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# Crash forecasting in the Korean stock market based on the log-periodic structure and pattern recognition

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## HIGHLIGHTS

- The bubble forming period of domestic financial crisis is different from the one of global financial crisis in Korea.
- The fitting period is added as the new parameter to distinguish the domestic crisis from global crisis.
- The proposed model shows statistically better performance of crash forecasting than that traditional alarm index.
- The trading strategy based on our alarm index shows better performance than other trading strategies.

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## ABSTRACT

The aim of this research is to propose an alarm index to forecast the crash of the Korean financial market in extension to the idea of Johansen–Ledoit–Sornette model, which uses the log-periodic functions and pattern recognition algorithm. We discover that the crashes of the Korean financial market can be classified into domestic and global crises where each category requires different window length of fitted datasets. Therefore, we add the window length as a new parameter to enhance the performance of alarm index. Distinguishing the domestic and global crises separately, our alarm index demonstrates more robust forecasting than previous model by showing the error diagram and the results of trading performance.



## 1. Introduction

A financial bubble is an artificial growth of an asset price incurred by irrationally aggressive expectations among market participants, which can be seen as the state of over-valued financial market [1–3]. Moreover, it usually bursts at the zenith of the economic cycle. The outbreak of such event appears with the large drop of asset price, which is also known as the market crash [4,5]. If a bubble is created from a non-popular asset, the severity of market crash is insignificant since only limited number of market participants will be harmed. In contrast, if a bubble of important asset bursts, the damage can be devastating not only to the market participants but also to the soundness of the entire financial system. Hence, the bubble in the financial market can be a threatening factor to various domains [6].

It has been popular to apply the models and methods of modern Physics to explain the economic phenomenon [7,8]. Specifically, it has been discovered that the financial market behaves like the non-equilibrium dynamics of the statistical Physics [9–11]. Based on this discovery, the Johansen–Ledoit–Sornette (JLS) model was proposed to detect market crashes in the financial market by simulating the herding behavior of noise traders [12,13]. The model follows the step-by-step scenario

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of market crash, which assumes that the price evolution of the financial asset during the bubble-forming period exhibits log-periodic oscillation. That is, the market crash is expected when the log-periodic power law evolution is converged. Furthermore, Sornette and Zhou [14] utilized the pattern recognition method to develop an alarm index for monitoring the possibility of market crash. This pattern recognition method was originally designed by mathematicians to forecast earthquakes whose algorithm enables the model to forecast a systematic breakdowns by learning historical patterns [15]. In the same manner, the pattern recognition of the JLS model has been discovered in the way of separating learning and forecasting periods [16,17]. In addition, the back testing process is also progressed on the learning period.

Like that of other countries, the Korean financial market has experienced many crashes in the past followed by the renowned financial crises [18–20]. Zhou and Sornette [21] attempts to fit the log-period function to the Korean stock market index to forecast the major market crashes. However, they did not consider the distinct systemic aspects of Korean financial market, and used the same parameters for the Korean stock market index as those for the US stock market indices, which did not consider the distinct systemic aspects of the Korean financial market. Unlike the U.S. financial market whose burst of bubble develops into the global financial crisis, the crash of the Korean financial market can be occurred not only by the global crisis but also by its domestic (regional) reasons. Considering two different causes of the market crash, we improve the previous JLS model in the following two folds:

1. We classify the crashes of the Korean financial market into the global and domestic crises so that the aspects of each category.
2. We add new parameter for the length of window of the learning period to improve the performance of pattern recognition.

The parameter for the window length aims to reflect the characteristics of the Korean stock market. The idea of new parameter lies on the assumption that the domestic crisis is more like an endogenous incident which requires the shorter bubble forming period than the global crisis. It means that the different learning sets should be used in different types of financial crises for better forecasting tailored.

This paper is organized as follows: Section 2 explains the mathematical background of the JLS model and pattern recognition algorithm; Section 3 summarizes the data and past major crashes in the Korean stock market; Section 4 discusses the empirical findings of log-periodic function and analyzes the performance of alarm index; and Section 5 concludes.

## 2. Methodology

### 2.1. JLS model

The JLS model was initially developed to forecast the point of market crash by fitting the log-periodic function [12,13,17]. This model assumes that the asset price follows the martingale property, such that

$$\mathbb{E}[p(t+s)|\mathscr{F}_t] = p(t), \quad s > 0 \tag{1}$$

where $p(t)$ represents the asset price at time $t$ and $\mathscr{F}_t$ is the filtration of $p(t)$. Then, the corresponding stochastic differential equation of the JLS model is,

$$dp = \mu(t)p(t)dt + \sigma(t)p(t)dW - \kappa p(t)dJ \tag{2}$$

where $\mu(t), \sigma(t), dW$, and $\kappa \in (0, 1)$ denote a trend function, a volatility function, an increment of standard Wiener process, and the percentage of price drop due to market crash, respectively. Furthermore, $dJ$ denotes a discontinuous jump process for the market crash where $J = 0$ prior to a market crash and $J = 1$ after a market crash.

Due to the martingale property in Eq. (1) and that of Wiener process,

$$\mathbb{E}[dp|\mathscr{F}_t] = \mu(t)p(t)dt - \kappa p(t)\mathbb{E}[dJ|\mathscr{F}_t] = 0. \tag{3}$$

Let $h(t)$ be the hazard rate representing the probability of market crash, then we can obtain $\mu(t) = \kappa h(t)$ since $\mathbb{E}[dJ|\mathscr{F}_t]$ can be expressed in terms of $h(t)$ where $\mathbb{E}[dJ|\mathscr{F}_t] = 1 \times h(t)dt + 0 \times (1 - h(t)dt)$.

Under the assumption of no market crash, dividing both sides of Eq. (2) by $p(t)$, conducting integration, and taking its expectation value yields,

$$\mathbb{E}\left[\ln\left(p(t)\right)\right] = \text{constant} + \kappa \int h(t)dt. \tag{4}$$

The JLS model assumes the herding behavior of noise traders that destabilizes the financial market. The aggregate effect of noise traders can be explained by the following dynamics of the crash hazard rate, such that

$$h(t) \approx B_0(\tau^c - t)^{-\alpha} + B_1(\tau^c - t)^{-\alpha}\cos[\omega \ln(\tau^c - t) + \psi'] \tag{5}$$

where $\tau^c$ is a critical time and $0 < \alpha < 1$. Note that $\alpha$ is the exponential parameter, whereas $\omega$ roles as a frequency parameter in the JLS model. The critical point defines the burst of well-behaved quantity in complex dynamic systems [12]. From an economic point of view, this critical time $\tau^c$ can be considered as the end of the economic bubble.

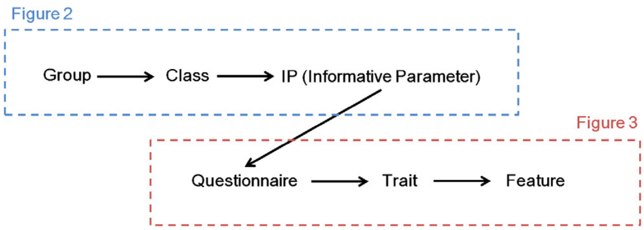



**Fig. 1.** The flow diagram of pattern recognition methodology.

Plugging Eq. (5) into Eq. (4) leads to,

$$\mathbb{E}\left[\ln[p(t)]\right] \approx \ln[p(\tau^c)] - \frac{\kappa B_0}{\beta}(\tau^c - t)^{\beta} - \frac{\kappa B_1}{\sqrt{\beta^2+\omega^2}}(\tau^c - t)^{\beta}\cos[\omega \ln(\tau^c - t) + \phi] \tag{6}$$

where $\beta = 1 - \alpha > 0$ and $\phi$ is a phase parameter which is constant.

Let $A = \ln[p(\tau^c)]$, $B = -(\kappa B_0)/\beta$, and $C = -\kappa B_1/\sqrt{\beta^2+\omega^2}$, then $A$, $B$, and $C$ are linear parameters of the JLS model. Therefore, Eq. (6) can be simplified to $\ln[p(t)] \approx A + Bf(t) + Cg(t)$ where $f(t) = (\tau^c - t)^{\beta}$ and $g(t) = (\tau^c - t)^{\beta}\cos(\omega \log(\tau^c - t) + \phi)$ as discussed in the work of [22].

