Inflation-linked Korea treasury bonds as a strategic asset

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Abstract

The Korean government introduced its first inflation indexed bond, the Inflation-Linked Korea Treasury Bond (hereafter KTBi), in March 2007. This paper investigates the role of KTBi as a strategic asset in a nominal asset portfolio by estimating a bivariate GARCH model with conditional correlation and by conducting spanning tests. Estimation of the bivariate GARCH model reveals that market information such as the yield curve slope and yield spread between KTBi and KTB are useful in predicting the correlation between the returns of KTBi and KTB as well as the level of the returns of these two assets.

Unconditional and conditional spanning tests produce different results regarding the potential role of KTBi as a strategic asset. While unconditional spanning tests do not reject the null hypothesis that existing assets span KTBi, the same null hypothesis is strongly rejected by conditional spanning tests. Such a result means that KTBi is capable of improving the mean-variance efficiency when added to existing investment portfolios.

Keywords: inflation-linked bond, spanning test, strategic asset, conditional correlation

JEL Classification: G11, G19, E43

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1 Introduction

The Korean government introduced its first inflation indexed bond, the Inflation-Linked Korea Treasury Bond (hereafter KTBi), in March 2007. In general, bonds pay principal and interest fixed in nominal terms. As a result, real returns from holding bonds decline when unexpected inflation occurs. To counter this, inflation indexed bonds are designed to offer a fixed real interest rate by linking principal and coupon payments to actual inflation rate.

Inflation indexed bonds can be beneficial to investors and issuers in a few ways. They provide investors with a means of hedging inflation risk and securing a fixed real rate of return. They can also help governments reduce their debt service cost by satisfying investors' demand for debt instruments whose returns are fixed in real terms. In addition, they are helpful in estimating inflation expectation as the yield difference between nominal bonds and inflation indexed bonds with adjustment for risk premium and liquidity premium can serve as a measure of inflation expectation of investors.

In addition, researchers and investors have also been paying attention to another benefit of inflation-indexed bonds, that is, their role as a strategic asset to be included in nominal asset portfolios. If inflation-indexed bonds have a risk profile quite different from that of nominal bonds, portfolio managers can achieve a statistically significant expansion in the mean-variance frontier by incorporating inflation-indexed bonds into their diversified asset portfolios. If this is the case, inflation-indexed bonds are useful as a new asset class, which implies that they should not be grouped together with nominal assets but should be treated as an entirely separate asset class.

The possibility of inflation indexed bonds to have a meaningful role as a new asset class depends on the correlation of their returns with those of nominal bonds included in the nominal asset portfolio. In particular, a low correlation between returns of inflation indexed bonds and returns of nominal bonds may provide a strategic opportunity to construct portfolios that have superior mean-variance characteristics. As a result, this paper starts its empirical analysis by investigating the predictability of the correlation between the returns of KTBi and KTB (Korea Treasury Bonds) using readily available market information such as the slope of the nominal yield curve and the spread between KTB and KTBi yields.

The paper employs spanning tests to investigate if KTBi is capable of enhancing the mean-variance efficient frontier of a diversified asset portfolio. If we summarize the findings from the spanning tests, the unconditional spanning test reveals that adding KTBi to the benchmark asset portfolio consisting of equities, KTBs, corporate bonds, certificates of deposits (CDs) and real estate cannot enhance the mean-variance frontier. The conditional spanning test, however, demonstrates that the spanning hypothesis is rejected for every benchmark portfolio examined, implying that KTBi constitutes a meaningful new asset class for investors who want to achieve mean-variance efficiency in nominal terms.

The organization of the paper is as follows. Section 2 provides a brief survey of the literature that has explored the role of inflation-indexed bonds as a strategic asset. Section 3 presents a simple introduction to the Inflation-Linked Korea Treasury Bond. Section 4 investigates the predictability of the correlation between the returns of KTBi and KTB using a bivariate GARCH model. In Section 5, we conduct spanning tests to examine whether the mean-variance frontier can be expanded by adding KTBi to existing asset portfolios. Section 6 concludes.

2 Previous literature

Since the introduction of Treasury Inflation-Protected Securities (TIPS) in the United States in 1997, several studies have tried to explore the diversification benefits of TIPS. Depending on the approach and the sample, these studies produced mixed results:

Phoa (1999) adopts a portfolio approach to investigate whether inflation-indexed bonds are capable of serving as a new asset class. More specifically, he calculates what weight TIPS should have in an asset portfolio comprising of equites and what effect inclusion of TIPS has on the expected return of the portfolio. He finds that TIPS has little asset allocation effect in the sense that incorporating TIPS into an efficient nominal asset portfolio fails to expand the efficient frontier significantly even under a scenario in which the TIPS returns are assumed to have low volatility. The fact that TIPS returns have a high correlation with stock returns rather than bond returns while their returns are lower than those of nominal bonds can explain the insignificant asset allocation effect of TIPS.

