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Robust design in multibody dynamics - application to vehicle ride-comfort optimization

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Abstract

This research is devoted to the robust design of multibody dynamical systems, that is to say to the optimal design of a multibody system which is carried out using an uncertain computational model. The probabilistic model of uncertainties is constructed using a probabilistic approach yielding a stochastic differential equation with random initial conditions. Then the robust design of the multibody system, in presence of uncertainties, is performed using the least-square method for optimizing the cost function, and the Monte Carlo simulation method as stochastic solver. The application consists in a simple multibody model of an automotive vehicle crossing a rough road and for which the suspensions have to be designed in order to optimize the comfort of the passengers.

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Keywords: Multibody dynamics; uncertainty quantification; robust design

1. Introduction

This paper deals with the robust design of a multibody dynamical system using a computational model for which some parameters are uncertain. The uncertainties are induced by natural variability or by a lack of knowledge existing on these parameters. For a multibody system, uncertainties can affect (1) the bodies themselves (inertia properties), (2) the joints between the bodies, (3) the external forces. If they are not negligible, theses uncertainties have to be taken into account in order to predict the response of the uncertain multibody system with a good robustness. In the context of the design of a multibody system, these uncertainties are propagated into the performance function that has to be optimized. Then the robust design consists in searching the optimal design of the multibody system taking into account uncertainties, and then allows the robustness of this optimal design o be analyzed with respect to uncertainties.

In the context of multibody dynamics, the parametric probabilistic approach of uncertainties consists in modeling the uncertain parameters of the multibody dynamical systems by random variables [1,4,10,12,15,18,19,22]. Therefore, the quantities of interest and the performance function become random variables, and a probabilistic robust design method [3,5,7,9,11,13,14,17,25] has to be used.

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The first objective of this paper consists in constructing a probabilistic model of uncertainties yielding a stochastic differential equation with random initial conditions. The second one is related to the robust design of a multibody system in presence of uncertainties by using the least-square method for constructing the cost function, and the Monte Carlo simulation method as stochastic solver.

In Section 2, the nominal model is presented for the rigid multibody system. Then, the stochastic model of uncertainties and the robust design method are presented in Sections 3 and 4. Finally, Section 4 presents an application that consists in a simple multibody model of an automotive vehicle crossing a rough road, and for which the suspensions have to be designed in order to optimize the comfort of the passengers [9].

2. Nominal model for the rigid multibody dynamical system

The mean model is constructed as follows [21,20]. Let RB_i be one of the rigid bodies, occupying a bounded domain Ω_i with a given geometry. Each rigid body RB_i is represented by its mass m_i, the position vector $\mathbf{r}_i$ of its center of mass, and by the matrix $[J_i]$ of its tensor of inertia defined in the local frame. The multibody dynamical system is made up of n_b rigid bodies and of some ideal joints. The interactions between the rigid bodies are realized by these ideal joints, but also by springs, dampers, and actuators, which produce forces between the bodies. Let $\mathbf{u}$ be the vector in $\mathbb{R}^{6n_b}$ such that $\mathbf{u} = (\mathbf{r}_1, ..., \mathbf{r}_{n_b}, \mathbf{s}_1, ..., \mathbf{s}_{n_b})$ in which $\mathbf{s}_i = (\alpha_i, \beta_i, \gamma_i)$ is the rotation vector. The n_c constraints induced by the joints are given by n_c implicit equations which are globally written as $\boldsymbol{\varphi}(\mathbf{u}, t) = 0$. Then the function $\{\mathbf{u}(t)\,, \in [0\,, T]\}$ is the solution of the following differential equation

$$\begin{bmatrix} [M] & [\boldsymbol{\varphi}_{\mathbf{u}}]^T \\ [\boldsymbol{\varphi}_{\mathbf{u}}] & [0] \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{q} - \mathbf{k} \\ -\frac{d}{dt}\boldsymbol{\varphi}_t - [\frac{d}{dt}\boldsymbol{\varphi}_{\mathbf{u}}]\,\dot{\mathbf{u}} \end{bmatrix}, \tag{1}$$

with the initial conditions

$$\mathbf{u}(0) = \mathbf{u}_0 \quad , \quad \dot{\mathbf{u}}(0) = \mathbf{v}_0 \quad , \tag{2}$$