Finally, we can estimate the parameters $A$, $B$, and $C$ based on the following equation.

$$\sum_{i=1}^{N}\begin{bmatrix}\ln[p(t_i)]\\ \ln[p(t_i)]f(t_i)\\ \ln[p(t_i)]g(t_i)\end{bmatrix} = \sum_{i=1}^{N}\begin{bmatrix}1 & f(t_i) & g(t_i)\\ f(t_i) & f(t_i)^2 & f(t_i)g(t_i)\\ g(t_i) & f(t_i)g(t_i) & g(t_i)^2\end{bmatrix}\cdot\begin{bmatrix}A\\ B\\ C\end{bmatrix}. \tag{7}$$

Note that the unknown set of parameters, $\boldsymbol{\theta} = (\tau^c, \phi, \omega, \beta)$, can be estimated by minimizing the following residual sum of squares,

$$\min_{\boldsymbol{\theta}} \sum_{i=1}^{N}(y_{\boldsymbol{\theta}}(t_i) - y(t_i))^2 \tag{8}$$

where $y_{\boldsymbol{\theta}}(t_i) = A + Bf(t_i) + Cg(t_i)$ and $y(t_i) = \ln[p(t_i)]$.

To search the optimal value for $\boldsymbol{\theta}$, we utilize the genetic algorithm to solve the minimization problem of Eq. (8) as described in [23].

### 2.2. Pattern recognition and alarm index

Before introducing the pattern recognition method, we briefly explain the related terminologies as follows.

- **Market crash**: The day when the market index is the highest within ±300 days.
- **Learning set**: The set of windows where the end and critical time $\tau^c$ are located before $t_d$, which is the date dividing the learning and forecasting set. In this study, $t_d$ is set to be 2002-01-02.
- **Classes**: Fits in a set is classified into two classes by measuring the closeness between the critical time of given fit and the actual market crash date. Sets are assigned to Class I if the time lag between the critical time $\tau_c$ and the crash date $d$ is shorter than pre-specified $D$ dates; otherwise sets are assigned to Class II.
- **Groups**: Fits are classified into 10 groups by comparing the window lengths. Window lengths vary from 100 days to the 5000 days.
- **Informative parameter (IP)**: IP is used to classify a Class I and II in a group.
- **Questionnaire**: Questionnaire consisting of values, $-1, 0, 1$, is a series of forecasting results corresponding each informative parameter.
- **Traits**: Traits are induced from Questionnaire in order to define an informative characteristics.
- **Features**: Feature of a class is the set of traits that exhibit a certain characteristic of a class.
- **Alarm Index**: Alarm index is the probability that a certain day is the market crash date, and the index is estimated by counting a number of traits included in features.

A brief flow diagram of pattern recognition methodology which might be helpful for understanding is demonstrated in Fig. 1. Furthermore, a detail explanation of each part is visualized in Figs. 2, 3 and 4.

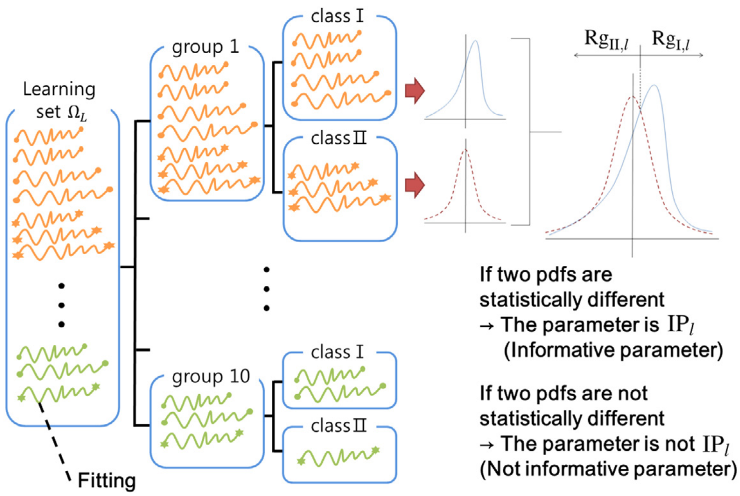



**Fig. 2.** The algorithm to classify group and class, and the selection of informative parameter.

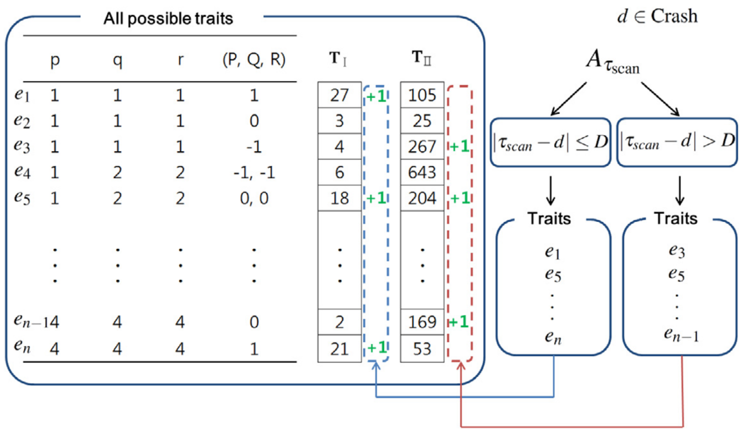



**Fig. 3.** An example of constructing the counting vectors from a questionnaire.

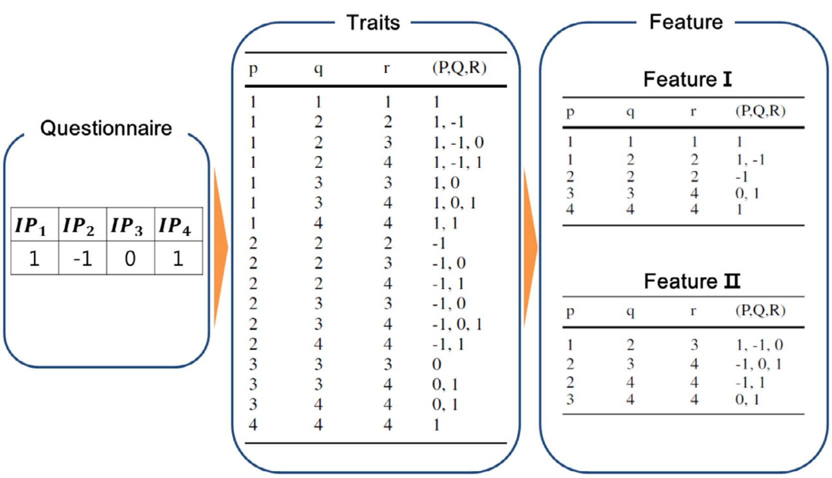



**Fig. 4.** An example of generating traits and feature from a questionnaire.

*2.2.1. Market crash, time interval and learning set*

In order to construct an alarm index based on the pattern recognition method used in [15], we define "the day of market crash" as follows. Let $d$ be a day of market crash if it is the highest value of market index during the period from $d-300$ day to $d+300$ day. Then, the set of crash days is

$$\text{Crash} = \{d : d = \arg\max_{t \in T_d} \ p(t)\} \tag{9}$$

where $T_d = \{t : d-300 \le t \le d+300\}$ and $p(t)$ is a closing price of day $t$.

As a result, we define seven dates of market crash in Korea from 1980-01-01 to 2012-12-28: 1981-07-07, 1989-04-01, 1994-11-08, 2000-01-04, 2002-04-18, 2007-10-31, and 2011-05-02. Later in Section 3, we classify the dates into either global or domestic financial crisis.

The window length plays an important role in the pattern recognition with log-periodic structures. Several time points are selected to setting various windows. The first time point $\tau_1$ is set to be 1980-03-03 and the period between two time points in KOSPI index data are set to be 25 (days). Now, every combination of these time points are used as the windows. We select the window whose length is between 100 to 5000 as the whole set of windows which are possible to fit. Now this whole set of windows is defined as $\Omega$. The window is denoted by $[\tau_k^s, \tau_k^e]$ and the corresponding length of interval is $\Delta\tau_k = \tau_k^s - \tau_k^e$ where $k$ is the index representing the window, and $\tau_k^s$ and $\tau_k^e$ are the start-date and the end-date of the window, respectively. For each window $[\tau_k^s, \tau_k^e]$, we get the critical time $\tau_k^c$, introduced in Section 2.1, by fitting log-periodic function on this period.

