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# Improving the accuracy of top-*N* recommendation using a preference model

Jongwuk Lee[a], Dongwon Lee[b,*], Yeon-Chang Lee[c], Won-Seok Hwang[c], Sang-Wook Kim[c]

[a] *Division of Computer and Electronic Systems Engineering, Hankuk University of Foreign Studies, Republic of Korea*
[b] *College of Information Sciences and Technology, The Pennsylvania State University, PA, USA*
[c] *Department of Electronics and Computer Engineering, Hanyang University, Republic of Korea*

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ABSTRACT

In this paper, we study the problem of retrieving a ranked list of *top-N items* to a target user in recommender systems. We first develop a novel *preference model* by distinguishing different rating patterns of users, and then apply it to existing *collaborative filtering* (*CF*) algorithms. Our preference model, which is inspired by a voting method, is well-suited for representing *qualitative* user preferences. In particular, it can be easily implemented with less than 100 lines of codes on top of existing CF algorithms such as user-based, item-based, and matrix-factorization-based algorithms. When our preference model is combined to three kinds of CF algorithms, experimental results demonstrate that the preference model can improve the accuracy of all existing CF algorithms such as ATOP and NDCG@25 by 3–24% and 6–98%, respectively.

## 1. Introduction

The goal of recommender systems (RS) [1] is to suggest appealing items (*e.g.*, movies, books, news, and music) to a target user by analyzing her prior preferences. As the number of applications using RS as a core component has increased rapidly, improving the quality of RS becomes a critically important problem. The quality of RS can be measured by various criteria such as accuracy, serendipity, diversity, relevance, and utility [29,38]. In measuring the accuracy of RS, in particular, two approaches have prevailed: *rating prediction* and *top-N recommendation* [36]. A typical practice of rating prediction is to minimize prediction errors for all *unobserved* (or *missing*) user-item pairs. In contrast, top-*N* recommendation provides users with the most appealing top-*N* items. In this paper, we mainly focus on improving the accuracy of top-*N* recommendation.

*Collaborative filtering* (*CF*) [37] has been widely used as one of the most popular techniques in RS. Basically, it is based on users' past behavior such as explicit user ratings and implicit click logs. Using the similarity between users' behavior patterns, CF suggests the most appealing top-*N* items for each user. In this process, existing CF algorithms [6,10,11,16,30,31] return top-*N* items with the highest ratings in terms of rating prediction, and ignore the *qualitative* order of items for top-*N* recommendation. Although some recent work [2,5,20,32,39] developed CF algorithms for optimizing top-*N* recommendation,

[*] Corresponding author. Tel.: +1 814 865 0687.
*E-mail addresses:* julee@hufs.ac.kr (J. Lee), dongwon@psu.edu (D. Lee), lyc0324@agape.hanyang.ac.kr (Y.-C. Lee), hws23@agape.hanyang.ac.kr (W.-S. Hwang), wook@agape.hanyang.ac.kr (S.-W. Kim).

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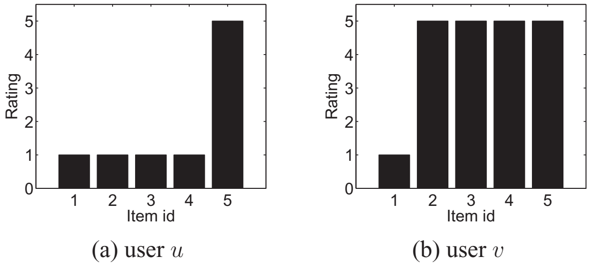


**Fig. 1.** The rating distribution of two users $u$ and $v$.

they still have the limitations in identifying qualitative user preferences, *i.e.*, relative preferences between items with explicit ratings.

Specifically, two key issues in existing top-*N* recommendation are: (1) how to distinguish *user experience* between observed and missing ratings and (2) how to model *latent* user preferences from observed ratings. As the solution to (1), for instance, we may impute missing ratings as *zero* (*i.e.*, *negative user feedback*), as used in [5]. However, as to the solution to (2), it is non-trivial to distinguish relative preferences between observed ratings (*i.e.*, *positive user feedback*) as user ratings often have different distributions per user.

To motivate our preference model, we illustrate a scenario in which two users $u$ and $v$ have rated for the same set of five items $\{i_1, \ldots, i_5\}$ (Fig. 1). Even if two users gave the same rating of 5 to item $i_5$, the meaning of the rating can be interpreted differently depending on rating distributions. For example, suppose that both $u$ and $v$ gave 5 to $i_5$. Because $v$ tends to give many ratings of 5, it is difficult to determine how satisfactory $v$ was really with $i_5$. In contrast, because $u$ rarely gives the rating of 5, it is plausible to assure that $u$ is very satisfied with $i_5$. Therefore, we argue that $u$ prefers $i_5$ more than $v$ does.

Our proposed preference model, which is inspired by a *voting method*, aims to discern subtle latent user preferences by considering different rating distributions. While existing algorithms are mainly based on the quantitative values among items to represent user preferences, our model exploits the qualitative order of items. Our preference model is able to convert raw user ratings into latent user preferences. In our formal analysis, it can be derived from *maximum likelihood estimation*, which is consistent with existing work [14].

We then develop a family of CF algorithms by embedding the proposed preference model into existing CF algorithms. As an orthogonal way, our preference model can be easily combined with existing CF algorithms such as user-based neighborhood, item-based neighborhood, and matrix-factorization-based algorithms. Observed user ratings are converted to user preference scores and missing ratings are imputed as zero values. After this conversion is performed, existing CF algorithms are applied with the converted user-item matrix. The key advantages of our proposed algorithms are two-fold: a simple algorithm design and a sizable accuracy improvement in various kinds of CF algorithms.

In order to validate our proposed algorithms, we lastly conduct comprehensive experiments with extensive settings. Because user ratings tend to be biased to item popularity and rating positivity [5,27,35], we employ two testing item sets: *All Items* (*i.e.*, All Items in a test item set) and *Long-tail Items* (*i.e.*, non-popular items with a few ratings) [5]. In case of *cold-start users* [21] who have rated only a few items, we also compare our proposed algorithms against existing CF algorithms.

To summarize, this paper makes the following contributions.

- We design a preference model in order to identify latent user preferences from user ratings. The idea of the preference model is intuitive to understand, and easy to implement, costing less than 100 lines of codes, on top of existing CF algorithms.
- We propose a family of CF algorithms by incorporating the proposed preference model into existing CF algorithms. Our preference model can be easily applied to existing CF algorithms.
- We evaluate our proposed CF algorithms applied to user-based, item-based, and matrix-factorization-based algorithms. Our proposed algorithms are validated over two test sets: *All Items* and *Long-tail Items*. It is found that our proposed algorithms improve three kinds of CF algorithms by 3–24% and 6–98% for ATOP and NDCG@25.

This paper is organized as follows. In Section 2, we survey existing work for top-*N* recommendation. In Section 3, we discuss two categories of CF algorithms and their variants for top-*N* recommendation. In Section 4, we design a preference model and propose a family of CF algorithms using our preference model. In Section 5, we show detailed evaluation methodology. In Section 6, we report and analyze the experimental results of our proposed algorithms. In Section 7, we finally conclude our work.

## 2. Related work

*Collaborative filtering* (*CF*) [37] is well-known as one of the most prevalent techniques in recommender systems (RS). (The broad survey for RS can be found in [1,29].) Conceptually, it collects the past behavior of users, and makes rating predictions based on the similarity between users' behavior patterns. The underlying assumptions behind CF are as follows:

(1) If users have similar behavior patterns in the past, they will also make similar behavior patterns in the future. (2) The users' behavior patterns are consistent over time. The user behavior is used to infer hidden user preferences and is usually represented by *explicit* (*e.g.*, user ratings) and *implicit* user actions (*e.g.*, clicks and query logs). In this paper, we assume that the user behavior is represented by explicit user ratings on items.