This result is in contrast to Lamm (1998) who shows that that depending on the assumptions about the return volatility and the correlation, it is possible for TIPS to have a stronger portfolio effect than nominal bonds. Kothari and Shanken (2004) and Roll (2004) also investigate asset allocation among stocks, TIPS, Treasury bonds, and a riskless asset using a mean-variance framework.

Other researchers take a different approach of employing spanning tests to explore if inflation-indexed bonds constitute a new asset class for investors. Hunter and Simon (2002) point out, based on their estimation of the conditional Sharpe ratios, that the U.S. TIPS has superior volatility-adjusted returns relative to nominal Treasury bonds through their first four and a half years of introduction. Such a finding implies that there is room for improving the mean-variance frontier by adding TIPS to an asset portfolio consisting of nominal Treasury bonds only. Their spanning tests also confirm this conjecture. The results of both of unconditional and conditional spanning tests, however, display that investors cannot achieve significant diversification benefits when the U.S. TIPS is added to a well-diversified portfolio consisting of nominal bonds, Treasury bills, and stocks.

Adopting a longer sample covering the period from February 1997 to August 2005 and applying conditional spanning tests, however, Mamun and Visaltanachoti (2005), reach the opposite conclusion. They investigate if the U.S. TIPS and the U.K. Inflation-Linked Gilts (ILGs) can serve as meaningful strategic assets in a portfolio comprising S&P 500 stocks, Treasury bills, Treasury bonds, corporate bonds, and REITs, and find that unconditional spanning tests cannot reject the spanning hypothesis for both of the inflation-linked bonds. They argue that this result is in line with expectation given the high correlation between the TIPS returns and the Treasury bond returns. On the other hand, conditional spanning tests strongly reject the hypothesis of spanning, which means that investors experience statistically significant diversification benefits from including TIPS or ILGs in their asset portfolio and utilizing market information to manage the portfolio.

Although it is from a different perspective, spanning tests have also been adopted by Martellini and Milhau (2014) in analyzing the gains from including inflation indexed bonds in the portfolio of long-term investors facing inflation-linked liabilities. Using formal intertemporal spanning tests, they find that interest risk dominates inflation risk so much that introducing or removing inflation-index bonds from liability-hedging portfolios has relatively little impact on investors' welfare.

Previous empirical literature thus shows that depending on the sample period and the country spanning tests give different results regarding whether inflation indexed bonds can serve as a meaningful new asset class in a well-diversified portfolio. As a result, whether KTBi can serve as a meaningful strategic asset should be subject to empirical tests. Since the first KTBi was issued in 2007, empirical studies with KTBi are rather rare. So far, the only attempt has been made by Han (2010). His study shows that the hypothesis of unconditional spanning cannot be rejected but that the hypothesis of conditional spanning can be rejected. Such a result can be interpreted to imply KTBi is capable of expanding the efficient frontier depending on the composition of assets in the initial portfolio. This study, however, covers a sample period of only 38 months and only one issue of KTBi and as a result, the robustness of these findings need to be confirmed by studies covering a longer sample and different issues of KTBi.

In this study, we investigate the role of KTBi as a strategic asset using a longer sample period covering 115 months from March 2007 to September 2016 and using a KTBi index covering at least two different issues of KTBi.

3 Inflation-linked Korea treasury bonds

The Korean government introduced the Inflation-Linked Korea Treasury Bond (KTBi) for the first time in March 2007. KTBi is an inflation-linked bond whose principal and coupon payments are linked to inflation rate. Its principal is adjusted by the consumer price index (CPI) announced by Statistics Korea every month. Then, coupon payments are determined by multiplying the coupon rate to the inflation-adjusted principal amount. The amount of principal paid at maturity is the product of the face value of the bond multiplied by the cumulative change in CPI[1].

KTBi adopts the Canadian model to compute accrued interest. Ideally, interest payments for a given period have to be calculated reflecting the actual inflation rate during the same period. Inflation indexed bonds, however, are subject to indexation lags for the following two reasons. First, inflation statistics can be calculated and reported only with a delay. In Korean, for example, the CPI for a certain month is announced at the beginning of the next month. Second, by convention, bond trades in the secondary market made between the bond's coupon dates entail payment of accrued interest from the previous coupon date. In order to determine the amount of accrued interest, the amount of current coupon payment should be fixed in advance. The amount of current coupon payment for inflation indexed bonds is determined by the coupon rate (real interest rate) and the compensation for the current period inflation, which is not known until the end of the current coupon payment period. In consequence, in order to fix the nominal amount of the current coupon for indexed bonds with semi-annual coupon payment, at least a six-month time lag is unavoidable. Combining these two lags, conventional inflation-linked bonds used to carry an indexation lag of 8 months. 'The Canadian model' reduces the indexation lag to 3 months by adopting a different method of determining the accrued interest. (Price, 1997)

When KTBi was first introduced, its demand was lackluster perhaps possibly due to low market liquidity. In 2008, after experiencing failure in issuing KTBi as the bid to coverage ratio fell short of 1.0 in successive auctions, issuance of KTBi was temporarily halted until 2010. In June, 2010, issuance of KTBi resumed. This time, the government introduced a few measures to boost the demand for KTBi. One of the

1 Inflation-linked bonds whose principal and coupon payments are adjusted by inflation rates are termed as capital-indexed bonds. In contrast, in interest-indexed bonds, adjustment for inflation is achieved through changes in coupon payments only.

measures was introducing guarantee on the principal amount. Since the principal amount is adjusted based on changes in CPI, it is possible that the amount of principal paid at maturity to fall short of the face value if deflation persists.

