (G^\star, \lambda, \mu, T)$ be a soft covering vertex graph over the simple graph $G^\star$. The covering soft set (λ, T) over V is given in Table 1 such that $\lambda(\alpha_1) = \{v_3, v_4, v_5, v_6\}$, $\lambda(\alpha_2) = \{v_1, v_2\}$, $\lambda(\alpha_3) = \{v_3, v_5, v_6\}$, $\lambda(\alpha_4) = \{v_3, v_4\}$, $\lambda(\alpha_5) = \{v_1, v_2, v_3, v_7\}$, $\lambda(\alpha_6) = \{v_1, v_2, v_5, v_6\}$*

Then $\mathcal{S} = (V, \mathcal{C}_\mathcal{V})$ is a soft vertex covering approximation space. Let $X = \{v_1, v_2, v_4\} \subseteq V$. Then

$$\underline{appr}_\mathcal{S}(X) = \bigcup_{\alpha \in T} \{\lambda(\alpha) : \lambda(\alpha) \subseteq X\} = \{v_1, v_2\}$$

$$\overline{appr}_\mathcal{S}(X) = \bigcup \{Mdes_\mathcal{S}(v) : v \in V\} = \{v_1, v_2, v_3, v_4\}$$

Since $\underline{appr}_\mathcal{S}(X) \neq \overline{appr}_\mathcal{S}(X)$. So X is a soft vertex covering based rough set and $G_\mathcal{S} := (V, E)$ is an $\mathcal{S}$-soft rough vertex covering graph, where

$$\underline{G_\mathcal{S}} = \left(\underline{appr}_\mathcal{S}(X), E\right) = (\{v_1, v_2\}, \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}),$$

$$\overline{G_\mathcal{V}} = (\overline{appr}_\mathcal{V}(X), E) = (\{v_1, v_2, v_3, v_4\}, \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}) \text{ and}$$

$$\xi_{G_\mathcal{S}}(X) = 1 - \frac{1}{2}\left[1 + \frac{\left|\underline{appr}_\mathcal{S}(X)\right|}{\left|\overline{appr}_\mathcal{S}(X)\right|}\right] = 0.25.$$

Note if $X = \{v_1, v_2\} \subseteq V$, then X is $\mathcal{S}$-soft vertex covering based definable because $\underline{appr}_\mathcal{S}(X) = \overline{appr}_\mathcal{S}(X) = \{v_1, v_2\}$ and $G_\mathcal{S} := (V, E)$ is an $\mathcal{S}$-soft vertex covering definable graph, where

$$\begin{aligned}\underline{G_\mathcal{S}} &= \left(\underline{appr}_\mathcal{S}(X), E\right) = (\{v_1, v_2\}, \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}) \\ &= \overline{G_\mathcal{V}} = (\overline{appr}_\mathcal{V}(X), E)\end{aligned}$$

$$\xi_{G_{\mathcal{S}}}(X) = 1 - \frac{1}{2}\left[1 + \frac{\left|\underline{appr}_{\mathcal{S}}(X)\right|}{\left|\overline{appr}_{\mathcal{S}}(X)\right|}\right] = 0.$$

Thus there is no roughness.

Definition 10. *Let $\mathcal{G} = \left(G^{\star}, \lambda, \mu, T\right)$ be a full soft edge graph. Then $\mathcal{G}$ is called covering soft edge graph if $\mu(\alpha) \neq \emptyset$ for all $\alpha \in T$. In this case (μ, T) is called covering soft edge set over E, denoted by $\mathcal{C}_{\mathcal{E}}$. Denote by $\mathcal{Q} = (E, \mathcal{C}_{\mathcal{E}})$ and call it a soft edge covering approximation space.*

Definition 11. *Let $\mathcal{Q} = (E, \mathcal{C}_{\mathcal{E}})$ be a soft edge covering approximation space and $e \in E$. Then the set*

$$Mdes_{\mathcal{Q}}(e) = \left\{ \begin{array}{c} \mu(\alpha) : \alpha \in T \wedge e \in \mu(\alpha) \wedge \\ (\text{for all } \beta \in T \wedge e \in \mu(\beta) \subseteq \mu(\alpha) \text{ implies } \mu(\beta) = \mu(\alpha)) \end{array} \right\}$$

is called soft minimal edge description of $e \in E$.

Definition 12. *Let $\mathcal{Q} = (E, \mathcal{C}_{\mathcal{E}})$ be a soft edge covering approximation space. Based on $\mathcal{Q} = (E, \mathcal{C}_{\mathcal{E}})$, the sets defined by*

$$\underline{appr}_{\mathcal{Q}}(Y) = \bigcup_{\alpha \in T} \{\mu(\alpha) : \mu(\alpha) \subseteq Y\},$$

and

$$\overline{appr}_{\mathcal{Q}}(Y) = \bigcup \{Mdes_{\mathcal{Q}}(e) : e \in Y\}$$

for the subset $Y \subseteq E$, are called the lower $\mathcal{Q}$-soft edge covering and upper $\mathcal{Q}$-soft edge covering approximations of Y, respectively.
Also,

$$Post_{\mathcal{Q}}(Y) = \underline{appr}_{\mathcal{Q}}(Y),$$

$$Negt_{\mathcal{Q}}(Y) = E - \overline{appr}_{\mathcal{Q}}(Y)$$

$$\text{and } Bnd_{\mathcal{Q}}(Y) = \overline{appr}_{\mathcal{Q}}(Y) - \underline{appr}_{\mathcal{Q}}(Y).$$

are called $\mathcal{Q}$-soft positive edge covering region, $\mathcal{Q}$-soft negative edge covering region and $\mathcal{Q}$-soft boundary edge covering region respectively. If $\overline{appr}_{\mathcal{Q}}(Y) = \underline{appr}_{\mathcal{Q}}(Y)$, for $Y \subseteq E$, then Y is called $\mathcal{Q}$-soft edge covering definable set and $G_{\mathcal{Q}} = (V, E)$ is called $\mathcal{Q}$-soft edge covering definable graph. On the other hand if $\overline{appr}_{\mathcal{Q}}(Y) \neq \underline{appr}_{\mathcal{Q}}(Y)$ then Y is called $\mathcal{Q}$-soft edge covering based rough set and $G_{\mathcal{Q}}$ is called $\mathcal{Q}$-soft rough edge covering graph. Define and denote the lower and upper Q-soft edge covering approximations of $G_{\mathcal{Q}}$ by

$$\underline{G_{\mathcal{Q}}} = \left(V, \underline{appr}_{\mathcal{Q}}(Y)\right)$$

and

$$\overline{G_{\mathcal{Q}}} = (V, \overline{appr}_{\mathcal{Q}}(Y))$$

for any $Y \subseteq E$.

Definition 13. *Let $G_{\mathcal{Q}}$ be a $\mathcal{Q}$-soft rough edge covering graph. Then the $\mathcal{Q}$-roughness membership function of $Y \subseteq E$ is given by*

$$\xi_{G_{\mathcal{Q}}}(Y) = 1 - \frac{1}{2}\left[1 + \frac{\left|\underline{appr}_{\mathcal{Q}}(Y)\right|}{\left|\overline{appr}_{\mathcal{Q}}(Y)\right|}\right].$$

Thus if $\overline{appr}_{\mathcal{Q}}(Y) = \underline{appr}_{\mathcal{Q}}(Y)$ then $\xi_{G_{\mathcal{Q}}}(Y) = 0$, and so the graph $G_{\mathcal{Q}}$ is soft edge covering definable, i.e., no roughness.

