Habit formation, asymmetric price adjustment, and real exchange rate persistence

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Abstract

This paper studies the eect of habit formation in consumption on the real exchange persistence under monetary policy shocks. Closed economy models of monetary policy have emphasized the ability of habit formation to generate endogenous persistence, which has encouraged the open economy literature to incorporate habit formation into theoretical models to generate greater persistence in the real exchange rate. However, there has not yet been a thorough evaluation of the habit formation eect in the real exchange rate context. By incorporating habit formation into the model of Benigno (2004), it is shown that the habit formation eect depends on the degree of price stickiness assumed within and across countries as well as the design of the monetary policy rule. Habit formation does not aect the dynamics of the real exchange rate under symmetric price adjustment and does not signicantly contribute to the persistence of the real exchange rate for more general cases.

Keywords: habit formation, real exchange rate persistence, closed-economy output gap persistence, PPP puzzle

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1 Introduction

The purchasing power parity (PPP) puzzle is that in the data, the real exchange rate defined as deviations from PPP is enormously volatile and stubbornly persistent. Each of its volatility and persistence can be explained by monetary and technology shocks, respectively, so that the key of the PPP puzzle is to explain them in a common framework. One possible way to address this issue relies on sticky prices. Monetary shocks lead the real exchange rate to be so volatile at the short term while sticky prices cause it to die out at a very slow rate after monetary shocks. Following this reasoning, many previous papers (see Chari et al. (2002) among others) have attempted to resolve the PPP puzzle in the sticky price framework. Although it is shown that it is quite successful in generating the observed volatility of the real exchange rate, an implausibly high degree of price stickiness is needed to match the persistence observed in the data. As a result, other model ingredients that help prices to be more persistent endogenously are added to improve the real exchange rate persistence: for instance, a translog in preference in Bergin and Feenstra (2001) and sticky wages in Chari et al. (2002). Such elements, however, turn out to have marginal effects on the persistence of the real exchange rate.

In this paper, we investigate the feasibility of habit formation in consumption as an endogenous persistence mechanism for the real exchange rate under monetary shocks. Habit formation has been employed in closed economy monetary policy models (e.g., Fuhrer (2000)), improving the predicted dynamics of aggregate variables such as output, consumption, and inflation in response to monetary shocks. In particular, habit formation as an endogenous persistence mechanism significantly improves the persistence of output gap in the closed economy under monetary shocks. The closed economy output gap is a measure of real economic activity within a country and the real exchange rate is a measure of relative real economic activity across two countries. Hence, in a symmetric two-country open economy setting, the real exchange rate is sometimes considered isomorphic to the closed economy output gap. From this point of view, habit formation is considered a promising element to generate a more persistent real exchange rate. In addition, one possible channel through which habit persistence directly influences the real exchange rate is the sensitivity of the real marginal cost to monetary shocks. Habit formation generates the endogenous persistence of consumption. In equilibrium, the marginal rate of substitution between labor and consumption is equal to the real wage. Consequently, the real marginal costs faced by price-setting firms are less sensitive to monetary shocks. As a result, prices also respond slowly to monetary shocks, producing a persistent deviation from PPP. For these reasons, many researchers in the open economy area have incorporated habit formation into their models to generate a more persistent real exchange rate. A rigorous evaluation of the role of habit formation in the PPP context, however, has not been made.

For the purpose of this paper, we begin by reviewing the roles of three endogenous persistence mechanisms, that is, interest rate smoothing, backward-looking pricing, and habit formation in consumption, in resolving the well-known persistence problem in a closed economy model with a standard Calvo pricing. We focus on their implications for the persistence of output gap in the closed economy model since it is shown that in its simple extension without such inertias to a symmetric two-country open economy model (i.e., the model of Benigno (2004)), the real exchange rate is isomorphic to the closed economy output gap. This might imply that habit formation helps improve the real exchange rate persistence significantly as like its effect on the closed economy output gap persistence. However, we show that when habit formation is incorporated in the symmetric two-country open economy model, the seemingly isomorphic relationship between the closed economy output gap and real exchange rate breaks down and then habit formation does not affect the dynamics of the real exchange rate under monetary shocks. More importantly, this breakdown predicts that in more general open economy cases where price adjustment speed is allowed to vary within and across countries, habit formation does not significantly contribute to the persistence of the real exchange rate. We also show that the relative price inertia emphasized by Benigno (2004) is not able to generate a more persistent real exchange rate, showing that relative price inertia significantly increases the real exchange rate persistence only under implausibly high degrees of price stickiness.

The rest of the paper is organized as follows. Section 2 reviews the endogenous persistence problem in the closed economy and describes the implications of endogenous persistence inertias suggested in the literature. In Section 3, the structure of the two-country open economy model is described and the linear system of equilibrium equations is provided. Section 4 analyzes the effect of habit formation on the real exchange rate persistence, depending on the assumption of the degree of price stickiness as well as the design of the monetary policy rule. If analytical solutions are available, they are provided. Otherwise, the analysis is done through simulation. Section 5 has concluding remarks.

2 Endogenous persistence problem in the closed economy model

This section reviews the persistence problem in a closed economy model with a standard Calvo pricing and describes the roles of three endogenous persistence mechanisms: interest rate smoothing, backward-looking pricing, and habit formation in consumption. In a general equilibrium where the monetary policy is designed as an interest-rate feedback rule, the following system of three equations determines the dynamics of the nominal interest rate $\widehat{i_t}$, inflation π_t, and output gap x_t under monetary policy shocks ε_t:[2]

$$\widehat{i_t} = \phi\pi_t + \psi x_t + \varepsilon_t, \tag{1}$$

$$\pi_t = \beta\pi_t E_t[\pi_{t+1}] + \kappa x_t, \tag{2}$$

$$x_t = E_t[\pi_{t+1}] - \rho^{-1}(\widehat{i_t} - E_t[\pi_{t+1}]), \tag{3}$$

where $\kappa = \xi(\eta+\rho)$ with $\xi = (1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$ and each of three equations is called a Taylor rule, a new Keynesian Phillips curve (NKPC), and an intertemporal IS curve (ISC), respectively.[3] The system, however,

[2] $\widehat{i_t}$ is the deviation of the logarithmic gross nominal interest rate from its steady state value and x_t is defined as the logarithmic difference between the actual output level and output level under flexible prices.

[3] All derivation is provided in the technical appendix, which is available upon request. Also, see Woodford (2003). In Section 3, structural parameters η, ρ, α, β, and σ are defined as well.

cannot generate any persistence in any of the three endogenous variables if monetary policy shocks are serially uncorrelated. While the nominal price adjustment is sluggish, the endogenous variables return to the initial equilibrium after one period in response to a one-period monetary shock, and thus the degree of price stickiness does not affect their persistence. To resolve this persistence problem, some endogenous persistence mechanisms are incorporated into the original system (i.e., Equations (1), (2), and (3)).

One of endogenous persistence mechanisms is the interest rate smoothing behavior of the monetary policy authority, which is represented by:

$$\widehat{i_t} = \lambda \widehat{i}_{t-1} + (1-\lambda)(\phi\pi_t + \psi x_t) + \varepsilon_t,$$

where $0 \leq \lambda < 1$ is a measure of the partial adjustment of the interest rate. In the system with this modified Taylor rule, the interest rate, inflation, and output gap all exhibit persistence which now is proportional to the degree of price stickiness denoted by α that determines κ defined in Equation (2). For reasonable values of α and λ, however, the system can neither generate the observed persistence of inflation nor output gap in terms of the autocorrelation coefficient or impulse response function.

In response, backward-looking pricing behavior and habit formation in consumption are suggested to generate more persistent inflation and output gap, respectively. In a partial equilibrium sense, a fully forward-looking Phillips curve (i.e., Equation (2)) is unlikely to be supported by the data, even though the forward-looking pricing is considered as a dominant component in the dynamics of inflation (see Gali and Gertler (1999) among others). Incorporating a backward-looking pricing behavior into the standard Calvo mechanism modifies Equation (2) as a hybrid NKPC:

$$\pi_t - \gamma\pi_{t\text{-}1} = \beta E_t[\pi_{t+1} - \gamma\pi_t] + \kappa x_t,$$

where $0 \leq \gamma \leq 1$ is a measure of indexation scheme (see Christiano et al. (2005)). The hybrid NKPC, the modified Taylor rule, and the original ISC are able to generate more persistent inflation rate, but do not improve the output gap persistence.