Then we set $\tau_d$ be 2002-01-02 in order to measure a performance of an alarm index. Forecasting a market crash is achieved by learning patterns of the parameters that successively forecasted the previous market crash. Let $\Omega_L$ be a set of windows during the learning process where the end and critical time are located before $\tau_d$, then

$$\Omega_L = \{k \in \Omega : \tau_k^e \le \tau_k^c < \tau_d\}. \tag{10}$$

Again, the aspect, discovered by fitting log-periodic function on the windows in learning set is used to forecast the date of the future crash.

*2.2.2. Classes and groups*

We organize the fitted learning sets into two classes based on their forecasting abilities which measure the closeness between the critical time and the actual date of market crash. As described in Eq. (11), a set is assigned to class I if the distance from the critical time, $\tau_k^c$, to the crash date, $d$, is shorter than allowed $D$ dates; otherwise a set is assigned to class II. If $D$ is too large, the forecasting ability of the market becomes too low. Therefore, we set $D = 10$ to avoid the noise within the class, such that

$$\begin{aligned} \text{Class I} &= \{k : |\tau_k^c - d| \le D, \quad k \in \Omega_L, \quad d \in \text{Crash}\} \\ \text{Class II} &= \{k : |\tau_k^c - d| > D, \quad k \in \Omega_L, \quad d \in \text{Crash}\} \end{aligned} \tag{11}$$

where Class I and Class II are mutually exclusive. In this paper, the window length is used as additional classification variable and divides the learning sets into 10 groups. The window lengths vary from 100 to 5000 days in 25 days of time unit. We set 10 groups, and the $i$th group for $i = 1, \dots, 10$, is defined as

$$\text{Group}_i = \{k : \Delta\tau_k \in (\ 500(i-1)),\ 500i\ ],\ \ k \in \Omega_L\} \tag{12}$$

where $\Delta\tau_k = \tau_k^s - \tau_k^e$ is the length of $k$th window for fitting.

*2.2.3. Informative parameters*

Parameters used in the JLS model are $\beta$, $\omega$, $\phi$, $B$, $b$ and $q$. $\beta$, $\omega$ and $\phi$ can be achieved from Eq. (6) where $B = -\kappa B_0/\beta$. Note that parameter $b$ can be solved based on the non-negative restrictive condition of the hazard rate, $b = -B\beta - |C|\sqrt{\beta^2 + \omega^2} \ge 0$, whereas parameter $q$ is simply the residuals of the fitting. Each fitting has 6 parameters, and these are used to assign the Classes and Groups.

As we mentioned earlier, we employ additional parameter for window length defined as follows.

$$I = \Delta\tau_k = \tau_k^s - \tau_k^e. \tag{13}$$

By using different values of $I$, the accuracy of forecasting both domestic and global crises can be improved. Now the pattern recognition contains 140 clusters (= 7 parameters × 2 classes × 10 groups). We use adaptive kernel method to generate the probability density function (pdf) of each cluster. Then, we compare the pdf of Class I and Class II in each parameter and group. We set the parameter to be informative if pdfs in different classes are statistically different. The Anderson–Darling test [24] are used to examine the statistical difference between the probability distribution of two classes with 5% significance. Now let us define the informative parameter in unique term, $\text{IP}_l$, as follows.

$$\text{IP}_l = 7 \times (i-1) + j \tag{14}$$

where $i \in \{1, 2, \ldots, 10\}$ and $j \in \{1, 2, \ldots, 7\}$ for each group and parameter, $l \in \{1, 2, \ldots, L\}$, and $L$ is the number of informative parameters. We can define $\mathrm{Rg}_{\mathrm{I},l}$ as a region where a pdf of $l$th informative parameter, $\mathrm{IP}_l$, in Class I is higher than the pdf of parameters in Class II. Also $\mathrm{Rg}_{\mathrm{II},l}$ is a region where a pdf of the parameter in Class I is lower. Thus, if a parameter is in $\mathrm{Rg}_{\mathrm{I},l}(\mathrm{Rg}_{\mathrm{II},l})$, the probability of being in Class I (Class II) also increases.

### 2.2.4. Questionnaires

Questionnaire of the pattern recognition can be established with the informative parameters. One can ask whether certain date is real market crash date or not. The answer of this question can be achieved by referring to each informative parameter. Questionnaire is a series of these answers and each number in questionnaire is one of values, −1, 0, and 1. Questionnaire allows the model to learn the range of parameter values to forecast the future market crash. We follow the algorithm in [17] to get questionnaire of the pattern recognition. Let $\Omega_T$ be a test set which is a set of windows with possible 'Learning' or 'Prediction' set, then the algorithm to generate the questionnaire is as follows:

1. **FIND** the max($\tau^c_{Max}$) and min($\tau^c_{Min}$) of crash dates forecasted from the $\Omega_T$.
2. **OBSERVE** $\tau^c_k$ in the $k$th window by increasing $\tau_{scan}$ from $\tau^c_{Min}$ to $\tau^c_{Max}$, then derive $S_{\tau_{scan}}$ which is a set of windows having $\tau^c$ close to $\tau_{scan}$ as the critical time. ($\tau_{scan}$ is a moving variable from $\tau^c_{Min}$ to $\tau^c_{Max}$)

$$S_{\tau_{scan}} = \{k : |\tau^c_k - \tau_{scan}| \leq D,\ \ k \in \Omega_T\},\ \ \text{where } D = 10. \tag{15}$$

3. **FIND** the groups of each window in $S_{\tau_{scan}}$.
4. **ASSIGN** −1, 0 or 1 to $a_{\mathrm{IP}}$ according to the values of informative parameters in $S_{\tau_{scan}}$ to generate the questionnaire. $a_{\mathrm{IP}_l} = 1$ if more numbers of windows are included in $\mathrm{Rg}_{\mathrm{I},l}$ (Class I), $a_{\mathrm{IP}_l} = -1$ if more numbers of windows are included in $\mathrm{Rg}_{\mathrm{II},l}$ (Class II), and $a_{\mathrm{IP}_l} = 0$ if $m = 0$ or the number of windows assigned in $\mathrm{Rg}_{\mathrm{I},l}$ and $\mathrm{Rg}_{\mathrm{II},l}$ are the same.
5. **DEFINE** the questionnaire, $A_{\tau_{scan}}$, based on $a_{\mathrm{IP}_l}$ ($l = 1, 2, \ldots, L$) at each $\tau_{scan}$. Length of each $A_{\tau_{scan}}$ is the same as the number of informative parameters, $L$, such that,

$$A_{\tau_{scan}} = a_{\mathrm{IP}_1} a_{\mathrm{IP}_2} \cdots a_{\mathrm{IP}_L},\qquad a_{\mathrm{IP}_l} \in \{-1, 0, 1\}. \tag{16}$$

### 2.2.5. Traits and features

Traits are created to define the characteristics of questionnaire. We follow the process in [17] to generate traits from a questionnaire. $A_{\tau_{scan}}$ can be classified by traits whose series of integers are limited between 4 and 6. Each trait has a series composed of two different types: 1) $p, q, r$ and 2) $(P, Q, R)$ - series of integers among $p, q, r$. Integers, $p, q, r \in \{1, 2, \ldots, L\}$ such that $1 \leq p \leq q \leq r \leq L$ represent the numeric position in the questionnaire, and $(P, Q, R)$ represent the value of the position as follows.

i. If $p = q = r$: $(P, Q, R) = a_{\mathrm{IP}_p}$
ii. If $p \neq q = r$: $(P, Q, R) = a_{\mathrm{IP}_p}, a_{\mathrm{IP}_q}$
iii. If $p \neq q \neq r$: $(P, Q, R) = a_{\mathrm{IP}_p}, a_{\mathrm{IP}_q}, a_{\mathrm{IP}_r}$.

