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New Operations of Totally Dependent-Neutrosophic Sets and Totally Dependent-Neutrosophic Soft Sets

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Abstract: The purpose of the paper is to study new algebraic operations and fundamental properties of totally dependent-neutrosophic sets and totally dependent-neutrosophic soft sets. First, the in-coordination relationships among the original inclusion relations of totally dependent-neutrosophic sets (called type-1 and typ-2 inclusion relations in this paper) and union (intersection) operations are analyzed, and then type-3 inclusion relation of totally dependent-neutrosophic sets and corresponding type-3 union, type-3 intersection, and complement operations are introduced. Second, the following theorem is proved: all totally dependent-neutrosophic sets (based on a certain universe) determined a generalized De Morgan algebra with respect to type-3 union, type-3 intersection, and complement operations. Third, the relationships among the type-3 order relation, score function, and accuracy function of totally dependent-neutrosophic sets are discussed. Finally, some new operations and properties of totally dependent-neutrosophic soft sets are investigated, and another generalized De Morgan algebra induced by totally dependent-neutrosophic soft sets is obtained.

Keywords: neutrosophic set; soft set; totally dependent-neutrosophic set; totally dependent-neutrosophic soft set; generalized De Morgan algebra

1. Introduction

In the real world, uncertainty exists universally, so uncertainty becomes the research object of many branches of science. In order to express and deal with uncertainty, many mathematical tools and methods have been put forward, for example, probability theory, fuzzy set theory [1], intuitionistic fuzzy set [2], and soft set theory [3], and these theories have been widely used in many fields [4–17].

As a general framework, F. Smarandache proposed the concept of a neutrosophic set to deal with incomplete, indeterminate, and inconsistent decision information [18]. A neutrosophic set includes truth membership, falsity membership, and indeterminacy membership. In this paper, we only discuss single-valued neutrosophic sets [19]. Recently, the neutrosophic set theory has been applied to many scientific fields (see [20–25]).

In 2006, F. Smarandache introduced, for the first time, the degree of dependence (and consequently the degree of independence) between the components of the fuzzy set, and also between the components of the neutrosophic set [26]. In 2016, the refined neutrosophic set was generalized to the degree of dependence or independence of subcomponets [26]. In this paper, we will discuss a special kind of neutrosophic set, that is, a totally dependent-neutrosophic set. A neutrosophic set A

on the universe X is called totally dependent if T_A, I_A, F_A are 100% dependent, that is $T_A(x) + I_A(x) + F_A(x) \leq 1$ for any x in X.

It should be noted that a "totally dependent-neutrosophic set" is also known as a picture fuzzy set (see [27–29]) or standard neutrosophic set (see [30]). But, F. Smarandache, for the first time, used the name "totally dependent", so this name will be used from the beginning of this article.

This paper tried to prove the new ordering relation on D^* that is given in paper [29] (it is named as type-3 ordering relation in this paper) as a partial ordering relation and consider some new operations on totally dependent-neutrosophic sets and totally dependent-neutrosophic soft sets. In Section 2, we first review some basic notions of intuitionistic fuzzy sets, fuzzy soft sets, totally dependent-neutrosophic sets, and so on. Moreover, we analyze the in-coordination relationships among the original inclusion relations of totally dependent-neutrosophic sets (picture fuzzy sets), called type-1 inclusion relation, and type-2 inclusion relation in this paper; and union (intersection) operations. In Section 3, we prove that the type-3 ordering relation is a partial ordering relation and D^* makes up a lattice about type-3 intersection and type-3 union relations. In Section 4, new algebraic operations (called type-3 union and type-3 intersection) of totally dependent-neutrosophic sets are given with their operations rules. Additionally, we point out that all totally dependent-neutrosophic sets on a certain universe make up generalized De Morgan algebra about the type-3 intersection operation, type-3 union operation, and complement operation. In Section 5, we study some new operations and properties of totally dependent-neutrosophic soft sets (that is, picture fuzzy soft sets) and show that, for the appointed parameter set, totally dependent-neutrosophic soft sets over a certain universe make up generalized De Morgan algebra about type-3 intersection, type-3 union, and complement operations.

2. Preliminaries and Motivation

2.1. Some Basic Concepts

We will now review several basic concepts of intuitionistic fuzzy sets, fuzzy soft sets, standard neutrosophic sets (picture fuzzy sets), and so on.

Definition 1 [2]. *Let X be a nonempty set (universe). An intuitionistic fuzzy set A on X is an object of the form:*

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\},$$

where $\mu_A(x), \nu_A(x) \in [0, 1]$, $\mu_A(x) + \nu_A(x) \leq 1$ for all x in X. $\mu_A(x) \in [0, 1]$ is named the "degree of membership of x in A", and $\nu_A(x)$ is named the "degree of non-membership of x in A".

Definition 2 [6]. *Assume that $F(U)$ is the set of all fuzzy sets on U, and E is a set of parameters, $A \subseteq E$. If F is a mapping given by $F{:}A \to F(U)$, then the pair $\langle F, A\rangle$ is known as a fuzzy soft set over U.*

Definition 3 [26,27]. *Let X be a nonempty set (universe). A totally dependent-neutrosophic set (or picture fuzzy set) A on X is an object of the form:*

$$A = \{(x, \mu_A(x), \eta_A(x), \nu_A(X)) \mid x \in X\},$$

where $\mu_A(x), \eta_A(x), \nu_A(x) \in [0, 1]$, $\mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1$, for all x in X. $\mu_A(x)$ is named as the "degree of positive membership of x in A", $\eta_A(x)$ is named as the "degree of neutral membership of x in A", and $\nu_A(x)$ is named the "degree of negative membership of x in A".

Let $TDNS(X)$ denote the set of all totally dependent-neutrosophic sets (or picture fuzzy sets) on X.

Definition 4 [26,28]. *Assume that U is an initial universe set and E is a set of parameters, $A \subseteq E$. If F is a mapping given by F:A $\to$ TDNS(U), then the pair (F, A) is called a totally dependent-neutrosophic soft set (or picture fuzzy soft set) over U.*

Obviously, a totally dependent-neutrosophic soft set (*TDNSSs*) is a mapping from parameters to *TDNS*(*U*). It is a parameterized family of totally dependent-neutrosophic sets of *U*. Clearly, $\forall e \in A$, $F(e)$ can be written as a totally dependent-neutrosophic set such that:

$$F(e) = \left\{ \left(x, \mu_{F(e)}(x), \eta_{F(e)}(x), \nu_{F(e)}(x) \right) | x \in U \right\},$$

where $\mu_{F(e)}(x)$, $\eta_{F(e)}(x)$, and $\nu_{F(e)}(x)$ are the positive membership, neutral membership, and negative membership functions, respectively.

Remark 1. *For inclusion relation and basic algebraic operations of totally dependent-neutrosophic sets (or picture fuzzy sets), we can define them as special simple-valued neutrosophic sets, that is (see [18,19], we will call them type-1 operations): for every two totally dependent-neutrosophic sets (TDNSs) A and B,*

(1) $A \subseteq_1 B$ *if* $\forall x \in X$, $\mu_A(x) \le \mu_B(x)$, $\eta_A(x) \ge \eta_B(x)$, $\nu_A(x) \ge \nu_B(x)$;
(2) $A = B$ *if* $A \subseteq_1 B$ *and* $B \subseteq_1 A$;
(3) $A \cup_1 B = \{(x, \max(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x)), \min(\nu_A(x), \nu_B(x))) \mid x \in X\}$;
(4) $A \cap_1 B = \{(x, \min(\mu_A(x), \mu_B(x)), \max(\eta_A(x), \eta_B(x)), \max(\nu_A(x), \nu_B(x))) \mid x \in X\}$;
(5) $co(A) = A^c = \{(x, \nu_A(x), \eta_A(x), \mu_A(x)) \mid x \in X\}$.

In [27], inclusion relation and basic algebraic operations of totally dependent-neutrosophic sets (or picture fuzzy sets) are defined using another approach, and we will call them type-2 operations.

Definition 5 [27]. For every two totally dependent-neutrosophic sets (TDNSs) A and B, type-2 inclusion relation, union, intersection operations, and the complement operation are defined as follows:

(1) $A \subseteq_2 B$ *if* $\forall x \in X$, $\mu_A(x) \le \mu_B(x)$, $\eta_A(x) \le \eta_B(x)$, $\nu_A(x) \ge \nu_B(x)$;
(2) $A = B$ *if* $A \subseteq_2 B$ *and* $B \subseteq_2 A$;
(3) $A \cup_2 B = \{(x, \max(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x)), \min(\nu_A(x), \nu_B(x))) \mid x \in X\}$;
(4) $A \cap_2 B = \{(x, \min(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x)), \max(\nu_A(x), \nu_B(x))) \mid x \in X\}$;
(5) $co(A) = A^c = \{(x, \nu_A(x), \eta_A(x), \mu_A(x)) \mid x \in X\}$.