Definition 14. *A full soft graph $\mathcal{G} = \left(G^{\star}, \lambda, \mu, T\right)$ is called covering soft graph if $\lambda(\alpha) \neq \emptyset$ and $\mu(\alpha) \neq \emptyset$ for all $\alpha \in T$.*

Table 2

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
α_1	1	0	1	0	1	0	0	0	0	1
α_2	0	0	0	1	0	0	0	0	0	0
α_3	0	0	0	1	1	0	0	1	0	0
α_4	1	1	0	0	1	1	0	1	1	0
α_5	1	0	1	0	0	0	0	0	0	0
α_6	0	0	0	0	0	1	1	0	1	0

Example 2. *(Continued from Example 1) Let (μ, T) be a covering soft set over E and $\mathcal{Q} = (E, \mathcal{C}_{\mathcal{E}})$ be a soft edge covering approximation space such that $\mu(\alpha_1) = \{e_1, e_3, e_5, e_{10}\}$, $\mu(\alpha_2) = \{e_4\}$, $\mu(\alpha_3) = \{e_4, e_5, e_8\}$, $\mu(\alpha_4) = \{e_1, e_2, e_5, e_6, e_8, e_9\}$, $\mu(\alpha_5) = \{e_1, e_3\}$ and $\mu(\alpha_6) = \{e_6, e_7, e_9\}$ as shown in the Table 2*

Let $Y = \{e_1, e_3, e_4, e_{10}\} \subseteq E$. Then

$$\underline{appr}_{\mathcal{Q}}(Y) = \bigcup_{\alpha\in T} \{\mu(\alpha) : \mu(\alpha) \subseteq Y\} = \{e_1, e_3, e_4\}$$

$$\overline{appr}_{\mathcal{Q}}(Y) = \bigcup \{Mdes_{\mathcal{Q}}(e) : e \in E\} = \{e_1, e_3, e_5, e_{10}\}.$$

As $\underline{appr}_{\mathcal{Q}}(Y) \neq \overline{appr}_{\mathcal{Q}}(Y)$, so Y is a soft edge covering based rough set and $G_{\mathcal{Q}} := (V, E)$ is $\mathcal{Q}$-soft rough edge covering graph, where

$$\underline{G_{\mathcal{Q}}} = \left(V, \underline{appr}_{\mathcal{Q}}(Y)\right) = (\{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}, \{e_1, e_3, e_4\})$$

$$\overline{G_{\mathcal{Q}}} = (V, \overline{appr}_{\mathcal{Q}}(Y)) = (\{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}, \{e_1, e_3, e_5, e_{10}\}) \text{ and}$$

$$\xi_{G_{\mathcal{Q}}}(Y) = 1 - \frac{1}{2}\left[1 + \frac{\left|\underline{appr}_{\mathcal{S}}(Y)\right|}{\left|\overline{appr}_{\mathcal{S}}(Y)\right|}\right] = 0.125.$$

Definition 15. *The $G^{\star}$-roughness membership function of any subgraph graph $G^{\star\star} = (X, Y)$ of $G^{\star}$ is given by*

$$\mu_{G^{\star}}\left(G^{\star\star}\right) = 1 - \frac{1}{2}\left[\frac{\left|\underline{appr}_{\mathcal{S}}(X)\right|}{\left|\overline{appr}_{\mathcal{S}}(X)\right|} + \frac{\left|\underline{appr}_{\mathcal{Q}}(Y)\right|}{\left|\overline{appr}_{\mathcal{Q}}(Y)\right|}\right].$$

Definition 16. *A soft graph $\mathcal{G} = \left(G^{\star}, \lambda, \mu, T\right)$ is called soft covering based definable if*
(1) *X is $\mathcal{S}$-soft vertex covering definable i.e., $\overline{appr}_{\mathcal{S}}(X) = \underline{appr}_{\mathcal{S}}(X)$ for $X \subseteq V$ and*
(2) *Y is $\mathcal{Q}$-soft edge covering definable i.e., $\overline{appr}_{\mathcal{Q}}(Y) = \underline{appr}_{\mathcal{Q}}(Y)$ for $Y \subseteq E$.*

Definition 17. *A soft graph $\mathcal{G} = \left(G^{\star}, \lambda, \mu, T\right)$ is called soft covering based rough graph if*
(1) *X is $\mathcal{S}$-soft vertex covering based rough set i.e., $\overline{appr}_{\mathcal{S}}(X) \neq \underline{appr}_{\mathcal{S}}(X)$*
(2) *Y is $\mathcal{Q}$-soft edge covering based rough set i.e., $\overline{appr}_{\mathcal{Q}}(Y) \neq \underline{appr}_{\mathcal{Q}}(Y)$.*
A soft covering based rough graph is denoted by $G = (G_{\mathcal{S}}, G_{\mathcal{Q}})$.

Definition 18. *The lower and upper approximations of the soft covering based rough graph $G = (G_{\mathcal{S}}, G_{\mathcal{Q}})$, is denoted and defined as $\underline{appr}(G) = \left(\underline{appr}_{\mathcal{S}}(X), \underline{appr}_{\mathcal{Q}}(Y)\right)$ and $\overline{appr}(G) = (\overline{appr}_{\mathcal{S}}(X), \overline{appr}_{\mathcal{Q}}(Y))$ respectively, for any $X \subseteq V$ and $Y \subseteq E$.*

Proposition 1. *Let $\mathcal{G} = \left(G^{\star}, \lambda, \mu, T\right)$ be a covering soft graph and $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$ and $\mathcal{Q} = (E, \mathcal{C}_{\mathcal{E}})$ be soft vertex covering approximation space and soft edge covering approximation space, respectively. Then*

(1) $\underline{appr}(G) = \bigcup_{\alpha\in T} \{(\lambda(\alpha), \mu(\alpha)) : \lambda(\alpha) \subseteq X, \mu(\alpha) \subseteq Y\}$

(2) $\overline{appr}(G) = \bigcup\{(Mdes_{\mathcal{S}}(v), Mdes_{\mathcal{Q}}(e)) : v \in X \subseteq V \text{ and } e \in Y \subseteq E\}$
(3) $\underline{appr}(\emptyset) = \overline{appr}(\emptyset) = \emptyset$
(4) $\underline{appr}(G^{\star}) = \overline{appr}(G^{\star}) = G^{\star} = \left(\bigcup_{\alpha\in T}\lambda(\alpha), \bigcup_{\alpha\in T}\mu(\alpha)\right)$
(5) $\underline{appr}(G_1 \cap G_2) \subseteq \underline{appr}(G_1) \cap \underline{appr}(G_2)$
(6) $\underline{appr}(G_1 \cup G_2) \supseteq \underline{appr}(G_1) \cup \underline{appr}(G_2)$
(7) $\overline{appr}(G_1 \cup G_2) = \overline{appr}(G_1) \cup \overline{appr}(G_2)$
(8) $\overline{appr}(G_1 \cap G_2) \subseteq \overline{appr}(G_1) \cap \overline{appr}(G_2)$
(9) $G_1 \subseteq G_2$ *implies* $\underline{appr}(G_1) \subseteq \underline{appr}(G_2)$ *and* $\overline{appr}(G_1) \subseteq \overline{appr}(G_2)$ *where* G_1 *and* G_2 *are subgraphs of* $G^{\star}$.

Proof. Straightforward. □

Proposition 2. *Let* $\mathcal{G} = (G^{\star}, \lambda, \mu, T)$ *be a covering soft graph and let* $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$ *and* $\mathcal{Q} = (E, \mathcal{C}_{\mathcal{E}})$ *be soft vertex covering approximation space and soft edge covering approximation space, respectively. Then*

(1) *For any* $X \subseteq V$, X *is* $\mathcal{S}$ – *soft vertex covering definable iff* $\overline{appr}_{\mathcal{S}}(X) \subseteq X$.
(2) *For any* $Y \subseteq E$, Y *is* $\mathcal{Q}$ – *soft edge covering definable iff* $\overline{appr}_{\mathcal{Q}}(Y) \subseteq Y$.

