

Symmetry energy of dense matter in holographic QCD

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ABSTRACT: We study the nuclear symmetry energy of dense matter using holographic QCD. To this end, we consider two flavor branes with equal quark masses in a D4/D6/D6 model. We find that at all densities the symmetry energy monotonically increases. At small densities, it exhibits a power law behavior with the density, $E_{\text{sym}} \sim \rho^{1/2}$.

KEYWORDS: Gauge/gravity duality, Dense matter.

Contents

1. Introduction	1
2. Nuclear symmetry energy in D4/D6/D6 model	2
3. Summary and Discussion	6

1. Introduction

Nuclear symmetry energy is one of key words in nuclear physics as well as in astrophysics. Its density dependence is a core quantity of asymmetric nuclear matter which has important effects on heavy nuclei and is essential to understand neutron star properties. Although much efforts have been given, it is still very poorly understood especially in the supra-saturation density regime, see [1, 2, 3, 4, 5, 6, 7] for a review and for a recent discussion.

From experimental side, the available data do not constrain much the value of the symmetry energy at supra-saturation densities. Recently, using the FOPI data on π^-/π^+ ratio in central heavy ion collisions, Xiao et al. [8] obtained a circumstantial evidence for a soft nuclear symmetry energy at $\rho \geq 2\rho_0$, where the nuclear symmetry energy increases with the density up to the saturation density ρ_0 and then starts to decrease afterwards. Theoretically, almost all possible tools were employed to study the density dependence of the symmetry energy. While they showed similar behaviors up to the nuclear saturation density, at supra-saturation densities, all possible results one can imagine were predicted and no consensus could be reached: some showed stiff dependence (increasing monotonically with density), while others showed soft one, see Fig. 1 for a typical example. See also [3] for a review. Given this situation, it would be very interesting if we can examine the behavior of the nuclear symmetry energy at high densities with a reliable calculational tool.

The gauge/gravity duality [10, 11, 12] provides a new tool to study strongly interacting dense matter, and a few models for QCD [14, 15] based on the duality were constructed. Although the true holographic dual of QCD is yet to be constructed, it is worthwhile to find out what the new tool says about QCD using available models mimicking the dual of QCD. A way to treat the dense matter in confined phase was suggested in [16], and a model for transition from nuclear matter to strange matter was proposed in [17]. The purpose of this paper is to calculate the symmetry energy of nuclear matter in this model. We will find that the symmetry energy is increasing with the total charge Q , showing that the symmetry energy of our system has a stiff dependence on the density. Also, we will explicitly calculate the density dependence of the symmetry energy at low density to show $E_{\text{sym}} \sim \rho^{1/2}$.

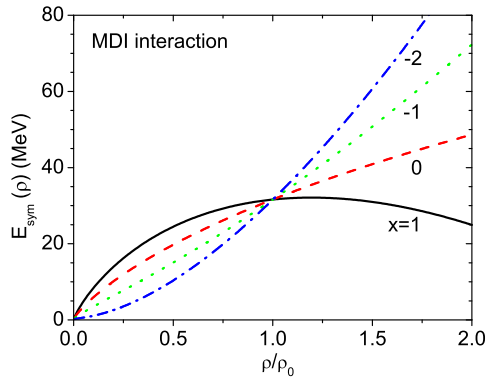


Figure 1: (Color online) Example of density dependence of the nuclear symmetry energy, taken from [9]. Depending on the value of the parameter x , various high density behaviors are possible.

2. Nuclear symmetry energy in D4/D6/D6 model

The nuclear symmetry energy is defined as the energy per nucleon required to change isospin symmetric nuclear matter to pure neutron matter. In the Bethe-Weizsäcker mass formula for the nuclear binding energy, it represents the amount of binding energy that a nucleus has to lose when the numbers of protons and neutrons are not equal. The semi-empirical mass formula based on the liquid drop model has the form:

$$E_B = a_v A - a_a (N - Z)^2 / A - a_c Z^2 / A^{1/3} - a_s A^{2/3} \pm a_\delta / A^{3/4}. \quad (2.1)$$

Here Z (N) is the number of protons (neutrons) in a nucleus. The first term is called the volume energy since the volume of a nucleus is proportional to A , where A is the total nucleon number. The origin of this volume term is the strong nuclear force. The second is known as the asymmetry term, which defines the symmetry energy. If there were no Coulomb repulsions between protons, we would expect to have equal number of neutrons and protons in nuclei in general. The term with a_c accounts for the Coulomb interaction of all pairs of protons in the nucleus. The last two terms represent the surface energy and pairing effect, respectively. Using data for nucleus binding energies, one can determine a set of coefficients in Eq. (2.1).