Remark 2. *It should be noted that the type-2 operations here (for totally dependent-neutrosophic sets) are not the same as in the literature [25] (for neutrosophic sets).*

Proposition 1 [27]. *For every TDNS's A, B, and C, the following assertions are true:*

(1) *If* $A \subseteq_2 B$ *and* $B \subseteq_2 C$, *then* $A \subseteq_2 C$;
(2) $(A^c)^c = A$;
(3) $A \cap_2 B = B \cap_2 A$, $A \cup_2 B = B \cup_2 A$;
(4) $(A \cap_2 B) \cap_2 C = A \cap_2 (B \cap_2 C)$, $(A \cup_2 B) \cup_2 C = A \cup_2 (B \cup_2 C)$;
(5) $(A \cap_2 B) \cup_2 C = (A \cup_2 C) \cap_2 (B \cup_2 C)$, $(A \cup_2 B) \cap_2 C = (A \cap_2 C) \cup_2 (B \cap_2 C)$;
(6) $(A \cap_2 B)^c = A^c \cup_2 B^c$, $(A \cup_2 B)^c = A^c \cap_2 B^c$.

Definition 6 [31]. *Assume that* $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha, \rho_\alpha)$ *is a totally dependent-neutrosophic number (picture fuzzy number), where* $\mu_\alpha + \eta_\alpha + \nu_\alpha \le 1$ *and* $\rho_\alpha = 1 - \mu_\alpha - \eta_\alpha - \nu_\alpha$. *The mapping* $S(\alpha) = \mu_\alpha - \nu_\alpha$ *is called the score function, and the mapping* $H(\alpha) = \mu_\alpha + \eta_\alpha + \nu_\alpha$ *is called the accuracy function, where* $S(\alpha) \in [-1, 1]$, $H(\alpha) \in [0, 1]$. *Moreover, for any two totally dependent-neutrosophic numbers (picture fuzzy number)* α *and* β,

(1) *when* $S(\alpha) > S(\beta)$, *we say that* α *is superior to* β, *and it is expressed by* $\alpha \succ \beta$;
(2) *when* $S(\alpha) = S(\beta)$, *then*

(i) *when* $H(\alpha) = H(\beta)$, *we say that* α *is equivalent to* β, *and it is expressed by* $\alpha \sim \beta$;
(ii) *when* $H(\alpha) > H(\beta)$, *we say that* α *is superior to* β, *and it is expressed by* $\alpha \succ \beta$.

Definition 7 [32]. *Let* $(M, \vee, \wedge, ^{-}, 0, 1)$ *be a universal algebra. Then* $(M, \vee, \wedge, ^{-}, 0, 1)$ *is called a generalized De Morgan algebra (or GM-algebra), if* $(M, \vee, \wedge, 0, 1)$ *is a bounded lattice and the unary operation satisfies the identities:*

(1) $(x^{-})^{-} = x$;
(2) $(x \wedge y)^{-} = x^{-} \vee y^{-}$;
(3) $1^{-} = 0$.

2.2. On Inclusion Relations of Totally Dependent-Neutrosophic Sets (Picture Fuzzy Sets)

In Ref. [29], the set D^* is defined as follows:

$$D^* = \left\{ x = (x_1, x_2, x_3) \middle| x \in [0,1]^3, x_1 + x_2 + x_3 \leq 1 \right\}.$$

When $x \in D^*$, it is denoted by $x = (x_1, x_2, x_3)$, that is, the first component of x is expressed by x_1, the second component of x is expressed by x_2, and the third component of x is expressed by x_3. Moreover, the units of D^* are expressed by $1_{D^*} = (1, 0, 0)$ and $0_{D^*} = (0, 0, 1)$, respectively.

It can easily be seen that a totally dependent-neutrosophic set

$$A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) | x \in X\},$$

can be regarded as a D^*-fuzzy set, that is, a mapping of:

$$A : X \to D^* : x \to (\mu_A(x), \eta_A(x), \nu_A(x)).$$

By Definition 5(1), the original inclusion relation of totally dependent-neutrosophic sets is built on the following order relation on D^* (it is named a type-2 inclusion relation in this paper):

$$\forall x, y \in D^*,\ x \leq_2 y \Leftrightarrow (x_1 \leq y_1) \wedge (x_2 \leq y_2) \wedge (x_3 \geq y_3).$$

The above "$\wedge$" denotes "and". Then,

$A \subseteq_2 B$ if and only if $(\forall x \in X)$ $(\mu_A(x), \eta_A(x), \nu_A(x)) \leq_2 (\mu_B(x), \eta_B(x), \nu_B(x))$.

Accordingly, type-2 union, intersection, and complement operations in Definition 5 are denoted as the following:

$$\begin{aligned} A \cup_2 B &= \{(\max(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x)), \min(\nu_A(x), \nu_B(x))) \mid x \in X\} \\ &= \{(\mu_A(x), \eta_A(x), \nu_A(x)) \vee_2 (\mu_B(x), \eta_B(x), \nu_B(x)) \mid x \in X\}; \end{aligned}$$

$$\begin{aligned} A \cap_2 B &= \{(\min(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x)), \max(\nu_A(x), \nu_B(x))) \mid x \in X\} \\ &= \{(\mu_A(x), \eta_A(x), \nu_A(x)) \wedge_2 (\mu_B(x), \eta_B(x), \nu_B(x)) \mid x \in X\}; \end{aligned}$$

$$A^{c_2} = \{ (\nu_A(x), \eta_A(x), \mu_A(x)) | x \in X\} = \left\{ (\mu_A(x), \eta_A(x), \nu_A(x))^{c_2} \middle| x \in X \right\}.$$

Now, we discuss the in-coordination relationships among type-2 inclusion relations of totally dependent-neutrosophic sets and type-2 union (intersection) operations. Consider the following examples.

Example 1. *Let x = (0.3, 0.4, 0.1), y = (0.4, 0.3, 0.2) ∈ D*. Then,*

$$x \vee_2 y = (0.4, 0.3, 0.1),\ x \wedge_2 y = (0.3, 0.3, 0.2).$$

Therefore, $x \nleq_2 x \vee_2 y$. *This means that* $x \vee_1 y$ *is not an upper bound of x and y. Moreover,*

$$x \vee_2 (x \wedge_2 y) = (0.3, 0.3, 0.1) \neq x.$$

It follows that the absorption law is not true for $\vee_2$ *and* $\wedge_2$.

Example 2. *Let x = (0.3, 0.4, 0.2), y = (0.4, 0.35, 0.1) ∈ D*. Then,*

$$x \leq_2 y,\ x \vee_2 y = (0.4, 0.35, 0.1) \neq y.$$

This means that $x \leq_2 y \nRightarrow x \vee_2 y = y$.

The above examples show that the type-2 inclusion relation of totally dependent-neutrosophic sets is inconsistent with the union and intersection operations. Now, we introduce a new inclusion of totally dependent-neutrosophic sets.

Definition 8. *Assume that A and B are two totally dependent-neutrosophic sets on X. Then, the relation* $\subseteq_3$ *defined as the following is called a type-3 inclusion relation:* $A \subseteq_3 B$ *when and only when*

$$\text{for all } x \text{ in } X,\ (\mu_A(x) < \mu_B(x),\ \nu_A(x) \geq \nu_B(x)), \text{ or } (\mu_A(x) = \mu_B(x),\ \nu_A(x) > \nu_B(x)),$$
$$\text{or } (\mu_A(x) = \mu_B(x),\ \nu_A(x) = \nu_B(x) \text{ and } \eta_A(x) \leq \eta_B(x)).$$

It should be noted that the relation $\subseteq_3$ is built on the following order relation on D^* (see [29], it is named a type-3 order relation):

$$x \leq_3 y \Leftrightarrow ((x_1 < y_1) \wedge (x_3 \geq y_3)) \vee ((x_1 = y_1) \wedge (x_3 > y_3)) \vee ((x_1 = y_1) \wedge (x_3 = y_3) \wedge (x_2 \leq y_2)).$$

Here, "∧" represents the logic and operation, and "∨" represents the logic or operation.

Remark 3. *To avoid confusion, type-3 order relation on D* is represented by the symbol "*$\leq_3$*". The strict proof process of the basic properties of the order relation "*$\leq_3$*" is not given in the literature [29], and these proofs are presented in this article (see next section). In addition, if* $x \leq_3 y$ *and* $y \leq_3 x$ *are not true for x, y ∈ D*, then x is not comparable to y, which is expressed by* $x||_{\leq_3}y$.

3. On Type-3 Ordering Relation

In this section, we first prove that (D^*, $\leq_3$) is a partial ordered set. Then, we prove that D^* makes up a lattice through type-3 intersection and type-3 union operations.

Proposition 2. *Let* $D^* = \left\{ x = (x_1, x_2, x_3) \middle| x \in [0,1]^3, x_1 + x_2 + x_3 \leq 1 \right\}$. *Then (D*,* $\leq_3$*) is a partial ordered set.*

Proof. Suppose $x, y, z \in D^*$.

(1) By the definition of $\leq_3$, we have $x \leq_3 x$.
(2) Assume that $x \leq_3 y$ and $y \leq_3 x$, then

Case 1: $x_1 < y_1$ and $x_3 \geq y_3$. According to the definition of $y \leq_3 x$, we can get $x_1 \geq y_1$, which is contradictory.

Case 2: $x_1 = y_1$ and $x_3 > y_3$. According to the definition of $y \leq_3 x$, we can get $x_3 \leq y_3$, which is contradictory.

Case 3: $x_1 = y_1$ and $x_3 = y_3$. From $x \leq_3 y$, we have $x_2 \leq y_2$; also from $y \leq_3 x$, we have $x_2 \geq y_2$. Thus, $x_2 = y_2$.