The approaches for CF can be categorized as two models: neighborhood models and latent factor models [37]. First, the neighborhood models make predictions based on the similarity between users or items such as user-based algorithms [10] and item-based algorithms [31]. Second, the latent factor models learn hidden patterns from observed ratings by using matrix factorization techniques [11,17]. In particular, *singular value decomposition* (*SVD*) is widely used as one of the well-established techniques [13,15,16,30]. In this paper, we mainly handle CF algorithms with explicit user ratings.

**Evaluation of top- *N* recommendation:** Top-*N* recommendation provides users with a ranked set of *N* items [6], which is also involved to the "who rated what" problem [4]. To evaluate top-*N* recommendation, we have to take the characteristics of observed ratings into account. That is, ratings are distributed *not at random* [22,23,34]. The ratings are biased to *item popularity* [5,35] and tend to be more preferred than missing ratings, *i.e.*, *rating positivity* [27]. In this paper, we thus employ two testing items such as *All Items* and *Long-tail Items*, and adopt various metrics for fair evaluation of top-*N* recommendation.

**One-class collaborative filtering:** When user ratings are represented by binary values, observed user ratings are simply set as one (positive feedback). Meanwhile, because missing ratings can be seen as a mixture of unknown and negative feedback, they need to be considered carefully. First, [26] introduced this problem to distinguish different semantics of missing ratings. In addition, [25] proposed an improved weight scheme to discern missing ratings. As the simplest way, we imputed all missing ratings as zero (*i.e.*, negative feedback). We compare our proposed algorithms with the one class collaborative filtering method, where missing values are considered negative feedback with uniform weights. In our future work, we will discuss an alternative method for handling missing ratings, *e.g.*, a imputation method [21] for missing ratings.

**Ranking-oriented collaborative filtering:** For top-*N* recommendation, it is important to consider the ranking of items. Weimer et al. [39] proposed *CoFiRank* that uses maximum margin matrix factorization to optimize the ranking of items. Liu and Yang [20] developed *EigenRank* to decide the rankings of items using neighborhood-based methods and Hu [12] proposed a preference-relation-based similarity measure for multi-criteria dimensions. In addition, [2,32] combined CF with *learning to rank* methods to optimize the ranking of items. Shi et al. [33] combined rating- and ranking-oriented algorithms with a linear combination function, and Liu et al. [19] extended probabilistic matrix factorization with list-wise preferences. Rendle et al. [28] proposed *Bayesian personalized ranking* that maximizes the likelihood of pair-wise preferences between observed and unobserved items in implicit datasets. In particular, [28] proposed a new objective function that aims to achieve higher accuracy for top-*N* recommendation. Ning and Karypis [24] delveloped a *sparse linear method* (*SLIM*) to learn the similarity between items for improving the accuracy of top-*N* recommendation. Recently, Tejeda-Lorente et al. [38] developed a hybrid approach to combining the content-based the and collaborative filtering method in which the user participates in the feedback process by ranking her interests in the user profiles. Liu et al. [18] adopted *boosted regression trees* to represent conditional user preferences in CF algorithms. In this paper, we propose an alternative user preference model, and combine it with a family of CF algorithms.

## 3. Collaborative filtering algorithms

In this section, we first introduce basic notations used throughout this paper. The domain consists of a set of $m$ users, $\mathcal{U} = \{u_1, \ldots, u_m\}$, and a set of $n$ items, $\mathcal{I} = \{i_1, \ldots, i_n\}$. All user-item pairs can be represented by an $m$-by-$n$ matrix $\mathcal{R} = \mathcal{U} \times \mathcal{I}$, where an entry $r_{ui}$ indicates the rating of user $u$ to item $i$. If $r_{ui}$ has been *observed* (or *known*), it is represented by a positive value in a specific range. Otherwise, $r_{ui}$ is *empty*, implying a *missing* (or *unknown*) rating. Let $\mathcal{R}^+ \subseteq \mathcal{R}$ denote a subset of user-item pairs for which ratings are observed, *i.e.*, $\mathcal{R}^+ = \{r_{ui} \in \mathcal{R} | r_{ui} \text{ is observed}\}$. For the sake of representation, $u$ and $v$ indicate arbitrary users in $\mathcal{U}$, and $i$ and $j$ refer to arbitrary items in $\mathcal{I}$.

In the following, we explain existing CF algorithms. They can be categorized into: (1) *neighborhood-based models* and (2) *latent-factor-based models*.

### 3.1. Neighborhood models

The neighborhood models are based on the similarity between either users or items. There are two major neighborhood algorithms, *i.e.*, *user-based* and *item-based* algorithms, depending on which sides are used. The user-based algorithms make predictions based on users' rating patterns similar to that of a target user $u$. Let $U(u; i)$ be a set of users who rate item $i$ and have similar rating patterns to the target user $u$. The predicted rating $\hat{r}_{ui}$ of user $u$ to item $i$ is computed by:

$$\hat{r}_{ui} = b_{ui} + \frac{\sum_{v \in U(u;i)} (r_{vi} - b_{vi}) w_{uv}}{\sum_{v \in U(u;i)} w_{uv}} \tag{1}$$

where $r_{vi}$ is a rating of user $v$ to item $i$, $b_{vi}$ is a biased rating value for $v$ to $i$, and $w_{uv}$ is the similarity weight between two users $u$ and $v$. In other words, $\hat{r}_{ui}$ is calculated by a weighted combination of neighbors' residual ratings. Note that $b_{ui}$ and $w_{uv}$ can be different depending on the rating normalization scheme and the similarity weight function.

On the other hand, the item-based algorithms predict the rating for target item $i$ of user $u$ based on the rating patterns between items. Let $I(i;\, u)$ be a set of items that have rating patterns similar to that of $i$ and have been rated by $u$. Let $w_{ij}$ denote the similarity weight between two items $i$ and $j$. Specifically, the predicted rating $\hat{r}_{ui}$ is calculated by:

$$\hat{r}_{ui} = b_{ui} + \frac{\sum_{j \in I(i;u)} (r_{uj} - b_{uj}) w_{ij}}{\sum_{j \in I(i;u)} w_{ij}} \tag{2}$$

We discuss how the neighborhood models can be adapted for top-$N$ recommendation. It is observed that predicting the exact rating values is unnecessary for top-$N$ recommendation. Instead, it is more important to distinguish the importance of items that are likely to be appealing to the target user. Toward this goal, items are ranked by ignoring the denominator used for rating normalization. By simply removing the denominators in Eqs. (1) and (2), it can only consider the magnitude of observed ratings. This modification can help discern the *user experience* between observed and missing ratings. Formally, the predicted score is computed by:

$$\hat{r}_{ui} = b_{ui} + \sum_{v \in U(u;i)} (r_{vi} - b_{vi}) w_{uv} \tag{3}$$

Similarly, the predicted score in the item-based algorithms is calculated by:

$$\hat{r}_{ui} = b_{ui} + \sum_{j \in I(i;u)} (r_{uj} - b_{uj}) w_{ij} \tag{4}$$

Note that $\hat{r}_{ui}$ in Eqs. (3) and (4) means the score for quantifying the importance of item $i$ to user $u$. For top-$N$ recommendation, it is observed that the non-normalized algorithms outperform conventional neighborhood algorithms that minimize prediction errors. The results are also consistently found in existing work [5].

### 3.2. Latent factor models

The latent factor model is an alternative way to infer hidden characteristics of rating patterns by using matrix factorization techniques. In this paper, we adopt *singular value decomposition* (*SVD*) as the well-established matrix factorization method [17]. A key idea of SVD is to factorize an $m$-by-$n$ matrix $\mathcal{R}$ into an inner product of two low-rank matrices with dimension $f$. That is, one low-rank matrix is called an $m$-by-$f$ *user-factor* matrix and the other is called an $n$-by-$f$ *item-factor* matrix. Each user $u$ is thus associated with an $f$-dimensional vector $p_u \in \mathbb{R}^f$, and each item $i$ is involved with an $f$-dimensional vector $q_i \in \mathbb{R}^f$. Formally, the predicted rating $\hat{r}_{ui}$ is calculated by:

$$\hat{r}_{ui} = b_{ui} + p_u q_i^T \tag{5}$$

In this process, a key challenge is how to handle the presence of missing ratings. Recent work adopts an objective function that only minimizes prediction errors for observed ratings with regularization. Formally, the objective function is represented by:

$$\min_{p,q,b} \frac{1}{2} \sum_{r_{ui} \in \mathcal{R}^+} (r_{ui} - b_{ui} - p_u q_i^T)^2 + \frac{\lambda}{2} (||p_u||^2 + ||q_i||^2) \tag{6}$$

where $\lambda$ is a parameter for regularization. In order to improve the accuracy of rating predictions, other information such as implicit user feedback [13] and temporal effects [15] can be used together in the objective function.