Due to the invariance of nuclear forces under neutron-proton interchange, iso-scalar quantities in a nuclear system are function of only even powers of the asymmetry factor $\tilde{\alpha}$ defined by $\tilde{\alpha} \equiv (N - Z)/A$. Then we can express the energy density per nucleon $E(\rho, \tilde{\alpha})$ as

$$E(\rho, \tilde{\alpha}) \simeq E(\rho, 0) + S_2(\rho) \tilde{\alpha}^2, \quad (2.2)$$

where ρ is the nucleon number density and $S_2(\rho) = \frac{1}{2} \frac{\partial^2 E}{\partial \tilde{\alpha}^2} |_{\tilde{\alpha}=0}$ is the symmetry energy.

Now we study the symmetry energy in the D4/D6/D6 model with baryon vertices which consist of compact D4 branes and fundamental strings [17]. The gluon dynamics

is replaced by the gravity sourced by the N_c color D4 branes, and two probe D6 branes are used to describe the up and down quarks. The bare quark masses are the distances between the N_c color D4 and two D6's in the absence of the string coupling.

We can write the metric of the confining D4 background as

$$ds^2 = (U/R)^{3/2} (-dt^2 + d\vec{x}^2 + f(U)dx_4^2) + (R/U)^{3/2} (U/\xi)^2 (d\xi^2 + \xi^2 d\Omega_4^2),$$

where $f(U) = 1 - (U_{KK}/U)^3$ and $(U/U_{KK})^{3/2} = (\xi^{3/2} + \xi^{-3/2})/2 \equiv \xi^{3/2}\omega_+/2$.

We wrap the compact D4 brane on S^4 which is transverse to the color D4 brane. Due to the Chern-Simons interaction with RR-field, $U(1)$ gauge field is induced on the D4 brane world volume. The source of gauge field is interpreted as the end point of fundamental strings. Substituting the equation of motion for gauge field to the Dirac-Born-Infeld action of D4 brane with the Chern-Simons, we get the Hamiltonian for the compact D4 brane as

$$\mathcal{H}_{D4} = \tau_4 \int d\theta \sqrt{\omega_+^{4/3} (\xi^2 + \xi'^2)} \sqrt{D(\theta)^2 + \sin^6 \theta}, \quad (2.3)$$

where $\tau_4 = \frac{1}{2^{2/3}} \mu_4 \Omega_3 g_s^{-1} R^3 U_{KK}$, $D(\theta) = -2 + 3 \cos \theta - \cos^3 \theta$ and the prime denotes the derivative with respect to θ . We assume that the radial coordinate ξ depends only on the polar angle θ of S^4 .

The fundamental strings out of the compact D4 branes are attached to two D6 branes, and they provide the source of $U(1)$ gauge field on the D6 brane. By taking the Legendre transformation for the gauge field, we obtain the Hamiltonian which controls the brane configuration with fixed charge.

$$\mathcal{H}_{D6} = \tau_6 \int d\rho \sqrt{1 + \dot{y}^2} \sqrt{\omega_+^{4/3} (\tilde{Q}^2 + \rho^4 \omega_+^{8/3})}, \quad (2.4)$$

where $\tau_6 = \frac{1}{4} \mu_6 V_3 \Omega_2 g_s^{-1} U_{KK}^3$. \tilde{Q} is dimensionless and related to the number of fundamental strings Q by $\tilde{Q} = \frac{U_{KK} Q}{2 \cdot 2^{2/3} \pi \alpha' \tau_6}$. Baryons are represented by compact D4 branes, and each of them has N_c fundamental strings attached. Such configuration of compact D brane plus fundamental strings are called baryon vertex [13]. The other ends of fundamental strings are attached to D6 branes. Therefore, D6 branes are pulled down and compact D4 brane is pulled up. As discussed in [16], the length of the fundamental strings becomes zero since the tension of the fundamental strings is always larger than that of D-branes. Finally, the position of the cusp of D6 branes should be joined to that of the compact D4 brane. We consider Q_1 fundamental strings attached to one of the D6 branes and Q_2 strings to another D6 brane. The final configuration is drawn in Fig. (2). We denote the slope at the cusp of each brane as $\dot{y}_c^{(1)}$ and $\dot{y}_c^{(2)}$. The force at the cusp of D6 branes can be calculated to give