Regarding the role of habit formation in consumption, it has been emphasized in monetary policy models, since it yields better fit to estimated impulse response functions of output, consumption, and inflation in response to identified monetary policy shocks (see Bouakez et al. (2005) among others). In particular, habit formation helps generate a more persistent output gap. Adding habit formation into the original system modifies Equations (2) and (3) simultaneously so that each of them becomes:

$$\pi_t = \beta E_t[\pi_{t+1}] + \xi(\eta + \frac{\rho}{1-b})x_t - \xi\frac{\rho b}{1-b}x_{t-1},$$

and

$$x_t = \frac{1}{1+b}E_t[\pi_{t+1}] + \frac{b}{1+b}x_{t-1} - \frac{1-b}{1+b}\rho^{-1}(\widehat{i_t} - E_t[\pi_{t+1}]),$$

where $0 \leq b < 1$ is a measure of the importance of the habit stock, that is, the last period's aggregate consumption level. In particular, the ISC is modified in a way that the current output gap now depends on a weighted average of the expected future and past output gaps and is less sensitive to the real interest rate. As the habit parameter increases, a more weight is put to the past output gap. In this way, habit formation helps generate a more persistent output gap. The NKPC is also modified so that the cur-

Table 1. Structural papmeter values

Panel A: The closed economy model in section 2

Parameter	Description	Value
β	subjective discount factor	0.99
ρ	relative risk aversion	6
η	inverse of elasticity of labor supply	2
σ	elasticity of substitution of goods	10
α	degree of price stickiness	0.75
ϕ	inflation targeting coefficient	1.5
ψ	output gap coefficient	0.5
λ	interest rate smoothing (IS)	If IS is considered, 0.8; otherwise, 0
γ	backward-looking indexation (BL)	If BL is considered, 1; otherwise, 0
b	habit formation (HF)	If HF is considered, 0.8 otherwise, 0

Panel B: The open economy model in section 3

Parameter	Description	Value
ϕ	relative size of home country	0.5
θ	elasticity of substitutuin of home and foreign sub-indices	1.5

Note: The values of the closed economy structural parameters are still used for the open economy. When dierent values are used, they are specied in the text.

rent inflation rate depends on the past output gap as well as the current one. However, the effect on the inflation persistence is much smaller than the effect on the output gap persistence. Table 2 summarizes the preceding arguments. Note that the structural parameter values used in simulation are provided in Table 1.

Table 2. Persistence of inflation rate, output gap, and interest rate in the closed economy model

Endogenous inertia	Inflation rate	Output gap	Interest rate
interest smoothing (IS)	0.757	0.757	0.757
backward-looking (BL)	0.891	0.016	0.024
habit formation (HF)	0.029	0.772	0.017
IS & BL	0.975	0.731	0.738
IS & HF	0.765	0.963	0.776
IS, BL & HF	0.976	0.954	0.759

Note: The number represents the rst-order autocorrelation coefficient of the long-period series obtained by simulating the closed economy model considered in Section 2.

Typically, all of three inertia elements are incorporated into the original system (i.e., Equations (1), (2), and (3)) to fit the observed dynamics of the interest rate, inflation, and output gap.[4] Regarding the persistence of output gap, the system even with all three endogenous inertias cannot generate the observed persistence in the data (see McCallum (2001)). In a symmetric two-country open setting, the real exchange rate as a measure of relative real economic activity across two countries is isomorphic to the closed economy output gap as a measure of real economic activity within a country and in the data, the real exchange rate is as much persistent as output gap.[5] These observations and the roles of three endogenous inertia mechanisms discussed above would imply that at least gains of the output gap persistence due to interest rate smoothing and habit formation still apply to the real exchange rate to match its observed persistence.

[4] When all three inertias are added, the system consists of the modified Taylor rule, the modified ISC, and the hybrid NKPC that depends the past output gap as well as the current one.

[5] see Moosa and Kim (2001) and Ahn (2009), among others.

3 Structure of the model

The model is a simple extension of the closed economy considered in the previous section to an open economy. There are two countries that produce differentiated goods in each country. All goods are tradable and their prices are sticky under a standard Calvo mechanism. The new prices set by firms are denominated in the local currency so that local currency pricing is only a source of deviations from PPP (i.e., fluctuations in the real exchange rate). Moreover, the price adjustment speed (i.e., the degree of price stickiness) is allowed to vary within and across countries. Asset markets are complete both domestically and internationally.

3.1 Households

There are two countries, home and foreign, denoted by H and F, respectively. The home country produces goods on $[0, n]$ and the foreign country produces goods on $(n, 1]$. In each country with its population size set equal to the range of the produced goods, there is a continuum of economic agents indexed by i for a home agent and by j for a foreign agent, and each individual owns a domestic firm indexed by h for a home firm and indexed by f for a foreign firm.

Under the functional forms of instantaneous utility from consumption and disutility from labor:

$$U\left(C_t(i), C_{t-1}\right) = \frac{\left(C_t(i) - bC_{t-1}\right)^{1-\rho}}{1-\rho}, \quad V\left(L_t(i)\right) = \frac{L_t(i)^{1+\eta}}{1+\eta}$$

where $0 \leq b < 1$ indicates the importance of the habit stock which is the last period's aggregate consumption level C_{t-1}, ρ is a measure of relative risk aversion, and η is the inverse of the elasticity of labor supply, the lifetime utility of each agent is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U\left(C_t(i),\, C_{t-1}\right) + N\left(\frac{M_t(i)}{P_t}\right) - V\left(L_t(i)\right)\right], \tag{4}$$

where $\frac{M_t(i)}{P_t}$ is an agent's real balances, $L_t(i)$ is his labor supply, and $C_t(i)$

is his consumption index defined as[6]

$$C=\left[n^{\frac{1}{\theta}}C_H^{\frac{\theta-1}{\theta}}+(1-n)^{\frac{1}{\theta}}C_F^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

where $\theta>0$ is the elasticity of substitution between home and foreign consumption sub-indices. Their intratemporal optimal demands are

$$C_H=n\left(\frac{P_H}{P}\right)^{-\theta}C,\ C_F=(1-n)\left(\frac{P_F}{P}\right)^{-\theta}C,$$

where the aggregate price index is defined as

$$P=\left[nP_H^{1-\theta}+(1-n)\,P_H^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

by which money is deflated. The consumption sub-indices C_H and C_F are given as

$$C_H=\left(\left(\frac{1}{n}\right)^{\frac{1}{\sigma}}\int_0^n c(h)^{\frac{\sigma-1}{\sigma}}dh\right)^{\frac{\sigma}{\sigma-1}},\ C_F=\left(\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}}\int_n^1 c(h)^{\frac{\sigma-1}{\sigma}}df\right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma>1$ is the elasticity of substitution for goods produced within a country, and the demand for each differentiated good is

$$c(h)=\left(\frac{p(h)}{P_H}\right)^{-\sigma}\left(\frac{P_H}{P}\right)^{-\theta},\ c(f)=\left(\frac{p(f)}{P_F}\right)^{-\sigma}\left(\frac{P_F}{P}\right)^{-\theta}C,$$

where two price sub-indices P_H and P_F expressed in the domestic currency equals

$$P_H=\left(\frac{1}{n}\int_0^n p(h)^{1-\theta}dh\right)^{\frac{1}{1-\sigma}},\ P_F=\left(\frac{1}{1-n}\int_n^1 p(h)^{1-\theta}df\right)^{\frac{1}{1-\sigma}}.$$

In each period t, the economy faces one of finitely many events $s^t\in\Gamma$, where Γ is the set of finitely many states. The history of events up through and including period t is denoted by p^t and the conditional probability of occurrence of state s^{t+1} at period t is $\mu(s^{t+1}|p^t)$, where the initial realization s^0 is given. There are complete markets in this economy

[6] In what follows, the index i for each agent is omitted. Thus, the consumption index of an individual without i is different from the habit stock of his instantaneous utility from consumption.

at both domestic and international levels. $B(i,\ s^{t+1})$ denotes the home agent's holdings of a one-period contingent claim that pays one unit of the home currency if state s^{t+1} occurs and 0 otherwise, and $Q(s^{t+1}|p^t)$ denotes the price of one unit of such a claim at period t and state s^t in units of the home currency. Foreign agents' holdings of the contingent claim are indexed with j. Thus, the budget constraint at time t is

$$P_tC_t(i)+M_t(i)+\sum_{s^{t+1}\in\Gamma}Q(s^{t+1}|p^t)B(i,\ s^{t+1})$$
$$=W_t(i)L_t(i)+M_{t-1}(i)+B(i,\ s^t)+PR_t(i)+TR_t(i), \tag{5}$$

where $W_t(i)$ is the nominal wage, $PR_t(i)$ are nominal profits of his own firm, and $TR_t(i)$ is a nominal transfer from the government. The government's budget is balanced every period so that total transfers are equal to seigniorage revenues:

$$\int_0^n (M_t(i)-M_{t-1}(i))di=\int_0^n TR_t(i)di.$$

Each agent maximizes his preferences, Equation (4), subject to the sequence of budget constraints, Equation (5), so that the first-order conditions with respect to consumption, labor, holdings of state contingent claims, and money balances imply: the labor supply condition is

$$\frac{V_L(L_t(i))}{U_C(C_t,\ C_{t-1})}=\frac{W_t(i)}{P_t}, \tag{6}$$

where $U_C(C_t,\ C_{t-1})=(C_t-bC_{t-1})^{-\rho}$, $V_L(L_t(i))=L_t(i)^{\eta}$, and $N_{M/p}(\cdot)$ below denote the derivative with respect to its argument, the price for a one period risk-free nominal bond is

$$\frac{1}{1+i_t}=\beta E_t\left[\frac{U_C(C_{t+1},\ C_t)}{U_C(C_t,\ C_{t-1})}\frac{P_t}{P_{t+1}}\right], \tag{7}$$

and the demand for balances is

$$\frac{N_{M/p}\left(\frac{M_t(i)}{Pt}\right)}{U_C(C_t,\ C_{t-1})}=\frac{i_t}{1+i_t}. \tag{8}$$

Equation (6) equates the marginal rate of substitution between consumption and labor to the real wage; Equation (7) is the Euler equation; Equation (8) equates the marginal rate of substitution between consumption and real balances with the opportunity cost of holding balances.