Then the conditions of traits, $p, q, r \in \{1, 2, \ldots, L\}$ such that $1 \leq p \leq q \leq r \leq L$ and $a_{\mathrm{IP}_p} \in \{-1, 0, 1\}$, suggest the number of trait types as $3^k \binom{L}{k}$ where $k = 1 + \mathbf{1}_{\{p<q\}} + \mathbf{1}_{\{q<r\}}$. Note that $\mathbf{1}_{\{x\}}$ is an indicator function having value 1 if $x$ is true and 0, otherwise.

Hence the maximum number of trait types at $\tau_{scan}$ is $3L + 3^2\binom{L}{2} + 3^3\binom{L}{3}$. If the questionnaire, $A_{\tau_{scan}}$, has $L$ length, then the number of traits derived from a questionnaire is limited to $L + \binom{L}{2} + \binom{L}{3}$. Now we can use traits to define the features which determine a characteristic of each window in Class I or Class II.

Let $\vec{T}_{\mathrm{I}}$ and $\vec{T}_{\mathrm{II}}$ be vectors of which elements count the number of corresponding traits. An example constructing the counting vectors, $\vec{T}_{\mathrm{I}}$ and $\vec{T}_{\mathrm{II}}$, is presented in Fig. 3. In this figure, $e_i$ denotes $i$th trait from some questionnaire. Also let $\alpha$ denotes the threshold that the questionnaire generating the trait accurately forecasts the crash date (within D days) and $\beta$ be the threshold that the questionnaire generating the trait inaccurately forecasts $\tau_{scan}$, then a feature can be defined as,

i. $e_i \in$ Feature I where $i$ is the index satisfying,

$$\vec{T}_{\mathrm{I}}(\,i\,) \geq \alpha,\ \ \vec{T}_{\mathrm{II}}(\,i\,) \leq \beta$$

where $(\cdot)$ represent the corresponding element of the vector.

ii. $e_j \in$ Feature II where $j$ is the index satisfying,

$$\vec{T}_{\mathrm{I}}(\,j\,) < \alpha,\ \ \vec{T}_{\mathrm{II}}(\,j\,) > \beta.$$

Then, we can adjust the values of $\alpha$ and $\beta$ to control the qualification of the Feature I and Feature II. An example of generating traits and feature from a questionnaire is illustrated in Fig. 4

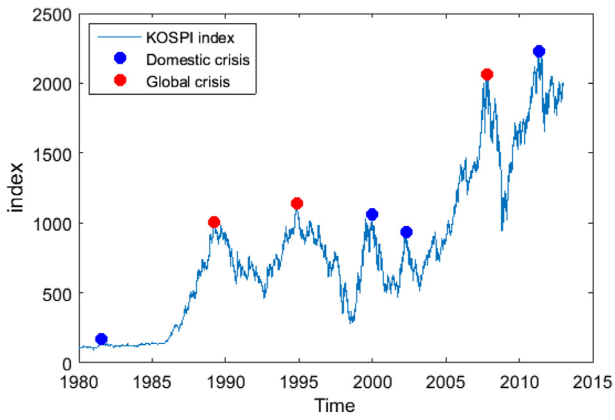



**Fig. 5.** Evolution of KOSPI and major crashes.

*2.2.6. Alarm index (Back testing & forecasting)*

We have defined that $\tau^c$ is assigned to a single class, a feature characterizes the windows in each class, and a qualification pairs $(\alpha, \beta)$ decides the accuracy of the feature. As mentioned before, the windows in the learning set, $\Omega_L$, are investigated to find out whether it belongs to Feature I or Feature II. Both back testing and forecasting of market crash can be achieved by learning the Feature I and Feature II. Also the results of the back testing can be used to improve the forecasting performance. We created an "Alarm index" that measures the performance of back testing and forecasting. Every trait from the questionnaire $A_{\tau_{scan}}$ is considered to generate the alarm index value on the date $\tau$. Let $N_{\tau,\mathrm{I}}$ be the number of traits on the date $\tau$ in Feature I and $N_{\tau,\mathrm{II}}$ be the number of traits on the date $\tau$ in Feature II, then an "Alarm Index", $\mathrm{AI}_\tau$, can be defined as follows,

$$\mathrm{AI}_\tau = \begin{cases} \dfrac{N_{\tau,\mathrm{I}}}{N_{\tau,\mathrm{I}} + N_{\tau,\mathrm{II}}} & (\text{ if } N_{\tau,\mathrm{I}} + N_{\tau,\mathrm{II}} \neq 0\ ) \\ 0 & (\text{ if } N_{\tau,\mathrm{I}} + N_{\tau,\mathrm{II}} = 0\ ). \end{cases} \tag{17}$$

Therefore, $\mathrm{AI}_\tau$ is the value between 0 and 1, and represents the probability of market crash occurrence. if $N_{\tau,\mathrm{I}}$ is much greater than $N_{\tau,\mathrm{II}}$, the alarm index, $\mathrm{AI}_\tau$ approaches to 1. On the contrary to this, if $N_{\tau,\mathrm{I}} + N_{\tau,\mathrm{II}} = 0$ or $N_{\tau,\mathrm{I}}$ is much smaller than $N_{\tau,\mathrm{II}}$, the alarm index, $\mathrm{AI}_\tau$ approaches to 0.

## 3. Data

Korea Composite Stock Price Index (KOSPI) is the market capitalization index of the Korean financial market. Hence, we choose KOSPI as the representation of the financial market in Korea. The time period of study is considered from 1980-01-01 to 2013-12-31; we select seven major crashes (four cases of domestic and three cases of global crises) as shown in Fig. 5. The domestic crises are Lee–Chang's Bill Fraud (1981-07-07), Daewoo Crisis (2000-01-04), Credit Card Crisis (2002-04-18), and Samsung Crisis (2011-05-02), whereas the global crises are Northern Europe Crisis (1989-04-01), Asian Currency Crisis (1994-11-08), and Sub-prime Mortgage Crisis (2007-10-31).

As previously stated, this paper focuses on the length of window which is a new parameter for the JLS model. Therefore, we apply our model to other emerging markets including China (SSE), India (SENSEX), and Brazil (IBOVESPA) to obtain a broader acceptance of different window length in emerging markets. The dates of market crashes for the other emerging markets also follow the same rule used in the case of Korea. Note that a crash is classified as the global crisis if its date is within $\pm 252$ days(a year) of the global crises in Korea. The time periods for China, India, and Brazil are available from 1990-12-19 to 2013-12-31 (with five domestic crises and two global crises), 1997-07-01 to 2013-12-31 (with two domestic crises and one global crises), and 1993-04-27 to 2013-12-31 (with three domestic crises and two global crises), respectively.

## 4. Results

As mentioned in Section 1, this paper aims to discover the different aspects between the domestic and global crises. By comparing the best length of fitting data in forecasting each market crash, we find that the domestic and global crises can be discriminated based on the relative size of window length.

**Table 1**
The best forecasting accuracy for KOSPI (Korea) and its corresponding length of window for the JLS model.

| | Crisis | Forecasted date (Crisis date) | Error in days | Start date End date | Best window length |
|---|---|---|---|---|---|
| Domestic | Lee–Chang Bill Fraud | 1981-06-29 (1981-07-07) | −8 | 1981-04-02 1980-06-15 | 74 |
| | Daewoo | 1999-08-12 (2000-01-04) | −145 | 1998-08-06 1999-07-19 | 347 |
| | Credit Card | 2002-03-27 (2002-04-18) | −83 | 2001-08-02 2001-12-27 | 147 |
| | Samsung Crisis | 2011-02-08 (2011-05-02) | −12 | 2010-05-06 201-01-21 | 172 |
| Global | Northern Europe Crisis | 1988-12-15 (1989-04-01) | −107 | 1985-02-13 1988-07-13 | 1246 |
| | Asian Currency | 1994-12-17 (1994-11-08) | +39 | 1991-12-14 1994-08-10 | 970 |
| | Subprime Mortgage | 2007-11-14 (2007-10-31) | +14 | 2001-08-02 2007-01-08 | 1985 |

### *4.1. Fitting log-periodic function*

The JLS model is applied to seven major crashes in the past evolution of KOSPI; the forecasting results are summarized in Table 1. The best forecasting result of Lee–Chang Bill Fraud crisis under the JLS model yields the date of 1981-06-29, which actually broke out in 1981-07-07, with −8 days error. Also, the dates of market crash for Credit Card and Samsung Crisis are also well forecasted with −83 and −12 days error, respectively. The above results are fairly encouraging since the errors are relatively small. On the other hand, the forecasting result of Daewoo crisis yields −145 days error, which is relatively larger that those of other domestic crises.