Proof. (1) Suppose X is an $\mathcal{S}$-soft vertex covering definable. Then $\underline{appr}_{\mathcal{S}}(X) = \overline{appr}_{\mathcal{S}}(X)$ and so $\overline{appr}_{\mathcal{S}}(X) = \underline{appr}_{\mathcal{S}}(X) \subseteq X$. Conversely suppose that $\overline{appr}_{\mathcal{S}}(X) \subseteq X$ for $X \subseteq V$. To prove X is an $\mathcal{S}$-soft vertex covering definable , we have to prove only $\overline{appr}_{\mathcal{S}}(X) \subseteq \underline{appr}_{\mathcal{S}}(X)$. Let $v \in \overline{appr}_{\mathcal{S}}(X) = \bigcup\{Mdes_{\mathcal{S}}(v) : v \in X\}$. Then $v \in Mdes_{\mathcal{S}}(v)$ and $Mdes_{\mathcal{S}}(v) \cap X \neq \emptyset$, showing that $v \in Mdes_{\mathcal{S}}(v) \subseteq \overline{appr}_{\mathcal{S}}(X) \subseteq X$. So $v \in \lambda(\alpha)$ for some $\alpha \in T$. Hence $v \in \underline{appr}_{\mathcal{S}}(X) = \bigcup_{\alpha\in T}\{\lambda(\alpha) : \lambda(\alpha) \subseteq X\}$ and so $\overline{appr}_{\mathcal{S}}(X) \subseteq \underline{appr}_{\mathcal{S}}(X)$.
(2) Can be proved in similar way. □

The above proposition is illustrated in the following example:

Example 3. *Let* $\mathcal{G} = (G^{\star}, \lambda, \mu, T)$ *be a covering soft graph over the simple graph* $G^{\star} = (V, E)$, *where* $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ *is a set of vertices,* $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ *is the set of edges and* $T = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$ *is the set of parameters. Let* (λ, T) *be a covering soft set over* V *such that* $\lambda(\alpha_1) = \{v_2, v_3, v_5, v_6\}$, $\lambda(\alpha_2) = \{v_2, v_6\}$, $\lambda(\alpha_3) = \{v_1, v_2, v_3, v_6\}$, $\lambda(\alpha_4) = \{v_1, v_5\}$, $\lambda(\alpha_5) = \{v_1, v_3, v_4, v_5\}$. *Then* $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$ *is soft vertex covering approximation space. For* $X = \{v_2, v_6\} \subseteq V$, *we have* $\underline{appr}_{\mathcal{V}}(X) = \{v_2, v_6\} = \overline{appr}_{\mathcal{V}}(X)$. *Hence* $\underline{appr}_{\mathcal{S}}(X) \subseteq X$ *and* X *is* $\mathcal{S}$*-soft vertex covering definable.*
(2) *Let* (μ, T) *is called covering soft set over* E *such that* $\mu(\alpha_1) = \{e_1, e_3, e_6\}$, $\mu(\alpha_2) = \{e_2, e_3, e_4, e_5\}$, $\mu(\alpha_3) = \{e_1, e_2, e_4, e_6\}$, $\mu(\alpha_4) = \{e_2, e_4\}$, $\mu(\alpha_5) = \{e_1, e_2, e_4, e_5, e_6\}$. *Then* $\mathcal{Q} = (E, \mathcal{C}_{\mathcal{E}})$ *is a soft edge covering approximation space. Let* $Y = \{e_2, e_4\} \subseteq E$. *Then we can see that* $\underline{appr}_{\mathcal{Q}}(Y) = \{e_2, e_4\} = \overline{appr}_{\mathcal{Q}}(Y)$ *showing that* Y *is* $\mathcal{Q}$*-soft edge covering definable. Clearly,* $\mathcal{G} = (G^{\star}, \lambda, \mu, T)$ *is soft covering based definable graph.*

Proposition 3. *Let* $G = (G_{\mathcal{S}}, G_{\mathcal{Q}})$ *be a soft covering based rough graph. Then*
(1) $\underline{appr}(\overline{appr}(G)) = \overline{appr}(G)$
(2) $\overline{appr}(\underline{appr}(G)) \supseteq \underline{appr}(G)$
(3) $\underline{appr}(\underline{appr}(G)) = \underline{appr}(G)$
(4) $\overline{appr}(\overline{appr}(G)) \supseteq \overline{appr}(G)$.

Proof. Let $G = (G_{\mathcal{S}}, G_{\mathcal{Q}})$ be a soft covering based rough graph . Let $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$ be a soft vertex covering approximation space and $\mathcal{Q} = (E, \mathcal{C}_{\mathcal{E}})$ be a soft edge covering approximation space.
(1) Let $\mathcal{L} = \overline{appr}(G) = (\overline{appr}_{\mathcal{S}}(X), \overline{appr}_{\mathcal{Q}}(Y))$ for every $X \subseteq V$, $Y \subseteq E$. Let $(l_1, l_2) \in \mathcal{L}$ be such that $l_1 \in \overline{appr}_{\mathcal{S}}(X)$ and $l_2 \in \overline{appr}_{\mathcal{Q}}(Y)$. As $l_1 \in \overline{appr}_{\mathcal{S}}(X)$, so $l_1 \in \lambda(\alpha)$ and $Mdes_{\mathcal{S}}(v) \cap X \neq \emptyset$ for some $\alpha \in T$. Since $l_2 \in \overline{appr}_{\mathcal{Q}}(Y)$ so $l_2 \in \mu(\alpha)$ and $Mdes_{\mathcal{Q}}(e) \cap Y \neq \emptyset$ for some $\alpha \in T$. But

$$\overline{appr}(G) = \bigcup\{(\mathrm{Mdes}_{\mathcal{S}}(v), \mathrm{Mdes}_{\mathcal{Q}}(e)) : v \in X \subseteq V \text{ and } e \in Y \subseteq E\}$$

so there exists $\alpha \in T$ such that $(l_1, l_2) \in (\lambda(\alpha), \mu(\alpha)) \subseteq \mathcal{L}$. Hence $(l_1, l_2) \in \underline{appr}(\mathcal{L})$, and so $\mathcal{L} \subseteq \underline{appr}(\mathcal{L})$. But $\underline{appr}(\mathcal{L}) \subseteq \mathcal{L}$ which shows $\mathcal{L} = \underline{appr}(\mathcal{L})$. Therefore $\underline{appr}(\overline{appr}(G)) = \overline{appr}(G)$.

(2) Let $\mathcal{L} = \underline{appr}(G) = \left(\underline{appr}_{\mathcal{S}}(X), \underline{appr}_{\mathcal{Q}}(Y)\right)$ for every $X \subseteq V$, $Y \subseteq E$. Let $(l_1, l_2) \in \mathcal{L}$ such that $l_1 \in \underline{appr}_{\mathcal{S}}(X)$ and $l_2 \in \underline{appr}_{\mathcal{Q}}(Y)$. As $l_1 \in \underline{appr}_{\mathcal{S}}(X)$, so $l_1 \in \lambda(\alpha)$ and $\mathrm{Mdes}_{\mathcal{S}}(l_1) \cap X \neq \emptyset$ for some $\alpha \in T$. Since $l_2 \in \underline{appr}_{\mathcal{Q}}(Y)$, so $l_2 \in \mu(\alpha)$ and $Mdes_{\mathcal{Q}}(l_2) \cap Y \neq \emptyset$ for some $\alpha \in T$. But

$$\underline{appr}(G) = \bigcup_{\alpha \in T} \{(\lambda(\alpha), \mu(\alpha)) : \lambda(\alpha) \subseteq X, \mu(\alpha) \subseteq Y\},$$

$\mathrm{Mdes}_{\mathcal{S}}(l_1) \cap \underline{appr}_{\mathcal{S}}(X) = Mdes_{\mathcal{S}}(l_1) \neq \emptyset$ and $\mathrm{Mdes}_{\mathcal{Q}}(l_2) \cap \underline{appr}_{\mathcal{Q}}(Y) = Mdes_{\mathcal{Q}}(l_2) \neq \emptyset$. Then $(\mathrm{Mdes}_{\mathcal{S}}(l_1), \mathrm{Mdes}_{\mathcal{Q}}(l_2)) \cap \mathcal{L} = (\mathrm{Mdes}_{\mathcal{S}}(l_1), \mathrm{Mdes}_{\mathcal{Q}}(l_2))$, $\alpha \in T$, which shows $(l_1, l_2) \in \overline{appr}(\mathcal{L})$. So $\mathcal{L} \subseteq \overline{appr}(\mathcal{L})$ or $\overline{appr}\left(\underline{appr}(G)\right) \supseteq \underline{appr}(G)$.