Finally, under the assumption that home and foreign agents are allowed to trade in the contingent one-period nominal claims denominated in the home currency, the following risk-sharing condition holds

$$\frac{U_{C^*}\left(C_t^*,\ C_{t-1}^*\right)}{U_{C^*}(C_t,\ C_{t-1})}=\kappa RS_t, \tag{9}$$

where the real exchange rate is denoted by $RS_t = {S_t P_t^*}/{P_t}$ with the nominal exchange rate S_t defined as the home currency price of one unit of the foreign currency and a constant κ depends on initial conditions.

3.2 Firms and price setting

All of produced goods are assumed to be tradable. The prices set by firms are assumed to be sticky through a standard Calvo pricing mechanism and to be denominated in terms of the local currency.

The production function of a home firm h is given by

$$y_t(h)=A_t L_t(h),$$

where A_t is the country specific technology shock at period t and $L_t(h)$ is the labor force employed by a firm h. When a home firm receives a signal to change its price with the probability of $1-\alpha$, it sets the prices of its good in the home and foreign markets at period t to maximize the expected discounted value of its profits:

$$\max_{\tilde{p}_t(h),\tilde{p}_t^*(h)} E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s}\left[\tilde{p}_t(h)\tilde{y}_{t+s}^h(h)+S_{t+s}\tilde{p}_t^*(h)\tilde{y}_{t+s}^f(h)-W_{t+s}(h)\frac{\tilde{y}_{t+s}^d(h)}{A_{t+s}}\right]$$

$$s.t.\ \tilde{y}_{t+s}^h(h)=\left(\frac{\tilde{p}_t(h)}{p_{Ht+s}}\right)^{-\sigma} C_{Ht+s},\ \tilde{y}_{t+s}^f(h)=\frac{1-n}{n}\left(\frac{\tilde{p}_t^*(h)}{P_{Ht+s}^*}\right)^{-\sigma} C_{Ht+s}^*,$$

$$\tilde{y}_{t+s}^d(h)=\tilde{y}_{t+s}^h(h)+\tilde{y}_{t+s}^f(h),$$

where $Q_{t,\ t+s}=\beta^s\frac{U_C(C_{t+s},\ C_{t+s-1})}{U_C(C_t,\ C_{t-1})}\frac{P_t}{P_{t+s}}$ is the nominal stochastic discount factor, two prices $\tilde{p}_t(h)$ and $\tilde{p}_t^*(h)$ are those charged on the home and foreign markets in the local currency and still apply to period $t+s$, respectively, and $\tilde{y}_{t+s}^h(h)$ and $\tilde{y}_{t+s}^f(h)$ are the total demands of home and foreign markets, respectively. Since the production function is linear, this maximization problem can be separated for the home and foreign markets. In addition, when a firm sets its prices charged on two markets, the different length of contracts is allowed across home and foreign markets as well as across home and foreign firms. To put it differently, the degree of price stickiness can vary within and across countries: that is, α^H and α^{H*} for the home and foreign markets of home goods and α^F and α^{F*} for the home and foreign markets of foreign goods.

For each degree of price stickiness, the first-order conditions of the maximization problem imply:

$$\tilde{p}_t(h)=\mu\frac{E_t\sum_{s=0}^{\infty}\left(\alpha^H\right)^s Q_{t,t+s}\tilde{y}_{t+s}^h(h)MC_{t+s}(h)}{E_t\sum_{s=0}^{\infty}(\alpha^H)^s Q_{t,t+s}\tilde{y}_{t+s}^h(h)},$$

$$\tilde{p}_t^*(h)=\mu\frac{E_t\sum_{s=0}^{\infty}\left(\alpha^H\right)^s Q_{t,t+s}\tilde{y}_{t+s}^f(h)MC_{t+s}(h)}{E_t\sum_{s=0}^{\infty}(\alpha^H)^s Q_{t,t+s}\tilde{y}_{t+s}^f(h)S_{t+s}},\tag{10}$$

where $\mu=\sigma/(\sigma-1)$ is the markup over prices and MC_{t+s} represents the nominal marginal cost in period $t+s$. For foreign firms, the similar first-order conditions hold:

$$\tilde{p}_t(f)=\mu\frac{E_t\sum_{s=0}^{\infty}(\alpha^F)^s Q_{t,t+s}^*\tilde{y}_{t+s}^h(f)MC_{t+s}^*(f)}{E_t\sum_{s=0}^{\infty}(\alpha^F)^s Q_{t,t+s}^*\tilde{y}_{t+s}\frac{\tilde{y}_{t+s}(f)}{S_{t+s}}},$$

$$\tilde{p}_t^*(f)=\mu\frac{E_t\sum_{s=0}^{\infty}(\alpha^{F^*})^s Q_{t,t+s}^*\tilde{y}_{t+s}^f(f)MC_{t+s}^*(f)}{E_t\sum_{s=0}^{\infty}(\alpha^{F^*})^s Q_{t,t+s}^*}.\tag{11}$$

Then, the evolution of the price sub-indices is expressed as:

$$P_{H_t}^{1-\sigma}=\alpha^H\tilde{p}_t^{1-\sigma}(h)+(1-\alpha^H)P_{Ht-1}^{1-\sigma},$$

$$P_{Ft}^{1-\sigma}=\alpha^{F}\tilde{p}_{t}^{1-\sigma}(f)+(1-\alpha^{F})P_{Ft-1}^{1-\sigma}, \tag{12}$$

and

$$P_{H_t}^{*1-\sigma}=\alpha^{H^*}\tilde{p}_{t}^{*1-\sigma}(h)+(1-\alpha^{H^*})P_{Ht-1}^{*1-\sigma}$$

$$P_{Ft}^{*1-\sigma}=\alpha^{F^*}\tilde{p}_{t}^{*1-\sigma}(f)+(1-\alpha^{F^*})P_{Ft-1}^{*1-\sigma}$$

Finally, let's define the following relative price indices:

$$T\equiv\frac{P_F}{P_H}\text{ and }T^*\equiv\frac{P_H^*}{P_F^*}.$$

Each of these relative price indices represents the ratio of the imported good price to the domestically-produced good price expressed in the local currency so that it is the market rate at which consumers are willing to substitute one unit of imported good for one unit of domestically-produced good. Given these relative price indices, the aggregate price indices imply the following:

$$(P_H/P)^{1-\theta}=[n+(1-n)T^{1-\theta}]^{-1},\ (P_F/P)^{1-\theta}=[nT^{*1-\theta}+(1-n)]^{-1},$$

and

$$(P_H^*/P^*)^{1-\theta}=[n+(1-n)T^{*-(1-\theta)}]^{-1},\ (P_F^*/P^*)^{1-\theta}=[nT^{*1-\theta}+(1-n)]^{-1}.$$

As the prices of foreign goods are expensive relative to those of home goods at the home country (i.e., the increase in T), the ratio of the price sub-index of home goods to the aggregate price index (i.e., P_H/P) decreases while the ratio of the price sub-index of foreign goods (i.e., P_F/P) increases. As a result, the movements of the relative price indices" influence the total demands of home and foreign goods.

3.3 System of equations

This section provides the linear system of equilibrium equations that are obtained by log-linearizing the equilibrium conditions around the deterministic steady state.[7] In the system, the monetary policy rule is designed as an interest-rate feedback rule. We denote the logarithmic deviation of a variable X from its steady state value under sticky prices as $\widehat{X}=\log X-\log\overline{X}$ and the corresponding one under flexible prices as $\widetilde{X}=\log X-\log\overline{X}$.