The forecasting results of global crises also exhibit the decent overall performance. The result of Northern Europe crisis suggests 1988-12-15 as the date of market crash, which allows −107 days error. Asian crisis shows only +39 days gap between the forecasted and actual date. In contrast, the forecasting of Subprime Mortgage crisis only allows +14 days error.

Figs. 6 and 7 illustrate the fitting and forecasting result of each crisis. The blue dots represent trained datasets for fitting the Log-Periodic-Power-Law, whereas the green dots are days left to the forecasted crisis date. More importantly, the red dots show the evolution of KOSPI after the date of crash forecasting. As explained in Table 1, Lee–Chang Bill Fraud and Subprime Mortgage crisis show the definite price drops after the forecasted crisis date. In contrast, Daewoo crisis shows an interesting evolution that once the price drops after the forecasted crisis date but it rebounds back to the actual date of the crisis and crashes again. Based on this phenomenon, we can conclude that the model detects the first small crash rather than the major crash which leads to the relatively large error. In other words, the JLS model well forecasted the bubble burst of the market in the earlier stage, but the timeline was not just right for the Daewoo crisis.

Comparing the best window length of domestic and global crises, we find that the best fit length of domestic crisis is around 50 to 350 days, whereas that of global crisis is around 950 to 2,000. We assume that the different size of window length are inherited from the unique characteristics of an import- and export-oriented economy (e.g. Korea). Since the global crisis severely impacts the world-wide economic indicators, the price drop and macro-economic damage to such country would be more catastrophic in the global crisis than the domestic ones. It is also analogous to state that the bubble in the financial market is larger in the global crisis since the size of price drop is larger. The worldwide co-movement of economic cycle is a well-known fact. Therefore, the large-size bubble needs a long period of world-wide boom cycle. In contrast, a domestic crisis is usually a structural problem of internal economy. Eventually, it can be assumed that the financial bubble for domestic crisis bursts in a shorter period.

Regarding that the convincing result of one country could be an idiosyncratic feature of this particular market, we try to extend our model to other emerging markets as described in Section 3. The forecasting results of China, India, and Brazil are summarized in Tables 2–4, respectively. In general, the most of results support the findings in Korean financial market by providing the best window lengths of domestic crises to be within 50 to 150, whereas those of global crises to be within 1000 to 1700. The only exception can be found in the forecasting of Brazilian Asian Currency crisis whose best window length is much smaller than those of others. In fact, such an unpleasant result seems to be caused by a short period of its dataset for the Asian Currency crisis since the best and maximum window lengths are the same. Also, the forecasting error is 467 days, which is much larger than those of other countries. We expect that the forecasting performance would be more precise if a longer window length is provided.

### *4.2. Alarm index*

As mentioned in Section 2.2.5, the feature qualification $(\alpha, \beta)$ can control the number of elements in the sets, Feature I and Feature II. If $\alpha$ is too large and $\beta$ is too small, the number of features in Feature I would be too small and those in Feature II would be too large. Therefore, the numerical value of alarm index would be inconclusive and fail to detect the financial

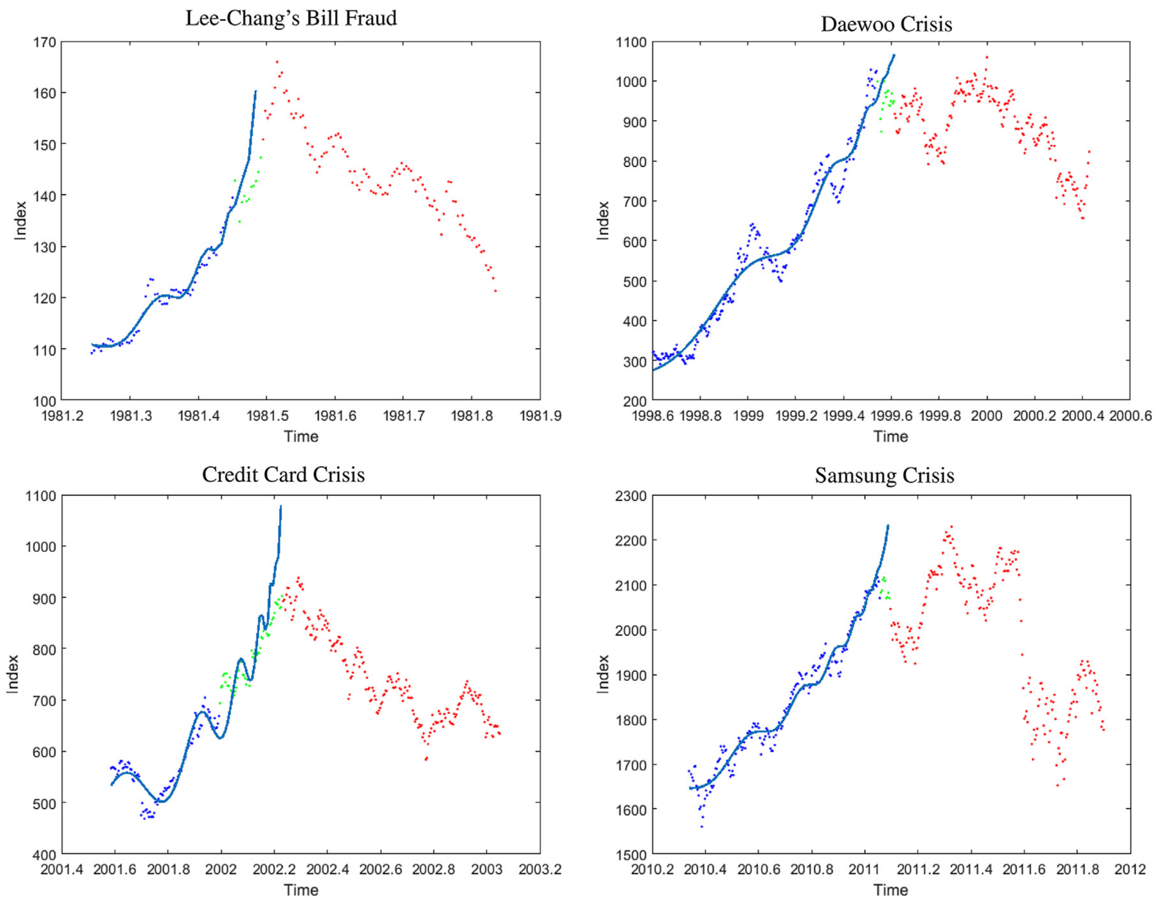



**Fig. 6.** Best fits of JLS Model in domestic crises.

**Table 2**
The best forecasting accuracy for SSE (China) and its corresponding length of window for the JLS model.

| | Crisis | Forecasted date (Crisis date) | Error in days | Start date End date | Best window length |
|---|---|---|---|---|---|
| Domestic | Domestic Crisis 1 | 1993-04-22 (1993-02-15) | +66 | 1992-11-16 1993-01-22 | 67 |
| | Domestic Crisis 2 | 1997-01-12 (1997-05-12) | −120 | 1997-01-07 1997-04-11 | 94 |
| | Domestic Crisis 3 | 2001-08-14 (2001-06-13) | +62 | 2000-12-11 2001-04-27 | 137 |
| | Domestic Crisis 4 | 2003-12-09 (2004-04-06) | −119 | 2003-12-09 2004-02-24 | 77 |
| | Domestic Crisis 5 | 2009-08-03 (2009-08-04) | −1 | 2008-12-02 2009-06-18 | 198 |
| Global | Asian Currency | 1994-12-04 (1994-09-13) | +82 | 1991-06-11 1994-04-14 | 1038 |
| | Subprime Mortgage | 2007-09-30 (2007-10-16) | −16 | 2002-08-29 2007-03-23 | 1667 |

crisis. On the contrary, if $\alpha$ is too small and $\beta$ is too large, the number of features in Feature I would be too large and those in Feature II would be too small. In this case, the alarm index carries a lot of noises with frequent false alarms. Therefore, choosing the adequate feature qualification of $(\alpha, \beta)$ is critical to optimize the detection ability.