(3) Let $\mathcal{L} = \underline{appr}(G) = \left(\underline{appr}_{\mathcal{S}}(X), \underline{appr}_{\mathcal{Q}}(Y)\right)$ for every $X \subseteq V$, $Y \subseteq E$. Let $(l_1, l_2) \in \mathcal{L}$ be such that $l_1 \in \underline{appr}_{\mathcal{S}}(X)$ and $l_2 \in \underline{appr}_{\mathcal{Q}}(Y)$. Since $l_1 \in \underline{appr}_{\mathcal{S}}(X)$, so $l_1 \in \lambda(\alpha)$ and $\lambda(\alpha) \subseteq X$ for some $\alpha \in T$. Since $l_2 \in \underline{appr}_{\mathcal{Q}}(Y)$, so $l_2 \in \mu(\alpha)$ and $\mu(\alpha) \subseteq Y$ for some $\alpha \in T$.
Since

$$\underline{appr}(G) = \bigcup_{\alpha \in T} \{(\lambda(\alpha), \mu(\alpha)) : \lambda(\alpha) \subseteq X, \mu(\alpha) \subseteq Y\}.$$

So there exists $\alpha \in T$ such that $(l_1, l_2) \in (\lambda(\alpha), \mu(\alpha)) \subseteq \underline{appr}(\mathcal{L})$. Thus $\mathcal{L} \subseteq \underline{appr}(\mathcal{L})$. But $\underline{appr}(\mathcal{L}) \subseteq (\mathcal{L})$. So $\mathcal{L} = \underline{appr}(\mathcal{L})$. Hence $\underline{appr}(G) = \underline{appr}\left(\underline{appr}(G)\right)$.

(4) To prove $\overline{appr}(\overline{appr}(G)) \supseteq \overline{appr}(G)$. Let $\mathcal{L} = \overline{appr}(G) = (\overline{appr}_{\mathcal{S}}(X), \overline{appr}_{\mathcal{Q}}(Y))$ for every $X \subseteq V$, $Y \subseteq E$. Let $(l_1, l_2) \in \mathcal{L}$ such that $l_1 \in \overline{appr}_{\mathcal{S}}(X)$ and $l_2 \in \overline{appr}_{\mathcal{Q}}(Y)$. Since $l_1 \in \overline{appr}_{\mathcal{S}}(X)$, so $l_1 \in \lambda(\alpha)$ and $\mathrm{Mdes}_{\mathcal{S}}(l_1) \cap X \neq \emptyset$ for some $\alpha \in T$. Since $l_2 \in \overline{appr}_{\mathcal{Q}}(Y)$ so $l_2 \in \mu(\alpha)$ and $\mathrm{Mdes}_{\mathcal{Q}}(l_2) \cap Y \neq \emptyset$ for some $\alpha \in T$.
Since

$$\overline{appr}(G) = \bigcup \{(\mathrm{Mdes}_{\mathcal{S}}(l_1), \mathrm{Mdes}_{\mathcal{Q}}(l_2)) : l_1 \in X \subseteq V \text{ and } l_2 \in Y \subseteq E\},$$

$$\overline{appr}_{\mathcal{S}}(X) = \bigcup \{\mathrm{Mdes}_{\mathcal{S}}(l_1) : l_1 \in X \subseteq V\} \text{ and}$$

$$\overline{appr}_{\mathcal{Q}}(Y) = \bigcup \{(\mathrm{Mdes}_{\mathcal{Q}}(l_2)) : l_2 \in Y \subseteq E\}.$$

Therefore $(\mathrm{Mdes}_{\mathcal{S}}(l_1), \mathrm{Mdes}_{\mathcal{Q}}(l_2)) \cap \mathcal{L} = (\mathrm{Mdes}_{\mathcal{S}}(l_1), \mathrm{Mdes}_{\mathcal{Q}}(l_2))$ with $\mathrm{Mdes}_{\mathcal{S}}(l_1) \neq \emptyset$ and $\mathrm{Mdes}_{\mathcal{Q}}(l_2) \neq \emptyset$. So $(l_1, l_2) \in (\mathrm{Mdes}_{\mathcal{S}}(l_1), \mathrm{Mdes}_{\mathcal{Q}}(l_2)) \subseteq \overline{appr}(\mathcal{L})$ showing $(l_1, l_2) \in \overline{appr}(\mathcal{L})$. Thus $\mathcal{L} \subseteq \overline{appr}(\mathcal{L})$. Hence $\overline{appr}(\overline{appr}(G)) \supseteq \overline{appr}(G)$. □

Corollary 1. *Let $\mathcal{G} = \left(G^{\star}, \lambda, \mu, T\right)$ be a soft graph and $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$ and $\mathcal{Q} = (E, \mathcal{C}_{\mathcal{E}})$ be soft vertex covering approximation space and soft edge covering approximation space, respectively. Then for every $X \subseteq V$ and $Y \subseteq E$, the following hold.*

(1) $\underline{appr}_{\mathcal{S}}(\overline{appr}_{\mathcal{S}}(X)) = \overline{appr}_{\mathcal{S}}(X)$.

(2) $\overline{appr}_{\mathcal{S}}\left(\underline{appr}_{\mathcal{S}}(X)\right) \supseteq \underline{appr}_{\mathcal{S}}(X)$

(3) $\underline{appr}_{\mathcal{S}}\left(\underline{appr}_{\mathcal{S}}(X)\right) = \underline{appr}_{\mathcal{V}}(X)$

(4) $\overline{appr}_{\mathcal{S}}(\overline{appr}_{\mathcal{S}}(X)) \supseteq \overline{appr}_{\mathcal{S}}(X)$ *for every* $X \subseteq V$.

(5) $\underline{appr}_{\mathcal{Q}}(\overline{appr}_{\mathcal{Q}}(Y)) = \overline{appr}_{\mathcal{Q}}(Y)$.

(6) $\overline{appr}_{\mathcal{Q}}\left(\underline{appr}_{\mathcal{Q}}(Y)\right) \supseteq \underline{appr}_{\mathcal{Q}}(Y)$

(7) $\underline{appr}_{\mathcal{Q}}\left(\underline{appr}_{\mathcal{Q}}(Y)\right) = \underline{appr}_{\mathcal{Q}}(Y)$

(8) $\overline{appr}_{\mathcal{Q}}(\overline{appr}_{\mathcal{Q}}(Y)) \supseteq \overline{appr}_{\mathcal{Q}}(Y)$ *for every* $Y \subseteq E$.

Example 4. *Let $\mathcal{G} = \left(G^{\star}, \lambda, \mu, T\right)$ be a soft graph as in Example 1 with $\mu(\alpha_1) = \{v_1, v_5, v_6, v_7\}$, $\mu(\alpha_2) = \{v_3, v_5, v_6\}$, $\mu(\alpha_3) = \{v_1, v_3, v_4\}$, $\mu(\alpha_4) = \{v_5, v_6\}$, $\mu(\alpha_5) = \{v_2, v_4, v_6, v_7\}$ and $\mu(\alpha_6) = \{v_1, v_2, v_3, v_6, v_7\}$. Let $X = \{v_2, v_3, v_5, v_6\} \subseteq V$. Then*

$$\underline{appr}_{\mathcal{S}}(X) = \bigcup_{\alpha \in T} \{\lambda(\alpha) : \lambda(\alpha) \subseteq X\} = \{v_3, v_5, v_6\},$$

and

$$\overline{appr}_{\mathcal{S}}(X) = \bigcup \{Mdes_{\mathcal{S}}(v) : v \in X\} = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$$

(1) *Let* $P = \overline{appr}_{\mathcal{S}}(X) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$. *Then* $\overline{appr}_{\mathcal{S}}(P) = \overline{appr}_{\mathcal{S}}(\overline{appr}_{\mathcal{S}}(X)) = P$.
(2) *Let* $R = \underline{appr}_{\mathcal{S}}(X) = \{v_3, v_5, v_6\}$. *Then* $\overline{appr}_{\mathcal{S}}(R) = \overline{appr}_{\mathcal{S}}\left(\underline{appr}_{\mathcal{S}}(X)\right) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\} = \overline{appr}_{\mathcal{S}}(X) \supseteq R$.
(3) $\underline{appr}_{\mathcal{S}}\left(\underline{appr}_{\mathcal{S}}(X)\right) = \underline{appr}_{\mathcal{S}}(R) = \{v_3, v_5, v_6\} = R = \underline{appr}_{\mathcal{S}}(X)$. *Similarly rest of the above results can be verified.*

Proposition 4. *Let* $\mathcal{G} = (G^\star, \lambda, \mu, T)$ *be a soft graph. Then* $\mathcal{G} = (G^\star, \lambda, \mu, T)$ *is a full soft graph if and only if* $\underline{appr}(G^\star) = G^\star = \overline{appr}(G^\star)$.