$$\begin{aligned} F_{D6} &= \left. \frac{\partial \mathcal{H}(Q_1)_{D6}}{\partial U_c} \right|_{\partial} + \left. \frac{\partial \mathcal{H}(Q_2)_{D6}}{\partial U_c} \right|_{\partial} \\ &\equiv F_{D6}^{(1)}(Q_1) + F_{D6}^{(2)}(Q_2). \end{aligned} \quad (2.5)$$

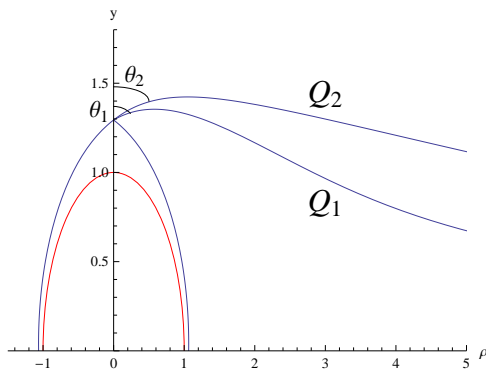


Figure 2: (Color online) Embedding of D-branes with $\alpha \neq 0.5$. The asymptotic heights of two branes are the same ($m_1 = m_2 = 0.1$). The two branes meet at infinity since $m_1 = m_2$. Red curve denotes to the position of U_{KK} .

To make the system stable, following force balancing condition should be satisfied;

$$\frac{Q}{N_c} F_{D4} = F_{D6}^{(1)}(Q_1) + F_{D6}^{(2)}(Q_2), \quad (2.6)$$

where $Q_1 = (1 - \alpha)Q$ and $Q_2 = \alpha Q$ with $0 \leq \alpha \leq 1$, and F_{D4} is the force at the cusp due to the compact D4 brane. Note that $\alpha = (1 - \tilde{\alpha})/2$.

To find the ground state of our system, we need to consider the energy minimization together with the force balancing condition. The total energy of our system is

$$E_{tot} = \frac{Q}{N_c} \mathcal{H}_{D4} + \mathcal{H}_{D6}(Q_1) + \mathcal{H}_{D6}(Q_2). \quad (2.7)$$

With given Q and α we can calculate the configuration satisfying the force balancing condition. The energy density per nucleon $E(\rho, \alpha)$ is given by E_{tot} in Eq. (2.7) divided by the total baryon number Q/N_c .

The symmetry energy is the coefficient of leading term of $\tilde{\alpha}$ and is a function of Q . Since Q is proportional to the density of the quark or baryon, this way we can calculate the the symmetry energy as function of density. If we further impose minimization of the total energy, we can determine the value of α as a function of Q . For $m_2/m_1 \neq 1$, there exists a transition from a matter with $\alpha = 0$ to $\alpha \neq 0$ at a finite value of Q . This is identified as a transition from nuclear to strange matter. For very large Q , α saturates to 0.5, as expected. If we take $m_1 = m_2$ and if we do not consider isospin violating interactions or electromagnetic interactions, then the ground state of the matter would be always with $\alpha = 1/2$.

The explicit form of symmetry energy per nucleon can be written as

$$S_2 = \frac{2\tau_6}{N_B} \int d\rho \frac{\sqrt{1 + \dot{y}^2} \tilde{Q}^2 \omega_+^{10/3} \rho^4}{(\tilde{Q}^2 + 4\omega_+^{8/3} \rho^4)^{3/2}}, \quad (2.8)$$

where y is the embedding solution of D6 brane with $\alpha = 1/2$. Notice that $N_B = Q/N_c$ and so the symmetry energy (2.8) contains N_c factor. We need to factor this N_c out for the reason we discuss in Section 3. Our results are given in Fig. 3.