Before evaluating the performance of new approach, the result of previous alarm index is presented in Fig. 8. Note that the previous alarm index does not distinguish the domestic and global financial crises, and it uses the same learning set to predict the market crashes. Fig. 8(a) illustrates the back testing result, which includes the information before 2002 to generate the informative parameters and features. In other words, the back testing only uses the learning set as defined in

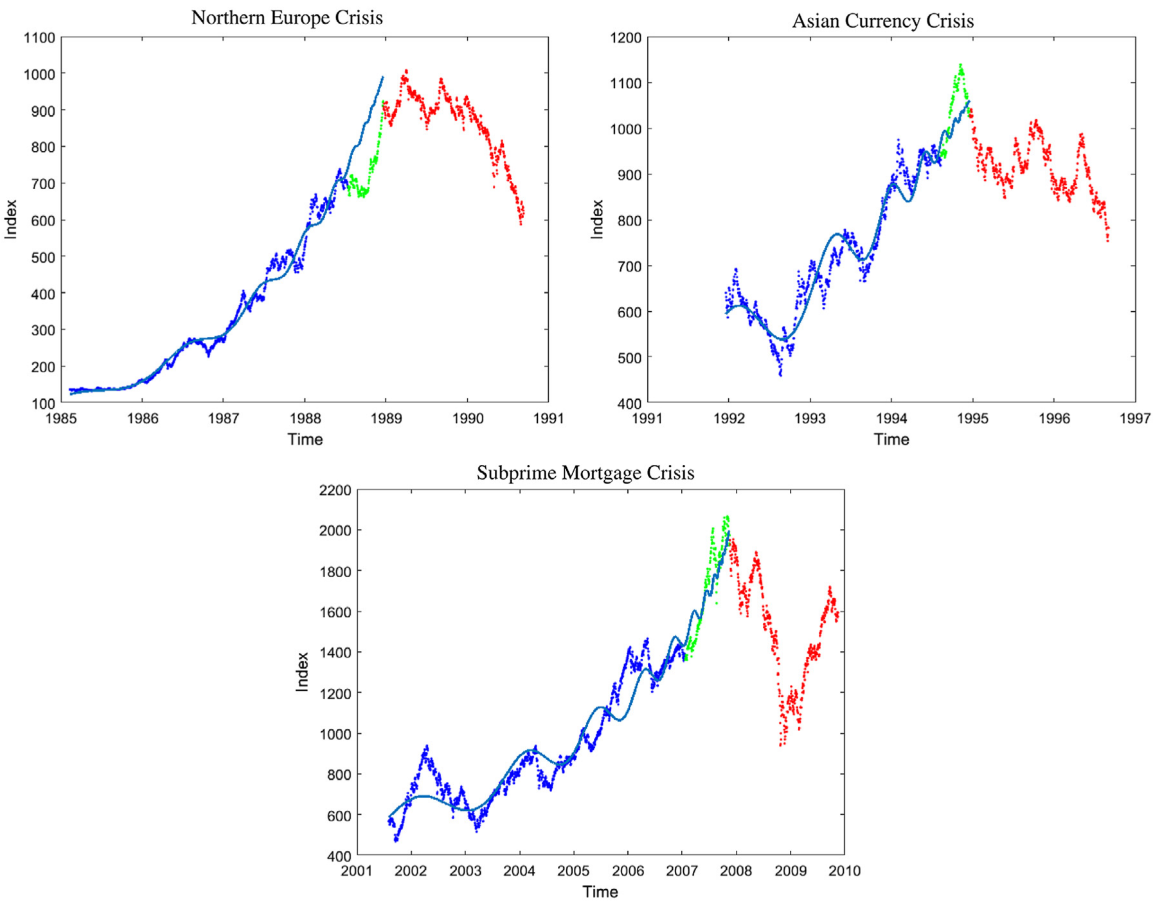


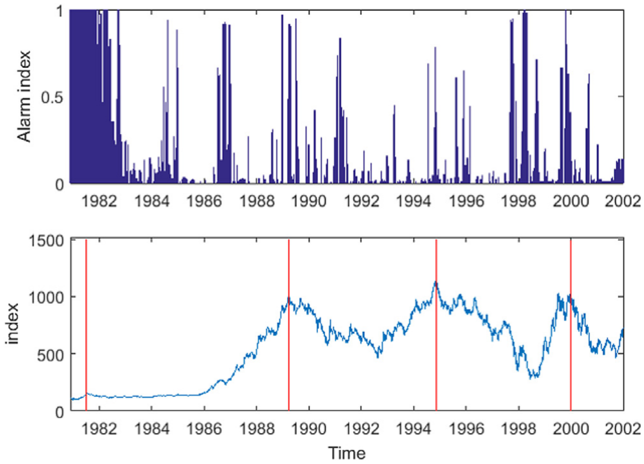

**Fig. 7.** The best fits of JLS Model in global crises.

**Table 3**
The best forecasting accuracy for SENSEX (India) and its corresponding length of window for the JLS model.

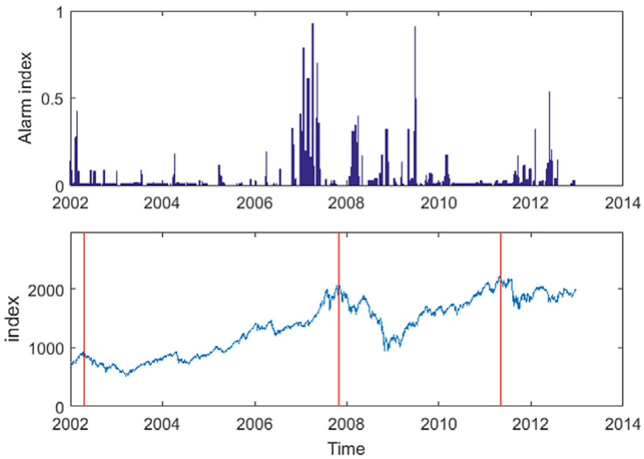
| | Crisis | Forecasted date (Crisis date) | Error in days | Start date End date | Best window length |
|---|---|---|---|---|---|
| Domestic | Domestic Crisis 1 | 2000-02-04 (2000-02-11) | −7 | 1999-10-15 2000-01-07 | 94 |
| | Domestic Crisis 2 | 2010-10-11 (2010-11-05) | −25 | 2010-05-21 2010-09-30 | 132 |
| Global | Subprime Mortgage | 2007-07-22 (2008-01-08) | −170 | 2003-12-27 2007-09-06 | 1349 |

**Table 4**
The best forecasting accuracy for IBOVESPA (Brazil) and its corresponding length of window for the JLS model.

| | Crisis | Forecasted date (Crisis date) | Error in days | Start date End date | Best window length |
|---|---|---|---|---|---|
| Domestic | Domestic Crisis 1 | 1997-09-07 (1997-07-08) | +61 | 1997-02-25 1997-06-05 | 100 |
| | Domestic Crisis 2 | 1999-12-18 (2000-03-27) | −100 | 1999-08-27 2000-01-21 | 147 |
| | Domestic Crisis 3 | 2010-08-27 (2010-11-04) | −89 | 2010-03-18 2010-06-04 | 78 |
| Global | Asian Currency | 1995-12-24 (1994-09-13) | +467 | 1993-04-27 1994-05-16 | 384 |
| | Subprime Mortgage | 2008-05-08 (2008-05-20) | −12 | 2004-06-09 2008-02-20 | 1351 |




(a) Back testing ($\alpha = 15$, $\beta = 500$).




(b) Forecasting ($\alpha = 5$, $\beta = 500$).