Proof. Suppose $\mathcal{G} = (G^\star, \lambda, \mu, T)$ is a full soft graph. Then $G^\star = \left(\bigcup_{\alpha \in T} \lambda(\alpha), \bigcup_{\alpha \in T} \mu(\alpha)\right)$. Since

$$\underline{appr}_{\mathcal{S}}(X) = \bigcup_{\alpha \in T} \{\lambda(\alpha) : \lambda(\alpha) \subseteq X\},$$

so $\underline{appr}_{\mathcal{S}}(V) = \bigcup_{\alpha \in T} \{\lambda(\alpha) : \lambda(\alpha) \subseteq V\} = \bigcup_{\alpha \in T} \lambda(\alpha) = V$. Hence $\underline{appr}_{\mathcal{S}}(V) = V$. Now $\underline{appr}_{\mathcal{Q}}(Y) = \bigcup_{\alpha \in T} \{\mu(\alpha) : \mu(\alpha) \subseteq Y\}$, so $\underline{appr}_{\mathcal{Q}}(E) = \bigcup_{\alpha \in T} \{\mu(\alpha) : \mu(\alpha) \subseteq E\} = \bigcup_{\alpha \in T} \mu(\alpha) = E$. Therefore $\underline{appr}(G^\star) = \left(\underline{appr}_{\mathcal{S}}(V), \underline{appr}_{\mathcal{Q}}(E)\right) = (V, E) = G^\star$. Also $\overline{appr}_{\mathcal{S}}(X) = \bigcup \{\mathrm{Mdes}_{\mathcal{S}}(v) : v \in X\}$, so $\overline{appr}_{\mathcal{S}}(V) = \bigcup \{\mathrm{Mdes}_{\mathcal{S}}(v) : v \in V\} = \bigcup Mdes_{\mathcal{S}}(v) = V$
Similarly $\overline{appr}_{\mathcal{Q}}(Y) = \bigcup \{\mathrm{Mdes}_{\mathcal{Q}}(e) : e \in Y\}$, so $\overline{appr}_{\mathcal{Q}}(E) = \bigcup \{\mathrm{Mdes}_{\mathcal{Q}}(e) : e \in E\} = \bigcup Mdes_{\mathcal{Q}}(e) = E$.
Hence $G^\star = \overline{appr}(G^\star)$.

Conversely suppose that $\underline{appr}(G^\star) = G^\star = \overline{appr}(G^\star)$. To show $\mathcal{G} = (G^\star, \lambda, \mu, T)$ is a full soft graph. Since $\underline{appr}(G^\star) = \left(\underline{appr}_{\mathcal{S}}(V), \underline{appr}_{\mathcal{Q}}(E)\right) = G^\star$ so $\underline{appr}_{\mathcal{S}}(V) = V$ and $\underline{appr}_{\mathcal{Q}}(E) = E$.
So $\underline{appr}_{\mathcal{S}}(V) = \bigcup_{\alpha \in T} \{\lambda(\alpha) : \lambda(\alpha) \subseteq V\} = V$ and $\underline{appr}_{\mathcal{Q}}(E) = \bigcup_{\alpha \in T} \{\mu(\alpha) : \mu(\alpha) \subseteq E\} = E$, which shows that $\bigcup_{\alpha \in T} \lambda(\alpha) = V$ and $\bigcup_{\alpha \in T} \mu(\alpha) = E$. Hence $G^\star = (V, E) = \left(\bigcup_{\alpha \in T} \lambda(\alpha), \bigcup_{\alpha \in T} \mu(\alpha)\right)$ showing that $\mathcal{G} = (G^\star, \lambda, \mu, T)$ is a full soft graph. □

Proposition 5. *Let* $\mathcal{G} = (G^\star, \lambda, \mu, T)$ *be a covering soft graph and* $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$ *be a soft vertex covering approximation space. Then the following are equivalent:*
(1) (λ, T) *is a full soft set.*
(2) $\underline{appr}_{\mathcal{S}}(V) = V$
(3) $\overline{appr}_{\mathcal{S}}(V) = V$
(4) $X \subseteq \overline{appr}_{\mathcal{S}}(X)$ *for all* $X \subseteq V$
(5) $\overline{appr}_{\mathcal{S}}(\{v\}) \neq \emptyset$ *for all* $v \in V$.

Proof. Using Proposition 4, it is easy to show that the conditions (1) and (2) are equivalent. Similarly the conditions (1) and (3) are equivalent.
Now to prove conditions (4), (5) and (1) are equivalent. Suppose condition (4) holds. To prove that (5) is true. For $v \in V$, by condition (4), $\{v\} \subseteq \underline{appr}_{\mathcal{S}}(\{v\})$. Thus $\underline{appr}_{\mathcal{S}}(\{v\}) \neq \emptyset$ because $v \in \underline{appr}_{\mathcal{S}}(\{v\})$. Hence (4) implies(5). Now to show (5) implies (1). Suppose $\overline{appr}_{\mathcal{S}}(\{v\}) \neq \emptyset$ for all $v \in V$. Let $l \in \underline{appr}_{\mathcal{S}}(\{v\})$. Then by definition of $\underline{appr}_{\mathcal{S}}(\{v\})$, there exists some α in T with $l \in \lambda(\alpha)$ and $\lambda(\alpha) \cap \{v\} \neq \emptyset$. It follows that $v = l \in \lambda(\alpha)$ and so $v \in \bigcup_{\alpha \in T} \lambda(\alpha)$. Hence (λ, T) is a full soft set showing that (5) implies (1). Now to complete the proposition, it is left to show (1) implies (4). Suppose (λ, T) is a full soft set with $X \subseteq V$. For any $p \in X$,

since (λ, T) is a full soft set so there exists some $\alpha \in T$ such that $p \in \lambda(\alpha)$. Also $X \cap \mathrm{Mdes}_{\mathcal{S}}(p) \neq \emptyset$ because $p \in X \cap Mdes_{\mathcal{S}}(p)$. Hence $p \in \overline{appr}_{\mathcal{S}}(X)$. Therefore $X \subseteq \overline{appr}_{\mathcal{S}}(X)$ for all $X \subseteq V$. □

Proposition 6. *Let $\mathcal{G} = (G^\star, \lambda, \mu, T)$ be a covering soft graph and $\mathcal{Q} = (E, \mathcal{C}_{\mathcal{E}})$ be a soft edge covering approximation space. Then the following are equivalent:*
(1) *(μ, T) is a full soft set.*
(2) *$\underline{appr}_{\mathcal{Q}}(E) = E$.*
(3) *$\overline{appr}_{\mathcal{Q}}(E) = E$.*
(4) *$Y \subseteq \overline{appr}_{\mathcal{Q}}(Y)$ for all $Y \subseteq E$.*
(5) *$\overline{appr}_{\mathcal{Q}}(\{e\}) \neq \emptyset$ for all $e \in E$.*

Proof. Similar to the proof of the Proposition 5. □

4 Applications of Soft Covering Based Rough Graphs

One of the most important applications of rough sets is the decision making and after combining with soft sets, it has promoted to multicriteria group decision making. Many applications of multicriteria group decision making are available in literature which can be seen in [26, 28, 31, 34, 39]. In such applications, the decision making has not been involved by the interaction of the objects, while their individual performance/characteristics have been used. Initial evaluation results have been used to perform the algorithms of decision making, which are prescribed to a few number of fields. In this section, we use the soft covering based rough graphs to settle a real life medical diagnosis problem. The algorithm is described as follows:
Let $V = \{V_1, V_2, V_3, \ldots, V_k\}$ be the set of objects(universe) and $\mathcal{T} = \{\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \ldots, \mathfrak{d}_r\}$ be the set of (diseases)parameters. Let $G^\star$ be a simple graph with V as a set of vertices and E as a set of edges. Let $(\lambda, \mathcal{T})$ and $(\mu, \mathcal{T})$ be two covering soft sets on V and E respectively, defined by;

$$\lambda(\mathfrak{d}_i) = \{V_P \in V : \text{Vertex } V_P \text{ possesses the attribute } \mathfrak{d}_i\}$$

and

$$\mu(\mathfrak{d}_i) = \{V_p V_q \in E : \text{Vertex } V_p \text{ with attribute } d_i \text{ has interaction with vertex } V_q\},$$

so that $\mathcal{G} = (G^\star, \lambda, \mu, T)$ is a soft graph, i.e., for each i, $G_i = (\lambda(\mathfrak{d}_i), \mu(\mathfrak{d}_i))$ is a subgraph of $G^\star$. For basic evaluation, suppose $\mathfrak{D} = \{S_1, S_2, S_3, \ldots, S_m\}$ is the set of m medical specialist and $(\pi, \mathfrak{D})$ be a soft set over V defined by

$$\pi(S_t) = \left\{ \begin{array}{c} V_P \in V : \text{the medical specialist } S_t \text{ suggested } V_P \text{ possessing} \\ \text{some attribute } \mathfrak{d}_i \end{array} \right\}.$$