It follows that, ($x \leq_3 y$ and $y \leq_3 x$) $\Rightarrow$ ($x_1 = y_1$, $x_2 = y_2$ and $x_3 = y_3$) $\Rightarrow x = y$.

(3) Assume that $x \leq_3 y$, $y \leq_3 z$, then

Case 1: ($x_1 < y_1$, $x_3 \geq y_3$) and ($y_1 < z_1$, $y_3 \geq z_3$). It follows that $x_1 < z_1$ and $x_3 \geq z_3$, thus $x \leq_3 z$.

Case 2: ($x_1 = y_1$, $x_3 > y_3$) and ($y_1 = z_1$, $y_3 > z_3$). It follows that $x_1 = z_1$ and $x_3 > z_3$, thus $x \leq_3 z$.

Case 3: ($x_1 = y_1$, $x_3 = y_3$, $x_2 \leq y_2$) and ($y_1 = z_1$, $y_3 = z_3$, $y_2 \leq z_2$). It follows that $x_1 = z_1$, $x_3 = z_3$ and $x_2 \leq z_2$, thus $x \leq_3 z$.

Case 4: ($x_1 < y_1$, $x_3 \geq y_3$) and ($y_1 = z_1$, $y_3 \geq z_3$). It follows that $x_1 < z_1$ and $x_3 \geq z_3$, thus $x \leq_3 z$.

Case 5: ($x_1 < y_1$, $x_3 \geq y_3$) and ($y_1 = z_1$, $y_3 = z_3$, $y_2 \leq z_2$). It follows that $x_1 < z_1$, $x_3 \geq z_3$ and $y_2 \leq z_2$, thus $x \leq_3 z$.

Case 6: ($x_1 = y_1$, $x_3 \geq y_3$) and ($y_1 < z_1$, $y_3 \geq z_3$). It follows that $x_1 < z_1$ and $x_3 \geq z_3$, thus $x \leq_3 z$.

Case 7: ($x_1 = y_1$, $x_3 \geq y_3$) and ($y_1 = z_1$, $y_3 = z_3$, $y_2 \leq z_2$). It follows that $x_1 = z_1$, $x_3 \geq z_3$ and $y_2 \leq z_2$, thus $x \leq_3 z$.

Case 8: ($x_1 = y_1$, $x_3 = y_3$, $x_2 \leq y_2$) and ($y_1 < z_1$, $y_3 \geq z_3$). It follows that $x_1 < z_1$, $x_3 \geq z_3$ and $x_2 \leq y_2$, thus $x \leq_3 z$.

Case 9: ($x_1 = y_1$, $x_3 = y_3$, $x_2 \leq y_2$) and ($y_1 = z_1$, $y_3 > z_3$), then $x_1 = z_1$, $x_3 > z_3$ and $x_2 \leq y_2$, thus $x \leq_3 z$.

It follows that, ($x \leq_3 y$ and $y \leq_3 z$) $\Rightarrow x \leq_3 z$.

Therefore, (D^*, $\leq_3$) is a partial ordered set. □

Remark 4. *It is important to note that the set D^* in this paper is different from the set D^* of the literature [25]. It follows that the corresponding type-3 intersection, type-3 union, and type-3 complement operations in this paper and related operations in [25] are not the same, respectively. So, the relevant results of this paper are not the direct inference of the results in [25] (although the research ideas are similar), and the readers must pay attention to it.*

Proposition 3. *Two operations are defined on D^* as follows: $\forall x, y \in D^*$,*

$$x \wedge_3 y = \begin{cases} x, & \text{when} x \leq_3 y \\ y, & \text{when} y \leq_3 x \\ (\min(x_1, y_1), 1 - \min(x_1, y_1) - \max(x_3, y_3), \max(x_3, y_3)), & \text{otherwise} \end{cases}$$

$$x \vee_3 y = \begin{cases} y, & \text{when} x \leq_3 y \\ x, & \text{when} y \leq_3 x \\ (\max(x_1, y_1), 0, \min(x_3, y_3)), & \text{otherwise} \end{cases}$$

Then, $x \wedge_3 y$ = inf(x, y), $x \vee_3 y$ = sup(x, y), and (D^*, $\leq_3$) is a lattice.

Proof. Suppose that $x \leq_3 y$ or $y \leq_3 x$, then, by the definition of "$\wedge_3$", $x \wedge_3 y$ is the largest lower bound of x, y, i.e., $x \wedge_3 y$ = inf(x, y). Moreover, suppose that $x \leq_3 y$ or $y \leq_3 x$, then $x \vee_3 y$ is the smallest upper bound of x, y, i.e., $x \vee_3 y$ = sup(x, y).

Next, assume that $x||_{\leq_3}y$. Then, from the definitions of "$\wedge_3$" and "$\vee_3$", we have:

$$x \wedge_3 y = (\min(x_1, y_1), 1 - \min(x_1, y_1) - \max(x_3, y_3), \max(x_3, y_3)),$$

$$x \vee_3 y = (\max(x_1, y_1), 0, \min(x_3, y_3)).$$

(1) To prove $x \wedge_3 y = \inf(x, y)$: Let

$$z = (z_1, z_2, z_3) = (\min(x_1, y_1), 1 - \min(x_1, y_1) - \max(x_3, y_3), \max(x_3, y_3)).$$

Then, $x_1 \geq \min(x_1, y_1) = z_1$, $x_3 \leq \max(x_3, y_3) = z_3$.
If $x_1 > z_1$ and $x_3 \leq z_3$, then $z \leq_3 x$.
If $x_1 = z_1$ and $x_3 < z_3$, then $z \leq_3 x$.
If $x_1 = z_1$ and $x_3 = z_3$, then $y_1 \geq x_1$, $y_3 \leq x_3$, and $x \leq_3 y$ or $y \leq_3 x$. This is contradictory to the assumed condition $x||_{\leq_3}y$.

Therefore, $z \leq_3 x$. In the same way, we can obtain $z \leq_3 y$. That is, z is a lower bound of x and y.
The next goal is to prove that z is the largest lower bound of x and y.
Suppose $a = (a_1, a_2, a_3) \in D^*$ such that $a \leq_3 x$ and $a \leq_3 y$.

Case 1: $(a_1 < x_1, a_3 \geq x_3)$ and $(a_1 < y_1, a_3 \geq y_3)$. It follows that $a_1 < \min(x_1, y_1) = z_1$ and $a_3 \geq \max(x_3, y_3) = z_3$, thus $a \leq_3 z$.
Case 2: $(a_1 = x_1, a_3 > x_3)$ and $(a_1 = y_1, a_3 > y_3)$. It follows that $a_1 = \min(x_1, y_1) = z_1$ and $a_3 > \max(x_3, y_3) = z_3$, thus $a \leq_3 z$.
Case 3: $(a_1 = x_1, a_3 = x_3, a_2 \leq x_2)$ and $(a_1 = y_1, a_3 = y_3, a_2 \leq y_2)$. It follows that $a_1 = \min(x_1, y_1) = z_1$, $a_3 = \max(x_3, y_3) = z_3$ and $a_2 \leq \min(x_2, y_2)$. Since $a_1 + a_2 + a_3 \leq 1$, so $a_2 \leq 1 - \min(x_1, y_1) - \max(x_3, y_3) = z_2$, thus $a \leq_3 z$.
Case 4: $(a_1 = x_1, a_3 > x_3)$ and $(a_1 < y_1, a_3 \geq y_3)$. It follows that $a_1 \leq \min(x_1, y_1)$ and $a_3 \geq \max(x_3, y_3)$. If $(a_1 < \min(x_1, y_1), a_3 \geq \max(x_3, y_3))$ or $(a_1 = \min(x_1, y_1), a_3 > \max(x_3, y_3))$, then $a \leq_3 z$; If $a_1 = \min(x_1, y_1)$ and $a_3 = \max(x_3, y_3)$, from this and the hidden condition $a_1 + a_2 + a_3 \leq 1$, we get $a_2 \leq 1 - \min(x_1, y_1) - \max(x_3, y_3) = z_2$, hence $a \leq_3 z$.
Case 5: $(a_1 < x_1, a_3 \geq x_3)$ and $(a_1 = y_1, a_3 > y_3)$. It follows that $a_1 \leq \min(x_1, y_1) = z_1$ and $a_3 \geq \max(x_3, y_3) = z_3$. Similar to Case 4, we can get $a \leq_3 z$.
Case 6: $(a_1 < x_1, a_3 \geq x_3)$ and $(a_1 = y_1, a_3 = y_3, a_2 \leq y_2)$. It follows that $y_1 < x_1$ and $y_3 \geq x_3$, so $y \leq_3 x$, it is a contradiction with hypothesis $x||_{\leq_3}y$.
Case 7: $(a_1 = x_1, a_3 > x_3)$ and $(a_1 = y_1, a_3 = y_3, a_2 \leq y_2)$. It follows that $y_1 = x_1$ and $y_3 > x_3$, so $y \leq_3 x$, which is a contradiction with hypothesis $x||_{\leq_3}y$.
Case 8: $(a_1 = x_1, a_3 = x_3, a_2 \leq x_2)$ and $(a_1 < y_1, a_3 \geq y_3)$. It follows that $x_1 < y_1$ and $x_3 \geq y_3$, so $x \leq_3 y$, which is a contradiction with hypothesis $x||_{\leq_3}y$.
Case 9: $(a_1 = x_1, a_3 = x_3, a_2 \leq x_2)$ and $(a_1 = y_1, a_3 > y_3)$. It follows that $x_1 = y_1$ and $x_3 > y_3$, so $x \leq_3 y$, which is a contradiction with hypothesis $x||_{\leq_3}y$.