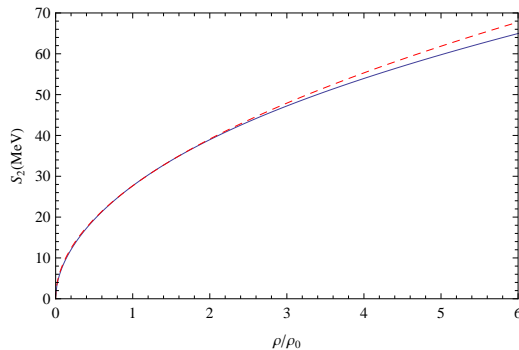


Figure 3: (Color online) Solid line is the calculated symmetry energy as a function of the density. For illustration purpose, we take $\lambda = 6$ and $M_{KK} = 1.04$ GeV. The dotted line is for $S_2 \sim \rho^{1/2}$.

Note that so far we use ρ for both the coordinate and the density. Hereafter ρ is only for the density. Although our main point here is the calculational scheme of the symmetry energy rather than the exact numerical numbers, it is better to see how the numbers fit to the reality. To fix the energy scale, we rewrite the energy density per nucleon as

$$E(\rho, \alpha) = \frac{\lambda N_c M_{KK} \tilde{E}}{2^{2/3} (9\pi) \tilde{Q}}, \quad \rho = \frac{2 \cdot 2^{2/3}}{81 (2\pi)^3} \lambda M_{KK}^3 \tilde{Q}, \quad (2.9)$$

where $\tilde{E} = E_{tot}/\tau_6$. One may determine the values of λ and M_{KK} by using the quark mass and meson mass as inputs into two basic relations: $m_q = \lambda M_{KK} y_\infty / (2^{2/3} 9\pi)$ and $M_{\eta'}^2 = 0.46 M_{KK}^2 y_\infty$ with our coordinate choice. From the the non-anomalous η' mass, ~ 390 MeV [20], and the quark mass, $M_q \sim 41$ MeV,¹ we can determine the parameters of the D4/D6 model: $M_{KK} = 1.04$ GeV [22] and $\lambda = 6$. In this case, $\tilde{Q} \sim 1.2$ corresponds to the normal nuclear matter density ρ_0 .

Now, we lay emphasis on two aspects of our results, which are rather insensitive to the choice of λ and M_{KK} . One is the stiffness of the symmetry energy S_2 in supra-saturation density regime, and another is its low density power law behavior $S_2 \sim \rho^{1/2}$.

The power law behavior of S_2 in low density can be understood by calculating analytically in a special limit, $m_q \rightarrow \infty$ and $\rho \rightarrow 0$. In this case, the solution of D6 brane embedding becomes trivial, $y = 0$ and we can integrate (2.8) analytically to have

$$S_2 = \left(\Gamma\left(\frac{5}{4}\right) \right)^2 \sqrt{\frac{\lambda \rho_0}{2 M_{KK}}} \sqrt{\frac{\rho}{\rho_0}}. \quad (2.10)$$

The current experimental result of the symmetry energy can be summarized by a fitting formula

$$S_2(\rho) = c(\rho/\rho_0)^\gamma \quad (2.11)$$

¹As well-known, the quark mass in D4/D6 model could be different from that in QCD by a constant factor. To obtain the constant we need to compare the scalar two point function obtained in D4/D6 model with that in the operator product expansion of QCD, for example see [21].

with $c \simeq 31.6$ MeV and $\gamma = 0.5 - 0.7$ in the low density regime, $0.3\rho_0 \leq \rho \leq \rho_0$, see [5] for example. With our choice of λ, M_{KK} , we obtain $\gamma \simeq 0.5$ and $c \simeq 27.7$ MeV from Eqs. (2.8) and (2.9). Notice that the value of γ in our results is rather insensitive to the value of λ and M_{KK} , while the value of c depends on them.