When issuance of KTBi resumed in 2010, the demand for KTBi was strong for two reasons. First, global commodity price hikes and expansionary monetary policies were driving inflation expectation higher. Second, investors were attracted by the tax benefit provided by KTBi. The tax benefit from KTBi arises from the fact that adjustments in the principal amount is not subject to taxation.

Reflecting its popularity, issuance of KTBi has been active since its issuance was resumed in 2010. The outstanding amount of KTBi, which stood at less than 2 trillion won during the first three years since its introduction in 2007 has increased steadily since 2010 to reach 11.1 trillion won at the end of 2016.

Figure 1. Yields to Maturity and Break-even Inflation Rate

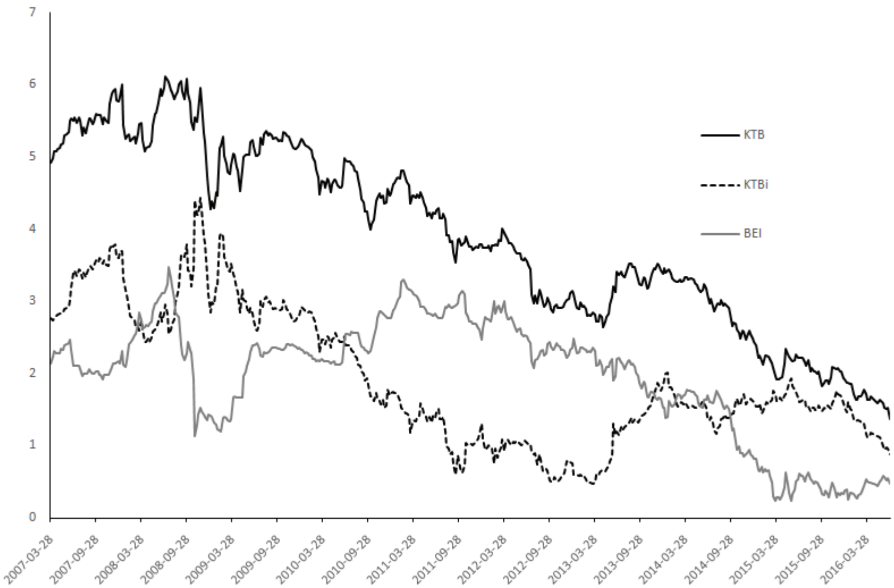

Figure 1 shows the yield to maturity of KTBi and that of KTB with 10-year maturity. The KTBi yield can be regarded as the real interest rate whereas the KTB yield is the nominal interest rate. Figure 1 also shows the difference between these two yields, which is also known as the break-even inflation (BEI) rate. The yields to maturity of both KTB and KTBi in general show a downward trend during the sample period but their movements do not necessarily coincide with each other, suggesting that there may be room for risk diversification by including KTBi in the asset portfolio that includes KTB. Changes in inflation expectations and changes in tax benefits may be responsible for the discrepancies in their movements.

4 Time varying correlation between KTBi and KTB returns

We use a bivariate conditional correlation GARCH model to estimate the conditional means and variances of KTBi and KTB returns and their conditional correlations (Bollerslev, 1990). The model is specified as follows.

$$R_{Nt} = \beta_{N0} + \beta_{N1}R_{Nt-1} + \beta_{N2}R_{Pt-1} + \beta_{N3}YC_{t-1} + \beta_{N4}SP_{t-1} + \varepsilon_{Nt} \quad (1)$$

$$R_{Pt} = \beta_{P0} + \beta_{P1}R_{Pt-1} + \beta_{P2}R_{Nt-1} + \beta_{P3}YC_{t-1} + \beta_{P4}SP_{t-1} + \varepsilon_{Pt} \quad (2)$$

$$\begin{bmatrix}\varepsilon_{Nt}\\ \varepsilon_{Pt}\end{bmatrix} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\sigma_{Nt}^2 & \sigma_{NPt}\\ \sigma_{NPt} & \sigma_{Pt}^2\end{bmatrix}\right) \quad (3)$$

$$\sigma_{Nt}^2 = \alpha_{N0} + \alpha_{N1}\varepsilon_{Nt-1}^2 + \alpha_{N2}\sigma_{Nt-1}^2 \quad (4)$$

$$\sigma_{Pt}^2 = \alpha_{P0} + \alpha_{P1}\varepsilon_{Pt-1}^2 + \alpha_{P2}\sigma_{Pt-1}^2 \quad (5)$$

$$\sigma_{NPt} = (\gamma_0 + \gamma_1 YC_{t-1} + \gamma_2 SP_{t-1})\sigma_{Nt}\sigma_{Pt} \quad (6)$$