Although the objective function is effective for minimizing prediction errors, it does not obtain high accuracy for top-$N$ recommendation [5]. This is because it does not differentiate between observed ratings and missing ratings. As a simplest way, all missing ratings are considered *negative* user feedback, and they are imputed as zero, *i.e.*, $\forall r_{ui} \in \mathcal{R}/\mathcal{R}^+ : r_{ui} = 0$. The imputation for missing ratings enables us to discern user experience between observed ratings and missing ratings. Furthermore, because this modification can form a complete $m$-by-$n$ matrix $\mathcal{R}$, the conventional SVD method can be applied to $\mathcal{R}$.

$$\mathcal{R} \approx U \Sigma V^T \tag{7}$$

where a low-rank matrix approximation is used with dimension $f$. That is, $U$ is an $n$-by-$f$ orthonormal matrix, $V$ is an $m$-by-$f$ orthonormal matrix, and $\Sigma$ is an $f$-by-$f$ diagonal matrix. The existing work [5] reports that the pure SVD using Eq. (7) improves the accuracy of top-$N$ recommendation.

## 4. Proposed preference model

Unlike the rating prediction that focuses on minimizing prediction errors of unrated items, top-$N$ recommendation aims to identify a sorted list of $N$ items that a user would prefer. The key challenges in top-$N$ recommendation are two-fold: (1) how to distinguish the user experience between observed and missing ratings and (2) how to model user preferences for observed ratings. Basically, the observed and missing ratings can be represented as positive/negative user feedback, *i.e.*, 1 if the rating is observed and 0 otherwise. Because the observed ratings are at a different scale, we can exploit them for deciding the relative preferences of items. In the following, we propose a *preference model* in order to address the challenges for top-$N$ recommendation.

### 4.1. Basic intuition

Our main question for designing a preference model is: *how does it effectively derive latent user preferences from observed ratings?* One simple way is to adopt existing rating normalization schemes. Let $\mathcal{R}_u^+$ denote a set of observed ratings for target user $u$. For instance, *mean-centered* normalization can be used as one of the popular rating normalization schemes. After the average $\mu$ of $\mathcal{R}_u^+$ is computed, each rating $r_{ui}$ in $\mathcal{R}_u^+$ is replaced by $r_{ui} - \mu$. In that case, because normalized ratings with negative values can be confused as negative feedback, it is difficult to distinguish user experience between observed and missing ratings. In addition, when the relative preferences between items can be quantified by computing *absolute distance* between normalized ratings, it does not discern different rating distributions for each user.

As depicted in Fig. 1, it is important to distinguish different rating distributions of two users $u$ and $v$, where they have rated for the same set of five items. When the mean-centered normalization is used, item $i_1$ rated by 1 can be interpreted as negative feedback. In addition, the difference of normalized ratings between $i_1$ and $i_5$ is equal for two users $u$ and $v$, even if the rating distributions of users are vastly different. This is because the existing normalization schemes are based on *quantitative* ratings instead of *qualitative* ordering of items.

To address this problem, we adopt an alternative preference model, which is inspired by a voting method, *Borda count method* [7]. When items are sorted by the ascending order of ratings, the preference score *pref*($r_{ui}$) of item $i$ for target user $u$ is computed by accumulating the number of items that are ranked lower than or equal to item $i$. Let $pref_{op}(r_{ui})$ denote the number of items in $\mathcal{R}_u^+$ that are satisfied with the condition for *op*, *i.e.*, $pref_{op}(r_{ui}) = |\{r_{uj} \in \mathcal{R}_u^+ | r_{ui}\ op\ r_{uj}\}|$, where *op* can be replaced with any comparison operator, *e.g.*, $>$, $<$, and $=$. Specifically, there are two cases for computing the preference score.

1. Given user rating $r_{ui}$, we simply count the number of items that are ranked lower than $i$. Let $pref_>(r_{ui})$ is the preference score of $r_{ui}$, which is the number of items with lower rankings, *i.e.*, $pref_>(r_{ui}) = |\{r_{uj} \in \mathcal{R}_u^+ | r_{ui} > r_{uj}\}|$.
2. Let $pref_=(r_{ui})$ is the preference score of $r_{ui}$, which is the number of items with the same rankings, *i.e.*, $pref_=(r_{ui}) = |\{r_{uj} \in \mathcal{R}_u^+ | r_{ui} = r_{uj}\}|$.

By combining the two cases, the preference score of $r_{ui}$ can be computed as a weighted linear combination, where $|\mathcal{R}_u^+|$ is used as a normalization parameter.

$$pref(r_{ui}) = \alpha \cdot \frac{pref_>(r_{ui})}{|\mathcal{R}_u^+|} + \beta \cdot \frac{pref_=(r_{ui})}{|\mathcal{R}_u^+|} \tag{8}$$

where $\alpha$ and $\beta$ are weight parameters for $pref_>(r_{ui})$ and $pref_=(r_{ui})$, respectively. Using this equation, we can convert user ratings into qualitative user preference scores depending on different rating distributions.

We now explain how to compute the preference scores of items with categories. When a set of user ratings is aggregated into a set of rating categories, it is easier to handle the set of rating categories with a smaller size, *e.g.*, $r_{ui} \in \{1, \ldots, 5\}$. Let $\{C_1, \ldots, C_k\}$ be a set of rating categories in a specific range. Intuitively, the items in a high rating category are ranked higher than those in a low rating category. Suppose that $r_{ui}$ belongs to category $C$, *i.e.*, $r_{ui} \in C$. To compute the preference score of $r_{ui}$, we simply count the number of items that are in low rating category $C'$, *i.e.*, $C > C'$, and the number of items in the same category $C$. The preference score *pref*($C$) of $C$ is computed as:

$$pref(C) = \alpha \cdot \sum_{C' \in \{C_1, \ldots C_k\}} \frac{pref_>(C, C')}{|\mathcal{R}_u^+|} + \beta \cdot \frac{pref_=(C)}{|\mathcal{R}_u^+|} \tag{9}$$

where $pref_>(C, C')$ is the number of items that belong to lower rating category $C'$ and $pref_=(C)$ is the number of items that belong to $C$. Based on the formal analysis in Section 4.3, parameters $\alpha$ and $\beta$ are set as 1.0 and 0.5, respectively.

**Example 1** (Qualitative preference)**.** Consider two users $u$ and $v$ who have rated the same set of 10 items. Table 1 describes the observed ratings of two users, where the items are simply numbered in ascending order of ratings. Assuming the rating categories range in $\{1, \ldots, 5\}$, Table 2 shows the preference scores of rating categories for $u$ and $v$. That is, we count the number of items for each category, and compute the preference score for each category by using Eq. (9), where $\alpha = 1.0$ and $\beta = 0.5$. After the preference scores are computed, user ratings are converted to the corresponding preference scores. Table 3 depicts the preference scores of items for users $u$ and $v$. When the mean-center scheme is used, both users $u$ and $v$ have the same average value, *i.e.*, 3.2. The relative preference gap between two items $i_4$ and $i_{10}$ is thus equally computed as 2 for both $u$ and $v$. In contrast, because our preference model is based on the qualitative order of items, we can identify user preference scores in a more sophisticated manner. While $u$ rated 5 for both $i_9$ and $i_{10}$, $v$ rated 5 for only $i_{10}$. For $i_{10}$,

**Table 1**
Observed ratings for users $u$ and $v$.

| | $i_1$ | $i_2$ | $i_3$ | $i_4$ | $i_5$ | $i_6$ | $i_7$ | $i_8$ | $i_9$ | $i_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $u$ | 1 | 2 | 3 | 3 | 3 | 3 | 3 | 4 | 5 | 5 |
| $v$ | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 |

**Table 2**
Preference scores for rating categories of users $u$ and $v$.

|  | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| $u$ | 0.05 | 0.15 | 0.45 | 0.75 | 0.9 |
| $v$ | 0.0 | 0.15 | 0.45 | 0.75 | 0.95 |

**Table 3**
Converting ratings to preference scores for users $u$ and $v$.

|  | $i_1$ | $i_2$ | $i_3$ | $i_4$ | $i_5$ | $i_6$ | $i_7$ | $i_8$ | $i_9$ | $i_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $u$ | 0.05 | 0.15 | 0.45 | 0.45 | 0.45 | 0.45 | 0.45 | 0.75 | 0.9 | 0.9 |
| $v$ | 0.15 | 0.15 | 0.15 | 0.45 | 0.45 | 0.45 | 0.75 | 0.75 | 0.75 | 0.95 |

the preference scores of $u$ and $v$ are thus different, *i.e.*, 0.9 and 0.95. In addition, the preference gap between $i_4$ and $i_{10}$ is different as well, *i.e.*, 0.45 and 0.5.