**Fig. 8.** Alarm index generated by the original(existing) methodology before including window length parameter $I$.

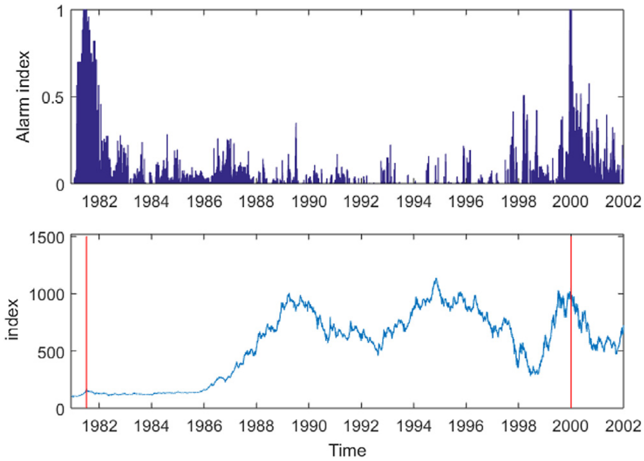
Eq. (10). Fig. 8(b) shows the forecasting result as it uses the information after 2002 to forecast the crashes in the future. Again, the data before $Q$ = 2002-01-02 is used for deriving the learning set.

Nevertheless the most of financial crises are well detected, we also find many false alarms through the alarm index. Furthermore, the alarm index shows much poorer performance in detecting the market crashes in the forecasting than that in the back testing.

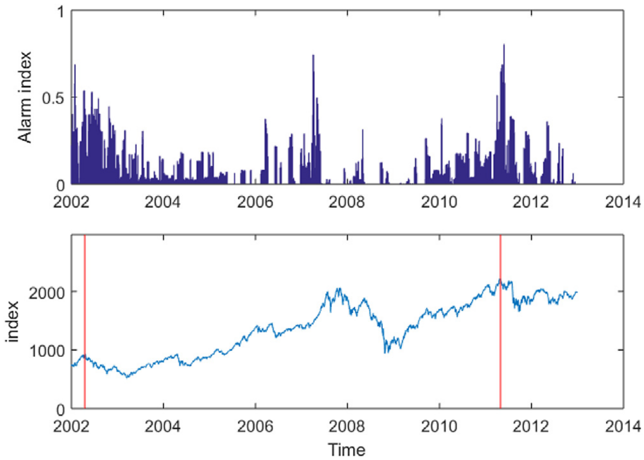
Fig. 9(a) illustrates the back testing result for domestic crises for the proposed alarm index. Here, the domestic crises are only used for the learning set. As shown in Fig. 9(a), the alarm index of back testing presents the high values for Lee–Chang's bill fraud and Daewoo crisis. Furthermore, Fig. 9(b) shows the forecasting result for domestic crises, which exhibits decent detection of the Credit Card crisis and Samsung crisis. Obviously, it can be concluded that the proposed model shows more definite alarming of market crash than the previous one for the domestic crises.

Fig. 10(a) indicates the back testing result for global crises for the proposed alarm index. Likewise, the information of global crises is only used for the learning set. Fig. 10(a) shows that the model adequately alarms the Northern Europe crisis and Asian currency crisis. Even though the alarm for the global crisis is turned on for Daewoo crisis, which in fact is defined as domestic one, only global crises are detected in view of numerical value of the alarm index. The last result is the forecasting of global crises. As in Fig. 10(b), the alarm index shows a decent performance for forecasting the sub-prime mortgage crisis by showing the high values around the date of market crash. Similar to the back testing result, the alarm responds to the Samsung crisis, which is one of domestic crisis, but still the numerical value of the alarm index for sub-prime mortgage crisis is much higher. It refers that our purpose of designing the alarm index merely for the global crisis is accomplished.

It is now clear that the proposed alarm index is created to enhance the forecasting performance of market crashes by differentiating the domestic and global crises. Therefore, we numerically compare the performances of the previous and




(a) Back testing ($\alpha = 15, \beta = 500$).



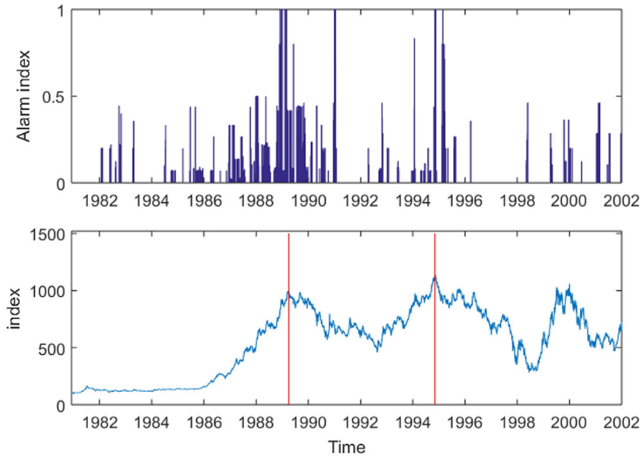

(b) Forecasting ($\alpha = 5, \beta = 500$).

**Fig. 9.** Alarm index of domestic crisis after including window length parameter $l$.

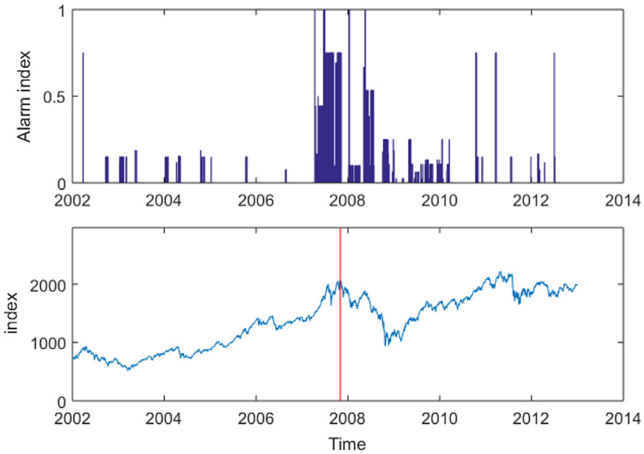
proposed alarm index based on the error diagram introduced in [17]. The algorithm for applying the error diagram is as follows.

1. **COUNT** the number of market crashes in the total period. In this paper, there are seven market crashes.
2. **SORT** the alarm index series by the values and make a new series.
3. **SET** the largest value in the series as the first threshold.
4. **FINE** the time point $d$ at which the alarm index value is over the current threshold. Success or failure of forecasting at the time point $d$ is decided whether the real market crash is in the time period $[d - D, d + D]$ or not. In this research, we set $D = 10$.
5. **SET** the next value of the series in step 2 to be new threshold if there is no time point at which the forecasting of market crash is accomplished.
6. **CALCULATE** the market crash rate (the number of failed-to-forecast market crashes divided by the number of total market crashes) and the alarm rate (the length of the combined total period in step 4/the length of the total period).
7. **DRAW** the point at the coordinate (the market crash rate, the alarm rate).

There are total 8 points marked in error diagram. Note that they are ranged in $(0, 1)$. It can be used to check whether the proposed method exhibits a better performance than randomly forecasted ones. The random forecasting lies on the line of slope of $-1$. Note that this line represents $y = 1-x$. If the point is marked under the line, it can be concluded that the model's forecasting performance is statistically better than that of random forecasting. Furthermore, the performance of proposed and previous method can be compared. The forecasting error is rated considering both false alarm and missed market crash.




(a) Back testing ($\alpha = 15$, $\beta = 500$).




(b) Forecasting ($\alpha = 10$, $\beta = 500$).

**Fig. 10.** Alarm index of global crisis after including fit length parameter $l$.

The false alarm is the case when the alarm index represents the high value at no market crash circumstances, whereas the missed market crash is the case when the alarm index does not represent high values although there is a real market crash.

The error diagram in Fig. 11 consists of black, red, and blue lines representing the error of random, proposed, and previous forecasting methods, respectively. The proposed forecasting method includes the parameter in the process of pattern recognition, which uses the different length of learning set for domestic and global crises. Note that the error of proposed forecasting method is calculated based on the maximum value of domestic alarm index and global alarm index. As in Fig. 11, the blue and red lines evolve under the diagonal black line $y = 1 - x$ except one point at the bottom-right of blue line. It means that both forecasting methods show a reasonable forecasting of market crashes. Interestingly, the proposed model proves its statistically stronger forecasting ability than that of previous one since the red line evolves closer to the $y$-axis than blue line. Hence, we can cross-check that the proposed model shows its novelty in the crash forecasting of the Korean financial market.

### *4.3. Trading strategy*

We see the performance of alarm index based on trading strategies versus a simple buy and hold one as well as random strategies as explained for instance in [25].