In these equations, subscripts N and P represent KTB and KTBi, respectively. Thus, R_{Nt} and R_{Pt} stand for the holding period rate of return for KTB and KTBi, σ_{Nt}^2 and σ_{Pt}^2 stand for the time-varying variance of ε_{Nt} and ε_{Pt}, respectively, and σ_{NPt} stands for the covariance between of ε_{Nt} and ε_{Pt}.

Equations (1) and (2) correspond to the conditional mean equations for KTB and KTBi. The conditional mean is specified as a linear function of a constant, the first lag of the dependent variable, the first lag of the other dependent variable, the first lag of the slope of the yield curve (*YC*) and the first lag of the yield spread between KTB and KTBi (*SP*). *YC* is computed as the yield difference between the 10-year maturity KTB and the 1-year maturity KTB while *SP* is computed as the yield difference between 10-year maturity KTB and 10-year maturity KTBi.

The yield curve slope is included as an explanatory variable in the mean equation because previous studies on term structure have found that a steeper yield curve is followed by higher returns on longer maturity nominal bonds driven by increase in prices. To the extent that the subsequent increase in nominal bond prices is driven by the real interest component of nominal interest rates, a steeper yield curve may be associated with higher subsequent returns of inflation-linked bonds. In consequence, by examining the coefficients of the yield curve slope in both of the mean equations, we can determine whether changes in KTB returns are mainly driven by changes in real interest rates or changes in inflation expectations.

The spread between KTB and KTBi corresponds to the break-even inflation rate, which can be interpreted as the market expectation of future inflation rate adjusted for inflation risk premium and liquidity premium. The effect of a rise in the level of break-even inflation rate on the returns of KTB and KTBi returns depends on the direction of subsequent changes in the real interest rate component of the nominal interest rate. For example, if the subsequent movement in nominal interest rates is mostly due to change in real interest rates rather than inflation expectation, the effect of changes in the level of break-even inflation rate on KTB and KTBi returns will be of the same sign.

Equations (4) and (5) model the time-varying variance of the returns of KTB and KTBi as a standard GARCH(1,1) process. The conditional covariance between the returns of KTB and KTBi is specified by equation (6), which states that the time varying component of the correlation between the returns of KTB and KTBi depends on the slope of the yield curve and the yield spread between KTB and KTBi. The correlation between the returns of KTB and KTBi depends on the extent to which changes in nominal interest rates are driven by real interest rate changes and inflation expectation changes. When inflation expectation plays a more dominant role, the correlation between KTB and KTBi will be lower. On the other hand, when real interest changes play a dominant role in nominal interest rate fluctuations, the returns of KTB and KTBi tend to move together, resulting in a higher level of correlation.

The effect of a change in the slope of the yield curve on the correlation between the returns of KTB and KTBi, on the other hand, depends on whether nominal interest changes reflect real interest rate changes to greater extent when the yield curve is flatter. This in turn relies on whether the Bank of Korea adjusts short-term interest rates more aggressively when its monetary policy is in the tightening mode. Since yield curves tend to be flatter at the peak of business cycle booms, a flatter yield curve is likely to be followed by a contractionary monetary policy that raises both short-term nominal interest rates and real interest rates. Thus, a flatter yield curve should be associated with a higher correlation between KTB and KTBi returns if the Bank of Korea tries to change real interest rates more aggressively when the monetary policy is in the tightening mode, leading to a negative value for the coefficient of the *YC* in equation (6).

We use daily data from March 21, 2007 to June 30, 2016 to estimate the bivariate GARCH model. The KTBi returns are computed from the KTBi index, which is constructed as the market price of the KTBi portfolio that is designed to include at least two different series of KTBi at any time. The market price data was obtained from Korea Asset Pricing[2].

Table 1 shows the summary statistics for the returns of KTB and KTBi and the instrumental variables included in the GARCH model. The KTB returns and the KTBi returns averaged 7.2% and 6.7% respectively during the sample period. Besides the KTB returns have a

[2] The authors are thankful to Dr. Kinam Park at Korea Asset Pricing for constructing and providing the KTBi index and the corresponding KTB index.

lower standard deviation than the KTBi returns, resulting in higher unconditional Sharpe ratio for KTB. The contemporaneous unconditional correlation between KTBi and KTB returns is 0.41.

Table 1. Summary Statistics

	Mean	Std. Dev.	Min	Max
KTBi	0.0666	0.9071	-6.5488	5.9365
KTB	0.0722	0.8466	-5.5321	4.4386
YC	0.0086	0.0083	-0.0004	0.0326
SP	0.0197	0.0079	0.0020	0.0349

Note: Returns are annualized daily rates of returns.