Hence, $a \leq_3 z$. That is, $z = (\min(x_1, y_1), 1 - \min(x_1, y_1) - \max(x_3, y_3), \max(x_3, y_3))$ is the largest lower bound of x, y.

(2) To prove $x \vee_3 y = \sup(x, y)$: Let

$$w = (w_1, w_2, w_3) = (\max(x_1, y_1), 0, \min(x_3, y_3)).$$

Then $x_1 \leq \max(x_1, y_1) = w_1$, $x_3 \geq \min(x_3, y_3) = w_3$.
If $x_1 < w_1$ and $x_3 \geq w_3$, then $x \leq_3 w$.
If $x_1 = w_1$ and $x_3 > w_3$, then $x \leq_3 w$.
If $x_1 = w_1$ and $x_3 = w_3$, then $y_1 \leq x_1$, $y_3 \geq x_3$, so $y \leq_3 x$ or $x \leq_3 y$, which is contradictory to the assumed condition $x||_{\leq_3}y$. Thus, $x \leq_3 w$.

In the same way, we can obtain $y \leq_3 w$. Hence, w is an upper bound of x and y.
The next goal is to prove that w is the smallest upper bound of x and y.
Assume $a = (a_1, a_2, a_3) \in D^*$ such that $x \leq_3 a$, $y \leq_3 a$.

Case 1: $(x_1 < a_1, x_3 \geq a_3)$ and $(y_1 < a_1, y_3 \geq a_3)$. It follows that $a_1 > \max(x_1, y_1) = w_1$ and $a_3 \leq \min(x_3, y_3) = w_3$. Thus, $w \leq_3 a$.

Case 2: ($a_1 = x_1$, $x_3 > a_3$) and ($a_1 = y_1$, $y_3 > a_3$). It follows that $a_1 = \max(x_1, y_1) = w_1$, $a_3 < \min(x_3, y_3) = w_3$, thus $w \leq_3 a$.

Case 3: ($a_1 = x_1$, $a_3 = x_3$, $x_2 \leq a_2$) and ($a_1 = y_1$, $a_3 = y_3$, $y_2 \leq a_2$). It follows that $a_1 = \max(x_1, y_1) = w_1$, $a_3 = \min(x_3, y_3) = w_3$ and $a_2 \geq \max(x_2, y_2) \geq 0$, thus $w \leq_3 a$.

Case 4: ($a_1 = x_1$, $x_3 > a_3$) and ($y_1 < a_1$, $y_3 \geq a_3$). It follows that $a_1 \geq \max(x_1, y_1) = w_1$ and $a_3 \leq \min(x_3, y_3) = w_3$. If ($a_1 > \max(x_1, y_1) = w_1$, $a_3 \leq \min(x_3, y_3) = w_3$) or (($a_1 = \max(x_1, y_1) = w_1$, $a_3 < \min(x_3, y_3) = w_3$), then $w \leq {}_3a$; If $a_1 = \max(x_1, y_1) = w_1$ and $a_3 = \min(x_3, y_3) = w_3$, according the hidden condition $a_2 \geq 0$, we can get $w \leq_3 a$.

Case 5: ($x_1 < a_1$, $x_3 \geq a_3$) and ($a_1 = y_1$, $y_3 > a_3$). It follows that $a_1 \geq \max(x_1, y_1) = w_1$ and $a_3 \leq \min(x_3, y_3) = w_3$, similar to Case 4, so we can get $w \leq_3 a$.

Case 6: ($x_1 < a_1$, $x_3 \geq a_3$) and ($a_1 = y_1$, $a_3 = y_3$, $y_2 \leq a_2$). It follows that $x_1 < y_1$ and $x_3 \geq y_3$, so $x \leq_3 y$, which is a contradiction with hypothesis $x||_{\leq_3}y$.

Case 7: ($a_1 = x_1$, $x_3 > a_3$) and ($a_1 = y_1$, $a_3 = y_3$, $y_2 \leq a_2$). It follows that $y_1 = x_1$ and $y_3 < x_3$, so $x \leq_3 y$, which is a contradiction with hypothesis $x||_{\leq_3}y$.

Case 8: ($a_1 = x_1$, $a_3 = x_3$, $x_2 \leq a_2$) and ($y_1 < a_1$, $y_3 \geq a_3$). It follows that $y_1 < x_1$ and $y_3 \geq x_3$, so $y \leq_3 x$, which is a contradiction with hypothesis $x||_{\leq_3}y$.

Case 9: ($a_1 = x_1$, $a_3 = x_3$, $x_2 \leq a_2$) and ($a_1 = y_1$, $y_3 > a_3$). It follows that $y_1 = x_1$ and $y_3 > x_3$, so $y \leq_3 x$, which is a contradiction with hypothesis $x||_{\leq_3}y$.

Hence, $w \leq_3 a$. That is, $w = (\max(x_1, y_1), 0, \min(x_3, y_3))$ is the smallest upper bound of x, y.
Integrating (1) and (2), $x \wedge_3 y = \inf(x, y)$, $x \vee_3 y = \sup(x, y)$, and (D^*, $\leq_3$) is a lattice. □

4. New Operations and Properties of Totally Dependent-Neutrosophic Sets (Picture Fuzzy Sets)

In this section, we investigate the properties of the type-3 inclusion relation of totally dependent- neutrosophic sets, and give some new operations named type-3 union, type-3 intersection, and type-3 complement of totally dependent-neutrosophic sets and study their basic properties. Moreover, we discuss the relationship between type-3 ordering relation $\leq_3$ and the rank of totally dependent-neutrosophic sets determined by score function and accuracy function (see Definition 6).

For any totally dependent-neutrosophic sets A and B on X, applying Definition 8, we see that:

$$A \subseteq_3 B \text{ if and only if } (\mu_A(x), \eta_A(x), \nu_A(x)) \leq_3 (\mu_B(x), \eta_B(x), \nu_B(x)), \forall x \in X.$$

From this, using Proposition 2, we can get the following proposition.

Proposition 4. *If A, B, and C are totally dependent-neutrosophic sets on X, then*

(1) $A \subseteq_3 A$;
(2) $(A \subseteq_3 B, B \subseteq_3 A) \Rightarrow A = B$;
(3) $(A \subseteq_3 B, B \subseteq_3 C) \Rightarrow A \subseteq_3 C$.

Definition 9. *Assume that A and B are totally dependent-neutrosophic sets on X. The operations defined as follows are called a type-3 union, type-3 intersection, and type-3 complement, respectively:*

(1) $$(A \cup_3 B)(x) = \begin{cases} (\mu_A(x), \eta_A(x), \nu_A(x)), & \text{if} (\mu_B(x), \eta_B(x), \nu_B(x)) \leq_3 (\mu_A(x), \eta_A(x), \nu_A(x)) \\ (\mu_B(x), \eta_B(x), \nu_B(x)), & \text{if } (\mu_A(x), \eta_A(x), \nu_A(x)) \leq_3 (\mu_B(x), \eta_B(x), \nu_B(x)) \\ (\max(\mu_A(x), \mu_B(x)), 0, \min(\nu_A(x), \nu_B(x))), & \text{otherwise} \end{cases}$$

(2) $$(A \cap_3 B)(x) = \begin{cases} (\mu_A(x), \eta_A(x), \nu_A(x)), & \text{if } (\mu_A(x), \eta_A(x), \nu_A(x)) \leq_3 (\mu_B(x), \eta_B(x), \nu_B(x)) \\ (\mu_B(x), \eta_B(x), \nu_B(x)), & \text{if} (\mu_B(x), \eta_B(x), \nu_B(x)) \leq_3 (\mu_A(x), \eta_A(x), \nu_A(x)) \\ (\min(\mu_A(x), \mu_B(x)), 1 - \min(\mu_A(x), \mu_B(x)) - \max(\nu_A(x), \nu_B(x)), \max(\nu_A(x), \nu_B(x))), & \text{otherwise} \end{cases}$$

(3) $$A^{c_3} = \{(x, \nu_A(x), 1 - \mu_A(x) - \eta_A(x) - \nu_A(x), \mu_A(x)) \ |x \in X)\}.$$

By Definition 9 and Proposition 3, we have:

Proposition 5. *If A and B are totally dependent-neutrosophic sets on X, then*

(1) $A \cup_3 B = \{(\mu_A(x), \eta_A(x), \nu_A(x)) \vee_3 (\mu_B(x), \eta_B(x), \nu_B(x)) \mid x \in X\}$;
(2) $A \cap_3 B = \{(\mu_A(x), \eta_A(x), \nu_A(x)) \wedge_3 (\mu_B(x), \eta_B(x), \nu_B(x)) \mid x \in X\}$.

Proposition 6. *If A, B, and C are totally dependent-neutrosophic sets on X, then*

(1) $A \cap_3 A = A$, $A \cup_3 A = A$;
(2) $A \cap_3 B = B \cap_3 A$, $A \cup_3 B = B \cup_3 A$;
(3) $(A \cap_3 B) \cap_3 C = A \cap_3 (B \cap_3 C)$, $(A \cup_3 B) \cup_3 C = A \cup_3 (B \cup_3 C)$;
(4) $A \cap_3 (B \cup_3 A) = A$, $A \cup_3 (B \cap_3 A) = A$;
(5) $A \subseteq_3 B \Leftrightarrow A \cup_3 B = B$; $A \subseteq_3 B \Leftrightarrow A \cap_3 B = A$.