### 4.2. Applying our preference model

Now, we discuss how to apply our proposed preference model to existing CF algorithms. First, the explicit ratings for each user are converted to corresponding preference scores. Based on this conversion, user-item matrix $\mathcal{R}$ is updated. Let $\mathcal{R}_{pref}$ denote a transformed matrix in which each entry represents a preference score. In this process, all missing ratings are imputed as zero, as used in existing work [5]. Then, any existing CF algorithms can easily be applied to $\mathcal{R}_{pref}$.

We first discuss how the preference model is combined with neighborhood-based algorithms. In computing the prediction score, a key difference is to use *pref*($r_{ui}$) instead of $r_{ui} - b_{ui}$ in Eq. (3). The predicted score in the user-based algorithms is computed as:

$$\hat{r}_{ui} = \sum_{v \in U(u;i)} pref(r_{vi}) w_{uv} \tag{10}$$

Similarly, the predicted score in the item-based algorithms is calculated as:

$$\hat{r}_{ui} = \sum_{j \in I(i;u)} pref(r_{uj}) w_{ij} \tag{11}$$

In both cases, $\hat{r}_{ui}$ indicates the importance of items for top-$N$ recommendation.

Next, we explain how to apply the preference model for latent factor based algorithms. For top-$N$ recommendation, ranking-based models (*e.g.*, [2,32,39]) employ an objective function that minimizes the error of item ranking. However, because the models are mainly based on the ordering of pair-wise items, they may fail to reflect overall rating distributions. In addition, their computation overhead is prohibitively high if all pair-wise preferences of items are considered. To address this problem, we first transform $\mathcal{R}$ into $\mathcal{R}_{pref}$ (as done in neighborhood-based algorithms), and then factorize $\mathcal{R}_{pref}$ using the conventional SVD method.

$$\mathcal{R}_{pref} \approx U \Sigma V^T \tag{12}$$

### 4.3. Formal analysis

We now derive the preference model by using *maximum likelihood estimation*. For simplicity, suppose that a set of ratings is represented by a rating distribution with categories. Given a set of category $\{C_1, \ldots, C_k\}$, $X_i$ denotes a set of items assigned to the corresponding rating category $C_i$. All Items with ratings are associated with $\{X_1, \ldots, X_k\}$. We estimate a *preference score* $\theta_i$ for the corresponding rating category $C_i$. Let $\theta = \{\theta_1, \ldots, \theta_k\}$ denote probability scores that each category is preferred. When generating a ranked list $L$ of observed items for user preferences, we identify a parameter $\theta$ that maximizes the likelihood function.

$$\theta^* = \arg\max_\theta p(L|\theta) \tag{13}$$

In that case, $p(L|\theta)$ can be replaced by the product of probability of generating the correct ranked list for any two items. That is, $p(L|\theta)$ is represented by:

$$p(L|\theta) = \prod_{i>j} \left( \prod_{x \in X_i, x' \in X_j} p(x > x'|\theta) \right) \prod_{i} \left( \prod_{x \neq x' \in X_i} p(x > x'|\theta) \right) \tag{14}$$

where $p(x > x'|\theta)$ is the probability that item $x$ is preferred to item $x'$, following binary distribution for $p(x)$:

$$p(x > x'|\theta) = p(x)(1 - p(x')) \tag{15}$$

where $p(x)$ is the probability that item $x$ is preferred.

As discussed in Section 4.1, a set of user ratings is represented by a set of discrete rating categories $\{C_1, \ldots, C_k\}$. We can replace $p(X)$ with the probability $p(C_i)$ for rating category $C_i$, and combine Eqs. (14) with (15). Therefore, $p(L|\theta)$ can be computed by:

$$p(L|\theta) = \prod_{C_i > C_j} \left(p(C_i)^{n(C_i)}(1 - p(C_j))^{n(C_j)}\right) \prod_{C_i} (p(C_i)(1 - p(C_i)))^{n(C_i)(n(C_i)-1)/2} \tag{16}$$

where $n(C_i)$ is the number of items assigned to $C_i$.

By maximizing the log likelihood of $p(L|\theta)$, we can finally obtain the highest preference probability $\theta_i \in \theta$, which is derived as:

$$p(C_i) = \sum_{i>j} p(C_i > C_j) + \frac{1}{2} \times p(C_i = C_j) \tag{17}$$

where $p(C_i > C_j)$ is the probability that $C_i$ is preferred to $C_j$, and $p(C_i = C_j)$ is the probability that $C_i$ and $C_j$ are preferred equally. In our experiments, we set $\alpha = 1.0$ and $\beta = 0.5$ in Eq. (9). It is found that the derivation for our preference model is consistent with existing work [14].

## 5. Evaluation methodology

In this section, we first explain real-life datasets used for evaluation (Section 5.1), and then show various accuracy metrics for top-$N$ recommendation (Section 5.2).

### 5.1. Datasets

We employ two real-life datasets of *MovieLens 100K* and *MovieLens 1M*. These are publicly available at http://grouplens.org/datasets/movielens. The MovieLens 100K dataset includes 943 users, 1682 items, and 100,000 ratings, and the MovieLens 1M dataset includes 6040 users, 3952 items, and 1,000,209 ratings. The ratings take integer values ranging from 1 (worst) to 5 (best). Table 4 summarizes the statistics for the two datasets.

For fair evaluation, we run a 5-fold cross validation. The dataset is randomly split into two subsets, *i.e.*, 80% *training* and 20% *testing* data. Let $Tr \subset \mathcal{R}^+$ and $Te \subset \mathcal{R}^+$ denote training and testing subsets partitioned from $\mathcal{R}^+$, *i.e.*, $Tr \cap Te = \emptyset$, $Tr \cup Te = \mathcal{R}^+$. Each subset is used for executing and evaluating CF algorithms. Specifically, $Tr_u$ and $Te_u$ denote a set of items rated by target user $u$, *i.e.*, $Tr_u = \{i \in \mathcal{I} | r_{ui} \in Tr\}$ and $Te_u = \{i \in \mathcal{I} | r_{ui} \in Te\}$. In particular, the items with the highest ratings are taken as *relevant* items in $Te_u$. Let $Te_u^+$ denote a set of relevant items, *i.e.*, $Te_u^+ = \{i \in \mathcal{I} | r_{ui} \in Te_u, r_{ui} = 5\}$. That is, $Te_u^+$ is used as *ground truth* for measuring accuracy metrics. Meanwhile, the items with low ratings and unrated items are considered *non-relevant*. In other words, suggesting the most preferred items only is treated as an effective top-$N$ recommendation.

In order to evaluate top-$N$ recommendation, it is also important to select a set of candidate items that CF algorithms will rank. Let $L_u$ denote a set of candidate items to be ranked for user $u$. Given $L_u$, the CF algorithms sort $L_u$ by the descending order of prediction scores and generate a sorted list with top-$N$ items. Depending on the selection of $L_u$, different top-$N$ items can be created [3]. For a generalized setting, we form $L_u = \mathcal{I} - Tr_u$ as a set of whole items except for the items that have been rated by user $u$. In this paper, we call this setting *All Items*.