The bivariate GARH model is estimated by the maximum likelihood method and the results are presented in Table 2. The conditional mean equation displays that both of the returns of KTB and KTBi are significantly affected by the lagged own return and the lagged return of the other, and lagged yield curve slopes but not by lagged yield spreads. In particular, the results demonstrate that both of the KTB and KTBi returns tend to be higher when the yield curve is steeper. This means that

Table 2. Estimation Result of the bivariate GARCH model

			Conditional Mean Equation		
	Constant	KTB(-1)	KTBi(-1)	YC(-1)	SP(-1)
KTB	0.0948**	0.1041**	-0.0754**	2.4416*	-2.0187
	(2.6155)	(3.8321)	(-3.3515)	(1.1670)	(-1.2683)
KTBi	0.0569*	-0.1706**	0.2052**	5.1427**	-1.7146
	(1.8540)	(-6.8982)	(7.6559)	(2.6327)	(-1.1786)
			Variance Equation		
	Constant	Squared Error	Variance		
KTB	0.1099**	0.9538**	0.2652**		
	(19.4214)	(32.9609)	(25.5672)		
KTBi	0.1317**	0.9281**	0.3407**		
	(22.4281)	(27.1008)	(32.1739)		
			Conditional Correlation Equation		
	Constant	YC(-1)	SP(-1)		
KTB & KTBi	0.5193**	10.6830**	7.4358**		
	(28.9629)	(30.3529)	(14.0429)		

Note: * And ** denote that the coefficients are different from zero at 10% and 5% significance level.

a steeper yield curve is associated with higher returns on long-term nominal bonds as well as long-term inflation indexed bonds in Korea. Such a result can be interpreted to mean that during the sample period changes in the real interest rates rather than changes in inflation expectations was the driving force of nominal interest rates.

The estimates for conditional variance equations show the usual relations found in high-frequency financial data. The conditional volatility of the returns of both KTB and KTBi tends to rise when lagged squared return innovations increase, but their impact on current volatility is much less than that of the lagged variance.

The estimation results for the conditional correlation equation indicate that market information is useful in predicting the correlation between KTB and KTBi returns. Both the lagged slope of the yield curve and the lagged yield spread affect the correlation significantly and in a positive direction. A steeper yield curve is associated with a higher conditional correlation of returns, which implies that the real interest component of nominal rate changes is smaller when the yield curve is flatter. Such a result can be interpreted to mean that the Bank of Korea is perceived to be more active and ready to change real rates when it is in the easing mode and trying to stimulate the economy than when it is in the tightening mode.

An increase in the yield spread between KTB and KTBi is also associated with a higher correlation between KTB and KTBi returns. This may reflect the possibility that when the breakeven inflation rate is high and market participants and policy makers are more concerned about the threat of inflation, interest rate movements largely reflect real interest rates changes rather than changes in inflation expectations.

Table 3 reports the sample statistic of conditional Sharpe ratios computed using the coefficient estimates presented in Table 2. Despite the lower expected return of nominal bonds, the volatility adjusted conditional returns are higher for KTB than for KTBi. This is not striking because during most of the sample period, the inflation rate remained low leading to lower inflation expectation. If it were not for the benefit of KTBi tax exemption for the inflation linked part of the interest payment, KTBi may have been a worse deal for investors.

Table 3. Statistic based on the GARCH model

	Sample Mean	Standard Dev.	Minimum	Maximum
KTBi return	0.0686	0.1228	-1.0864	0.7057
KTB return	0.0785	0.0591	-0.1714	0.4850
KTBi standard deviation	0.8008	-	0.3893	3.4674
KTB standard deviation	0.7751	-	0.4305	2.3859
Conditional correlation	0.7575	-	0.5737	0.9810
KTBi conditional Sharpe ratio	0.0959	0.1445	-0.6805	0.7157
KTB conditional Sharpe ratio	0.1117	0.0786	-0.2414	0.4959

Although KTBi has a lower average conditional Sharpe ratio than KTB over the sample period, whether adding KTBi to a portfolio comprising of nominal securities significantly expands the investor's mean-variance frontier needs to be tested formally. In the next section, we address this question through conducting spanning tests.

5 Spanning tests

5.1 Unconditional spanning test

Assume that there are K benchmark assets and N new test assets. We can say that the K assets "span" the K + N assets when the efficient frontier of the K benchmark assets and that of the K + N assets are the same. If the spanning test results reject the hypothesis that the K + N assets are spanned by the K assets, we conclude that the N new assets can function as additional strategic assets.