One can understand the stiffness based on the property of the branes: suppose two D6 branes meet with compact D4 at a point with different polar angles θ_1 and θ_2 as drawn in Fig. 2. Here Q_1 and Q_2 with $Q_1 + Q_2 = Q$ fixed are the number of the strings attached to each D6 brane. The force balance condition is nothing but the minimization of the action with respect to the position of the contact point. To balance the pulling force of the compact D4, the position of the contact point is located when each D6 brane balances roughly half of the pulling force, which is a statement supported from the numerical analysis. Since the upward force is proportional to the product of effective tension, $\sim \tau_6 Q$, times the projection factor to the vertical direction, $\cos \theta$, we have $Q_1 \cos \theta_1 \sim Q_2 \cos \theta_2$. If $\theta_1 > \theta_2$, $Q_1 > Q_2$. Namely, more strings should be attached to the lower brane in the figure 2. Therefore, the asymmetry in the number of the attached string is due to the angle difference. Notice that each end point of the string provides the ‘electric’ flux which contributes to the energy of the brane. Since the flux of charge Q_i is confined in each brane, the total energy is $f(Q_1) + f(Q_2)$, where $f(Q) = \mathcal{H}_{D6}(Q)$. Since f is a monotonically increasing function, the minimization of the energy with respect to the variation of Q_1 requests $Q_1 = Q_2 = Q/2$. As Q increases, the effective brane tension increases and so maintaining the angle difference costs more and more energy. Furthermore, since $f''(Q) \sim 1/\sqrt{Q}$ is positive, the symmetry energy, $Q^2 f''(Q/2)/N_B$, is a increasing function.

The Coulomb repulsion discussed above is, of course, not the electromagnetic one. The local $U(1)$ charge is holographic dual to the the global baryon number of the boundary theory. However, the repulsion in the dual bulk theory means repulsion in 4 dimension as well. From the boundary theory point of view, such repulsion is simply due to the presence of the baryon charge. So the origin of the repulsive nature is mysterious from the boundary point of view. To understand this, we notice two facts: The first one is that the charge carriers are fermions since the charge is introduced by the D4/D6 fundamental string end points, not from the bulk R-charge of type IIA gravity. The second is that in the boundary theory the tendency of $N = Z$ by the Pauli principle, while in the holographic dual bulk theory it is the Coulomb interaction that requests $N = Z$. Since two origins should be the same, we may suggest the Coulomb repulsion as the holographic Pauli principle. See [18] and [19] for similar observations in a different context.

Finally, we study the effect of small isospin violation by considering $m_1 \neq m_2$. We find in this case that the symmetry energy is almost the same with the case with isospin invariance for the mass ratios of order one.

3. Summary and Discussion

In summary, we calculated the symmetry energy of dense matter in the D4/D6/D6 model. To obtain the symmetry energy in nuclear matter with charge symmetry, we considered the case with $m_1 = m_2$ and found that the symmetry energy is increasing with the total

charge Q , showing a stiff nuclear symmetry energy. It is universal in the sense that the result is independent of the value of λ and M_{KK} . We also studied the low density behavior with power γ to be $\sim 1/2$, which is again independent of the value of λ and M_{KK} and is close to the value suggested by experiments, $\gamma = 0.5 - 0.7$.

One subtle point we mentioned in the main text was about the factor N_c . The reason we divided this factor out is as follows. In our model, the same flavor quarks form a nucleon; for instance, proton in our model consists of $N_c m_1$ quarks and neutron has $N_c m_2$ quarks in it. Hence, the total number difference of quarks is N_c times the number difference of neutrons and protons, resulting in the overall N_c factor in the symmetry energy. However, in reality, where $N_c = 3$, proton consists of two up quarks and one down quark, and neutron contains one up quark and two down quarks. The total number difference of quarks is the same as the number difference of neutrons and protons. Therefore, in order to compare our result with the realistic case, we have to divide the symmetry energy (2.8) by N_c .

We also studied the effect of isospin violation by considering $m_1 \neq m_2$. However, the symmetry energy in this case turned out to be almost the same with the case with isospin invariance for the mass ratios of order one.

Now, we list some generic cautionary remarks. In holographic approaches, it is not clear how to encode attractive scalar contributions that are essential to describe nuclear matter. In addition, for extreme large density limit, the back-reaction effect from the dense matter is not negligible, and so it should modify our result.

Finally, we comment on a future investigation. In conventional approaches, for instance see [6, 25], at very low densities $\rho \ll \rho_0$, the dominant contribution to the symmetry energy is coming from the kinetic energy which encodes the Pauli principle. This is because the kinetic contribution to the symmetry energy is $\sim \rho^{2/3}$, while the one from interactions starts from $\sim \rho^1$ due to the linear density approximation which works well at very low density. The origin of the factor $\gamma = 2/3$ is the dispersion relation $E \sim p^2$ together with the sharp Fermi surface. In our case, the fact $\gamma = 1/2$ suggests that either the dispersion relation is anomalous like $E \sim p^{3/2}$ or Fermi surface is fuzzy [23, 24] due to the strong interaction. This poses an interesting future study.

Acknowledgments

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