When using *All Items*, it is observed that the majority of ratings is biased to only a small fraction of popular items. Fig. 2 depicts the distribution of items in MovieLens 100K, where $Y$-axis represents *popularity*, *i.e.*, the number of observed ratings. The items in $X$-axis are sorted by the descending order of popularity. In Fig. 2, the top-100 popular items (*i.e.*, about 6%) have 29,931 ratings (*i.e.*, about 30%). In that case, because the top-$N$ items tend to be biased to a few popular items, top-$N$ recommendation may not bring much benefit for users. Note that the popularity bias of rated items can be found in other datasets [5,27].

To address this problem, we design a more elaborated setting in selecting $L_u$. Given a set $\mathcal{I}$ of items, it is partitioned into two subsets in terms of popularity. That is, most popular 100 items and remaining items are called *top-head* and *Long-tail* Items, respectively. Let *Top* denote a set of top-head items. In that case, we deliberately form $L_u$ as Long-tail Items, *i.e.*, $L_u = (\mathcal{I} - Top) - Tr_u$ as shown in [5]. We call this setting *Long-tail Items*. The evaluation for Long-tail Items can be less biased toward the popular items. Compared to *All Items*, it can be a more difficult setting. For extensive evaluation, we will use both settings, *i.e.*, *All Items* and *Long-tail Items*.

**Table 4**
Detailed statistics of real-life datasets.

| | MovieLens 100K | MovieLens 1M |
|---|---|---|
| Density | 6.30% | 4.26% |
| Min. # of ratings of users | 20 | 20 |
| Max. # of ratings of users | 737 | 2,314 |
| Avg. # of ratings per user | 106.04 | 165.59 |

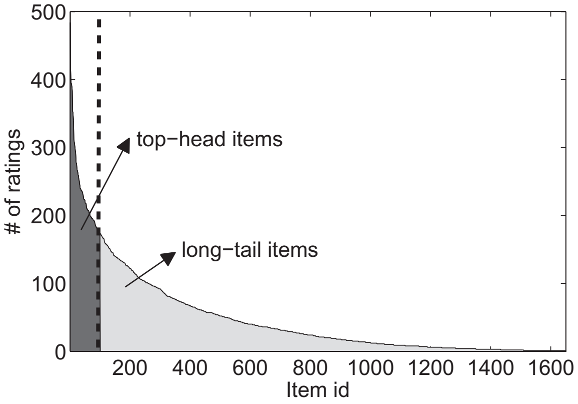


**Fig. 2.** The popularity distribution of items in MovieLens 100K.

Lastly, we evaluate top-*N* recommendation for *cold-start users* [21] who have rated only a few items. Because it is more challenging to identify users' latent preferences using only a small number of ratings, top-*N* recommendation results become less accurate. In this paper, the cold-start user setting is simulated in MovieLens 100K. Specifically, two subsets of users are randomly chosen: 500 users as training users (300, 400, and 500 users, respectively), and 200 users as testing users. For each target user, we vary the number of items rated by a target user from 5, 10 and 20 (Given 5, Given 10 and Given 20, respectively), where the rated items are randomly selected. By default, the testing items are selected from *Long-tail Items*.

### 5.2. Metrics

There are several accuracy metrics to evaluate top-*N* recommendation. A difficulty for evaluating top-*N* recommendation is that the number of observed ratings is usually small and missing ratings are simply ignored [6]. In addition, because ratings are *not at random* [22,23,34], top-*N* results can affect item popularity [5,35] and rating positivity [27]. To alleviate the problem, we adopt various metrics to complement their drawbacks. For each metric, we report the average of all users who have rated items in *Te*.

First, we employ traditional accuracy metrics such as *precision* and *recall* used in the IR community. Let $N_u$ denote a sorted list of *N* items as the result of top-*N* recommendation. The precision and recall at *N* are computed by:

$$\text{P@}N = \frac{|Te_u^+ \cap N_u|}{N} \tag{18}$$

$$\text{R@}N = \frac{|Te_u^+ \cap N_u|}{|Te_u^+|} \tag{19}$$

where $|Te_u^+|$ is the number of relevant observed items for user *u*. The two metrics consider the relevance of the top-*N* items, but neglect their ranked positions in $N_u$.

Second, we employ *normalized discounted cumulative gain* (*NDCG*). The ranked position of items in $N_u$ highly affects NDCG. Let $y_k$ represent a binary variable for the *k*th item $i_k$ in $N_u$, *i.e.*, $y_k \in \{0, 1\}$. If $i_k \in Te_u^+$, $y_k$ is set as 1. Otherwise, $y_k$ is set as 0. In this case, NDCG@*N* is computed by:

$$\text{NDCG@}N = \frac{\text{DCG@}N}{\text{IDCG@}N} \tag{20}$$

In addition, DCG@*N* is computed by:

$$\text{DCG@}N = \sum_{k=1}^{N} \frac{2^{y_k} - 1}{\log_2(k+1)} \tag{21}$$

IDCG@*N* means an *ideal DCG* at *N*, *i.e.*, for every item $i_k$ in $N_u$, $y_k$ is set as 1. Note that, because large values of *N* are useless for top-*N* recommendation, *N* is set to {5, 10, 15, 20, 25} for the three metrics.

Lastly, we employ an alternative measure that reflects all positions of items in $L_u$, *area under the recall curve*, also known as *ATOP* [34]. Because ATOP is calculated by the average rank of all relevant items in $Te_u$, it is more effective for capturing the overall distribution for the rankings of relevant items. Let rank($i_k$) denote the ranking score of item $i_k$ in $L_u$. If $i_k$ is ranked highest, rank($i_k$) has the largest ranking score, *i.e.*, $\text{rank}(i_k) = |L_u|$. Meanwhile, if $i_k$ is ranked lowest, rank($i_k$) has the smallest ranking score, *i.e.*, $\text{rank}(i_k) = 1$. In that case, rank($i_k$) is normalized by $\text{Nrank}(i_k) = (\text{rank}(i_k) - 1)/(|L_u| - 1)$. Formally, ATOP is computed by:

$$\text{ATOP} = \frac{1}{|Te_u^+|} \sum_{i_k \in Te_u^+} \text{Nrank}(i_k) \tag{22}$$

Note that all four metrics are maximized by 1.0, *i.e.*, the higher they are, the more accurate top-*N* recommendation is.

## 6. Experiments

In this section, we first explain state-of-the-art collaborative filtering algorithms to compare our proposed algorithm (Section 6.1). We then report our experimental results (Section 6.2).

### 6.1. Competing algorithms

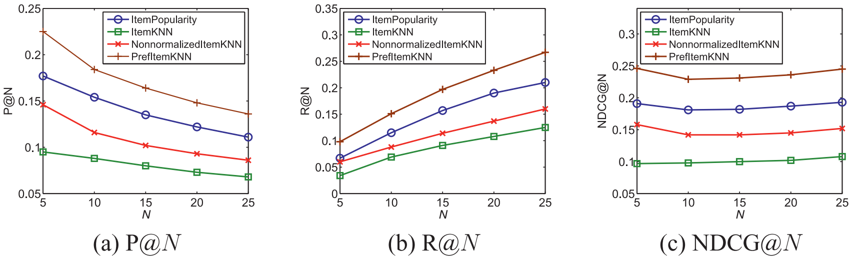
As the baseline method, we first compare the existing decoupled model [14], called Decoupled. Because it has focused on predicting the ratings of items, it is unsuitable for recommending top-*N* items. However, it is effective for evaluating the key differences between Decoupled and our proposed algorithms using the preference model. We also compare a non-personalized algorithm using *item popularity*, which is served as another baseline method. Specifically, it sorts items by the descending order of popularity, (*i.e.*, the number of ratings) and suggests most popular *N* items regardless of user ratings, called ItemPopularity. Despite its simple design, it shows even higher accuracy than existing CF algorithms for rating predictions (*i.e.*, UserKNN, ItemKNN, SVD, and SVD++) in *All Items* (as shown in Table 5) [5]. This is because it can directly capture the popularity bias of items.