in which $[M]$ is the $(6n_b \times 6n_b)$ mass matrix, $\mathbf{k}(\dot{\mathbf{u}})$ is the vector of the Coriolis forces, $[\boldsymbol{\varphi}_{\mathbf{u}}(\mathbf{u}(t), t)]_{ij} = \partial\varphi_i(\mathbf{u}(t), t)/\partial u_j(t)$, and $\boldsymbol{\varphi}_t = \partial\boldsymbol{\varphi}/\partial t$. The vector $\mathbf{q}(\mathbf{u}, \dot{\mathbf{u}}, t)$ is constituted of the applied forces and torques induced by springs, dampers, and actuators. The vector $\lambda(t)$ is the vector of the Lagrange multipliers.

The performance of the multibody system is measured by a performance function, $\mathbf{g}(\mathbf{u})$, with values in $\mathbb{R}^{n_g}$.

3. Stochastic model of uncertainties and robust design

For a multibody system, the possible sources of parametric uncertainties are the following.

(i)- Uncertainties for the spatial mass distribution inside a body. For instance, such a type of uncertainties can be encountered for a vehicle in which the passengers have a mass and a position that are variable. For each body, this type of uncertainties yields a random mass, a random position of the center of mass, and a random tensor of inertia. Consequently, the mass matrix $[\mathbf{M}]$ and the Coriolis forces $\mathbf{K}$ are random. The probability density functions (pdf) of these random masses, random positions of the centers of mass, and random tensors of inertia have been constructed [1] using the Maximum Entropy Principle, in which a special care has been devoted to the probabilistic modeling of the random tensor of inertia in following the methodology of the nonparametric probabilistic approach of uncertainties introduced in [23,24] for the construction of random matrices.

(ii)- Uncertainties in the joints. For the ideal joints, the directions and the points defining the joints can be uncertain. These uncertainties may be due to manufacturing tolerances, or due to the natural wear during the life cycle of the multibody system. Such uncertainties have to be taken into account in order to ensure a good accuracy for the prediction of the dynamical response of the multibody system. For non-ideal joints the friction coefficients can also be uncertain. The randomness in the joints between the bodies yields a random constraint vector $\boldsymbol{\Phi}$ in Eq. (1).

(iii)- Uncertainties in internal forces. Concerning internal forces, there may be uncertainties in the constitutive laws of the multidimensional springs and dampers. In such a case, uncertainties may be taken into account in the parameters of the constitutive laws, or directly in the stiffness and damping matrices using the nonparametric probabilistic approach of uncertainties (see[23]). The uncertainties in the internal forces induce a random vector $\mathbf{Q}$ in Eq. (1).

Let $\mathbf{x}$ be the vector of the n uncertain parameters of the multibody system. Since the uncertainties in the system parameters are taken into account using a probabilistic approach, then $\mathbf{x}$ is modeled by a random vector $\mathbf{X}$ with values in $\mathbb{R}^n$. Random vector $\mathbf{X}$ is written as $\mathbf{X} = (\mathbf{X}^d, \mathbf{X}^f)$ in which $\mathbf{X}^d$ is the random vector with values in $\mathbb{R}^{n_d}$ of the random design parameters, and $\mathbf{X}^f$ is the random vector with values in $\mathbb{R}^{n_f}$ of the fixed (but random) system parameters (we then have $n = n_d + n_f$). A *prior* probabilistic model of random vector $\mathbf{X}$ must be constructed and/or identified using an adapted methodology[24]. This *prior* probabilistic model depends on hyperparameters which are the mean values and other quantities allowing the statistical fluctuations (level of uncertainties) and the statistical dependencies between parameters to be controlled (coefficients of variation, correlation matrices, etc). In the context of uncertainty quantification, two cases can be considered. If experimental data are available, then the hyperparameters of the *prior* probabilistic model can be identified solving an inverse statistical problem. If there are no experimental data, then the mean values are chosen as the nominal values and the other hyperparameters can be considered as sensitivity parameters in order to analyze the robustness of the responses with respect to the level of uncertainties. In the context of robust design, the design parameters are chosen as the mean values of the *prior* probabilistic model of random design parameters, $\mathbf{X}^d$, while the other hyperparameters of the *prior* probabilistic model of $\mathbf{X}^d$ are treated as previously explained. In this paper, since no experimental data are available, the design parameters will be the mean values of $\mathbf{X}^d$ and all the other hyperparameters of the *prior* probabilistic model of $\mathbf{X}$ will be either fixed or used as sensitivity parameters. In this last case, the robustness of the optimal design point can be analyzed with respect to the level of uncertainties.