We first provide log-linearization of the aggregate demand side. Log-linearizing the Euler equations of the home and foreign countries and risk-sharing condition yields:

$$E_t[\widehat{C}_{t+1}]-(1+b)_{t-1}+b\widehat{C}_{t-1}=\rho^{-1}(1-b)(\widehat{i_t}-E_t[\pi_{t+1}], \tag{14}$$

and

$$E_t[\widehat{C}^*_{t+1}]-(1+b)\widehat{C}^*_t+b\widehat{C}^*_{t-1}=\rho^{-1}(1-b)(\widehat{i_t}^*-E_t[\pi^*_{t+1}]), \tag{15}$$

where $\widehat{i_t}\equiv\log(1+i_t)/(1+\bar{i})$ and $\pi_t\equiv\log P_t/P_{t-1}$ is the aggregate inflation rate, and

$$\frac{1}{1-b}\left(\widehat{C}_t-b\widehat{C}_{t-1}\right)=\frac{1}{1-b}\left(\widehat{C}^*_t-b\widehat{C}^*_{t-1}\right)+\rho^{-1}\widehat{RS}_t \tag{16}$$

Regarding the aggregate supply side, log-linearizing optimal sticky prices and price sub-indices yields four producers' Phillips curves:[8]

$$\pi_{Ht}=\beta E_t\left[\pi_{Ht+1}\right]+\left\{\begin{array}{l} k^H_{\pi_H}\pi_{Ht}+k^H_{\pi^*}\pi^*_{Ht}+k^H_C\left(\widehat{C}_t-\widetilde{C}_t\right)+k^H_{C_{-1}}\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right) \\ +k^H_{C^*}\left(\widehat{C}^*_t-\widetilde{C}^*_t\right)+k^H_{RS}\widehat{RS}_t+k^H_T\left(\widehat{T}_t-\widetilde{T}_t\right)+k^H_{T^*}\left(\widehat{T}^*_t-\widetilde{T}^*_t\right)\end{array}\right\}$$

$$\pi^*_{Ht}=\beta E_t\left[\pi^*_{Ht+1}\right]+\left\{\begin{array}{l} k^{H*}_{\pi_H}\pi_{Ht}+k^{H*}_{\pi^*}\pi^*_{Ht}+k^{H*}_C\left(\widehat{C}_t-\widetilde{C}_t\right)+k^{H*}_{C_{-1}}\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right) \\ +k^{H*}_{C^*}\left(\widehat{C}^*_t-\widetilde{C}^*_t\right)+k^{H*}_{RS}\widehat{RS}_t+k^{H*}_T\left(\widehat{T}_t-\widetilde{T}_t\right)+k^{H*}_{T^*}\left(\widehat{T}^*_t-\widetilde{T}^*_t\right)\end{array}\right\}$$

$$\pi_{Ft}=\beta E_t\left[\pi_{Ft+1}\right]+\left\{\begin{array}{l} k^F_{\pi_F}\pi_{Ft}+k^F_{\pi^*}\pi^*_{Ft}+k^F_C\left(\widehat{C}_t-\widetilde{C}_t\right)+k^F_{C_{-1}}\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right) \\ +k^F_{C^*}\left(\widehat{C}^*_t-\widetilde{C}^*_t\right)+k^F_{RS}\widehat{RS}_t+k^F_T\left(\widehat{T}_t-\widetilde{T}_t\right)+k^F_{T^*}\left(\widehat{T}^*_t-\widetilde{T}^*_t\right)\end{array}\right\}$$

$$\pi^*_{Ft}=\beta E_t\left[\pi^*_{Ft+1}\right]+\left\{\begin{array}{l} k^{F*}_{\pi_F}\pi_{Ft}+k^{F*}_{\pi^*}\pi^*_{Ft}+k^{F*}_C\left(\widehat{C}_t-\widetilde{C}_t\right)+k^{F*}_{C_{-1}}\left(\widehat{C}_{t-1}-\widetilde{C}_{t-1}\right) \\ +k^{F*}_{C^*}\left(\widehat{C}^*_t-\widetilde{C}^*_t\right)+k^{F*}_{RS}\widehat{RS}_t+k^{F*}_T\left(\widehat{T}_t-\widetilde{T}_t\right)+k^{F*}_{T^*}\left(\widehat{T}^*_t-\widetilde{T}^*_t\right)\end{array}\right\}$$

7 The technical appendix for details is available upon request.

8 Coefficients of each Phillips curve are provided in the technical appendix that is available upon request.

where $\pi_{jt}=\log P_{jt}/P_{jt-1}$ and $\pi^*_{jt}=\log P^*_{jt}/P^*_{jt-1}$ for $j=H$ and F. Note that the aggregate inflation rates are expressed as $\pi_t=n\pi_{Ht}+(1-n)\pi_{Ft}$ and $\pi^*_t=n\pi^*_{Ht}+(1-n)\pi^*_{Ft}$. Futhermore, the following identities are obtained from the definitions of the relative price indices and the real exchange rate, respectively:

$$\widehat{T}_t=\widehat{T}_{t-1}+\pi_{Ft}-\pi_{Ht} \text{ and } \widehat{T}^*_t=\widehat{T}^*_{t-1}+\pi^*_{Ht}-\pi^*_{Ft}, \tag{18}$$

and

$$\widehat{RS}_t=\widehat{RS}_{t-1}+\pi^*_t-\pi_t+\Delta s_t \tag{19}$$

Finally, the nominal interest rate is determined by the monetary policy rule:

$$\widehat{i}_t=\lambda\,\widehat{i}_{t-1}+(1-\lambda)(\phi\pi_t+\psi y_t{}^H)+\varepsilon_t$$

and

$$\widehat{i}_t{}^*=\lambda\,\widehat{i}_{t-1}{}^*+(1-\lambda)(\phi\pi_t{}^*+\psi y_t{}^F)+\varepsilon_t{}^*, \tag{20}$$

where ε_t and $\varepsilon_t{}^*$ are serially uncorrelated monetary policy shocks and each of home and foreign output gaps $y_t{}^H$ and $y_t{}^F$ defined as the logarithmic difference between the actual output level and the output level under flexible prices is expressed as:

$$y_t{}^H=(\widehat{C}_t{}^W-\widetilde{C}_t{}^W)+\theta(1-n)(\widehat{T}_t{}^W-\widetilde{T}_t)$$

and

$$y_t{}^F=(\widehat{C}_t{}^W-\widetilde{C}_t{}^W)-\theta n(\widehat{T}_t{}^W-\widetilde{T}_t),$$

where $\overline{C}_t{}^W\equiv n\widetilde{C}_t{}^*+(1-n)\widetilde{C}_t{}^*$ and $\widehat{T}_t{}^W\equiv n\widehat{T}_t-(1-n)\widehat{T}_t{}^*$.

Thus, the system consists of 12 equations above (i.e., Equations (14) through (20)) for 12 endogenous variables ($\widetilde{C}_t$, $\widetilde{C}_t{}^*$, $\widehat{RS}_t$, Δs_t, π_{Ht}, π_{Ft}, π^*_{Ht}, π^*_{Ft}, $\widehat{T}_t$, $\widehat{T}_t{}^*$, $\widehat{i}_t$, $\widehat{i}_t{}^*$) under sticky prices, and 3 equations below in Equation (21) for 3 endogenous variables ($\widetilde{T}_t$, $\widetilde{C}_t$, $\widetilde{C}_t{}^*$) \$ under flexible prices all of which depend only on the technology shocks:[9]

$$\widetilde{T}_t = \frac{\eta+1}{\eta\theta+1}\widehat{A}_t^R,$$

$$\eta\widetilde{C}_t^W + \frac{\rho}{1-b}\left(\widetilde{C}_t - b\widetilde{C}_{t-1}\right) = (\eta+1)\,\widehat{A}_t^W,$$

$$\widetilde{C}_t - b\widetilde{C}_{t-1} = \widetilde{C}_t^* - b\widetilde{C}_{t-1}^*, \tag{21}$$

where $\widehat{A}_t{}^R = \widehat{A}_t - \widehat{A}_t{}^*$ and $\widehat{A}_t{}^W = n\widehat{A}_t + (1-n)\widehat{A}_t{}^*$. Each country-specific technology shock is assumed to follow a AR(1) process, that is, $\widehat{A}_t = \rho_A\widehat{A}_{t-1} + v_{At}$ and $\widehat{A}_t{}^* = \rho_{A^*}\widehat{A}_{t-1}{}^* + v_{A^*t}$. As in the closed economy considered in Section 2, habit formation affects the Euler equation so that the current consumption level depends on a weighted average of the expected future and past consumption levels and is less sensitive to the real interest rate. In contrast, however, habit formation has a direct effect on producers' Phillips curves, not on the aggregate Phillips curves. To facilitate the analysis in the next section, we close this section by providing two lemmas.

Lemma 1: As long as $\alpha^H = \alpha^{H*}$ and $\alpha^F = \alpha^{F*}$, a Phillips curve of the relative aggregate inflation denoted by $\pi_t{}^R \equiv \pi_t - \pi_t{}^*$ is expressed as

$$\pi_t{}^R = \beta E_t[\pi_{t+1}{}^R] + (n\Phi^H + (1-n)\Phi^F)\widehat{RS}_t + (\Phi^H - \Phi^F)n(1-n)\widehat{T}_t{}^R,$$

where $\Phi^j = (1-\alpha^j\beta)(1-\alpha^j)/\alpha^j$ for $j = H, F$ and $\widehat{T}_t{}^R \equiv \widehat{T}_t + \widehat{T}_t{}^*$.