We then discuss detailed implementations of the CF algorithms using our preference model and existing CF algorithms. Note that our implementations are based on MyMediaLite [9], which is a well-known open-source code for evaluating RS. The key advantage of our proposed method is that existing CF algorithms can be easily extended by incorporating our preference model. We apply our preference model to both neighborhood models and latent factor models.

The neighborhood models can be categorized into two approaches: user-based and item-based algorithms. Broadly, the neighborhood models consist of three key components: (1) rating normalization, (2) similarity weight computation, and (3) neighborhood selection. First, the biased rating scheme [16] is used for rating normalization. Next, in order to quantify the similarity weight, *binary cosine similarity* is used for both user-based and item-based algorithms. Lastly, *k*-nearest-neighbor filtering is used for neighbor selection in $U(u;\ i)$ and $I(i;\ u)$. That is, the neighbors with negative weights are filtered out,

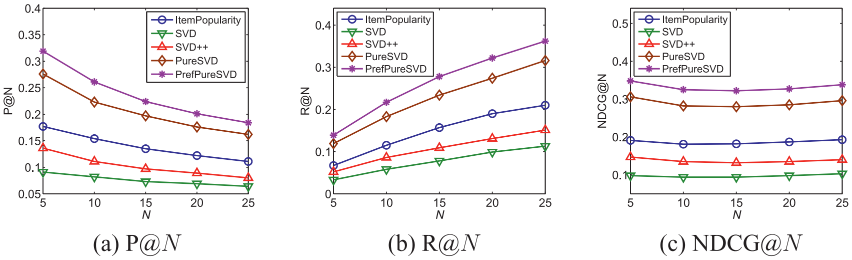
**Table 5**
Evaluation for the accuracy of top-*N* recommendation (MovieLens 100K).

| Algorithms | *All Items* | | | |
|---|---|---|---|---|
| | ATOP | P@25 | R@25 | NDCG@25 |
| Decoupled | 0.831 | 0.022 | 0.151 | 0.087 |
| ItemPopularity | 0.890 | 0.054 | 0.280 | 0.174 |
| UserKNN | 0.791 | 0.006 | 0.030 | 0.011 |
| NonnormalizedUserKNN | 0.786 | 0.060 | 0.280 | 0.191 |
| PrefUserKNN | 0.927 (18%) | 0.081 (35%) | 0.440 (57%) | 0.297 (56%) |
| ItemKNN | 0.812 | 0.042 | 0.184 | 0.106 |
| NonnormalizedItemKNN | 0.749 | 0.046 | 0.205 | 0.137 |
| PrefItemKNN | 0.925 (24%) | 0.077 (67%) | 0.411 (101%) | 0.272 (98%) |
| SVD | 0.752 | 0.037 | 0.166 | 0.102 |
| SVD++ | 0.803 | 0.046 | 0.215 | 0.124 |
| PureSVD | 0.888 | 0.078 | 0.425 | 0.272 |
| PrefPureSVD | **0.930 (5%)** | **0.097 (24%)** | **0.500 (18%)** | **0.339 (25%)** |
| WRMF | 0.942 | 0.092 | 0.490 | 0.320 |
| BPRSLIM | 0.915 | 0.066 | 0.383 | 0.238 |
| BPRMF | 0.942 | 0.083 | 0.449 | 0.282 |
| | *Long-tail Items* | | | |
| | ATOP | P@25 | R@25 | NDCG@25 |
| Decoupled | 0.785 | 0.007 | 0.066 | 0.033 |
| ItemPopularity | 0.855 | 0.022 | 0.145 | 0.069 |
| UserKNN | 0.752 | 0.002 | 0.014 | 0.004 |
| NonnormalizedUserKNN | 0.761 | 0.031 | 0.203 | 0.113 |
| PrefUserKNN | 0.904 (19%) | 0.044 (42%) | 0.334 (65%) | 0.188(66%) |
| ItemKNN | 0.774 | 0.025 | 0.158 | 0.081 |
| NonnormalizedItemKNN | 0.760 | 0.031 | 0.201 | 0.120 |
| PrefItemKNN | 0.900 (18%) | 0.041 (32%) | 0.309 (54%) | 0.175(46%) |
| SVD | 0.727 | 0.022 | 0.142 | 0.074 |
| SVD++ | 0.750 | 0.026 | 0.169 | 0.087 |
| PureSVD | 0.879 | 0.053 | 0.394 | 0.226 |
| PrefPureSVD | **0.908 (3%)** | **0.059 (11%)** | **0.424 (8%)** | **0.245 (8%)** |
| WRMF | 0.924 | 0.057 | 0.418 | 0.243 |
| BPRSLIM | 0.889 | 0.039 | 0.312 | 0.166 |
| BPRMF | 0.924 | 0.050 | 0.368 | 0.193 |



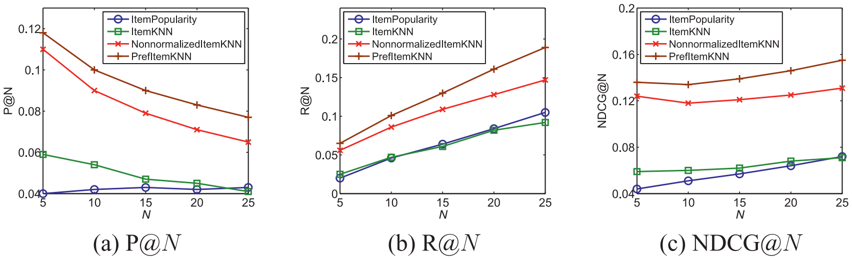
(a) P@$N$ (b) R@$N$ (c) NDCG@$N$

**Fig. 3.** Comparison results of item-based algorithms in *All Items* over varying *N* (MovieLens 1M).



(a) P@$N$ (b) R@$N$ (c) NDCG@$N$

**Fig. 4.** Comparison results of SVD-based algorithms in *All Items* over varying *N* (MovieLens 1M).



(a) P@$N$ (b) R@$N$ (c) NDCG@$N$

**Fig. 5.** Comparison results of item-based algorithms in *Long-tail Items* over varying *N* (MovieLens 1M).

and the highest similarity weights are chosen as $k$ neighbors. Empirically, we set $k = 80$ for both user-based and item-based algorithms.

We compare the following user-based neighborhood algorithms:

- UserKNN: user-based neighborhood algorithm [10] using Eq. (1)
- NonnormalizedUserKNN: user-based neighborhood algorithm with non-normalized ratings [5] using Eq. (3)
- PrefUserKNN: our proposed user-based neighborhood algorithm that incorporates the preference model using Eq. (10)

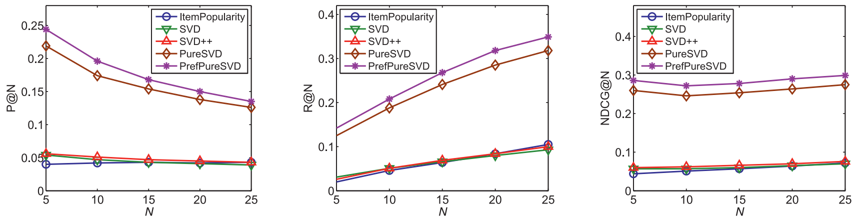
Similarly, we compare the following item-based neighborhood algorithms:

- ItemKNN: item-based neighborhood algorithm [31] using Eq. (2)
- NonnormalizedItemKNN: item-based neighborhood algorithm with non-normalized ratings [5] using Eq. (4)
- PrefItemKNN: our proposed item-based neighborhood algorithm that incorporates the preference model using Eq. (11)

We also explain how to implement latent-factor-based algorithms. Although there are various latent factor models, we focus on evaluating SVD-based algorithms and their variations [8]. The number of latent factors $f$ is fixed as 50. In case of SVD and SVD++, we set a regularized parameter $\lambda = 0.015$. Specifically, we compare the following latent-factor-based algorithms:

- SVD: typical SVD-based algorithm [30] with regularization for the incomplete matrix using Eq. (6)
- SVD++: state-of-the-art SVD-based algorithm [16] that shows the highest accuracy in terms of prediction errors, *i.e.*, root mean squired error (RMSE)
- PureSVD: SVD-based algorithm [5] for the complete matrix by filling missing ratings as zero using Eq. (7)
- PrefPureSVD: our proposed SVD-based algorithm that incorporates the preference model using Eq. (12)

Lastly, we compare our proposed algorithms with other latent-factor-based algorithms with implicit datasets. The ratings used in the algorithms are thus represented by binary data.