Let $\mathbf{U} = (\mathbf{R}_1, ..., \mathbf{R}_{n_b}, \mathbf{S}_1, ..., \mathbf{S}_{n_b})$ be the $\mathbb{R}^{6\,n_b}$-valued stochastic processes indexed by $[0, T]$, which model the $6\,n_b$ random coordinates. Let $\mathbf{\Lambda}$ be the $\mathbb{R}^{n_c}$-valued stochastic process indexed by $[0, T]$, which models the n_c random Lagrange multipliers. The deterministic Eq. (1) becomes the following stochastic equation

$$\begin{bmatrix} [\mathbf{M}] & \left[\boldsymbol{\varphi}_{\mathbf{u}}\right]^T \\ \left[\boldsymbol{\varphi}_{\mathbf{u}}\right] & [0] \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}} \\ \mathbf{\Lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q} - \mathbf{K} \\ -\frac{d}{dt}\boldsymbol{\varphi}_t - \left[\frac{d}{dt}\boldsymbol{\varphi}_{\mathbf{u}}\right]\dot{\mathbf{U}} \end{bmatrix} \quad , \tag{3}$$

$$\mathbf{U}(0) = \mathbf{U}_0 \quad , \quad \dot{\mathbf{U}}(0) = \mathbf{v}_0 \, , \tag{4}$$

in which $\mathbf{U}_0$ is a given random vector, and $\mathbf{v}_0$ is a given deterministic vector. Random Eqs. (3) and (4) are solved using the Monte Carlo simulation method.

The performance of the stochastic multibody system is measured by the $\mathbb{R}^{n_g}$-random variable, $\mathbf{G} = \mathbf{g}(\mathbf{U})$, which is assumed to be a second-order random variable.

4. Robust design

As explained in the previous section, the vector-valued design parameter is the vector $\mathbf{d} = E\{\mathbf{X}^d\}$ with values in $\mathbb{R}^{n_d}$, which is the mean vector of the random design parameter $\mathbf{X}^d$ with values in $\mathbb{R}^{n_d}$, and where E is the mathematical expectation. All the other hyperparameters of the probabilistic model of random vector $\mathbf{X} = (\mathbf{X}^d, \mathbf{X}^f)$ are assumed to be fixed (nevertheless, these other hyperparameters can be used for carrying out a sensitivity analysis of the optimal design point with respect to the level of uncertainties). Let $C_d \subseteq \mathbb{R}^{n_d}$ be the admissible set for vector $\mathbf{d}$. The performance random vector, $\mathbf{G}$, depends on $\mathbf{d}$, and is then rewritten as $\mathbf{G}_{\mathbf{d}} = \mathbf{g}(\mathbf{U}_{\mathbf{d}})$. We obtain a family of random variables $\{\mathbf{G}_{\mathbf{d}}\, , \, \mathbf{d} \in C_d\}$. Let $\underline{\mathbf{G}}_{\mathbf{d}} = E\{\mathbf{G}_{\mathbf{d}}\}$ be the mean value of $\mathbf{G}_{\mathbf{d}}$. Let $\mathbf{g}^*$ be the deterministic target performance vector associated with performance random vector $\mathbf{G}_{\mathbf{d}}$. The optimal value $\mathbf{d}^{\text{opt}}$ of vector $\mathbf{d}$ is calculated using the least square method, *i.e*,

$$\mathbf{d}^{\text{opt}} = \arg\min_{\mathbf{d} \in C_d} \mathcal{D}(\mathbf{d}) \, , \tag{5}$$

in which the cost function $\mathcal{D}(\mathbf{d})$ is written as

$$\mathcal{D}(\mathbf{d}) = E\{\|\mathbf{G_d} - \mathbf{g}^*\|^2\} = E\{\|\mathbf{G_d} - \underline{\mathbf{G}}_\mathbf{d}\|^2\} + \|\mathbf{g}^* - \underline{\mathbf{G}}_\mathbf{d}\|^2 \,, \tag{6}$$

which means that the minimization problem defined by Eq. (5) aims at minimizing both (i) the bias between the mean value of the random performance vector and the deterministic target performance vector and (2) the variance of the random performance vector.