Let R_{1t} be the vector of returns of the K benchmark assets and R_{2t} be the vector of returns of the N test assets. If R_t is the vector of returns of the K + N risky assets at time t, its mean and variance-covariance can be specified as follows:

$$R_t = \begin{bmatrix} R_{1t} \\ R_{2t} \end{bmatrix}$$

$$E[R_t] = \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \qquad (7)$$

$$Var[R_t] = V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

Mamun and Visaltanachoti (2005) set up the following regression equation to examine if the test assets can be spanned by the benchmark assets:

$$R_{2t} = \alpha + \beta R_{1t} + \varepsilon_t \ . \qquad (8)$$

In this equation, α is a size N vector of constants and β is an N×K matrix of coefficients. According to Huberman and Kandel (1987), the necessary and sufficient condition for spanning is satisfied when the following restrictions on the coefficients in equation (8) are met:

$$H_0 : \alpha = 0_N, \ \delta = 0_N, \qquad (9)$$

where 0_N is the size N vector of zero's and $\delta = \beta 1_K - 1_N$. Here, 1_K and 1_N denote the size K and size N vector of one's respectively. Therefore, the spanning test is a joint test of the hypothesis that the constant α is equal to 0 and the hypothesis that the sum of slope coefficients (the sum of the elements of β) is equal to 1. The null hypothesis indicates that the benchmark assets span the yield space of the combination of the benchmark assets and the test assets. Rejection of the null hypothesis implies that one can move the mean-variance efficient frontier significantly upwards by adding the test assets to the benchmark assets.

The coefficients of the regression model are estimated by the OLS method. The Wald test of coefficient restriction is adopted to test the null hypothesis.

5.2 Conditional spanning test

Unconditional spanning tests do not take into consideration the fact that investors try to make use of market information in managing their asset portfolio. However, there is evidence that market information is useful in predicting asset returns to some extent. In this case, improvement in mean-variance efficiency can be achieved by making use of such information. Ferson and Siegel (2001) and Hunter and Simon (2002) argue that conditional spanning tests are more consistent with the real life practice of active portfolio management by investors who construct and manage their portfolios using available market information.

Although adopting a conditional spanning approach is more appropriate, a disadvantage of this method lies with the fact that the dimension of the estimation and testing problem increases quickly. In order to circumvent the problem with dimension, this study adopts the method suggested by Shanken (1990) and Ferson and Schadt (1996). They assume that the variances and the covariances are constant and that α and β in equation (8) are linear functions of instrument variables. Denoting the vector of instrument variables as z_t, we can specify α and β as a linear function of as z_t as follows:

$$\alpha_i = a_{i0} + z'_{t-1}a_{i1}$$
$$\beta_i = b_{i0} + z'_{t-1}b_{i1}. \tag{10}$$

Then, substituting equation (10) for α and β in equation (8), the i-th row of equation (8) can be written as follows:

$$R_{2t,i} = a_{i0} + z'_{t-1}a_{i1} + b_{i0}R_{1t} + (z'_{t-1}b_{i1})R_{1t} + \varepsilon_t \tag{11}$$

DeRoon and Nijman (2001) show that tests of spanning under all economic conditions are equivalent to testing the following restrictions on the parameters in equation (11).

$$H_0: \ a_{i0} = 0, \ b_{i0}\iota_K = 1, \ a_{i1} = 0, \ b_{i1} = 0 \tag{12}$$

Conducting a conditional spanning test to find if the U.S. TIPS can serve as a strategic asset, Mamun and Visaltanachoti (2005) use the lagged yield curve slope, the lagged yield spread between Treasury bonds and TIPS, the lagged return of TIPS and the lagged return of the S&P 500 composite as instrumental variables. This study adopts similar instruments to perform conditional spanning tests.

5.3 Data

In this study, five assets are selected as the benchmark assets including stocks, CDs, KTBs, corporate bonds, and real estate[3]. Investors are assumed to pursue a buy-and-hold strategy with a monthly holding period. As a result, returns from assets are measured as one-month holding period rate of return.

The return from holding stocks is computed from the KOSPI 200 index and the returns from holding KTB and KTBi are calculated from the composite index of 10-year KTB and the composite index of KTBi, respectively. The return from holding corporate bonds is calculated from the composite corporate bond index. The Seoul apartment sale price index is used to compute the return from holding real estate. For CDs, 3-month CD yield is used to measure the return from investing in CDs.

The corporate bond composite index and the KOSPI 200 index are obtained from FN Guide while the KTBi index and the 10-year KTB index are constructed and provided by Korea Asset Pricing. The Seoul apartment price index is available from KB Kookmin Bank[4] . The time series for the yield of CDs is available from the Bank of Korea.

Among the assets included in the benchmark portfolio, CDs represent the risk-free asset. In the U.S., the Treasury bill is typically used as the risk-free asset. In Korea, however, the transaction amount of short-

[3] Mamun and Visaltanachoti (2005) included 5 assets in the benchmark portfolio, namely, Treasury Bills, Treasury Bonds, stocks, real estate, and corporate bonds.

[4] The Seoul apartment sale price index is selected as the real estate price index considering market liquidity.

term Korea Treasury bills is so small that their returns or yields cannot represent the risk-free rate properly. Instead, we use CDs as the risk-free asset because they reflect short-term financial market conditions relatively well. We also conduct spanning tests with CDs replaced by MSBs (Monetary Stabilization Bonds) in the benchmark portfolio.