Let $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$ be a soft vertex covering approximation space, then

$$\underline{appr}_{\mathcal{S}}(\pi(S_t)) = \left\{ \begin{array}{c} V_P \in V : V_P \text{ is an optimum candidate according to} \\ \text{medical specialist } S_t \end{array} \right\},$$

and

$$\overline{appr}_{\mathcal{S}}(\pi(S_t)) = \left\{ \begin{array}{c} V_P \in V : V_P \text{ is possibly an optimum candidate according} \\ \text{to medical specialist } S_t \end{array} \right\}.$$

Suppose $\Omega_{\underline{\pi(\mathfrak{D})}}(V_P)$ and $\Omega_{\overline{\pi(\mathfrak{D})}}(V_P)$ are two fuzzy sets for measure of optimality and possibly measure of optimality, respectively on E, of each object V_P such that

$$\Omega_{\underline{\pi(\mathfrak{D})}}(V_P) = \frac{1}{m}\sum_{k=1}^{m}\chi_{\underline{\pi(S_t)}}(V_P)$$

and

$$\Omega_{\overline{\pi(\mathfrak{D})}}(V_P) = \frac{1}{m}\sum_{k=1}^{m}\chi_{\overline{\pi(S_t)}}(V_P),$$

where $\chi_{\underline{\pi(S_t)}}$ and $\chi_{\overline{\pi(S_t)}}$ are a kind of indicator functions, defined by

$$\chi_{\underline{\pi(S_t)}}(V_P) = \begin{cases} 1 \text{ if } V_P \text{ is in } \underline{appr}_{\mathcal{S}}(\pi(S_t)) \\ 0 \quad \text{otherwise,} \end{cases}$$

and

$$\chi_{\overline{\pi(S_t)}}(V_P) = \begin{cases} 1 \text{ if } V_P \text{ is in } \overline{appr}_{\mathcal{S}}(\pi(S_t)) \\ 0 \quad \text{otherwise} \end{cases}.$$

Clearly $\Omega_{\underline{\pi(\mathfrak{D})}}(V_P)$ and $\Omega_{\overline{\pi(\mathfrak{D})}}(V_P)$ represents the optimality and possible optimality of each object according to each medical specialist. Now consider the interaction of vertex V_p with vertex V_q and vice versa. The marginal weight function ϕ for each V_p can be computed by;

$$\phi(V_p) = \frac{1}{k}[\phi_r(V_p) + \phi_c(V_p)]$$

for $p = 1, 2, 3, \dots, k$, where

$$\phi_r(V_p) = \sum_{i=1}^{k} \chi_E(V_p V_q),$$

is the measures the interaction of V_p with V_q, and

$$\phi_c(V_p) = \sum_{i=1}^{k} \chi_E(V_q V_p),$$

is the measures the interaction of vertex V_q with V_p, where χ_E is an indicator function on E, defined by

$$\chi_E(V_p V_q) = \begin{cases} 1 \text{ if } V_p V_q \text{ form an edge} \\ 0 \quad \text{otherwise} \end{cases}.$$

Henceforth the marginal weight function ϕ for each V_p, in both ways, actually measures the degree of interaction. Finally an evaluation function ψ is defined on V by

$$\psi(V_p) = \frac{1}{2}[\Omega_{\underline{\pi(\mathfrak{D})}}(V_p) + \Omega_{\overline{\pi(\mathfrak{D})}}(V_p)]\phi(V_p).$$

For a threshold $\gamma \in [0, 1]$, it can be seen that all persons V_j are at optimum for all j, in which $\psi(V_p) \geq \gamma$. The persons V_k is the best optimal if $\psi(V_k) = \max_p \{\psi(V_p)\}$. This algorithm involved both the indivisual's evaluations as well as the effects of interaction amongst the vertices/objects. This can be an interaction of two poles of transportation or network problems. One can apply this algorithm to other related problems. A real life application for diagnosing diseases from a group of people has been considered below.

Suppose during the annual medical checkup, four viral diseases found in a group of 20 people $V = \{V_1, V_2, V_3, \dots, V_{20}\}$, through different sources such as insect bite, eating contaminated food, having sex with an infected person and breathing air polluted by a virus. The above process of infection results in a diversity of symptoms that vary in severity and character, depending upon the individual factor and the kind of viral infection. Suppose $T = \{\eth_1, \eth_2, \eth_3, \eth_4\}$ is the set of parameters such that $\eth_1$ represents " entering of virus in human body through insect bite ", $\eth_2$ represents " entering of virus in human body through eating contaminated food ", $\eth_3$ represents " entering of virus in human body through having sex with an infected person and $\eth_4$ represents " entering of virus in human body through breathing air polluted by a virus. It is also assumed that a person V_j may have more than one viral disease. Suppose G^* is a simple digraph having vertex set V of 20 persons, (λ, T) be covering soft set on V indicating which member has what disease and $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$ be a soft vertex covering approximation space such that $\lambda(\eth_i) := \{V_p : V_p \text{ is infected through } \eth_i\}$ where

$$\lambda(\eth_1) = \{V_4, V_5, V_6, V_7, V_8, V_9, V_{16}, V_{18}, V_{19}, V_{20}\},$$

$$\lambda(\eth_2) = \{V_1, V_2, V_4, V_{10}, V_{12}, V_{13}, V_{14}, V_{15}, V_{19}\},$$

$$\lambda(\mathfrak{d}_3) = \{V_2, V_3, V_5, V_8, V_9, V_{11}, V_{17}, V_{20}\},$$

$$\lambda(\mathfrak{d}_4) = \{V_1, V_6, V_7, V_8, V_{13}, V_{17}, V_{18}\}.$$

Let $(\mu, \mathfrak{T})$ be a covering soft sets on E defined by;

$$\mu(\mathfrak{d}_i) = \{V_pV_q : V_p \text{ is infected by } V_q \text{ through } \mathfrak{d}_i\}$$

Let

$$\mu(\mathfrak{d}_1) = \left\{ \begin{array}{c} V_4V_6,\ V_5V_6,\ V_6V_7,\ V_7V_9,\ V_9V_8,\ V_8V_{16},\ V_{16}V_{20}, \\ V_7V_{20},\ V_{16}V_{18},\ V_{20}V_{19} \end{array} \right\},$$

$$\mu(\mathfrak{d}_2) = \left\{ \begin{array}{c} V_1V_4,\ V_1V_2,\ V_2V_4,\ V_4V_{19},\ V_{19}V_{10},\ V_{10}V_{12},\ V_{12}V_{15}, \\ V_{13}V_{14},\ V_1V_{10},\ V_{14}V_{15},\ V_{15}V_{10},\ V_4V_1,\ V_{19}V_1 \end{array} \right\},$$

$$\mu(\mathfrak{d}_3) = \left\{ \begin{array}{c} V_2V_5,\ V_3V_5,\ V_5V_8,\ V_2V_3,\ V_8V_9,\ V_9V_{11},\ V_2V_{17}, \\ V_8V_3,\ V_{17}V_{20},\ V_9V_{20},\ V_8V_{11},\ V_{17}V_2, V_{20}V_5, V_{11}V_{17}, V_{11}V_5 \end{array} \right\}$$

and

$$\mu(\mathfrak{d}_4) = \{V_1V_6,\ V_7V_{13},\ V_6V_7,\ V_{18}V_7,\ V_8V_{17},\ V_{13}V_{18},\ V_{13}V_{17},\ V_{17}V_6\}.$$