The relative aggregate inflation defined in Lemma 1 does not depend on the consumption levels. The reason is as follows. When firms of the home goods charges their prices on the home and foreign markets, they do not charge them differently under the circumstance that the price adjustment is the same across markets (e.g., $\alpha^H = \alpha^{H*}$). The only source of the difference between prices charged on two markets is the real exchange rate of the price sub-index of home goods denoted by $RSH_t \equiv S_t P_{Ht}{}^* / P_{Ht}$. When firms of home goods set their prices on the foreign market, they take RSH_t into account in a way that whenever RSH_t increases, the marginal revenue increases and thus firms decrease their price charged on the

[9] Note that $\widetilde{T}_t{}^* = -\widetilde{T}_t$.

foreign market (i.e., $\tilde{p}_t^*(h)/P_{Ht}^*$). Consequently, the difference between inflation rates of home goods across countries is expressed as:

$$\pi_{Ht}-\pi_{Ht}^*=\beta E_t[\pi_{Ht+1}-\pi_{Ht+1}^*]+\Phi^H\widehat{RSH}_t,$$

where as $\widehat{RSH}_t$ increases, the inflation rate of home goods at the home market increases relative to one at the foreign market. Similarly, for foreign goods, the following holds:

$$\pi_{Ft}-\pi_{Ft}^*=\beta E_t[\pi_{Ft+1}-\pi_{Ft+1}^*]+\Phi^F\widehat{RSF}_t.$$

Furthermore, each real exchange rate of the price sub-index is decomposed as:

$$RSH_t=\frac{S_tP_t^*}{P_t}\frac{P_{Ht}^*}{P_t^*}\frac{P_t}{P_{Ht}},\ RSF_t=\frac{S_tP_t^*}{P_t}\frac{P_{Ft}^*}{P_t^*}\frac{P_t}{P_{Ft}},$$

where $\widehat{RSH}_t=\widehat{RS}_t+(1-n)\widehat{T}_t^*+(1-n)\widehat{T}_t$ and $\widehat{RSF}_t=\widehat{RS}_t-n\widehat{T}_t^*-n\widehat{T}_t$. Thus, the relative aggregate inflation $\pi_t^R=\pi_t-\pi_t^*=n(\pi_{Ht}-\pi_{Ht}^*)+(1-n)(\pi_{Ft}-\pi_{Ft}^*)$ does not depend on consumption levels. It is worthwhile to note that the relative aggregate Phillips curve under no habit formation is the same as one in Lemma 1.

The relative output gap denoted by $y_t^R\equiv y_t^H-y_t^F$ is equal to $\theta(\widehat{T}_t^W-\widetilde{T}_t)$, where $\widehat{T}_t^W=n\widehat{T}_t-(1-n)\widehat{T}_t^*$ is a measure of the overall relative price of foreign goods in terms of the price of home goods, so that as $\widehat{T}_t^W$ increases, the prices of home goods are less expensive and the relative output gap increases. Regarding evolutions of two relative prices, the following lemma is provided.

Lemma 2: As long as $\alpha^H=\alpha^F$ and $\alpha^{H*}=\alpha^{F*}$, two relative prices are isolated from monetary policy shocks.

Under the assumption of Lemma 2, four producers Phillips curves imply the following:

$$(\pi_{Ft}-\pi_{Ht})=\beta E_t[(\pi_{Ft+1}-\pi_{Ht+1})]+k_1(\widehat{T}_t-\widetilde{T}_t)+k_2(\widehat{T}_t^*-\widetilde{T}_t^*),$$

and

$$({\pi_{Ft}}^* - {\pi_{Ht}}^*) = \beta E_t[({\pi_{Ft+1}}^* - {\pi_{Ht+1}}^*)] + k_3(\widehat{T}_t - \widetilde{T}_t) + k_4(\widehat{T}_t^* - \widetilde{T}_t^*),$$

where each coefficient k_j is a function of structural parameters, so that along with Equation (18), it follows that the dynamics of two relative prices depend only on (relative) technology shocks and so does $\widehat{T}_t^W$ or equivalently, the relative output gap ${y_t}^R$. It is also worthwhile to note that it is the case under no habit formation, that is, two equations above holds under no habit formation.

4 Analytical solution and simulation study

This section analyzes how habit formation affects the real exchange rate persistence, depending on the assumption of the degree of price stickiness. Three relevant cases are taken into account: symmetry within and across countries (i.e., $\alpha^H = \alpha^{H*} = \alpha^F = \alpha^{F*}$); asymmetry within a country (i.e., $\alpha^H = \alpha^{H*}$ and $\alpha^F = \alpha^{F*}$, but $\alpha^H \neq \alpha^F$); asymmetry across countries (i.e., $\alpha^H = \alpha^F$ and $\alpha^{H*} = \alpha^{F*}$, but $\alpha^H \neq \alpha^{F*}$). In each case, the role of habit formation is emphasized in terms of the design of the monetary policy rule, that is, through its inflation and output gap stabilization channels. If analytical solutions are available, they are provided. Otherwise, the analysis is done through simulation in which only white noise domestic monetary policy shocks are considered.[10]

4.1 Symmetry within and across countries: Seemingly isomorphic relationship

In case of the symmetric degree of price stickiness, we emphasize two important implications of endogenous persistence mechanisms discussed in Section 2 for the real exchange rate persistence: (1) interest rate smoothing is necessary to generate the persistent real exchange rate, but is not sufficient to match the observed persistence of the real exchange rate and (2) habit formation does not significantly contribute to the real exchange rate

10 The values of structural parameters used in simulation are provided in Table 1.

persistence for other cases where the degree of price stickiness is asymmetric. For the purpose, we introduce one of main findings in Benigno (2004) where neither backward-looking pricing nor habit formation is considered and then, show the breakdown of a seemingly isomorphic relationship between the real exchange rate and the closed economy output gap.

Benigno (2004) showed that when the degree of price stickiness is symmetric within and across countries, the system that determines the dynamics of the real exchange rate under monetary policy shocks consists of the following three equations:

$$\widehat{i}_t^R = \phi \pi_t^R + 2\vartheta \widehat{RS}_t + \varepsilon_t^R, \tag{22}$$

$$\pi_t^R = \beta E_t[\pi_{t+1}^R] + (1-\alpha\beta)(1-\alpha)/\alpha \widehat{RS}_t, \tag{23}$$

$$\widehat{RS}_t = E_t[\widehat{RS}_{t+1}] - (\widehat{i}_t^R - E_t[\pi_{t+1}^R]\ ), \tag{24}$$

where a variable with superscript R is the difference between the home and foreign variables (e.g., $\widehat{i}_t^R \equiv \widehat{i}_t - \widehat{i}_t^*$ is the nominal interest rate differential). The first equation is the relative monetary policy rule.[11] The second equation is the relative aggregate Phillips curve in which the real exchange rate is considered a measure of relative real economic activity across countries. The last equation is obtained by combining the UIP condition with the identity from the definition of the real exchange rate and states that the relative real interest rate is equal to the expected real depreciation. When the real interest rate of the home country is greater than that of the foreign country, it is expected that consumption of the home country increases in the next period, since goods in the home country will be less expensive (i.e. a real depreciation of the home country). This system is exactly the same as the original system of the closed economy model in Section 2 (i.e., Equations (1), (2), and (3)) up-to-scale and the real exchange rate as a measure of relative real economic activity across countries seems

[11] Under the symmetric degree of price stickiness within and across countries, the difference between output gaps of home and foreign countries is independent of monetary policy shocks, which is implied by Lemma 2 of Section 3.3. Thus, when the focus is on the dynamics of the real exchange rate under monetary policy shocks, omitting the relative output gap does not affect it. For comparison, it is assumed that the monetary policy authority responds to the real exchange rate with its weight ϑ as well as inflation and output gap.

to be isomorphic to the closed economy output gap as a measure of real economic activity within a country.

It follows from Section 2 that in the above system, the real exchange rate does not exhibit any persistence following monetary policy shocks and interest rate smoothing plays its important role in introducing the persistence into the system, but is not sufficient to match the observed persistence of the real exchange rate, which corresponds to one of main findings in Benigno (2004): there is no proportionality of the real exchange rate persistence to the degree of price stickiness that is against the findings of other previous work on the real exchange rate persistence (e.g., Chari et al. (2002) among others) and the role of interest rate smoothing is emphasized to match the observed persistence of the real exchange rate. At first glance, this finding appears to be surprising in the open economy context. The intuition behind this, however, is obvious. The fact that the pricing mechanism is fully forward-looking and there are no other endogenous inertias implies no persistence of the system and thus no persistence of the real exchange rate. To understand the contribution of habit formation to the persistence of the real exchange rate, we have the following proposition.

Proposition 1: Under $\alpha^{H}=\alpha^{H^*}=\alpha^{F}=\alpha^{F^*}$, habit formation does not affect the dynamics of the real exchange at all under monetary policy shocks.