By Definition 9(3), we have:

Proposition 7. *For any totally dependent-neutrosophic sets on X,* $(A^{c_3})^{c_3} = A$.

Proposition 8. *If A and B are totally dependent-neutrosophic sets on X, then:*

(1) $(A \cap_3 B)^{c_3} = A^{c_3} \cup_3 B^{c_3}$;
(2) $(A \cup_3 B)^{c_3} = A^{c_3} \cap_3 B^{c_3}$.

Proof. By Definition 9(3), we have:

$$A^{c_3} = \{(x, \nu_A(x), 1 - \mu_A(x) - \eta_A(x) - \nu_A(x), \mu_A(x)) | x \in X\}$$

$$B^{c_3} = \{(x, \nu_B(x), 1 - \mu_B(x) - \eta_B(x) - \nu_B(x), \mu_B(x)) | x \in X\}$$

(1) If $B \subseteq_3 A$, then:

Case 1: $\mu_B(x) < \mu_A(x)$ and $\nu_B(x) \geq \nu_A(x)$. It follows that $A^{c_3} \subseteq_3 B^{c_3}$. Thus $(A \cap_3 B)^{c_3} = A^{c_3} \cup_3 B^{c_3}$.
Case 2: $\mu_B(x) = \mu_A(x)$ and $\nu_B(x) > \nu_A(x)$. It follows that $A^{c_3} \subseteq_3 B^{c_3}$. Thus $(A \cap_3 B)^{c_3} = A^{c_3} \cup_3 B^{c_3}$.
Case 3: $\mu_B(x) < \mu_A(x)$, $\nu_B(x) = \nu_A(x)$ and $\eta_B(x) \leq \eta_A(x)$. Then $1 - \mu_A(x) - \eta_A(x) - \nu_A(x) \leq 1 - \mu_B(x) - \eta_B(x) - \nu_B(x)$. Thus $A^{c_3} \subseteq_3 B^{c_3}$, and $(A \cap_3 B)^{c_3} = B^{c_3} = A^{c_3} \cup_3 B^{c_3}$.

Similarly, if $A \subseteq_3 B$, then $(A \cap_3 B)^{c_3} = A^{c_3} \cup_3 B^{c_3}$.
If neither $B \subseteq_3 A$ nor $A \subseteq_3 B$, then:

$$A \cap_3 B = \{(x, \min(\mu_A(x), \mu_B(x), 1 - \min(\mu_A(x), \mu_B(x)) - \max(\nu_A(x), \nu_B(x)), \max(\nu_A(x), \nu_B(x))) | x \in X\},$$

$$A \cup_3 B = \{(x, \max(\mu_A(x), \mu_B(x)), 0, \min(\nu_A(x), \nu_B(x))) | x \in X\}.$$

Thus

$$(A \cap_3 B)^{c_3} = \{(x, \max(\nu_A(x), \nu_B(x)), 0, \min(\mu_A(x), \mu_B(x))) | x \in X\}.$$

$$A^{c_3} \cup_3 B^{c_3} = \{(x, \max(\nu_A(x), \nu_B(x)), 0, \min(\mu_A(x), \mu_B(x))) | x \in X\}.$$

Hence $(A \cap_3 B)^{c_3} = A^{c_3} \cup_3 B^{c_3}$.

(2) By (1) and Proposition 7, we can get that $(A \cup_3 B)^{c_3} = A^{c_3} \cap_3 B^{c_3}$. □

Theorem 1. *Let TDNS(X) be the set of all totally dependent-neutrosophic sets on X, and*

$$0_{TDNS} = \{(x, 0, 0, 1) \mid x \in X\},\ 1_{TDNS} = \{(x, 1, 0, 0) \mid x \in X\}.$$

Then, $(TDNS(X), \cup_3, \cap_3, {}^{c}{}_3, 0_{TDNS}, 1_{TDNS})$ is a GM-algebra (i.e., generalized De Morgan algebra).

Proof. Applying Proposition 6–8 and Definition 7, we can see that $(TDNS(X), \cup_3, \cap_3, {}^{c}{}_3, 0_{TDNS}, 1_{TDNS})$ is a GM-algebra.

We can verify that the distributive law is not true for $(TDNS(X), \cup_3, \cap_3, {}^{c}{}_3, 0_{TDNS}, 1_{TDNS})$, that is, it is not a De Morgan algebra. □

Example 3. *Let $X = \{a, b\}$ and*

$$A = \{(a, 0.1, 0.4, 0.3), (b, 0, 0, 1)\},\ B = \{(a, 0.3, 0.1, 0.5), (b, 0, 0, 1)\},\ C = \{(a, 0.2, 0.2, 0.4), (b, 1, 0, 0)\}.$$

Then:

$$(A \cap_3 B) \cup_3 C = \{(a, 0.2, 0.2, 0.4), (b, 1, 0, 0)\};\ (A \cup_3 C) \cap_3 (B \cup_3 C) = \{(a, 0.2, 0.4, 0.4), (b, 1, 0, 0)\};$$

$$(A \cup_3 B) \cap_3 C = \{(a, 0.2, 0.2, 0.4), (b, 0, 0, 1)\};\ (A \cap_3 C) \cup_3 (B \cap_3 C) = \{(a, 0.2, 0, 0.4), (b, 0, 0, 1)\}.$$

Therefore,

$$(A \cap_3 B) \cup_3 C \neq (A \cup_3 C) \cap_3 (B \cup_3 C),\ (A \cup_3 B) \cap_3 C \neq (A \cap_3 C) \cup_3 (B \cap_3 C).$$

Proposition 9. *If $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha, \rho_\alpha)$, $\beta = (\mu_\beta, \eta_\beta, \nu_\beta, \rho_\beta)$ are two totally dependent-neutrosophic numbers, and $(\mu_\alpha, \eta_\alpha, \nu_\alpha) \leq_3 (\mu_\beta, \eta_\beta, \nu_\beta)$, then $\alpha \prec \beta$ or $\alpha \sim \beta$.*

Proof. Assume $(\mu_\alpha, \eta_\alpha, \nu_\alpha) \leq_3 (\mu_\beta, \eta_\beta, \nu_\beta)$. By the definition of type-3 order relation $\leq_3$, we have

Case 1: $(\mu_\alpha < \mu_\beta, \nu_\alpha \geq \nu_\beta)$ or $(\mu_\alpha = \mu_\beta, \nu_\alpha > \nu_\beta)$. It follows that $S(\alpha) = \mu_\alpha - \nu_\alpha < \mu_\beta - \nu_\beta = S(\beta)$. Thus, $\alpha \prec \beta$.

Case 2: $(\mu_\alpha = \mu_\beta, \nu_\alpha = \nu_\beta$ and $\eta_\alpha < \eta_\beta$. It follows that $S(\alpha) = S(\beta)$, $H(\alpha) < H(\beta)$. Thus, $\alpha \prec \beta$.

Case 3: $\mu_\alpha = \mu_\beta, \nu_\alpha = \nu_\beta$ and $\eta_\alpha = \eta_\beta$. It follows that $S(\alpha) = S(\beta)$, $H(\alpha) = H(\beta)$. Thus, $\alpha \sim \beta$.

Therefore, the proof is completed. □

Example 4. *Let $\alpha = (0.3, 0.4, 0.1, 0.2)$ and $\beta = (0.5, 0.2, 0.2, 0.1)$ be two totally dependent-neutrosophic numbers, then $S(\alpha) < S(\beta)$, $\alpha \prec \beta$, but $(0.3, 0.4, 0.1) || _{\leq_3} (0.5, 0.2, 0.2)$. That is, the inverse of Proposition 9 is not true.*

5. New Operations and Properties of Totally Dependent-Neutrosophic Soft Sets

In this section, we investigate some new operations on totally dependent-neutrosophic soft sets (picture fuzzy soft sets), including type-3 intersection (type-3 union, type-3 complement).

The notions of intersection, union, and complement of totally dependent-neutrosophic (picture fuzzy) soft sets are introduced in [28]. To avoid confusion, these operations are called a type-2 intersection, type-2 union, and type-2 complement in this paper.

Remark 5. *Note that, for type-1 operations of totally dependent-neutrosophic soft sets (picture fuzzy soft sets), we denote the operations in neutrosophic soft sets (see [33,34]), that is, the corresponding operations of totally dependent-neutrosophic soft sets, as a kind of special neutrosophic soft sets.*

Definition 10 [28]. *The type-2 complement of a totally dependent-neutrosophic soft set (F, A) over U is denoted as* $(F, A)^{c_2}$ *and is defined by* $(F, A)^{c_2} = (F^{c_2}, A)$*, where* $F^{c_2}: A \to SNS(U)$ *is a mapping given by:*

$$F^{c_2}(e) = (F(e))^{c_2} = \left\{ \left(x, \nu_{F(e)}(x), \eta_{F(e)}(x), \mu_{F(e)}(x) \right) \middle| x \in U \right\}, \forall e \in A.$$

Definition 11 [28]. *The type-2 intersection of two totally dependent-neutrosophic soft sets (F, A) and (G, B) over a common universe U is a totally dependent-neutrosophic soft set (H, C), where* $C = A \cup B$ *and for all* $e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cap_2 G(e), & \text{if } e \in A \cap B \end{cases}$$

That is, $\forall e \in A \cap B$, $H(e) = \{(x, \min(\mu_{F(e)}(x), \mu_{G(e)}(x)), \min(\eta_{F(e)}(x), \eta_{G(e)}(x)), \max(\nu_{F(e)}(x), \nu_{G(e)}(x) \mid x \in U\}$*. This relation is denoted by* $(F, A) \cap_2 (G, B) = (H, C)$.