Clearly $\mathcal{G} = \left(G^{\star}, \lambda, \mu, T\right)$ is a soft graph, i.e., for each $i = 1, 2, 3, 4$, $G_i = (\lambda(\mathfrak{d}_i), \mu(\mathfrak{d}_i))$ is a subgraph of $G^{\star}$. Let $\mathfrak{D} = \{S_1, S_2, S_3\}$ be the set of 3 medical specialist's group who examine the patients with respect to the parameters $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3$ and $\mathfrak{d}_4$. Let $(\pi, \mathfrak{D})$ be a soft set over V showing whether a person is diagnosed by viral disease or not. Suppose

$$\pi(S_1) = \{V_1, V_2,\ V_3, V_4,\ V_5,\ V_6,\ V_7,\ V_8,\ V_9,\ V_{10},\ V_{13},\ V_{16},\ V_{17},\ V_{18},\ V_{19},\ V_{20}\},$$

$$\pi(S_2) = \{V_1,\ V_2,\ V_3,\ V_4,\ V_6,\ V_{10},\ V_{12},\ V_{13},\ V_{14},\ V_{15},\ V_{18}\},$$

and

$$\pi(S_3) = \{V_1,\ V_2,\ V_3,\ V_4,\ V_9,\ V_{10},\ V_{13},\ V_{18},\ V_{20}\}.$$

Let $\mathcal{S} = (V, \mathcal{C}_{\mathcal{V}})$ be a soft vertex covering approximation space. Then $\underline{appr}_{\mathcal{S}}(\pi(S_1)) = \{V_1, V_4, V_5, V_6, V_7, V_8, V_9, V_{13}, V_{17}, V_{18}, V_{19}, V_{20}\}$, $\underline{appr}_{\mathcal{S}}(\pi(S_2)) = \emptyset = \underline{appr}_{\mathcal{S}}(\pi(S_3))$ and $\overline{appr}_{\mathcal{S}}(\pi(S_i)) = V$, for $i = 1, 2, 3$. Suppose $\Omega_{\underline{\pi(\mathfrak{D})}}\left(V_j\right)$ and $\Omega_{\overline{\pi(\mathfrak{D})}}\left(V_j\right)$ be two fuzzy sets for measure of optimality and possibly measure of optimality, respectively on E of each object V_i such that

$$\Omega_{\underline{\pi(\mathfrak{D})}}(V_p) = \frac{1}{3}\sum_{k=1}^{3}\chi_{\underline{\pi(S_t)}}(V_p)$$

and

$$\Omega_{\overline{\pi(\mathfrak{D})}}(V_p) = \frac{1}{3}\sum_{k=1}^{3}\chi_{\overline{\pi(S_t)}}(V_p).$$

Here

$$\Omega_{\underline{\pi(\mathfrak{D})}}(V_p) = \frac{1}{3} \text{ for } j = 1, 4, 5, 6, 7, 8, 9, 13, 17, 18, 19, 20$$
$$\text{and } \Omega_{\underline{\pi(\mathfrak{D})}}(V_p) = 0 \text{ for rest of values of } p$$

and

$$\Omega_{\overline{\pi(\mathfrak{D})}}(V_p) = 1 \text{ for } p = 1, 2, 3, \ldots, 20.$$

The marginal weight function ϕ for each V_p can be computed by;

$$\phi(V_p) = \frac{1}{k}\left[\phi_r(V_p) + \phi_c(V_p)\right].$$

That is

$$\phi(V_1) = \frac{3}{10},\ \phi(V_2) = \frac{3}{10},\ \phi(V_3) = \frac{3}{20},\ \phi(V_4) = \frac{1}{4},$$

$$\phi(V_5) = \frac{3}{10},\ \phi(V_6) = \frac{1}{4},\ \phi(V_7) = \frac{1}{4},$$

$$\phi(V_8) = \frac{7}{20}, \phi(V_9) = \frac{1}{4},\ \phi(V_{10}) = \frac{1}{5},\ \phi(V_{11}) = \frac{1}{5},$$
$$\phi(V_{12}) = \frac{1}{10},\ \phi(V_{13}) = \frac{1}{5},\ \phi(V_{14}) = \frac{1}{10},$$

$$\phi(V_{15}) = \frac{3}{20},\ \phi(V_{16}) = \frac{3}{20},\ \phi(V_{17}) = \frac{7}{20},\ \phi(V_{18}) = \frac{3}{20},$$
$$\phi(V_{19}) = \frac{1}{5},\ \phi(V_{20}) = \frac{3}{10}.$$

Finally an evaluation function ψ is defined on V by

$$\psi(V_p) = \frac{1}{2}[\Omega_{\underline{\pi(\mathfrak{D})}}(V_p) + \Omega_{\overline{\pi(\mathfrak{D})}}(V_p)]\phi(V_p)$$

That is

$$\psi(V_1) = \frac{1}{5},\ \psi(V_2) = \frac{3}{10},\ \psi(V_3) = \frac{3}{40},\ \psi(V_4) = \frac{1}{6},\ \psi(V_5) = \frac{1}{5},$$
$$\psi(V_6) = \frac{1}{6},\ \psi(V_7) = \frac{1}{6},$$

$$\psi(V_8) = \frac{7}{30}, \psi(V_9) = \frac{1}{6},\ \psi(V_{10}) = \frac{1}{10},\ \psi(V_{11}) = \frac{1}{10},$$
$$\psi(V_{12}) = \frac{1}{20}, \psi(V_{13}) = \frac{2}{15}, \psi(V_{14}) = \frac{1}{20},$$

$$\psi(V_{15}) = \frac{3}{40},\ \psi(V_{16}) = \frac{3}{40}, \psi(V_{17}) = \frac{7}{30},\ \psi(V_{18}) = \frac{1}{10},$$
$$\psi(V_{19}) = \frac{2}{15}, \psi(V_{20}) = \frac{1}{5}.$$

Hence from the above calculations, the persons at highest risk is V_2.

The pseudocode of the above algorithm is presented below;

Pseudo code

(i) Consider $\mathcal{G} = \left(G^\star, \lambda, \mu, T\right)$ a soft graph and evaluation soft set $(\pi, \mathfrak{D})$.
(ii) Find lower and upper $\mathcal{S}$-soft vertex covering approximations of each $\pi(S_t)$.
(iii) Compute the fuzzy functions and $\Omega_{\underline{\pi(\mathfrak{D})}}(V_p)$ and $\Omega_{\overline{\pi(\mathfrak{D})}}(V_p)$ given by

$$\Omega_{\underline{\pi(\mathfrak{D})}}(V_p) = \frac{1}{m}\sum_{k=1}^{m}\chi_{\underline{\pi(S_t)}}(V_p)$$

and

$$\Omega_{\overline{\pi(\mathfrak{D})}}(V_p) = \frac{1}{m}\sum_{k=1}^{m}\chi_{\overline{\pi(S_t)}}(V_p).$$

(iv) Calculate the weights for each V_p, given by

$$\phi(V_p) = \frac{1}{k}[\phi_r(V_p) + \phi_c(V_p)]$$

(v) Finally calculate the evaluation function given by

$$\psi(V_p) = \frac{1}{2}[\Omega_{\underline{\pi(\mathfrak{D})}}(V_p) + \Omega_{\overline{\pi(\mathfrak{D})}}(V_p)]\phi(V_p)$$

The person V_k is at high risk if $\psi(V_k) = \max_p \{\psi(V_p)\}$.

5 Conclusion

The applications of soft sets and rough sets that are available in the literature are usually based upon the individ's properties of the members of the universe with the given attributes. In decision making problems, the diversity of attribute/behavior and characteristics with the member's interaction have not been considered so far. In the present work, we introduced the notion of soft covering based rough graphs. We not only discussed the basic properties of such graphs but also formulated a prediction system to optimize the diagnosis process of diagnosing some diseases among the members working in a factory. This interaction may cause the spreadness of disease among the staff members. Using the concepts of lower/upper $\mathcal{S}$-soft vertex covering approximations, the fuzzy sets $\Omega_{\underline{\pi(\mathfrak{D})}}$ and $\Omega_{\overline{\pi(\mathfrak{D})}}$ are introduced, while the marginal fuzzy sets $\phi_r(V_p)$ and $\phi_c(V_p)$ are used to find the measure of interaction of any staff member V_p with V_q and vice versa. Finally the evaluation function has pointed out the optimal carriers of diseases. We hope our results will prove a foundation for decision making problems. In future work we will be working on decision making problems in which lower/upper covering soft edge approximations are used to optimize the algorithm and will try to use different techniques to replace the marginal fuzzy sets.