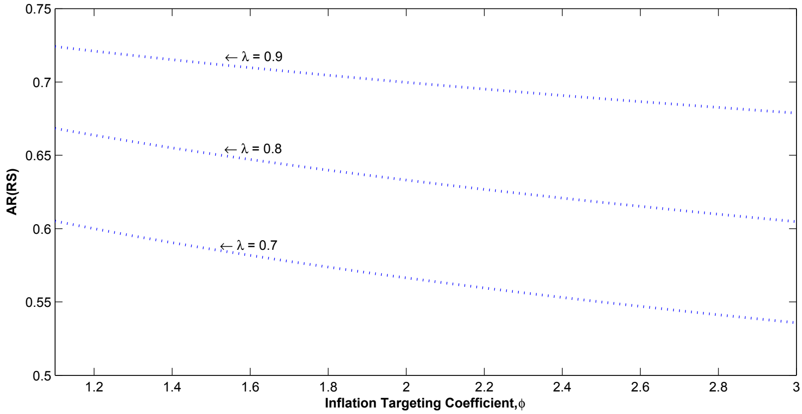
Proof: The system that determines the dynamics of the real exchange rate is the same as one under no habit formation (i.e., Equations (22), (23), and (23)). See the technical appendix.

Proposition 1 implies the breakdown of the seeming isomorphism of the real exchange rate to the closed economy output gap. While habit formation helps generate more persistent output gap in the closed economy, it does not affect the real exchange rate persistence so that there is no longer a one-to-one mapping of the real exchange rate to the closed economy output gap even under the symmetric two-country open economy. Seemingly isomorphic relationship is attributed to the complete asset market assumption. While Euler equations of the home and foreign countries are affected by habit formation, the UIP condition always holds under the internationally complete asset market assumption and thus habit formation does not change Equation (24), which is independent of the assumption of the degree of price stickiness. Along with this observation, since the improvement of the closed economy output gap persistence due to habit formation comes mainly from the change in the Euler equation (i.e., Equation (3)), the effect of habit formation on the real exchange rate persistence is expected to be limited for other cases where the degree of price stickiness is asymmetric. In addition, habit formation does not change the relative aggregate Phillips curve (that is, Equation (23) holds under habit formation), and the movements of relative prices (equivalently, the relative output gap) are still isolated from monetary policy shocks even under habit formation, which is verified in Lemmas 1 and 2 of Section 3.3. That is because when the relative Phillips curve is obtained, the dependence of producers' Phillips curves on consumption levels are completely offset by the risk-sharing condition.

We turn now to how the design of the monetary policy rule affects the real exchange rate persistence. The relative output gap is isolated from monetary shocks, so that the dynamics of the real exchange rate under monetary shocks is independent of the existence of the output gap stabilization.[12] When the aggregate inflation is stabilized more aggressively (i.e., the weight to inflation ϕ gets bigger), producers who choose new prices have less desire at any time to set their prices different from the average of existing prices. Thus, prices of goods return to the average of existing prices faster in response to monetary shocks so that prices are less sluggish endogenously. Consequently, the real exchange rate is less persistent. This is shown in Figure 1. Given a value of the interest rate smoothing coefficient, the persistence of the real exchange rate measured by its AR coefficient is monotonically decreasing with the inflation targeting coefficient.

12 Refer to the footnote 9.

Figure 1. Symmetric price stickiness within and across countries: Inflation stabilization channel



Note: This figure shows the first-order autocorrelation of the real exchange rate denoted by AR(RS) for dierent values of the in ation targeting coefficient, ϕ, given a value of the interest rate smoothing coefficient, λ. The parameter of price stickiness, α, is set equal to 0.75 and other structural parameters are set equal to those in Table 1.

4.2 Asymmetry within a country: Relative price inertia

In this case, two roles of the asymmetric price adjustment within a country are emphasized. First, the asymmetric degree of price stickiness within a country can be considered an endogenous inertia element *per se* to generate the persistence of the real exchange rate even when other endogenous persistence mechanisms including habit formation are not introduced. Second, it plays the role in transmitting habit persistence into the real exchange rate only through the output gap stabilization channel of the monetary policy rule. To distinguish these two roles, we provide the following proposition.

Proposition 2: Under $\alpha^H = \alpha^{H*}$ and $\alpha^F = \alpha^{F*}$, but $\alpha^H \neq \alpha^F$, when there is no weight to output gap in the monetary policy rule (i.e., $\psi = 0$), the real exchange rate is isolated from technology shocks and is not affected by habit formation.

Proof: In this case, the relative aggregate Phillips curve is that of Lemma

1 in Section 3.3:

$$\pi_t{}^R = \beta E_t[\pi_{t+1}{}^R] + (n\Phi^H + (1-n)\Phi^F)\widehat{RS}_t + (\Phi^H - \Phi^F)n(1-n)\hat{T}_t^R \qquad (25)$$

where $\Phi^j = (1-\alpha^j\beta)(1-\alpha^j)/\alpha^j$ for $j = H$, F and the sum of relative price indices denoted by $\hat{T}_t^R \equiv \hat{T}_t + \hat{T}_t^*$ is determined by:

$$\hat{T}_t^R - \hat{T}_{t-1}{}^R = \beta E_t[\hat{T}_{t+1}{}^R - \hat{T}_t^R] + (\Phi^F - \Phi^H)\widehat{RS}_t - (n\Phi^F + (1-n)\Phi^H)\hat{T}_t^R \qquad (26)$$

Thus, the system that determines the dynamics of the real exchange rate consists of two equations above and the following equations:

$$\widehat{RS}_t = E_t[\widehat{RS}_{t+1}] - (\hat{i}_t{}^R - E_t[\pi_{t+1}{}^R])$$

and

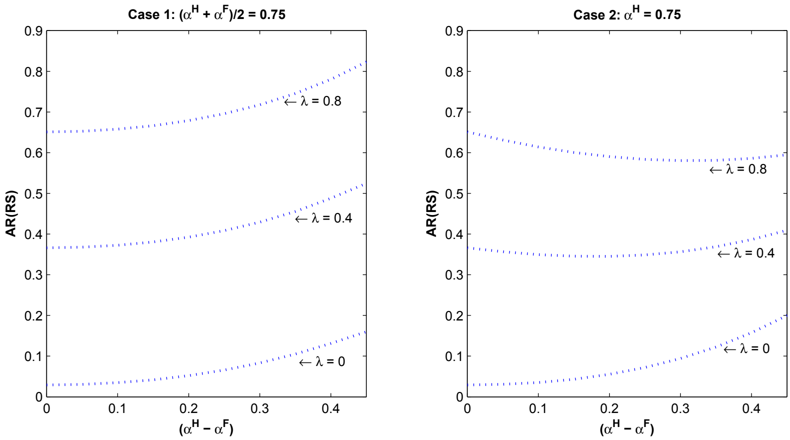
$$\hat{i}_t{}^R = \lambda\, \hat{i}_{t-1}{}^R + (1-\lambda)\phi\pi_t{}^R + \varepsilon_t{}^R$$

By the visual inspection, the system does not depend on variables under flexible prices so that the real exchange rate is isolated from technology shocks. Furthermore, this system is exactly the same as one under no habit formation. (See the technical appendix.)

For a moment, let us ignore the dependence of the relative inflation $\pi_t{}^R$ on $\hat{T}_t^R$ in Equation (25). Then, it is obvious that unless there is interest rate smoothing, the system that determines the dynamics of the real exchange rate is purely forward-looking so that there is no persistence in that system. However, Equation (26) is a second-order stochastic difference equation and thus it introduces the persistence into the system. That is, relative price inertia transmits into the dynamics of the real exchange rate so that it helps improve the persistence of the real exchange rate. Note that as long as $\alpha^H = \alpha^F$ (i.e., the price adjustment is symmetric), the term of $\hat{T}_t^R$ in Equation (26) disappears. Thus, the asymmetric degree of price stickiness within a country is considered an endogenous persistence mechanism in the sub-system that determines the dynamics of the real exchange rate.

Now, we pay attention to the benefit of the asymmetric price adjustment to the real exchange rate persistence. Figure 2 shows the AR coefficient of the real exchange rate as the difference between two degrees of price stickiness (i.e., $\alpha^H - \alpha^F$) varies. The left panel is made by keeping their average equal to 0.75 while the right panel is drawn by fixing one of them at 0.75. Note that in Figure 2, the output gap coefficient is set equal to zero to isolate the output gap stabilization channel that we discuss below.

Figure 2. Asymmetric price stickiness within a country: The role of relative price inertia



Note: This figure has two panels each of which shows the first-order autocorrelation of the real exchange rate denoted by AR(RS) as the dierence between two degrees of price stickiness, $(\alpha^H - \alpha^F)$, varies, given a value of the interest rate smoothing coefficient, λ. In both panels, the output gap coefficient, ψ, is set equal to zero to isolate the output gap stabilization channel, and other structural parameters are set equal to those in Table 1.