Definition 12 [28]. *The type-2 union of two totally dependent-neutrosophic soft sets (F, A) and (G, B) over a common universe U is a totally dependent-neutrosophic soft set (H, C), where* $C = A \cup B$ *and for all* $e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup_2 G(e), & \text{if } e \in A \cap B \end{cases}$$

That is, $\forall e \in A \cap B$, $H(e) = \{(x, \max(\mu_{F(e)}(x), \mu_{G(e)}(x)), \min(\eta_{F(e)}(x), \eta_{G(e)}(x)), \min(\nu_{F(e)}(x), \nu_{G(e)}(x) \mid x \in U\}$*. This relation is denoted by* $(F, A) \cup_2 (G, B) = (H, C)$.

Now, we discuss the type-3 complement, type-3 intersection, and type-3 union of totally dependent-neutrosophic soft sets. First, we introduce type-3 inclusion relation on totally dependent neutrosophic soft sets.

Definition 13. *Let (F, A) and (G, B) be two totally dependent-neutrosophic soft sets over U. Then, (F, A) is called a totally dependent-neutrosophic soft subset of (G, B), denoted by* $(F, A) \subseteq_3 (G, B)$*, if:*

(1) $A \subseteq B$;
(2) $\forall e \in A$, $F(e) \subseteq_3 G(e)$, *that is,* $\forall x \in U$, $(\mu_{F(e)}(x) < \mu_{G(e)}(x), \nu_{F(e)}(x) \geq \nu_{G(e)}(x))$, *or* $(\mu_{F(e)}(x) = \mu_{G(e)}(x), \nu_{F(e)}(x) > \nu_{G(e)}(x))$, *or* $(\mu_{F(e)}(x) = \mu_{G(e)}(x), \nu_{F(e)}(x) = \nu_{G(e)}(x)$ *and* $\eta_{F(e)}(x) \leq \eta_{G(e)}(x))$.

Example 5. *Let* $U = \{x_1, x_2, x_3, x_4\}$ *and* $E = \{e_1, e_2, e_3, e_4, e_5\}$*. Suppose that (F, A) and (G, B) are two SNSSs over U,* $A = \{e_1, e_2\}$, $B = \{e_1, e_2, e_5\}$ *and*

$$(F, A) = \left(\begin{array}{ccc} & e_1 & e_2 \\ x_1 & (0.1, 0.2, 0.5) & (0.2, 0.1, 0.6) \\ x_2 & (0.4, 0.2, 0.3) & (0.2, 0.2, 0.5) \\ x_3 & (0.2, 0.3, 0.4) & (0.1, 0.4, 0.2) \\ x_4 & (0.3, 0.3, 0.2) & (0.4, 0.0, 0.5) \end{array} \right),$$

$$(G, B) = \left(\begin{array}{cccc} & e_1 & e_2 & e_5 \\ x_1 & (0.3, 0.2, 0.4) & (0.2, 0.2, 0.4) & (0.7, 0.1, 0.2) \\ x_2 & (0.6, 0.0, 0.3) & (0.2, 0.1, 0.3) & (0.5, 0.3, 0.1) \\ x_3 & (0.3, 0.4, 0.2) & (0.1, 0.4, 0.2) & (0.8, 0.0, 0.1) \\ x_4 & (0.5, 0.2, 0.2) & (0.4, 0.1, 0.5) & (0.2, 0.5, 0.1) \end{array} \right)$$

Then, (F, A) is a totally dependent-neutrosophic soft subset of (G, B).

Definition 14. *Let (F, A) and (G, B) be two totally dependent-neutrosophic soft sets over U. (F, A) and (G, B) are said totally dependent-neutrosophic soft equals, denoted (F, A) = (G, B), if (F, A)* $\subseteq_3$ *(G, B) and (G, B)* $\subseteq_3$ *(F, A).*

By Proposition 4, we know that $(F, A) = (G, B) \Leftrightarrow A = B$ and $F(e) = G(e)$, $\forall e \in A$.

Definition 15. *Let (F, A) be a totally dependent-neutrosophic soft set over U. Type-3 complement of (F, A) is denoted as* $(F, A)^{c_3}$ *and is defined by* $(F, A)^{c_3} = (F^{c_3}, A)$*, where* $F^{c_3}\colon A \to TDNS(U)$ *is a mapping given by:*

$$F^{c_3}(e) = (F(e))^{c_3} = \left\{ \left(x, \nu_{F(e)}(x), 1 - \mu_{F(e)}(x) - \eta_{F(e)}(x) - \nu_{F(e)}(x), \mu_{F(e)}(x) \right) \Big| x \in U \right\}, \forall e \in A.$$

Definition 16. *Type-3 union of two totally dependent-neutrosophic soft sets (F, A) and (G, B) over U can be defined as (F, A)* $\cup_3$ *(G, B) = (H, C), where* $C = A \cup B$*, and* $\forall e \in C$*,*

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup_3 G(e), & \text{if } e \in A \cap B \end{cases}$$

Example 6. *Let* $U = \{x_1, x_2, x_3, x_4\}$*,* $E = \{e_1, e_2, e_3, e_4, e_5\}$*,* $A = \{e_1, e_2\}$*,* $B = \{e_1, e_3, e_5\}$*, and*

$$(F, A) = \left(\begin{array}{ccc} & e_1 & e_2 \\ x_1 & (0.1, 0.2, 0.6) & (0.4, 0.2, 0.3) \\ x_2 & (0.2, 0.1, 0.1) & (0.3, 0.1, 0.6) \\ x_3 & (0.7, 0.3, 0.0) & (0.5, 0.2, 0.3) \\ x_4 & (0.4, 0.0, 0.3) & (0.8, 0.0, 0.1) \end{array} \right),$$

$$(G, B) = \left(\begin{array}{cccc} & e_1 & e_3 & e_5 \\ x_1 & (0.4, 0.3, 0.2) & (0.2, 0.7, 0.1) & (0.6, 0.1, 0.2) \\ x_2 & (0.7, 0.1, 0.1) & (0.8, 0.0, 0.2) & (0.3, 0.2, 0.4) \\ x_3 & (0.3, 0.5, 0.1) & (0.2, 0.4, 0.2) & (0.5, 0.3, 0.0) \\ x_4 & (0.6, 0.1, 0.2) & (0.4, 0.3, 0.1) & (0.1, 0.1, 0.6) \end{array} \right).$$

Then, (F, A) $\cup_3$ *(G, B) = (H, C), where* $C = A \cup B = \{e_1, e_2, e_3, e_5\}$ *and*

$$(F, A) \cup_3 (G, B) = (H, C) =$$

$$\left(\begin{array}{ccccc} & e_1 & e_2 & e_3 & e_5 \\ x_1 & (0.4, 0.3, 0.2) & (0.4, 0.2, 0.3) & (0.2, 0.7, 0.1) & (0.6, 0.1, 0.2) \\ x_2 & (0.7, 0.1, 0.1) & (0.7, 0.1, 0.1) & (0.3, 0.1, 0.6) & (0.3, 0.2, 0.4) \\ x_3 & (0.7, 0.3, 0.0) & (0.5, 0.2, 0.3) & (0.2, 0.4, 0.2) & (0.5, 0.3, 0.0) \\ x_4 & (0.6, 0.1, 0.2) & (0.8, 0.0, 0.1) & (0.4, 0.3, 0.1) & (0.1, 0.1, 0.6) \end{array} \right).$$

Definition 17. *Assume that* $A, B \subseteq E$ *and* $A \cap B \neq \emptyset$*. Type-3 intersection of two totally dependent-neutrosophic soft sets (F, A) and (G, B) over U can be defined as (F, A)* $\cap_3$ *(G, B) = (H, C), where* $C = A \cap B$*, and* $\forall e \in C$*,* $H(e) = F(e) \cap_3 F(e)$*.*

Example 7. *Consider the totally dependent-neutrosophic soft sets (F, A), (G, B) in Example 6. We have (F, A) $\cap_3$ (G, B) = (H, C), where C = A $\cap$ B = {e_1} and*

$$(F,A) \cap_3 (G,B) = (H,C) = \begin{pmatrix} & e_1 \\ x_1 & (0.1, 0.2, 0.6) \\ x_2 & (0.2, 0.1, 0.1) \\ x_3 & (0.3, 0.5, 0.1) \\ x_4 & (0.4, 0.0, 0.3) \end{pmatrix}.$$

Proposition 10. *If (F, A), (G, B), and (H, C) are totally dependent-neutrosophic soft sets over U, then:*

(1) $\left((F,A)^{c_3}\right)^{c_3} = (F,A)$;
(2) *(F, A) $\cup_3$ (F, A) = (F, A), (F, A) $\cap_3$ (F, A) = (F, A);*
(3) *(F, A) $\cup_3$ (G, B) = (G, B) $\cup_3$ (F, A), (F, A) $\cap_3$ (G, B) = (G, B) $\cap_3$ (F, A);*
(4) *((F, A) $\cup_3$ (G, B)) $\cup_3$ (H, C) = (F, A) $\cup_3$ ((G, B) $\cup_3$ (H, C));*
(5) *((F, A) $\cap_3$ (G, B)) $\cap_3$ (H, C) = (F, A) $\cap_3$ ((G, B) $\cap_3$ (H, C)), when A $\cap$ B $\cap$ C $\neq \emptyset$.*

Proof. (1) It is easy to verify from Proposition 7 and Definition 15.
(2) and (3) It is obvious from Definitions 16 and 17.
(4) The proof is similar to Proposition 3.9 in [15].
(5) The proof is similar to Proposition 3.10 in [15].
□

Proposition 11. *If (F, A) and (G, A) are two totally dependent-neutrosophic soft sets over U, then:*

(1) *((F, A) $\cup_3$ (G, A))^{c3} = (F, A)^{c3} $\cap_3$ (G, A)^{c3};*
(2) *((F, A) $\cap_3$ (G, A))^{c3} = (F, A)^{c3} $\cup_3$ (G, A)^{c3}.*

Proof.