For each value of vector $\mathbf{d}$, function $\mathcal{D}(\mathbf{d})$ is estimated using the Monte Carlo simulation method.

5. Application

The application is devoted to the stochastic response of a vehicle (see its scheme in Fig. 1) at constant speed excited by the ground elevation that is assumed to be a homogeneous random field. Consequently, the imposed vertical displacement is a stationary stochastic process. The computational model of the multibody system is uncertain, and its input is a stationary stochastic excitation.

5.1. Definition of the nominal multibody system

The system is made up of 9 rigid bodies: The sprung mass RB_1 (the vehicle body), the two front unsprung masses RB_2^{fr}, RB_3^{fr} (front wheels), the two rear unsprung masses RB_2^{re}, RB_3^{re} (rear wheels), the two front massless bodies RB_4^{fr}, RB_5^{fr}, the two rear massless bodies RB_4^{re}, RB_5^{re}, the two front seats (with passengers) RB_6^{fr}, RB_7^{fr}, and the two rear seats (with passengers) RB_6^{re}, RB_7^{re}. The sprung mass RB_3 is linked to each seat by four springs and four dampers. The sprung mass RB_3 is linked to each unsprung mass by a spring and a damper. Each unsprung mass is linked to a massless body by a spring and a damper. Each massless body is linked to the ground by a prismatic joint following the vertical direction. Let L be the distance between the two axles and let V be the speed of the car. The responses are analyzed for t in $[0\,,T]$, in which $T = 100$ s is the final time. The front imposed displacements are modeled by independent stationary Gaussian stochastic processes denoted by $\{t \mapsto Z_1^{\text{fr}}(t), t \geq 0\}$ and $\{t \mapsto Z_2^{\text{fr}}(t), t \geq 0\}$. The corresponding rear imposed displacements stochastic processes are $\{t \mapsto Z_1^{\text{re}}(t), t \geq 0\}$ and $\{t \mapsto Z_2^{\text{re}}(t), t \geq 0\}$ such that (1) for $t \in [0, L/V[$, $Z_1^{\text{re}}(t) = 0$ and $Z_2^{\text{re}}(t) = 0$ and (2) for $t \in [L/V, T]$, $Z_1^{\text{re}}(t) = Z_1^{\text{fr}}(t - L/V)$ and $Z_2^{\text{re}}(t) = Z_2^{\text{fr}}(t - L/V)$. The power spectral density (PSD) functions of stochastic processes $Z_1^{\text{fr}}(t)$ and $Z_2^{\text{fr}}(t)$ are equal, and are plotted in