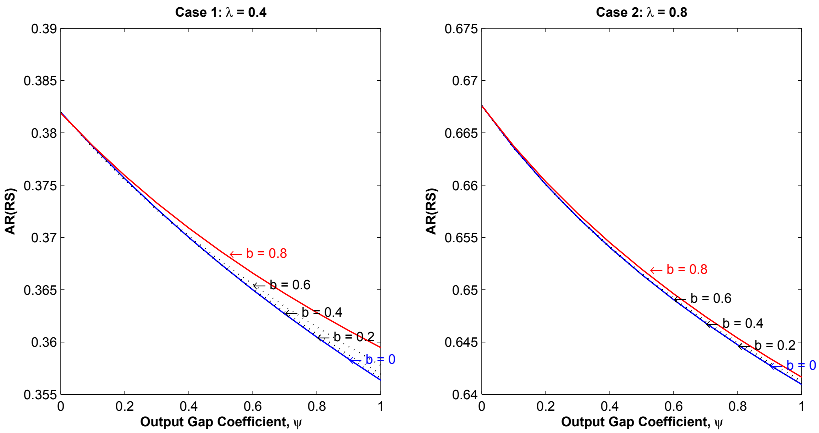
Each panel indicates that even when there is no interest rate smoothing (i.e., $\lambda = 0$), the real exchange rate persistence is monotonically increasing with the difference between two degrees of price stickiness. This is relative price inertia discussed above. Since a higher interest rate smoothing is necessary to match the observed persistence of the real exchange rate, it is more sensible to gauge the persistence gain from the asymmetric price adjustment for a higher value of interest rate smoothing. In case of $\lambda = 0.8$ at two panels, the left panel indicates that the persistence of the real exchange rate is strictly increasing over the range, but the right panel shows that it is not monotonic and is not improved, implying that the loss of the persistence due to the decrease in the average of two degrees dominates the gain from their difference. Thus, for a higher value of interest smoothing, what is really important is the average of degrees of price stickiness, which is consistent with one of findings in Section 2 that once the persistence is induced into the closed economy system by interest rate smoothing, the persistence of endogenous variables is proportional to the degree of price stickiness. Following this fact, the focus is on how much the real exchange rate persistence is improved by the difference of two degrees for their plausible values, holding the average of them fixed. In case of $\lambda = 0.8$ at the left panel, there is almost the persistence gain of 0.2 at $\alpha^H – \alpha^F = 0.45$, but at this difference, the degree of price stickiness of home goods α^H is equal to 0.975 which is an implausible degree. For more plausible values, say, $\alpha^H = 0.825$ and $\alpha^F = 0.675$ which correspond to $\alpha^H – \alpha^F = 0.15$, the gain is only 0.015. Hence, while the asymmetric price adjustment within a country is an endogenous inertia mechanism through which relative price inertia is transmitted into the real exchange rate, its effect is very small quantitatively under plausible values of degrees of price stickiness, which stands in contrast to Benigno (2004).

Next, we discuss the output gap stabilization channel through which the asymmetric price adjustment plays its role in transmitting habit persistence into the real exchange rate. Under the asymmetric price adjustment, the movements of the relative price indices $\widehat{T}_t$ and $\widehat{T}_t^{R*}$ affect the dynamics of the real exchange rate. Since output gap involves these relative price indices, stabilizing output gap yields stable relative prices, implying that as output gap is stabilized more aggressively (i.e., a higher value of ψ), the relative prices returns to the initial equilibrium faster after monetary shocks. Consequently, prices are less sluggish and so is the real exchange rate. In Figure 3 where the left and right panels are drawn for low and high interest rate smoothing, respectively, as the output gap weight increases, the real exchange rate persistence decreases, which is independent of habit formation.[13] Furthermore, each of the relative prices depends on consumption levels so that habit persistence transmits into the relative prices which subsequently contributes to the real exchange

[13] In Figure 3, the values of α^H and α^F are set equal to 0.825 and 0.675, implying their average of 0.75 and their difference of 0.15.

rate persistence.[14] Thus, Figure 3 indicates that for a positive value of the output gap coefficient (e.g., $\psi=0.5$), as the habit formation parameter increases, the real exchange rate exhibits more persistence. However, its effect is very small quantatively.[15] Regarding the inflation stabilization channel, as in the symmetric price adjustment case, as the inflation targeting coefficient increases, the real exchange rate persistence decreases.[16]

Figure 3. Asymmetric price stickiness within a country: The habit effect through output gap stabilization channel



Note: This figure has two panels each of which shows the first-order autocorrelation of the real exchange rate denoted by AR(RS) for dierent values of the output gap coefficient, ψ, given a value of the degree of habit formation, b. The values of α^H and α^F are set equal to 0.825 and 0.675, implying their average of 0.75 and their dierence of 0.15. Other structural parameters are set equal to those in Table 1.

In sum, under the asymmetric degree of price stickiness within a country, habit formation affects the real exchange rate only through the output

14 Refer to the technical appendix for the expression of the equation for each of the relative prices, $\widehat{T}_t$ and $\widehat{T}_t^*$.

15 There is also a trade-off: as the output gap coefficient gets bigger, the persistence gain of the real exchange rate due to habit formation through the output gap stabilization channgel gets larger, but at the same time, more aggressive output gap stabilization reduces the real exchange rate persistence by making relative prices less persistent. Figure 3 shows that the latter dominates the former.

16 To save the space, the figure that shows the inflation stabilization channel is not provided. The result is available upon request.

gap stabilization channel and its effect on the real exchange rate persistence is quantitatively trivial. In addition, the contribution of the asymmetric price adjustment as an endogenous inertia mechanism to the real exchange rate persistence is insignificant for plausible values of degrees of price stickiness.

4.3 Asymmetry across countries: Habit inertia

In case of the asymmetric price adjustment across countries under which there is a genuine pricing-to-market (PTM) component,[17] we emphasize the role of habit formation as an endogenous inertia mechanism as in the closed economy of Section 2. As long as there is no interest rate smoothing, the system does not generate any persistence under monetary shocks. The introduction of habit formation, however, induces the persistence into the system that determines the dynamics of the real exchange rate. Thus, PTM cannot generate any persistence endogenously, but is important for habit formation to induce the endogenous persistence, since for the symmetry case in Section 4.1 (no PTM component), habit formation does not affect the sub-system for the dynamics of the real exchange rate at all. We first provide the following proposition and then prove it by revisiting the endogenous persistence problem in the closed economy discussed in Section 2.

Proposition 3: Under $\alpha^{H}=\alpha^{F}$ and $\alpha^{H*}=\alpha^{F*}$, but $\alpha^{H}\neq\alpha^{H*}$, as long as there are no interest rate smoothing and no habit formation, the real exchange rate does not exhibit any persistence following monetary policy shocks.

Proof: Under no habit formation, as long as $\alpha^{H}=\alpha^{F}$ and $\alpha^{H*}=\alpha^{F*}$, but $\alpha^{H}\neq\alpha^{H*}$, the aggregate Phillips curves are expressed as

[17] The degree of price stickiness has a direct effect on new prices set by home firms. Thus, when price adjustment varies across the home and foreign markets, firms of home goods charge different prices on two markets. In this sense, the difference between prices at the home and foreign markets only due to the different degrees across markets is considered as a genuine pricing-to-market component.

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + k_\pi \pi_t + k_{\pi^*}\pi_t^* + k_C\left(\widehat{C}_t - \widetilde{C}_t\right) + k_{C^*}\left(\widehat{C}_t^* - \widetilde{C}_t^*\right) + k_{RS}\widehat{RS}_t$$

and

$$\pi_t^* = \beta E_t\left[\pi_{t+1}^*\right] + k_\pi^* \pi_t + k_{\pi^*}^*\pi_t^* + k_C^*\left(\widehat{C}_t^* - \widetilde{C}_t^*\right) + k_{C^*}^*\left(\widehat{C}_t^* - \widetilde{C}_t^*\right) + k_{RS}^*\widehat{RS}_t$$

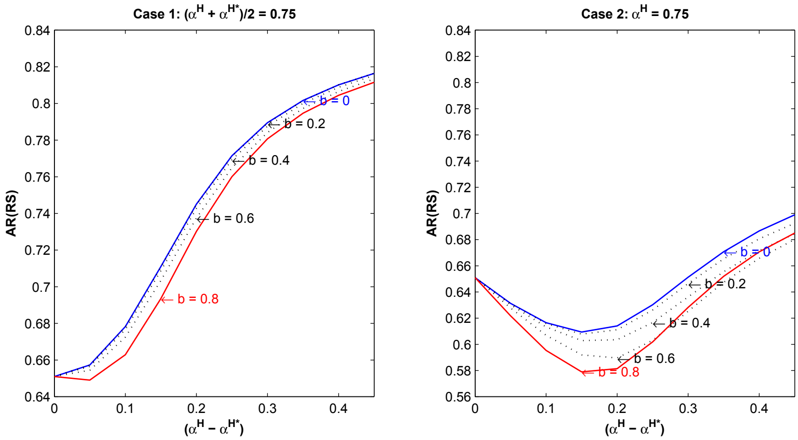
where each of coefficients is a function of structural parameters, which determine the aggregate inflation rates of the home and foreign countries simultaneously. Since consumption movements are purely forward-looking under standard Euler equations (i.e., no habit formation) and the real exchange rate is determined by the standard risk-sharing condition under no habit formation, if there is no endogenous inertia mechanism for the real exchange rate, then the aggregate inflation rates above do not exhibit any persistence at all. It follows from Lemma 2 of Section 3.3 that two relative prices, equivalently a measure of the overall relative price of foreign goods in terms of the home good price $\widehat{T}_t{}^R$, are isolated from monetary policy shocks, implying that output gap $y_t{}^H$ $(y_t{}^F)$ is equivalent to $\widehat{C}_t{}^W$ under monetary shocks. Thus, the system that determines the dynamics of the real exchange rate consists of two Phillips curve above, two standard Euler equations, two monetary policy rule without interest rate smoothing, and standard risk-sharing condition. This system is purely forward-looking as like the original system in Section 2 (i.e., Equations (1), (2), and (3)) in which there is no persistence at all and thus the real exchange rate does not exhibit any persistence following monetary shocks.