(1) Assume that (*F*, *A*) $\cup_3$ (*G*, *A*) = (*H*, *A*) and (*F*, *A*)^{c3} $\cap_3$ (*G*, *A*)^{c3} = (*I*, *A*). Then:

$$\forall e \in A,\ H(e) = F(e) \cup_3 G(e) \text{ (by Definition 16);}$$

$$\forall e \in A,\ I(e) = F^{c3}(e) \cap_3 G^{c3}(e) = (F(e))^{c3} \cap_3 (G(e))^{c3} = (F(e) \cup_3 G(e))^{c3}$$
$$\text{(by Definitions 15, 17 and Proposition 8).}$$

Thus $\forall e \in A$, $H^{c3}(e) = (H(e))^{c3} = (F(e) \cup_3 G(e))^{c3} = I(e)$. Since ((*F*, *A*) $\cup_3$ (*G*, *A*))^{c3} = (H^{c3}, *A*), it follows that ((*F*, *A*) $\cup_3$ (*G*, *A*))^{c3} = (H^{c3}, *A*) = (*I*, *A*) = (*F*, *A*)^{c3} $\cap_3$ (*G*, *A*)^{c3}.

(2) By (1), and using Proposition 10(1) we can get ((*F*, *A*) $\cap_3$ (*G*, *A*))^{c3} = (*F*, *A*)^{c3} $\cup_3$ (*G*, *A*)^{c3}. □

Proposition 12. *If (F, A) and (G, A) are two totally dependent-neutrosophic soft sets over U, then:*

(1) *(F, A) $\cap_3$ ((F, A) $\cup_3$ (G, A)) = (F, A);*
(2) *(F, A) $\cup_3$ ((F, A) $\cap_3$ (G, A)) = (F, A);*
(3) *(F, A) $\subseteq_3$ (G, A) $\Leftrightarrow$ (F, A) $\cup_3$ (G, A) = (G, A);*
(4) *(F, A) $\subseteq_3$ (G, A) $\Leftrightarrow$ (F, A) $\cap_3$ (G, A) = (F, A).*

Proof.

(1) Assume that $(F, A) \cup_3 (G, A) = (H, A)$ and $(F, A) \cap_3 ((F, A) \cup_3 (G, A)) = (I, A)$. Then:

$$\forall e \in A,\ H(e) = F(e) \cup_3 G(e) \text{ (by Definition 16);}$$

$$\forall e \in A,\ I(e) = F(e) \cap_3 H(e) = F(e) \cap_3 (F(e) \cup_3 G(e)) = F(e)$$
$$\text{(by Definition 17 and Proposition 6(3)).}$$

Thus, $\forall e \in A$, $I(e) = F(e)$, and it follows that $(F, A) \cap_3 ((F, A) \cup_3 (G, A)) = (F, A)$.

(2) The proof is similar to (1).

(3) From Proposition 6(4) and Definition 16, we get that $(F, A) \subseteq_3 (G, A) \Leftrightarrow (F, A) \cup_3 (G, A) = (G, A)$.

(4) From Proposition 6(4) and Definition 17, we get that $(F, A) \subseteq_3 (G, A) \Leftrightarrow (F, A) \cap_3 (G, A) = (F, A)$. □

Theorem 2. *Let TDNSS(U, A) be the set of all totally dependent-neutrosophic soft sets over universe U with a fixed parameter set A. Denote:*

$$(0_{TDNSS}, A) \in TDNSS(U, A);\ \forall e \in A,\ 0_{TDNSS}(e) = \{(x, 0, 0, 1) | x \in U\};$$

$$(1_{TDNSS}, A) \in TDNSS(U, A);\ \forall e \in A,\ 1_{TDNSS}(e) = \{(x, 1, 0, 0) | x \in U\}.$$

Then $(TDNSS(U, A), \cup_3, \cap_3, {}^{c3}, (0_{TDNSS}, A), (1_{TDNSS}, A))$ is a GM-algebra.

Proof. By Definition 13, we have $(F, A) \subseteq_3 (1_{TDNSS}, A)$ and $(0_{TDNSS}, A) \subseteq_3 (F, A)$, $\forall (F, A) \in TDNSS(U, A)$.

By Propositions 10 and 12, we know that $(TDNSS(U, A), \cup_3, \cap_3, (0_{TDNSS}, A), (1_{TDNSS}, A))$ is a bounded lattice. Therefore, by Propositions 10(1), 12 and Definition 7, we get that $(TDNSS(U, A), \cup_3, \cap_3, {}^{c3}, (0_{TDNSS}, A), (1_{TDNSS}, A))$ is a GM-algebra. □

We can verify that the distributive law (with respect to $\cup_3$ and $\cap_3$) in $TDNSS(U, A)$ is not satified.

Example 8. *Let $U = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2, e_3, e_4\}$, and $A = \{e_1, e_2\}$. Suppose that (F, A), (G, A), and (H, A) are totally dependent-neutrosophic soft sets over U, and*

$$F(e_1) = \{(x_1, 0.1, 0.4, 0.3), (x_2, 0.4, 0.2, 0.3), (x_3, 0.2, 0.3, 0.4), (x_4, 0.3, 0.3, 0.2)\},$$
$$F(e_2) = \{(x_1, 0.1, 0.1, 0.1), (x_2, 0.2, 0.2, 0.2), (x_3, 0.3, 0.3, 0.3), (x_4, 0.4, 0.4, 0.2)\};$$

$$G(e_1) = \{(x_1, 0.3, 0.1, 0.5), (x_2, 0.6, 0.0, 0.3), (x_3, 0.3, 0.4, 0.2), (x_4, 0.5, 0.2, 0.2)\},$$
$$G(e_2) = \{(x_1, 0.2, 0.2, 0.4), (x_2, 0.2, 0.1, 0.3), (x_3, 0.1, 0.4, 0.2), (x_4, 0.4, 0.1, 0.5)\};$$

$$H(e_1) = \{(x_1, 0.2, 0.2, 0.4), (x_2, 0.2, 0.1, 0.1), (x_3, 0.7, 0.3, 0.0), (x_4, 0.4, 0.0, 0.3)\},$$
$$H(e_2) = \{(x_1, 0.4, 0.2, 0.3), (x_2, 0.3, 0.1, 0.6), (x_3, 0.5, 0.2, 0.3), (x_4, 0.8, 0.0, 0.1)\}.$$

Then:

$$((F, A) \cup_3 (G, A)) \cap_3 (H, A) = \left(\begin{array}{ccc} & e_1 & e_2 \\ x_1 & (0.2, 0.2, 0.4) & (0.2, 0.5, 0.3) \\ x_2 & (0.2, 0.5, 0.3) & (0.2, 0.2, 0.6) \\ x_3 & (0.3, 0.4, 0.2) & (0.3, 0.4, 0.3) \\ x_4 & (0.4, 0.0, 0.3) & (0.4, 0.4, 0.2) \end{array}\right),$$

$$((F, A) \cap_3 (H, A)) \cup_3 ((G, A) \cap_3 (H, A) = \left(\begin{array}{ccc} & e_1 & e_2 \\ x_1 & (0.2, 0.0, 0.4) & (0.2, 0.0, 0.3) \\ x_2 & (0.2, 0.5, 0.3) & (0.2, 0.2, 0.6) \\ x_3 & (0.3, 0.4, 0.2) & (0.3, 0.3, 0.3) \\ x_4 & (0.4, 0.0, 0.3) & (0.4, 0.4, 0.2) \end{array}\right);$$

$$((F,A)\cap_3(G,A))\cup_3(H,A)=\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}\!\!\left(\begin{array}{cc} e_1 & e_2 \\ (0.2,0.2,0.4) & (0.4,0.2,0.3) \\ (0.4,0.0,0.1) & (0.3,0.0,0.3) \\ (0.7,0.3,0.0) & (0.5,0.2,0.3) \\ (0.4,0.0,0.2) & (0.8,0.0,0.1) \end{array}\right),$$

$$((F,A)\cup_3(H,A))\cap_3((G,A)\cup_3(H,A)=\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}\!\!\left(\begin{array}{cc} e_1 & e_2 \\ (0.2,0.4,0.4) & (0.4,0.2,0.3) \\ (0.4,0.0,0.1) & (0.3,0.0,0.3) \\ (0.7,0.3,0.0) & (0.5,0.2,0.3) \\ (0.4,0.0,0.2) & (0.8,0.0,0.1) \end{array}\right).$$

Hence

$$((F, A) \cup_3 (G, A)) \cap_3 (H, A) \neq ((F, A) \cap_3 (H, A)) \cup_3 ((G, A)) \cap_3 (H, A));$$

$$((F, A) \cap_3 (G, A)) \cup_3 (H, A) \neq ((F, A) \cup_3 (H, A)) \cap_3 ((G, A)) \cup_3 (H, A)).$$

6. Conclusions

In this paper, we prove that the type-3 ordering relation $\leq_3$ is a partial ordering relation and D^* makes up a lattice with respect to type-3 intersection and type-3 union operations. Then, we give some new operations of totally dependent-neutrosophic (picture fuzzy) sets and totally dependent-neutrosophic (picture fuzzy) soft sets, and their properties are presented. At the same time, we point out all of the totally dependent-neutrosophic (picture fuzzy) sets on X make up generalized De Morgan algebra with respect to type-3 intersection, type-3 union, and type-3 complement operations. Moreover, we prove that for appointed parameter sets, all of the totally dependent-neutrosophic (picture fuzzy) soft sets over U can also generate a generalized De Morgan algebra based on type-3 algebraic operations.