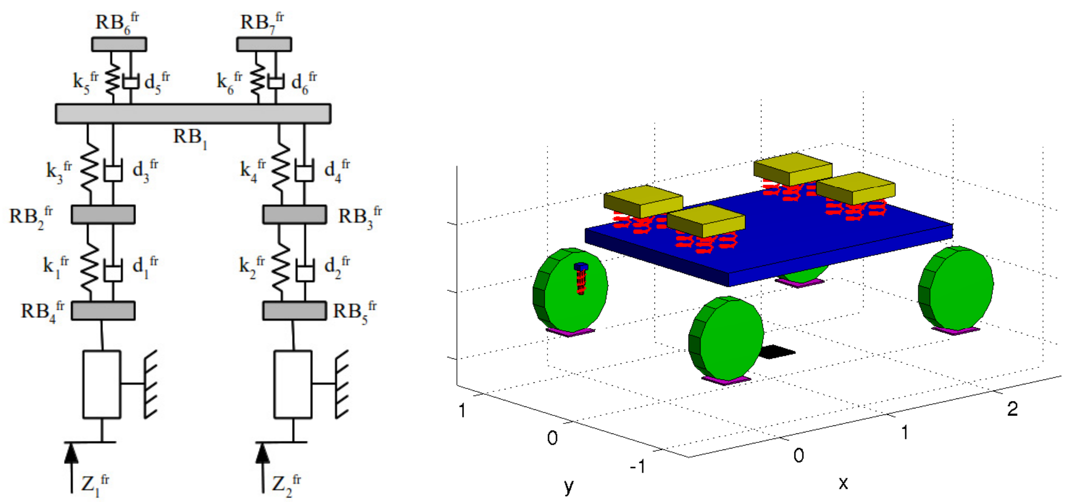

Fig. 1. Nominal multibody system: Schematic front view (left figure), 3D view (right figure).

Fig. 2. This PSD corresponds to a D-class road roughness in the ISO classification[8,9]. Figure 3 shows a realization

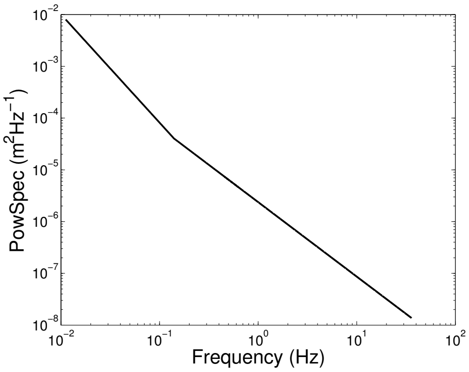

Fig. 2. Power spectral density function of stochastic processes Z_1^{fr} and Z_2^{fr}.

of stochastic process $Z_1^{\mathrm{fr}}(t)$. Let P_1 be an observation point belonging to the front-left seat and P_2 be an observation

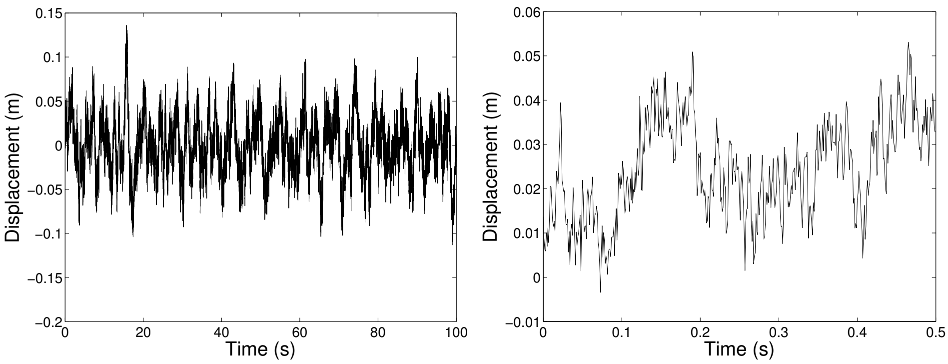

Fig. 3. Realization of stochastic process Z_1^{fr} (left figure), and its zoom in $[0, 5]$ s (right figure).

point belonging to the rear-left seat. The PSD function of the random transient vertical acceleration at point P_1 and at point P_2 are estimated using the periodogram method[16], and are plotted in Fig. 4.

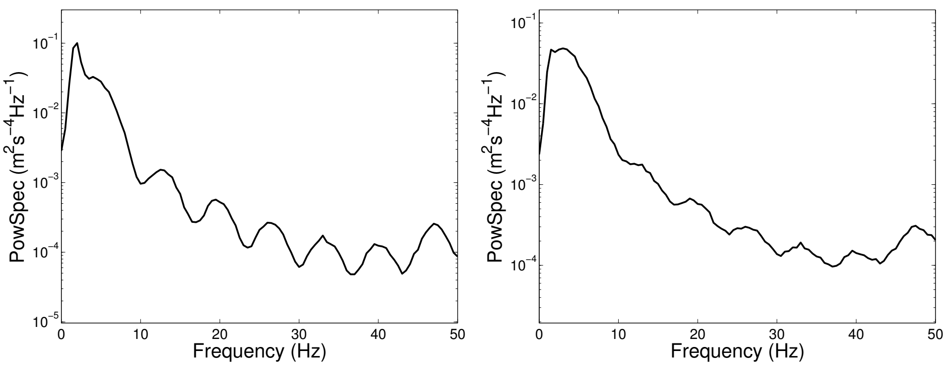

Fig. 4. Power spectral density function of the stationary stochastic acceleration at point P_1 (left figure) and at point P_2 (right figure).