The sample consists of monthly data from March 2007 when KTBi was first issued through to September 2016, for a total of 115 observations. Table 3 shows the summary statistics for the monthly return data of the assets used for the analysis. The average monthly returns of KTBi and KTB are 0.51% and 0.59%, respectively. These are higher than those of the other assets including stocks. Usually stocks show higher returns compared to other assets.

Although the average return of KTBi is lower than that of KTB, the return of KTBi is more volatile than that of KTB. As a result, KTBi has a lower return to volatility ratio (r/σ) compared to that of KTB, which means that KTBi may not be an attractive asset to be included in the investment portfolio in which KTB is already included. Yet, one needs to examine the correlation between the KTBi return with the returns from other assets before drawing such a conclusion.

Table 4. Sample Statistics of Monthly Returns

(%)

	KTBi	Stock	KTB	Corp. Bond	CD	Real Estate
Mean(r)	0.51	0.30	0.59	0.46	0.27	0.04
Median	0.54	-0.10	0.64	0.45	0.23	0.01
Maximum	8.80	16.47	10.59	4.76	0.50	1.98
Minimum	-9.36	-18.37	-3.13	-1.53	0.13	-1.64
Standard Deviation(σ)	2.10	5.59	1.62	0.81	0.10	0.46
(r/σ)	0.21	0.05	0.36	0.57	2.69	0.10

Table 5 presents the sample correlation of monthly returns for each pair of assets. According to the table, the return from KTBi seems be uncorrelated with those of stocks, CDs, and real estate. Even if KTBi return is positively correlated with the returns of KTB and corporate bonds, the level correlation is not so high, implying that there may be room for improving the mean-variance frontier by adding KTBi to the benchmark portfolio comprising of these assets.

Table 5. Correlation between Monthly Returns

	KTBi	Stock	KTB	Corporate Bond	CD	Real Estate
KTBi	1.00					
Stock	0.05	1.00				
KTB	0.44	-0.01	1.00			
Corporate Bond	0.46	-0.03	0.32	1.00		
CD	-0.02	-0.14	0.03	-0.06	1.00	
Real Estate	0.00	0.11	-0.22	-0.24	0.01	1.00

For the conditional spanning test, three variables are selected as instrumental variables: the yield spread between 10-year KTB and 1-year KTB, the yield spread of between KTBi and KTB, and the KTBi yield. All of the variables are one month lagged.

Table 6. ADF Test

Variable	Test Statistic	P-value
KTBi	-12.12	0.00
Stock	-10.11	0.00
KTB	-11.68	0.00
Corporate Bond	-8.40	0.00
CD	-5.47	0.00
MSB	-5.31	0.00
Real estate	-3.32	0.02
Yield Curve Slope*	-9.51	0.00
Yield Spread**	-11.32	0.00
KTBi Yield	-12.09	0.00

* Difference between 10-year KTB yield and 1-year KTB yield
** Spread between 10-year KTBi yield and 10-year KTB yield

To see if the variables used for the spanning tests are nonstationary, Table 6 shows the result of the ADF test of unit root. The result shows that all the variables used in spanning tests including the instrumental variables are stationary.

5.4 Results

The models are estimated by the ordinary least squares method. We then apply the Wald test to find if each of the null hypotheses given in equations (9) and (12) is accepted or not. If the null hypothesis is rejected, we can conclude that KTBi is capable of expanding the efficient frontier spanned by the benchmark portfolio.

Spanning tests are conducted with the benchmark portfolio consisting of all five assets namely CDs, stocks, real estate, KTB, and corporate bonds and the results are presented in Table 7. The Wald F- statistic shows that the null hypothesis of unconditional spanning cannot be rejected, which implies that the mean-variance efficient frontier cannot be improved by adding KTBi to the benchmark portfolio consisting of all of the five assets. On the other hand, Table 7 also shows the null hypothesis of conditional spanning can be rejected at 1% significance level, implying that the mean-variance efficient frontier can be expanded by adding KTBi to the benchmark portfolio if investors make use of market information.

Table 7 Results of the Spanning Test

Benchmark Portfolio	Test Statistics	Unconditional	Conditional
CD, stock, real estate, KTB, corporate bond,	F Statistic	0.28	4.57
	P-value	0.76	0.00
CD, stock, real estate, KTB	F Statistic	0.54	3.54
	P-value	0.58	0.01
CD, stock, real estate, corporate bond	F Statistic	0.35	3.29
	P-value	0.71	0.01
CD, stock, real estate	F Statistic	1.45	5.27
	P-value	0.21	0.00

Table 7 also displays the results of the spanning tests with alternative benchmark portfolios. The alternative benchmark portfolios are formed by excluding KTBs, corporate bonds, or both from the benchmark portfolio consisting of the entire five assets. The reason why alternative benchmark portfolios are constructed by excluding KTBs or corporate bonds or both is as follows.