(a) P@$N$ (b) R@$N$ (c) NDCG@$N$

**Fig. 6.** Comparison results of SVD-based algorithms in *Long-tail Items* over varying *N* (MovieLens 1M).

**Table 6**
Evaluation for the accuracy of top-*N* recommendation (MovieLens 1M).

| Algorithms | *All Items* | | | |
|---|---|---|---|---|
| | ATOP | P@25 | R@25 | NDCG@25 |
| Decoupled | 0.836 | 0.052 | 0.092 | 0.080 |
| ItemPopularity | 0.898 | 0.111 | 0.210 | 0.193 |
| UserKNN | 0.839 | 0.030 | 0.058 | 0.033 |
| NonnormalizedUserKNN | 0.802 | 0.099 | 0.187 | 0.180 |
| PrefUserKNN | 0.929 (16%) | 0.139 (40%) | 0.287 (54%) | 0.269 (50%) |
| ItemKNN | 0.845 | 0.068 | 0.125 | 0.108 |
| NonnormalizedItemKNN | 0.756 | 0.086 | 0.160 | 0.152 |
| PrefItemKNN | 0.919 (22%) | 0.136 (58%) | 0.267 (67%) | 0.245 (61%) |
| SVD | 0.818 | 0.064 | 0.113 | 0.103 |
| SVD++ | 0.842 | 0.080 | 0.151 | 0.140 |
| PureSVD | 0.869 | 0.162 | 0.316 | 0.296 |
| PrefPureSVD | **0.931 (7%)** | **0.184 (14%)** | **0.362 (15%)** | **0.338 (14%)** |
| WRMF | 0.940 | 0.201 | 0.319 | 0.324 |
| BPRSLIM | 0.909 | 0.136 | 0.229 | 0.224 |
| BPRMF | 0.937 | 0.185 | 0.288 | 0.296 |
| | *Long-tail Items* | | | |
| | ATOP | P@25 | R@25 | NDCG@25 |
| Decoupled | 0.798 | 0.021 | 0.052 | 0.037 |
| ItemPopularity | 0.867 | 0.043 | 0.105 | 0.072 |
| UserKNN | 0.800 | 0.009 | 0.019 | 0.011 |
| NonnormalizedUserKNN | 0.761 | 0.052 | 0.122 | 0.098 |
| PrefUserKNN | 0.912 (20%) | 0.079 (52%) | 0.200 (64%) | 0.165 (68%) |
| ItemKNN | 0.813 | 0.041 | 0.092 | 0.071 |
| NonnormalizedItemKNN | 0.767 | 0.065 | 0.147 | 0.131 |
| PrefItemKNN | 0.900 (17%) | 0.077 (19%) | 0.189 (29%) | 0.155(18%) |
| SVD | 0.790 | 0.039 | 0.093 | 0.070 |
| SVD++ | 0.796 | 0.043 | 0.100 | 0.076 |
| PureSVD | 0.899 | 0.126 | 0.318 | 0.275 |
| PrefPureSVD | **0.929 (3%)** | **0.135 (7%)** | **0.349 (10%)** | **0.299 (9%)** |
| WRMF | 0.929 | 0.134 | 0.251 | 0.230 |
| BPRSLIM | 0.889 | 0.089 | 0.181 | 0.157 |
| BPRMF | 0.923 | 0.117 | 0.219 | 0.197 |

- WRMF: one class collaborative filtering [25,26], where missing ratings are used as negative feedback with uniform weights
- BPRSLIM: sparse linear method [24] that leverages the objective function as Bayesian personalized ranking [28]
- BPRMF: matrix factorization using stochastic gradient method where the objective function is Bayesian personalized ranking for pair-wise preferences between observed and unobserved items in implicit datasets [28]

All experiments were conducted in Windows 7 running on Intel Core 2.67 GHz CPU with 8GB RAM. All algorithms were implemented with MyMediaLite [9]. For simplicity, the other parameters for each algorithm are set as default values provided in MyMediaLite.

### 6.2. Experimental results

We evaluate our proposed algorithms in extensive experimental settings. Specifically, our empirical study is performed in order to answer the following questions:

**Table 7**
ATOP comparison for cold-start user evaluation (MovieLens 100K).

| Algorithms | # of training users = 300 | | |
|---|---|---|---|
| | Given 5 | Given 10 | Given 20 |
| Decoupled | 0.755 | 0.761 | 0.765 |
| ItemPopularity | 0.826 | 0.834 | 0.846 |
| UserKNN | 0.714 | 0.717 | 0.728 |
| NonnormalizedUserKNN | 0.734 | 0.738 | 0.745 |
| PrefUserKNN | **0.872 (19%)** | **0.879 (19%)** | 0.888 (19%) |
| SVD | 0.61 | 0.645 | 0.684 |
| SVD++ | 0.721 | 0.725 | 0.73 |
| PureSVD | 0.723 | 0.772 | 0.792 |
| PrefPureSVD | 0.854 (18%) | 0.878 (14%) | **0.890 (12%)** |
| WRMF | 0.810 | 0.847 | 0.863 |
| BPRSLIM | 0.705 | 0.766 | 0.807 |
| BPRMF | 0.814 | 0.831 | 0.847 |
| | # of training users = 400 | | |
| | Given 5 | Given 10 | Given 20 |
| Decoupled | 0.760 | 0.759 | 0.770 |
| ItemPopularity | 0.831 | 0.837 | 0.850 |
| UserKNN | 0.722 | 0.725 | 0.736 |
| NonnormalizedUserKNN | 0.744 | 0.745 | 0.756 |
| PrefUserKNN | **0.877 (18%)** | 0.882 (18%) | 0.891 (18%) |
| SVD | 0.59 | 0.605 | 0.64 |
| SVD++ | 0.7 | 0.697 | 0.7 |
| PureSVD | 0.74 | 0.782 | 0.817 |
| PrefPureSVD | 0.863 (17%) | **0.884 (13%)** | **0.902 (10%)** |
| WRMF | 0.822 | 0.835 | 0.881 |
| BPRSLIM | 0.722 | 0.777 | 0.826 |
| BPRMF | 0.831 | 0.844 | 0.857 |
| | # of training users = 500 | | |
| | Given 5 | Given 10 | Given 20 |
| Decoupled | 0.769 | 0.767 | 0.781 |
| ItemPopularity | 0.834 | 0.838 | 0.852 |
| UserKNN | 0.736 | 0.738 | 0.749 |
| NonnormalizedUserKNN | 0.750 | 0.754 | 0.765 |
| PrefUserKNN | **0.880 (17%)** | 0.885 (17%) | 0.895 (17%) |
| SVD | 0.572 | 0.617 | 0.634 |
| SVD++ | 0.727 | 0.724 | 0.734 |
| PureSVD | 0.734 | 0.782 | 0.822 |
| PrefPureSVD | 0.872 (19%) | **0.893 (14%)** | **0.914 (11%)** |
| WRMF | 0.808 | 0.841 | 0.878 |
| BPRSLIM | 0.722 | 0.784 | 0.832 |
| BPRMF | 0.842 | 0.857 | 0.879 |

1. Do our proposed algorithms using the preference model outperform existing algorithms in both of *All Items* and *Long-tail Items*?
2. Do our proposed algorithms also outperform the existing algorithms in the cold-start user setting?
3. Which one of our proposed algorithms show the highest accuracy in extensive experimental settings?
4. Do our proposed algorithms show a consistent improvement for the four metrics?