5.2. Probability model of fixed system-parameter uncertainties

In this application, only the fixed system parameters are uncertain, and the design parameters are assumed to be deterministic. The uncertain fixed parameters of the multibody system are the inertia properties of the sprung mass, the inertia properties of the seats, the stiffnesses of the unsprung-masses/massless springs, the damping coefficient of the unsprung-masses/massless dampers. The random inertia properties (random mass, random center of mass, and random tensor of inertia) are constructed as follows[1]. The random stiffnesses and the random damping coefficients are Gamma random variables with coefficient of variation (ratio between the standard deviation and the mean value) equal to 0.1.

5.3. Robust design

The design parameters are (1) the damping coefficients of the dampers between the sprung mass and the unsprung masses, which are such that $d_3^{\text{fr}} = d_4^{\text{fr}} = d_3^{\text{re}} = d_4^{\text{re}}$, and (2) the damping coefficients of the dampers between the sprung mass and the seats, which are such that $d_5^{\text{fr}} = d_6^{\text{fr}} = d_5^{\text{re}} = d_6^{\text{re}}$. Therefore, there are $n_d = 2$ design parameters and $\mathbf{d} = (d_3^{\text{fr}}, d_5^{\text{fr}})$. Its initial value is $\mathbf{d}^{\text{init}} = (2217, 318)\,\text{N s m}^{-1}$.

For a given realization of the random vector $\mathbf{X}^f$ (modeling the uncertainties for the fixed system parameters), the performance function is related to the ride comfort index[8,9], which is expressed as a function of the PSD functions of the vertical accelerations at given points (which are stationary stochastic processes due to the stationarity of the stochastic input). Considering all the possible realizations (following the probability model) of the random vector $\mathbf{X}^f$, the ride comfort index, $\mathbf{G_d} = (G_{1,\mathbf{d}}, G_{2,\mathbf{d}})$, becomes a random vector such that

$$
\begin{aligned}
G_{1,\mathbf{d}} &= \sqrt{\int_{f_1}^{f_2} w(f)^2\, S_{\mathbf{d}}^1(2\pi f)\, df}\,, \\
G_{2,\mathbf{d}} &= \sqrt{\int_{f_1}^{f_2} w(f)^2\, S_{\mathbf{d}}^2(2\pi f)\, df}\,,
\end{aligned}
\tag{7}
$$

in which $f \mapsto S_{\mathbf{d}}^1(2\pi f)$ and $f \mapsto S_{\mathbf{d}}^2(2\pi f)$ are the random PSD functions of the stationary stochastic vertical acceleration at point P_1 and at point P_2, and where $f \mapsto w(f)$ is the weighting function plotted in Fig. 5.

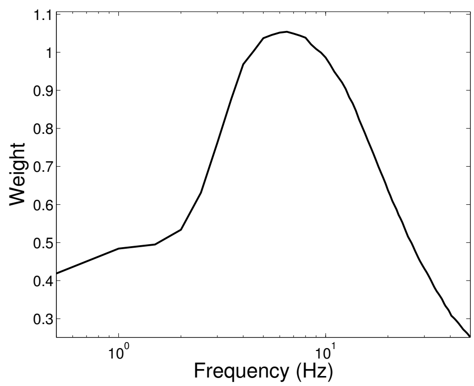

Fig. 5. Weighting function $f \mapsto w(f)$.

The cost function $\mathbf{d} \mapsto \mathcal{D}(\mathbf{d})$ is plotted in Fig. 6. In this figure, it can be seen that the optimal value is $\mathbf{d}^{\text{opt}} = (665, 669)\,\text{N s m}^{-1}$. The corresponding probability density functions of the random ride comfort indices, $G_{1,\mathbf{d}^{\text{opt}}}$ and $G_{2,\mathbf{d}^{\text{opt}}}$, are plotted in Fig. 7. The mean values and the variances of $G_{1,\mathbf{d}^{\text{opt}}}$ and of $G_{2,\mathbf{d}^{\text{opt}}}$ are reported in Table 1. As expected, it can be seen in Fig. 7 and in Table 1 that, compared to the initial design configuration, the mean values and the variances of the random ride comfort indices are lower for the optimal design configuration.