Investigating whether the U.S. TIPS is capable of functioning as a strategic asset, Mamun and Visaltanachoti (2005) find that the results of the unconditional spanning tests are mixed depending on the benchmark portfolio. In particular, they find that in all the cases where the unconditional spanning hypothesis cannot be rejected, the benchmark portfolio includes Treasury bonds, corporate bonds, or both of these assets. They argue that such a result is expected given the high correlation between TIPS and these two assets. According to Table 5, KTBi has relatively high correlation with KTB and corporate bonds making KTBi more likely to be spanned by benchmark portfolios including one or both of these assets.

The test statistics presented in Table 7 show that the results with the benchmark portfolio comprising the entire assets are still valid with benchmark portfolios excluding KTBs or corporate bonds or both. For all of the three alternative benchmark portfolios considered, the unconditional spanning hypothesis cannot be rejected whereas the conditional spanning hypothesis is strongly rejected.

Table 8 Results of the Spanning Test with MSB

Benchmark Portfolio	Test Statistics	Unconditional	Conditional
MSB, KTB, Corporate bond, Stock, Real estate	F Statistic	0.30	3.34
	p value	0.74	0.01
MSB, KTB, stocks, real estate	F Statistic	0.61	3.32
	p value.	0.54	0.01
MSB, corporate bond, stocks, real estate	F Statistic	0.30	2.80
	P-value	0.73	0.03
MSB, stocks, real estate	F Statistic	0.34	2.76
	P-value	0.28	0.03

Note: MSB substitutes CD as one of benchmark assets.

Table 8 reports the results of the spanning tests when CDs are substituted by MSBs with 91-day maturity in the benchmark portfolio. The issuance amount of CDs has decreased significantly since 2009 as CDs are no longer counted as deposits for the purpose of prudential regulation. As a result, it is argued that yields on CDs may not be able to properly reflect money market conditions. Table 8 shows that the results of the spanning tests are robust to the choice of the risk-free asset included in benchmark portfolios. While the unconditional spanning test cannot reject the null hypothesis, the conditional spanning test strongly rejects the null hypothesis.

In summary, this study finds that the hypothesis that investors experience statistically significant diversification benefits from KTBi is supported by conditional spanning tests. The study also shows that this conclusion is robust to the choice of benchmark portfolio.

The findings of this study are consistent with those of previous studies including Mamun and Visaltanachoti (2005) and Han (2010). Mamun and Visaltanachoti (2005), in a study of the role of the U.S. TIPS as a strategic asset, find that while the results of unconditional spanning tests are mixed depending on the benchmark portfolio, conditional spanning tests reject the spanning hypothesis for every benchmark portfolio considered in the study. The results are also consistent with Han (2010) who finds that conditional spanning tests support the role of the KTBi as a strategic asset.

Such a result implies that if portfolio managers build and adjust their asset portfolios based on the information gained in the market, they will be able to achieve improvement in the mean-variance efficient frontier by augmenting KTBi to their investment portfolios. As is argued by Ferson and Siegel (2001), investors and fund managers not only make asset allocation decisions based on an asset's risk return characteristics, but they also take into consideration the condition of the economy and the financial market. Although the role of KTBi as a strategic asset is not supported by the unconditional spanning test, it is the conditional spanning test that is more appropriate to the reality.

The analysis with the bivariate GARCH model presented in Section IV may explain why the role of KTBi as a strategic asset is supported by the conditional spanning test. The estimation result of the bivariate GARCH model presented in Table 2 demonstrates that market information such as the yield curve slope and the break-even inflation rate are useful in predicting not only the level of the KTB returns and the KTBi returns but the correlation between the returns of these two assets. Predictability of not only the level of the returns of the nominal and inflation indexed securities but also their correlation is important to portfolio managers who allocate funds in a non-passive manner. Investors and portfolio managers can utilize the correlation and the level of returns predicted by market information to adjust their asset portfolios to improve their risk-return characteristics.

6 Conclusion

This paper investigates the role of KTBi as a strategic asset in a nominal asset portfolio by estimating a bivariate GARCH model with conditional correlation and by conducting spanning tests. Estimation of the bivariate GARCH model display that market information such as the yield curve slope and yield spread between KTBi and KTB are useful in predicting the correlation between the returns of KTBi and KTB as well as the level of the returns of these two assets.

Unconditional and conditional spanning tests produce different results regarding the potential role of KTBi as a strategic asset. While unconditional spanning tests do not reject the null hypothesis that existing assets span KTBi, the same null hypothesis is strongly rejected by conditional spanning tests. Such a result means that KTBi is capable of improving the mean-variance efficiency when added to existing investment portfolios. This in turn implies that, when investors include KTBi in their investment portfolio, they can achieve a better risk-return profile by actively making investment adjustments based on available market information rather than sticking to a buy-and-hold strategy.

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