References

[1] Euler L., Solutio problematis ad geometriam situs pertinensis. Comm. Acad. Sci. Imper. Petropol., 8, (1736) , 128-140.

[2] Beineke L. W., Wilson, R. J. and Cameron, P.J. eds., Topics in algebraic graph theory, Camb. Uni. Press, 102, (2004).

[3] West D. B., Introduction to graph theory, Upper Saddle River: Prentice hall, 2, (2001).

[4] Atanassov K. T., Intuitionistic fuzzy sets, Fuzzy sets and Sys 20, (1986) , 87-96.

[5] Molodtsov D., Soft set theory – First results, Comput. Math. Appl., 37, (1999) , 19-31.

[6] Pawlak Z., Rough sets, Int. J. Comput. Inform. Sci., 11, (1982) , 341-356.

[7] Zadeh L. A., Fuzzy sets, Inform. Sci. 8 (1965) 338-353.

[8] Zhu W., Wang F. Y., On three types of covering-based rough sets, IEEE Trans. Knowledge Data Eng (2007) , 19-8.

[9] Zhu W., Relationship between generalized rough sets based on binary relation and covering, Inform. Sci. 179, (2009) , 210-225.

[10] Greco S., Matarazzo B. , Slowinski R. , Rough approximation by dominance relations, Int. J. Intell. Syst. 17 (2002), 153-171.

[11] Guo Z. L, Liu Y. L, Yang H. L, A novel rough set model in generalized single valued neutrosophic approximation spaces and its application. Symm, 9(7) (2017) , 1-19.

[12] Liu G., Sai Y., A comparison of two types of rough sets induced by coverings, Int. J. Approx. Reason. 50 (2009), 521-528.

[13] Liu G., Zhu W., The algebraic structures of generalized rough set theory, Inform. Sci. 178 (2008), 4105-4113.

[14] Pawlak Z., Rough Sets: Theoretical Aspects of Reasoning about Data, Klu. Acad. Pub, Dordrecht, (1991).

[15] Pawlak Z., Skowron A., Rudiments of rough sets, Inform. Sci. 177 (2007), 3-27.

[16] Pawlak Z., Skowron A.,, Rough sets: some extensions, Inform. Sci. 177 (2007), 28–40.

[17] Słowinski R., Vanderpooten D., A generalized definition of rough approximations based on similarity, IEEE Trans. Knowledge Data Eng. 12 (2000), 331-336.

[18] Ali M. I., et al, On some new operations in soft set theory, Comput & Math with Appl, 57-9 (2009), 1547-1553.

[19] Maji P. K., Biswas R., Roy A. R., Soft set theory, Comput. Math. Appl. 45 (2003), 555-562.

[20] P. K. Maji, A. R. Roy, R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44 (2002), 1077-1083.

[21] Chitcharoen D., and Pattaraintakorn P.,Towards theories of fuzzy set and rough set to flow graphs, IEEE Int. Conf. Fuzzy Sys, 4630596 (2008) , 1675-1682.

[22] Morsi N. N., and Yakout M. M, Axiomatics for fuzzy rough sets, Fuzzy sets and Sys, 100 (1998), 327-342.

[23] Pedrycz W., and Homenda W., From fuzzy cognitive maps to granular cognitive maps, IEEE Trans Fuzzy Sys 22.4 (2014), 859-869.

[24] Rolka L., and Rolka A. M., Labeled Fuzzy Rough Sets Versus Fuzzy Flow Graphs, IJCCI- Proceed. 8th Inter. Joint Conf. Comput. Intel, (2016) , 115-120.

[25] Chen D., Wang C. Z., and Hu Q. H., A new approach to attribute reduction of consistent and inconsistent covering decision systems with covering rough sets, Inform. Sci. 177 (2007), 3500-3518.

[26] Feng F., Liu X. , Fotea V. L., and Jun Y. B., Soft sets and soft rough sets, Inform. Sci. 181 (2011), 1125-1137.

[27] Feng F., Soft rough sets applied to multicriteria group decision making, Ann. Fuzzy Math. Inf 2 (2011), 69-80.

[28] Molodtsov D., The Theory of Soft Sets (in Russian), URSS Publishers, Moscow, (2004).

[29] Roy A. R, and Maji P. K, A fuzzy soft set theoretic approach to decision making problems, J. Comput. Appl. Math. 203 (2007), 412-418.
[30] Tozlu N., Yuksel S., and Simsekler T. H, A Topological Approach to Soft Covering Approximation Space, arXiv preprint arXiv:1503.07896 (2015).
[31] Yuksel S, Ergül Z. G., and Tozlu N., Soft covering based rough sets and their application, The Sci. World J. (2014), 1-9.
[32] Yuksel S, Tozlu N and Dizman T. H., An application of multicriteria group decision making by soft covering based rough sets, Filomat 29.1 (2015), 209-219.
[33] Ali M. I., A note on soft sets, rough soft sets and fuzzy soft sets, Appl. Soft. Comput.,11 (2011) , 3329-3332.
[34] Bonikowski Z., Bryniariski E., Skardowska V. W., Extension and intensions in the rough set theory, Inform. Sci. 107, (1998), 149-167.
[35] Faizi S., et al, Group decision-making for hesitant fuzzy sets based on characteristic objects method, Symm. 9 (8) (2017), 1-17.
[36] Feng, et al, Attribute analysis of information systems based on elementary soft implications, Know-Based Sys.70 (2014), 281-292.
[37] Li T. J., Leung Y., Zhang W.X. , Generalized fuzzy rough approximation operators based on fuzzy coverings, Int. J. Approx. Reason. 48 (2008), 836-856.
[38] Skowron A., Stepaniuk J. , Tolerance approximation spaces, Fundam. Inform. 27 (1996), 245-253.
[39] Xu W. X., Zhang W. X., Measuring roughness of generalized rough sets induced by a covering, Fuzzy Sets Sys. 158 (2007), 2443-2455.
[40] Ma X., et al, A survey of decision making methods based on two classes of hybrid soft set models, Artif. Intel Rev (2016), 1-19.
[41] Ma X., Liu Q., Zhan J., A survey of decision making methods based on certain hybrid soft set models, Artificial Intelligence Review 47, (2017), 507-530.
[42] Ma X., Zhan J., Ali M. I., and Mehmood N., A survey of decision making methods based on two classes of hybrid soft set models, Artificial Intelligence Review 49 (4) (2018), 511-529.
[43] Yao Y. Y., Lin T. Y., Generalization of rough sets using modal logic, Intell. Autom. Soft Comput. Int. J. 2 (1996), 103-120.
[44] Yao Y. Y., Constructive and algebraic methods of the theory of rough sets, Inform. Sci. 109 (1998), 21-47.
[45] Zhu W., Generalized rough sets based on relations, Inform. Sci. 177 (22) (2007), 4997-5011.
[46] Zhan J., Ali M. I., and Mehmood N., On a novel uncertain soft set model: Z-soft fuzzy rough set model and corresponding decision making methods, Appl. Soft Comput. 56 (2017), 446-457.
[47] Zhang H., and Zhan J., Rough soft lattice implication algebras and corresponding decision making methods, Int. J. Mach Lear. Cyber. 8 (2017), 1301-1308.
[48] Akram M., Nawaz S., Operations on Soft Graphs, Fuzzy Inf. Eng. 7 (2015) , 423-449.
[49] Akram M., Nawaz S., Fuzzy soft graphs with applications. J. Int Fuzzy Syst. 30.6 ((2016)) 3619-3632.
[50] Ali M. I., Shabir M., and Feng F., Representation of graphs based on neighborhoods and soft sets, International Journal of Machine Learning and Cybernetics 8 (5) (2017), 1525-1535.
[51] Firouzian S., and Jouybari M. N., Coloring fuzzy graphs and traffic light problem, TJMCS, 2 (3) (2011), 431-435.
[52] Mohinta S., and Samanta T. K., An introduction to fuzzy soft graph, Math. Mor 19-2 (2015), 35-48.
[53] Shah N., and Hussain A., Neutrosophic soft graphs, Neut. Sets and Sys. 11 (2016), 31-44.
[54] Shah N., Rehman N., Shabir M., and Ali M. I., Another approach to roughness of soft graphs with applications in decision making, Symmetry 10, 145 (2018).