It can be seen from the results of this paper that the type-3 algebraic operations of the totally dependent-neutrosophic (picture fuzzy) sets have good properties and a new property which is different from the fuzzy sets and the intuitionistic fuzzy sets (because the distribution law is not established). This theoretically shows that although the totally dependent-neutrosophic (picture fuzzy) set is a generalization of the fuzzy set and intuitionistic fuzzy set, it has different characteristics. In fact, the type-1 and type-2 algebraic operations of totally dependent-neutrosophic (picture fuzzy) sets (including the order relations, see Remark 1 and Definition 5) simply imitate the corresponding operations of the intuitionistic fuzzy sets, which cannot truly reflect the original idea of totally dependent-neutrosophic (picture fuzzy) sets. For example, for type-2 inclusion relation Definition 5, $A \subseteq_2 B$ if ($\forall x \in X$, $\mu_A(x) \leq \mu_B(x)$, $\eta_A(x) \leq \eta_B(x)$, $\nu_A(x) \geq \nu_B(x)$), which means that the first two membership functions (μ, η) have the same effect, but the three membership functions in the original definition of neutrosophic sets are completely independent, which is incongruous. For the type-1 inclusion relation, there is a similar problem. From Definition 8, we know that the type-3 inclusion relation has overcome this defect.

As further research topics, we will discuss the applications of the new algebraic operations in multiple attribute decision making and uncertainty reasoning. At the same time, it is a meaningful topic for reviewers to suggest developing new directions, such as drawing on new ideas in [35].

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References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96. [CrossRef]
3. Molodtsov, D.A. Soft set theory-first results. *Comput. Math. Appl.* **1999**, *37*, 19–31. [CrossRef]
4. Zhang, X.H.; Pei, D.W.; Dai, J.H. *Fuzzy Mathematics and Rough Set Theory*; Tsinghua University Press: Beijing, China, 2013.
5. Zhang, X.H. Fuzzy anti-grouped filters and fuzzy normal filters in pseudo-BCI algebras. *J. Intell. Fuzzy Syst.* **2017**, *33*, 1767–1774. [CrossRef]
6. Roy, A.R.; Maji, P.K. A fuzzy soft set theoretic approach to decision making problems. *J. Comput. Math. Appl.* **2007**, *203*, 412–418. [CrossRef]
7. Jun, Y.B. Soft BCK/BCI-algebras. *Comput. Math. Appl.* **2008**, *56*, 1408–1413. [CrossRef]
8. Feng, F.; Jun, Y.B.; Liu, X.; Li, L. An adjustable approach to fuzzy soft set based decision making. *J. Comput. Appl. Math.* **2010**, *234*, 10–20. [CrossRef]
9. Zhan, J.M.; Ali, M.I.; Mehmood, N. On a novel uncertain soft set model: Z-soft fuzzy rough set model and corresponding decision making methods. *Appl. Soft Comput.* **2017**, *56*, 446–457. [CrossRef]
10. Zhan, J.M.; Zhu, K.Y. A novel soft rough fuzzy set: Z-soft rough fuzzy ideals of hemirings and corresponding decision making. *Soft Comput.* **2017**, *21*, 1923–1936. [CrossRef]
11. Ma, X.L.; Zhan, J.M.; Ali, M.I.; Mehmood, N. A survey of decision making methods based on two classes of hybrid soft set models. *Artif. Intell. Rev.* **2018**, *49*, 511–529. [CrossRef]
12. Zhan, J.M.; Alcantud, J.C.R. A novel type of soft rough covering and its application to multicriteria group decision making. *Artif. Intell. Rev.* **2018**. [CrossRef]
13. Qin, K.Y.; Zhao, H. Lattice structures of fuzzy soft sets. In *Advanced Intelligent Computing Theories and Applications*; Springer: Berlin, Germany, 2010; pp. 126–133.
14. Yang, H.L.; Guo, Z.L. Kernels and closures of soft set relations and soft relation mapping. *Comput. Math. Appl.* **2011**, *61*, 651–662. [CrossRef]
15. Li, Z.W.; Zheng, D.W.; Hao, J. L-fuzzy soft sets based on complete Boolean lattices. *Comput. Math. Appl.* **2012**, *64*, 2558–2574. [CrossRef]
16. Zhang, X.H.; Wang, W.S. Lattice-valued interval soft sets-A general frame of many soft models. *J. Intell. Fuzzy Syst.* **2014**, *26*, 1311–1321.
17. Zhang, X.H.; Park, C.; Wu, S.P. Soft set theoretical approach to pseudo-BCI algebras. *J. Intell. Fuzzy Syst.* **2018**, *34*, 559–568. [CrossRef]
18. Smarandache, F. *A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability*; American Research Press: Rehoboth, DE, USA, 1999.
19. Wang, H.; Smarandache, F.; Sunderraman, R. Single-valued neutrosophic sets. *Rev. Air Force Acad.* **2013**, *17*, 10–13.
20. Ye, J. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. *Int. J. Gen. Syst.* **2013**, *42*, 386–394. [CrossRef]
21. Ye, J. Similarity measures between interval neutrosophic sets and their multicriteria decision-making method. *J. Intell. Fuzzy Syst.* **2014**, *26*, 165–172.
22. Liu, P.D.; Tang, G.L. Some power generalized aggregation operators based on the interval neutrosophic numbers and their application to decision making. *J. Intell. Fuzzy. Syst.* **2015**, *30*, 2517–2528. [CrossRef]
23. Liu, P.D.; Teng, F. Multiple attribute decision making method based on normal neutrosophic generalized weighted power averaging operator. *Int. J. Mach. Learn. Cybern.* **2015**, 1–13. [CrossRef]
24. Zhang, X.H.; Smarandache, F.; Liang, X.L. Neutrosophic duplet semi-group and cancellable neutrosophic triplet groups. *Symmetry* **2017**, *9*, 275. [CrossRef]
25. Zhang, X.H.; Bo, C.X.; Smarandache, F.; Dai, J.H. New inclusion relation of neutrosophic sets with applications and related lattice structrue. *Int. J. Mach. Learn. Cybern.* **2018**. [CrossRef]
26. Smarandache, F. Degree of dependence and independence of the (sub)components of fuzzy set and neutrosophic set. *Neutrosophic Sets Syst.* **2016**, *11*, 95–97.
27. Cuong, B.C. *Picture Fuzzy Sets-First Results, Part 1 and Part 2, Seminar "Neuro-Fuzzy Systems with Applications"*; Preprint 03/2013 and Preprint 04/2013; Institute of Mathematics: Hanoi, Vietnam, 2013.
28. Cuong, B.C. Picture fuzzy sets. *J. Comput. Sci. Cybern.* **2014**, *30*, 409–420.

29. Cuong, B.C.; Hai, P.V. Some fuzzy logic operators for picture fuzzy sets. In Proceedings of the IEEE Seventh International Conference on Knowledge and Systems Engineering, Ho Chi Minh City, Vietnam, 8–10 October 2015; pp. 132–137.
30. Cuong, B.C.; Phong, P.H.; Smarandache, F. Standard neutrosophic soft theory: Some first results. *Neutrosophic Sets Syst.* **2016**, *12*, 80–91.
31. Wang, C.; Zhou, X.; Tu, H.; Tao, S. Some geometric aggregation operators based on picture fuzzy sets and their application in multiple attribute decision making. *Ital. J. Pure Appl. Math.* **2017**, *37*, 477–492.
32. Daniel, S. Free non-distributive morgan-stone algebars. *N. Z. J. Math.* **1996**, *25*, 85–94.
33. Maji, P.K. Neutrosophic soft set. *Ann. Fuzzy Math. Inform.* **2013**, *5*, 157–168.
34. Şahin, R.; Küçük, A. On similarity and entropy of neutrosophic soft sets. *J. Intell. Fuzzy Syst.* **2014**, *27*, 2417–2430.
35. Bucolo, M.; Fortuna, L.; La Rosa, M. Network self-organization through "small-worlds" topologies. *Chaos Solitons Fractals* **2001**, *14*, 1059–1064. [CrossRef]