### 6.2.1. *Evaluation for All Items*

Fig. 3 reports the accuracy of item-based neighborhood algorithms over *All Items*. For all algorithms, P@$N$ decreases with $N$, R@$N$ increases with $N$, and NDCG@$N$ tends to stay constant with $N$. (For simplicity, we later report the experimental results for Decoupled that shows the worst accuracy.) We found three key observations. First, for the three metrics, PrefItemKNN shows the highest accuracy regardless of $N$. Specifically, PrefItemKNN improves NonnormalizedItemKNN by 0.093 (*i.e.*, 61%) in terms of NDCG@25. Second, NonnormalizedItemKNN shows higher accuracy than ItemKNN as observed in existing work [5]. Third, because of popularity bias of items, ItemPopularity outperforms item-based algorithms for the three metrics. (In the setting of *Long-tail Items*, however, we found an opposite result.) We omit to report the results for user-based algorithms because they show a similar tendency with item-based algorithms.

Fig. 4 reports the accuracy of SVD-based algorithms over *All Items*. It is clear that PrefPureSVD outperforms all existing SVD-based algorithms. For NDCG@25, PrefPureSVD improves PureSVD by 0.042 (*i.e.*, 14%). In addition, ItemPopularity shows higher accuracy than SVD and SVD++ for the three metrics. Unlike the item-based algorithms, the normalized variant PureSVD of SVD shows higher accuracy than ItemPopularity.

**Table 8**
NDCG@25 comparison for cold-start user evaluation (MovieLens 100K).

| Algorithms | # of training users = 300 | | |
|---|---|---|---|
| | Given 5 | Given 10 | Given 20 |
| Decoupled | 0.042 | 0.041 | 0.032 |
| ItemPopularity | 0.061 | 0.061 | 0.066 |
| UserKNN | 0.004 | 0.003 | 0.002 |
| NonnormalizedUserKNN | 0.08 | 0.091 | 0.086 |
| PrefUserKNN | **0.142 (78%)** | 0.149 (64%) | 0.149 (73%) |
| SVD | 0.045 | 0.053 | 0.065 |
| SVD++ | 0.073 | 0.076 | 0.069 |
| PureSVD | 0.079 | 0.114 | 0.145 |
| PrefPureSVD | **0.142 (80%)** | **0.151 (33%)** | **0.168 (16%)** |
| WRMF | 0.133 | 0.147 | 0.171 |
| BPRSLIM | 0.065 | 0.099 | 0.109 |
| BPRMF | 0.090 | 0.102 | 0.114 |
| | # of training users = 400 | | |
| | Given 5 | Given 10 | Given 20 |
| Decoupled | 0.032 | 0.032 | 0.030 |
| ItemPopularity | 0.066 | 0.069 | 0.074 |
| UserKNN | 0.003 | 0.003 | 0.002 |
| NonnormalizedUserKNN | 0.094 | 0.099 | 0.100 |
| PrefUserKNN | **0.155 (65%)** | **0.159 (61%)** | 0.153 (53%) |
| SVD | 0.033 | 0.040 | 0.050 |
| SVD++ | 0.070 | 0.068 | 0.070 |
| PureSVD | 0.083 | 0.114 | 0.157 |
| PrefPureSVD | 0.143 (72%) | 0.153 (34%) | **0.167 (6%)** |
| WRMF | 0.136 | 0.160 | 0.178 |
| BPRSLIM | 0.074 | 0.106 | 0.125 |
| BPRMF | 0.089 | 0.105 | 0.123 |
| | # of training users = 500 | | |
| | Given 5 | Given 10 | Given 20 |
| Decoupled | 0.028 | 0.028 | 0.023 |
| ItemPopularity | 0.068 | 0.062 | 0.070 |
| UserKNN | 0.004 | 0.004 | 0.004 |
| NonnormalizedUserKNN | 0.094 | 0.097 | 0.102 |
| PrefUserKNN | **0.152 (62%)** | **0.154 (59%)** | 0.160 (57%) |
| SVD | 0.029 | 0.036 | 0.043 |
| SVD++ | 0.082 | 0.080 | 0.080 |
| PureSVD | 0.078 | 0.109 | 0.148 |
| PrefPureSVD | 0.140 (80%) | 0.148 (36%) | **0.173 (17%)** |
| WRMF | 0.130 | 0.151 | 0.174 |
| BPRSLIM | 0.067 | 0.095 | 0.125 |
| BPRMF | 0.097 | 0.100 | 0.124 |

### *6.2.2. Evaluation for Long-tail Items*

Fig. 5 depicts the accuracy comparison of item-based algorithms over *Long-tail Items*. In particular, there are two key observations. First, PrefItemKNN still shows the highest accuracy among all item-based algorithms for the three metrics. PrefItemKNN apparently outperforms NonnormalizedItemKNN by 0.024 (*i.e.*, 18%) for NDCG@25. Second, NonnormalizedUserKNN outperforms ItemPopularity for the three metrics, unlike the results in *All Items*.

Fig. 6 depicts the accuracy of SVD-based algorithms. It is apparent that PrefPureSVD outperforms all existing SVD-based algorithms for the three metrics. Similar to *All Items*, PureSVD is higher than SVD and SVD++ in accuracy. Meanwhile, ItemPopularity and SVD++ are comparable in *Long-tail Items*.

In summary, Tables 5 and 6 report comparison results for all algorithms (including Decoupled, WRMF, BPRSLIM, and BPRMF) in MovieLens 100K and MovieLens 1M, respectively. Note that, in both tables, gray color rows indicate our proposed algorithms and bold font signifies the highest accuracy compared to previous baseline algorithms. Besides, the percent in parentheses is a relative improvement ratio of our proposed algorithms using the preference model, compared to the base CF algorithms: NonnormalizedUserKNN, NonnormalizedItemKNN, and PureSVD. For all metrics, our proposed algorithms outperform the existing algorithms in both settings. In both datasets, our proposed algorithms consistently outperform all existing algorithms. Compared to PrefUserKNN and PrefItemKNN, PrefPureSVD shows the highest accuracy. In addition, PrefPureSVD shows comparable and better accuracy than other latent-factor-based methods using ranking models. In particular, PrefPureSVD always outperforms the existing algorithms in terms of recall@25 and NDCG@25.

### 6.2.3. Evaluation for cold-start users

Tables 7 and 8 describe comparison results for user-based and SVD-based algorithms for ATOP and NDCG@25. Note that, since the data setting is based on cold-start users, item-based algorithms cannot be applied. (In case of cold-start items, it is found that the item-based algorithms show similar results with user-based algorithms.) As the number of training users and the number of items rated for testing increases, ATOP and NDCG@25 increase. Regardless of parameter settings, our proposed algorithms PrefUserKNN and PrefPureSVD outperform all existing algorithms. PrefUserKNN and PrefPureSVD show comparable results in both metrics. When the number of training users and the number of given ratings is small, PrefUserKNN is slightly better than PrefPureSVD. Meanwhile, as the parameter values increase, PrefPureSVD shows higher accuracy than PrefUserKNN. (Because our proposed algorithms show consistently the highest accuracy, we skip the results for other metrics.) Similar to *All Items* and *Long-tail Items* settings, PrefPureSVD always outperforms existing latent-factor-based algorithms in terms of the ATOP, and it shows a comparable accuracy in terms of the NDCG@25.

## 7. Conclusion

In this paper, we have studied how to improve the accuracy of top-*N* recommendation. To address this problem, two key challenges arise: (1) how to distinguish user experience between observed and missing ratings and (2) how to infer latent user preference for observed ratings. We first designed a preference model based on the qualitative order of items. We then proposed a family of CF algorithms that combine the preference model with existing CF algorithms: user-based neighborhood, item-based neighborhood, and matrix-factorization-based algorithms. The empirical study showed that our proposed algorithms improved the existing algorithms by 3–24%, 7–67%, 8–101%, and 6–98% for ATOP, P@25, R@25, and NDCG@25, respectively.

In future work, we plan to consider two directions to further improve the accuracy of our proposed algorithms. First, we considered all missing ratings as negative feedback. Inspired by one class collaborative filtering [25,26], we plan to distinguish the weight of missing ratings based on the confidence of negative feedback. We hope that such a fine-grained distinction scheme for negative feedback can contribute to the improvement of accuracy. Second, we plan to combine our proposed preference model with a pair-wise ranking model of items in Bayesian personalized ranking [28]. We expect that such an idea can represent relative preferences more effectively.

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