The effectiveness of our alarm index can be verified by comparing the returns with various investment strategies. Therefore, we create a simple strategy based on the alarm index and moving average as in [26].

Let $\theta$ be the proportion of investing in financial index. Then, it means that $(1 - \theta)$ refers to the proportion of investing in the U.S. treasury bill for obtaining a risk-free profit. To compute the proportion, we create the representative alarm index of

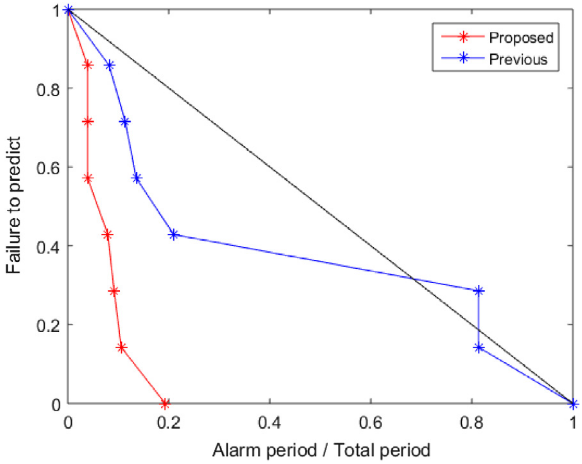



**Fig. 11.** Error diagram comparing previous and proposed models. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

**Table 5**
Investment results of the proposed, buy and hold, and random strategy during the back testing period. The performance of proposed strategy is calculated with different moving average 20, 30, 40, and 60.

| | Proposed strategy | | | | Buy and hold | Random strategy |
|---|---|---|---|---|---|---|
| | 20 | 30 | 40 | 60 | | |
| Cumulated log-return | 2.0159 | 2.0962 | 2.1494 | 2.1284 | 1.8430 | 1.6641±0.4535 |
| Cumulated excess return | 0.7246 | 0.8049 | 0.8581 | 0.8371 | 0.5517 | 0.3728±0.4535 |
| Sharpe ratio | 1.7968 | 1.7304 | 1.7620 | 1.7991 | 1.7629 | 1.7505±0.1867 |

**Table 6**
Investment results of the proposed, buy and hold, and random strategy during the forecasting period. The performance of proposed strategy is calculated with different moving average 20, 30, 40, and 60.

| | Proposed strategy | | | | Buy and hold | Random strategy |
|---|---|---|---|---|---|---|
| | 20 | 30 | 40 | 60 | | |
| Cumulated log-return | 1.1001 | 1.0847 | 1.0659 | 1.0962 | 0.8977 | 0.8099±0.3002 |
| Cumulated excess return | 0.5413 | 0.5258 | 0.5070 | 0.5374 | 0.3388 | 0.2510±0.3002 |
| Sharpe ratio | 1.5333 | 1.5272 | 1.5462 | 1.5264 | 1.4454 | 1.4109±0.4503 |

a financial index, $AI$, such that,

$$AI = \max(AI_D, AI_G) \tag{18}$$

where $AI_D$ and $AI_G$ are the domestic and global crisis, respectively.

If we set $N_{MA}$ be the length of moving average, then the moving average of the representative index, $AI_{MA}$, can be obtained: It can be considered as the relatively consistent proportion for the investment strategy. Therefore, the portfolio can be managed by re-defining $\theta$ be $1 - AI_{MA}$.

In order to examine the performance of trading strategy based on the alarm index, we select Korea Composite Stock Price Index (KOSPI) as investment and simple buy and hold strategy as benchmarks. Note that the random strategy only accepts long positions (buy first sell later) with an average leverage of 0.8 and an average duration per deal of nine days.

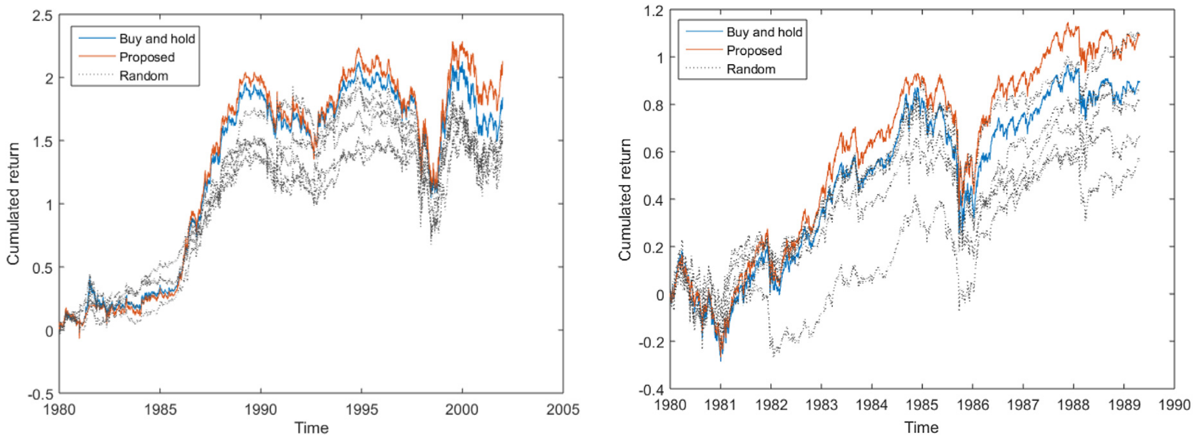
Tables 5 and 6 summarizes the performances of each trading strategies for $N_{MA} = 20, 30, 40, 60$ days. The Sharpe ratio, $S$, is a measure of the excess returns that can be achieved against the risk of an investment strategy. It can be simply defined as,

$$S = \frac{(\bar{R} - R_f)}{\sigma_R} \tag{19}$$

where $\bar{R}$, $R_f$, and $\sigma_R$ are the average returns of the investment, risk-free rate, and standard deviation of returns, respectively.

Since $S$ suggests the excess return on investment assets given the risks investors take, it refers that the higher the Sharpe ratio, the better the investment strategy is.

Table 5 summarizes the investment results of the proposed, buy and hold, and random strategy during the back testing period, whereas Table 6 summarizes the results during the forecasting period. Note that the random strategy is simulated




(a) Back testing period.

(b) Forecasting period.

**Fig. 12.** Cumulated log-return of buy and hold (gray dots), proposed (solid red line), and random strategy (solid blue line) for each period. 5 of 100 random strategy paths are represented and proposed strategy path is calculated with moving average duration $N_{MA} = 20$. Cumulated return of proposed strategy is higher than the others, In other words, proposed strategy outperform the others. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

for 100 times and its mean and standard deviations are higher than the other strategies regardless of the length of moving average, $N_{MA}$. Such results an be found in both back testing and forecasting periods. Furthermore, the Sharpe ratio of the proposed strategy during the forecasting period is higher than that of other strategies. However, the Sharpe ratio of the proposed strategy during the back testing period is only higher than that of other strategies for $N_{MA} = 20, 60$. the investment results of the proposed, buy and hold, and random strategy when $N_{MA} = 60$ are visualized in Fig. 12 .

## 5. Conclusion

In this paper, we propose the new method to forecast the market crash, which can be suitably applied to the Korean stock market. Unlike the financial market of the United States, we recognize that the financial crises, which causes the major market crashes, held in the Korean financial market can be classified in terms of domestic and global crisis. Based on this notion, we employ the JLS model to forecast the date of market crash. Interestingly, the JLS model shows a decent forecasting performance in most of seven market crashes. Furthermore, the model discovers that it requires different best window length to forecast the market crashes in domestic (short) and global crises (long). By incorporating this aspect, we develop the appropriate alarm index for the Korean stock market by utilizing the pattern recognition algorithm with different lengths of windows, and finally reached several findings. At first, the bubble forming period of domestic crisis is different from that of global crisis in case of the Korean stock market. Secondly, because of the first finding, the traditional alarm index requires the length of window as the new parameter to successfully distinguish the domestic and global crisis. Specifically, It aims to develop more robust forecasting of domestic crisis by learning domestic crisis and vice versa. Lastly, the error diagram and results of trading performance prove that the forecasting performance of proposed alarm index is robust and statistically better than that of previous one. We believe that our model can be easily implemented to prevent the upcoming market crashes and financial crises for the countries where the financial crises can be classified in terms of domestic and global crises.

## Acknowledgment

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