Proposition 3 implies that the system does not have an endogenous persistence mechanism inherent in the open economy like the relative price inertia mentioned in the previous section. So it is interest rate smoothing and habit formation that induce the persistence endogenously.[18] Interest rate smoothing is necessary to match the observed persistence of the real exchange rate. Thus, for a high value of interest rate smoothing of 0.8, Figure 4 shows the AR coefficient of the real exchange rate as the difference of two degrees (i.e., $\alpha^H - \alpha^{H^*}$) varies, where the left and right panels are drawn by keeping the average of two degrees equal to 0.75 and fixing one of them equal to 0.75, respectively. From two panels, it follows that the habit formation effect on the persistence of the real exchange rate is

[18] We do not consider backward-looking pricing.

trivial quantitatively and the average of two degrees of price stickiness matters for the real exchange rate persistence.

Figure 4. Asymmetric price stickiness across countries: The role of habit inertia



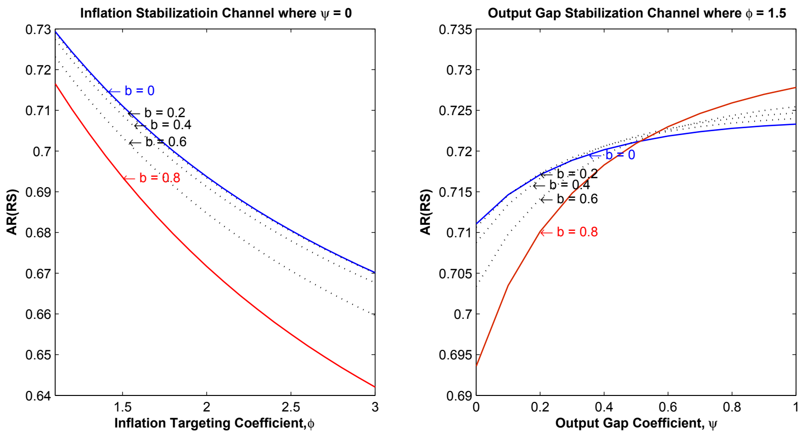
Note: This figure has two panels each of which shows the first-order autocorrelation of the real exchange rate denoted by AR(RS) as the dierence between two degrees of price stickiness, $(\alpha^H – \alpha^{H*})$, varies, given a value of the degree of habit formation, b. In both panels, the interest rate smoothing coefficient, λ, and output gap coefficient, ψ, are set equal to 0.8 and zero, respectively, and other structural parameters are set equal to those in Table 1.

We now pay attention to the effect of habit formation through the inflation and output gap stabilization channels, which are shown in Figure 5.[19] From the left panel where the output gap weight is equal to zero to isolate the output gap channel, it is shown that as the habit formation parameter increases, the real exchange rate is less persistent over the range of the inflation weight, indicating that habit formation reduces the persistence of the real exchange rate through the inflation stabilization channel. Under the asymmetric price adjustment across countries, an individual aggregate inflation affects the dynamics of the real exchange rate. Under the circumstance that aggregate inflation is more persistent due to habit inertia, stabilizing inflation yields more stable aggregate price level than before the introduction of habit formation, inducing a less persistent real exchange

[19] In Figure 5, the values of α^H and α^{H*} are set equal to 0.825 and 0.675, respectively, implying their average of 0.75 and their difference of 0.15.

rate. However, such habit effect is small quantitatively. The right panel of Figure 5 indicates the effect of habit formation through the output gap stabilization channel, which is not monotonic and depends on the output gap weight. Such effect is also trivial.

Figure 5. Asymmetric price stickiness across countries: Inflation and output gap stabilization channels

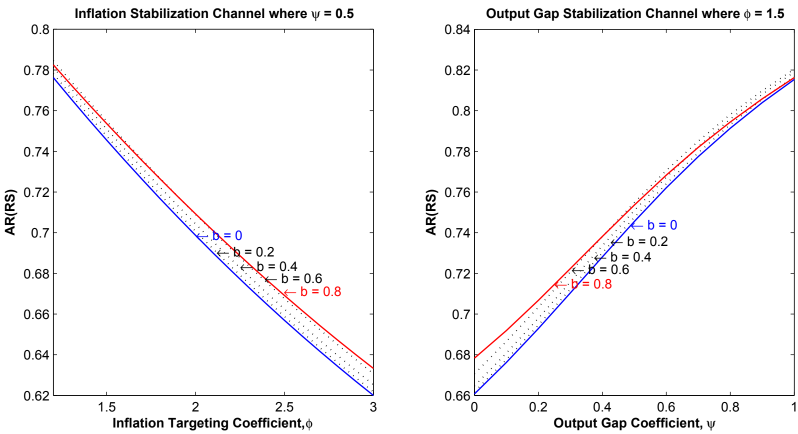


Note: This figure has two panels each of which shows the first-order autocorrelation of the real exchange rate denoted by AR(RS) for dierent values of the inflation targeting coefficient ϕ (in the left panel) or output gap coefficient ψ (the right panel), given a value of the degree of habit formation, b. In both panels, the values of α^H and α^{H*} are set equal to 0.825 and 0.675, respectively, implying their average of 0.75 and their dierence of 0.15, the interest rate smoothing coefficient, λ, is set equal to 0.8, and other structural parameters are set equal to those in Table 1.

4.4 Example in asymmetry within and across countries

To close the analysis, we provide an example in which the price adjustment varies within and across countries. The example reflects the findings in the previous sections. From Benigno (2004), we pick up the values of four degrees of price stickiness as well as the value of interest rate smoothing parameter: α^H=0.8, α^{H*}=0.66, α^F=0.67, α^{H*}=0.4; λ=0.85. Figure 6 indicates that habit formation improves the real exchange rate persistence, but its improvement is very small quantitatively over the ranges of the inflation and output gap weights, respectively.

Figure 6. Asymmetric price stickiness within and across countries: Inflation and output gap stabilization channels



Note: This figure has two panels each of which shows the first-order autocorrelation of the real exchange rate denoted by AR(RS) for dierent values of the in ation targeting coefficient ϕ (in the left panel) or output gap coefficient ψ (the right panel), given a value of the degree of habit formation, b. In both panels, the values of four dierent price stickiness is set such that α^H=0.8, α^{H*}=0.66, α^F=0.67, and α^{F*}=0.4, the interest rate smoothing coecient, λ, is set equal to 0.85, and other structural parameters are set equal to those in Table 1.

5 Conclusion

This paper provided the sharp understanding of the effect of habit formation on the real exchange rate persistence under monetary shocks. The role of habit formation as an endogenous inertia mechanism for the real exchange rate depends on the assumption of the price adjustment speed as well as the design of the monetary policy rule. In the symmetric price adjustment case, the breakdown of the seemingly isomorphic relationship between the real exchange rate and the closed economy output gap makes the strong prediction on how habit formation affects the real exchange rate persistence for asymmetric price adjustment cases. The habit formation effect on the persistence of the real exchange rate is very limited, which stands in contrast to the role of habit formation as a promising endogenous inertia mechanism in closed economy models of monetary policy. Futhermore, it is shown that the contribution of the relative price inertia inherent in the open economy to the persistence of the real exchange rate is trivial under plausible values of degrees of price stickiness within a country and thus most of the real exchange rate persistence relies on interest rate smoothing behavior. In the closed economy, interest rate smoothing is necessary to match the persistence of the output gap, but not sufficient (see McCallum (2001)). The real exchange rate is as much persistent as the closed economy output gap. Thus, the model with habit formation is not able to match the observed persistence of the real exchange rate under monetary shocks.[20]

The limited ability of habit formation to generate a more persistent real exchange rate is attributed to the internationally complete asset market assumption. Its effect on the real exchange rate can be different in a model with the incomplete asset market. Moreover, habit formation can affect the real exchange rate under real shocks in a different way from what was shown in this paper. Therefore, it would be interesting to study the effect of habit formation on the real exchange rate persistence either under incomplete asset market or under real shocks or under both.

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20 Steinsson (2008) emphasizes the hump-shaped impulse response function of the real exchange rate to shocks so as to match its observed persistence. The introduction of habit formation does not generate the hump-shaped impulse response of the real exchange rate to monetary policy shocks. So the limited ability of habit formation we found in this paper is in line with Steinsson.

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