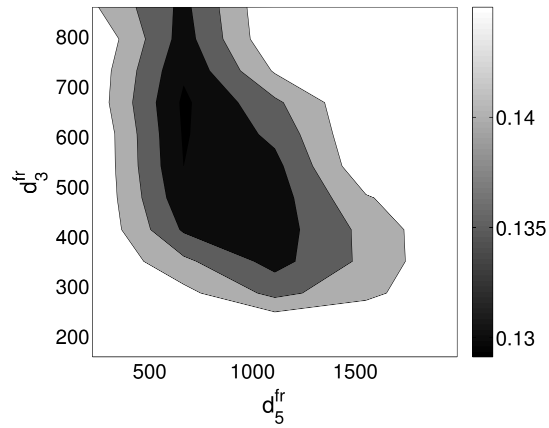

Fig. 6. Cost function $\mathbf{d} \mapsto \mathcal{D}(\mathbf{d})$.

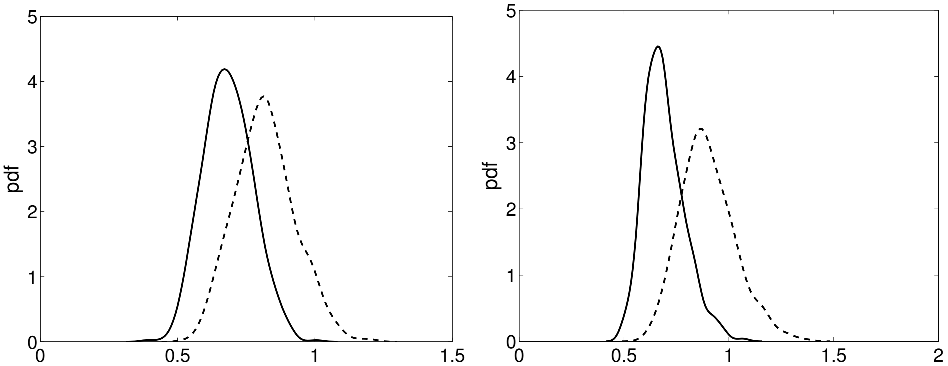

Fig. 7. Solid lines: Probability density functions of $G_{1,\mathbf{d}^{\text{opt}}}$ (left figure) and $G_{2,\mathbf{d}^{\text{opt}}}$ (right figure). Dashed lines: Probability density functions of $G_{1,\mathbf{d}^{\text{init}}}$ (left figure) and $G_{2,\mathbf{d}^{\text{init}}}$ (right figure).

	Initial configuration	Optimal configuration
Mean value of $G_{1,\mathbf{d}}$	$0.82\ \text{m}\,\text{s}^{-2}$	$0.68\ \text{m}\,\text{s}^{-2}$
Mean value of $G_{2,\mathbf{d}}$	$0.90\ \text{m}\,\text{s}^{-2}$	$0.69\ \text{m}\,\text{s}^{-2}$
Variance of $G_{1,\mathbf{d}}$	$1.2 \times 10^{-2}\ \text{m}^2\,\text{s}^{-4}$	$7.8 \times 10^{-3}\ \text{m}^2\,\text{s}^{-4}$
Variance of $G_{2,\mathbf{d}}$	$1.7 \times 10^{-2}\ \text{m}^2\,\text{s}^{-4}$	$9.1 \times 10^{-3}\ \text{m}^2\,\text{s}^{-4}$

Table 1. Statistics for $G_{1,\mathbf{d}^{\text{opt}}}$ and $G_{2,\mathbf{d}^{\text{opt}}}$.

6. Conclusions

A methodology has been presented for analyzing the robust design of multibody systems in presence of uncertainties. The uncertainties have been taken into account using a parametric probabilistic approach. The optimal design parameters have been calculated using the least-square method. The methodology has been validated on a multibody system representing an automotive vehicle crossing a rough road. The optimal damping coefficients have been calculated in order to optimize the ride comfort index.

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